

# Vub experimental measurement and theoretical interpretation

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FPCP 09, Lake Placid, 2009

# Introduction

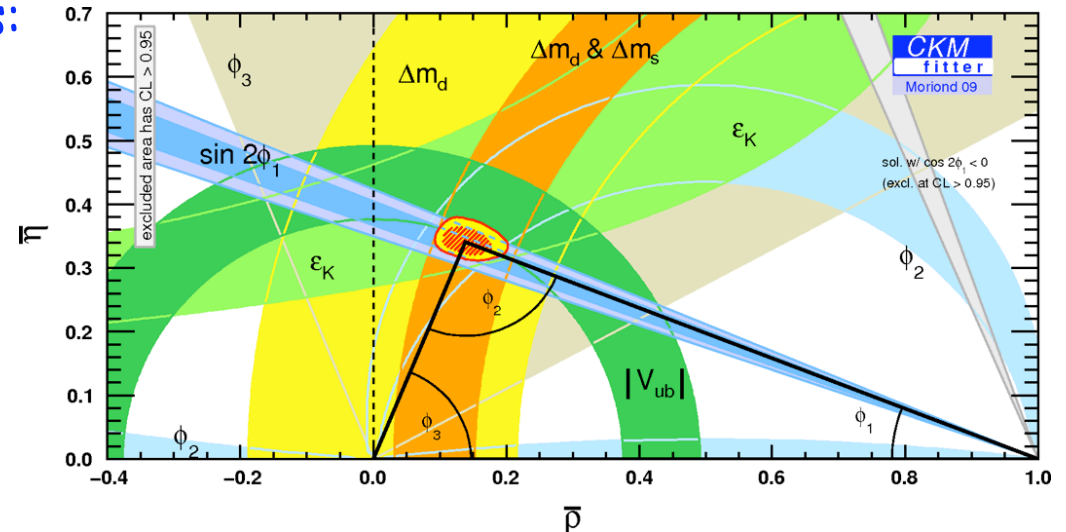
CKM matrix elements are fundamental parameter of the Standard Model and cannot be predicted

- exploit the unitarity constraint to look for new physics → geometrical

relation between CKM elements:

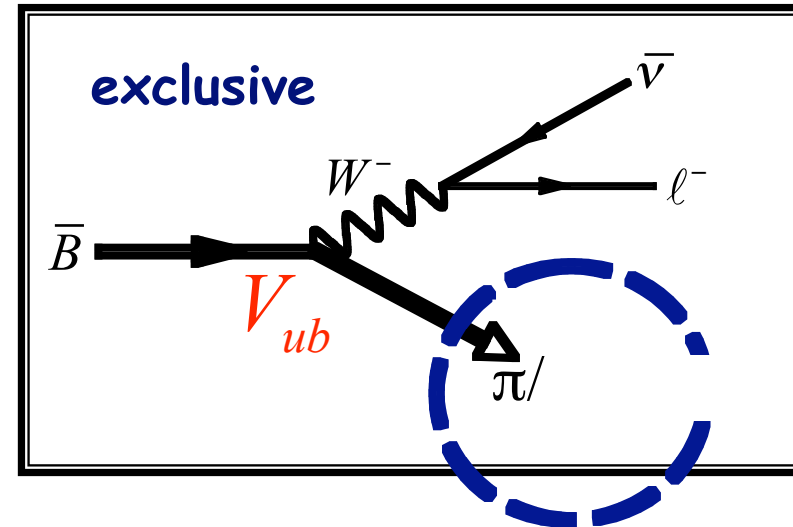
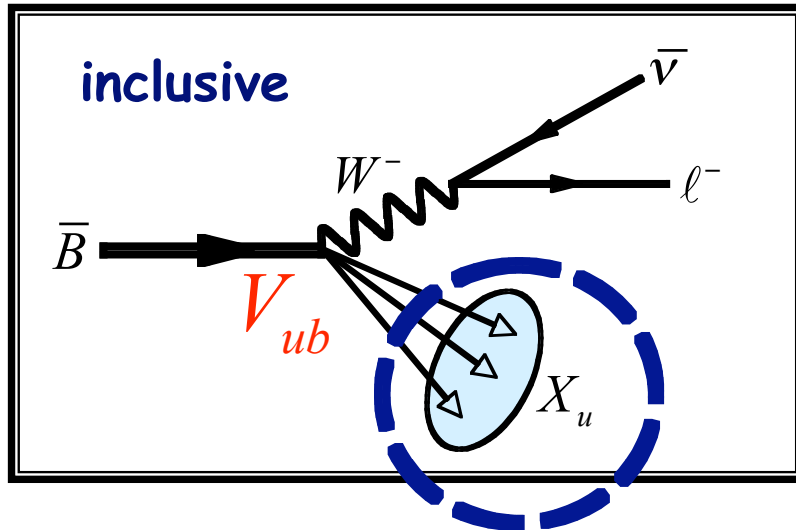
- ★ angle from CP asymmetries
- ★ size from V

- New precision era where new physics may appear as a few percent disagreement: large new physics contributions to penguins would have already been seen.



- We must make the green ring thinner → uncertainty dominated by  $|V_{ub}|$

# $V_{ub}$ determination

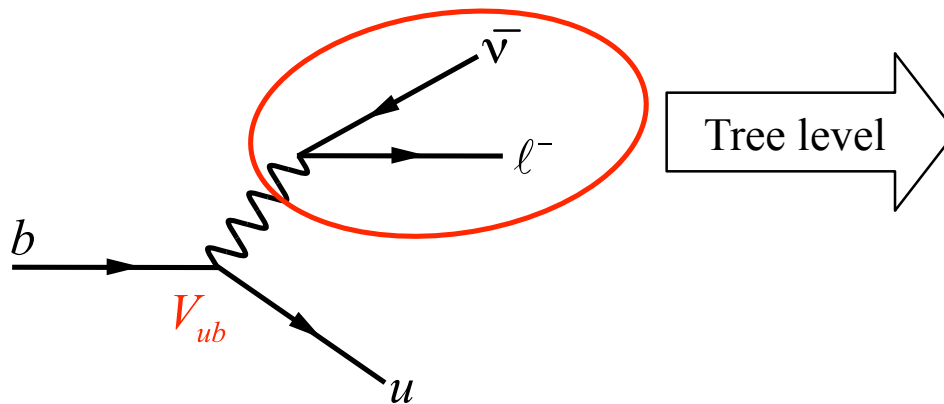


Theory errors dominates: measurements with different methods

- **Inclusive**  $B \rightarrow X_u \ell \nu$ 
  - Use difference in kinematics to separate  $u \ell \nu$  from  $c \ell \nu$
  - Theory must predict signal spectrum
- **Exclusive** mainly  $B \rightarrow \pi \ell \nu$ 
  - Better Signal/Noise
  - Theory must predict form factor

# $V_{ub}$ inclusive determination

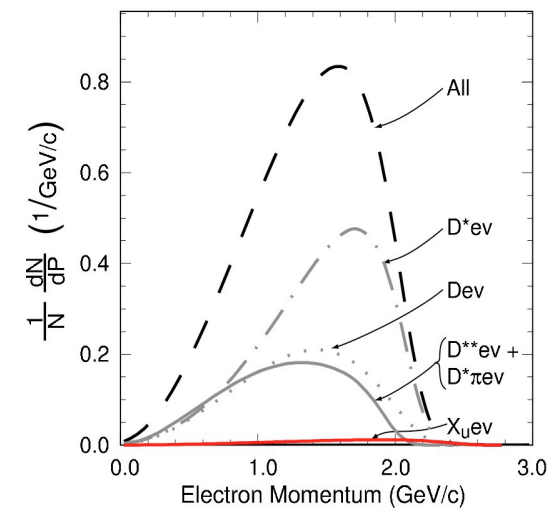
$B \rightarrow X_u | \nu$  rate is very small,  $V_{ub}$  is small, therefore very difficult to measure



$$\Gamma(b \rightarrow u l \bar{\nu}) = \frac{G_F^2}{192\pi^2} |V_{ub}|^2 m_b^5$$

the problem is the  $b \rightarrow cl\nu$  decay

$$\frac{\Gamma(b \rightarrow u l \bar{\nu})}{\Gamma(b \rightarrow c l \bar{\nu})} \approx \frac{|V_{ub}|^2}{|V_{cb}|^2} \approx \frac{1}{50}$$



Selection to remove background removes a sizeable part of the phase space.

Need theoretical extrapolation for the full phase space

# $V_{ub}$ inclusive determination

- Cut away  $b \rightarrow cl \nu \Rightarrow$  lose a part of the  $b \rightarrow ul \nu$  signal

- We measure  $\Gamma(B \rightarrow X_u \ell \nu) \times f_C = |V_{ub}|^2 \xi_C$

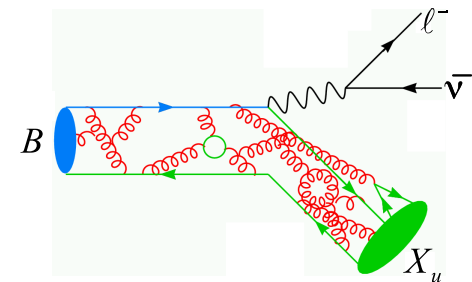
Total  $b \rightarrow ul \nu$

Cut-dependent constant predicted by theory

Fraction of the signal surviving: need knowledge of b quark's motion inside B meson

★ Must be corrected for QCD

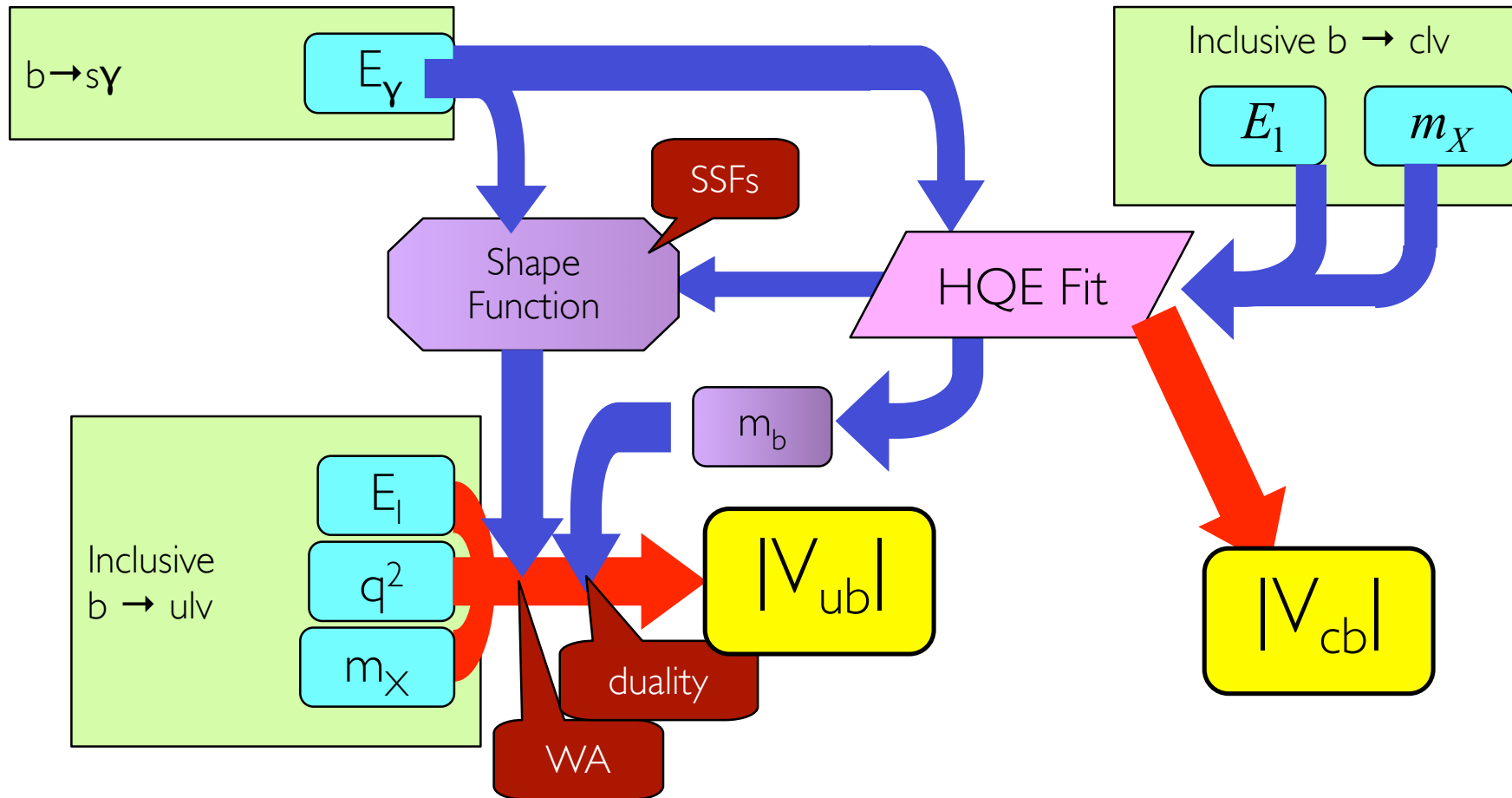
$$\Gamma(B \rightarrow X_u \ell \nu) = \frac{G_F^2 |V_{ub}|^2 m_b^5}{192\pi^3} \left[ 1 - O\left(\frac{\alpha_s}{\pi}\right) - \frac{9\lambda_2 - \lambda_1}{2m_b^2} + \dots \right]$$



In principle the main uncertainty is from  $m_b^5$  but we need a **reasonable fraction** (e.g.,  $E_\ell > 2 \text{ GeV}$ ) of the phase space.

# How Things Mesh Together

AKA: M. Morii's HQE plumbing diagram



# Shape Function

Limited phase space to reduce the  $B \rightarrow X_c l \nu$  background:

non-perturbative **Shape Function** to extrapolate to the full phase space

Detailed shape not constrained:

★ Fit the  $b \rightarrow s \gamma$  spectrum to constrain the shape

function

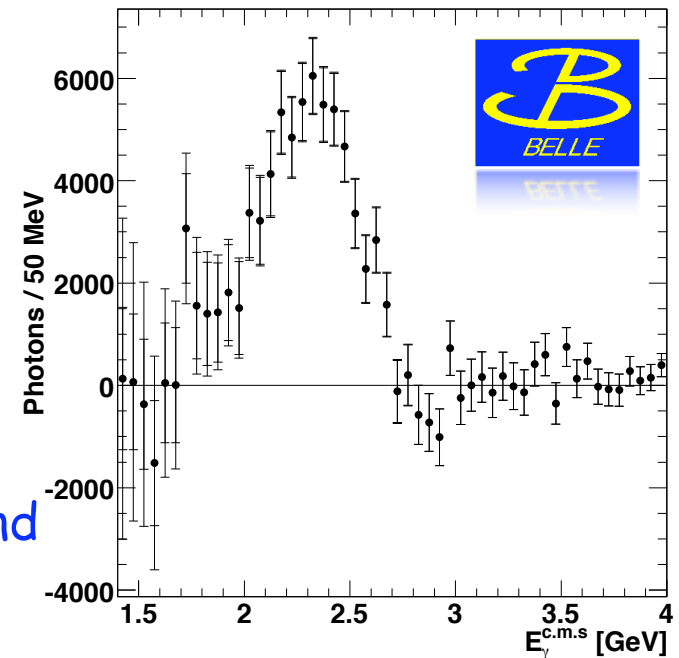
★ b-quark: mass and kinetic energy from  $b \rightarrow c l \nu$  and

$b \rightarrow s \gamma$  decays

★ Plug the SF into the  $b \rightarrow u l \nu$  spectrum calculations

Shape Function needs to be determined from experimental data.

alternatives: **cut out SF region**, or **factorise out the SF**



# Endpoint method

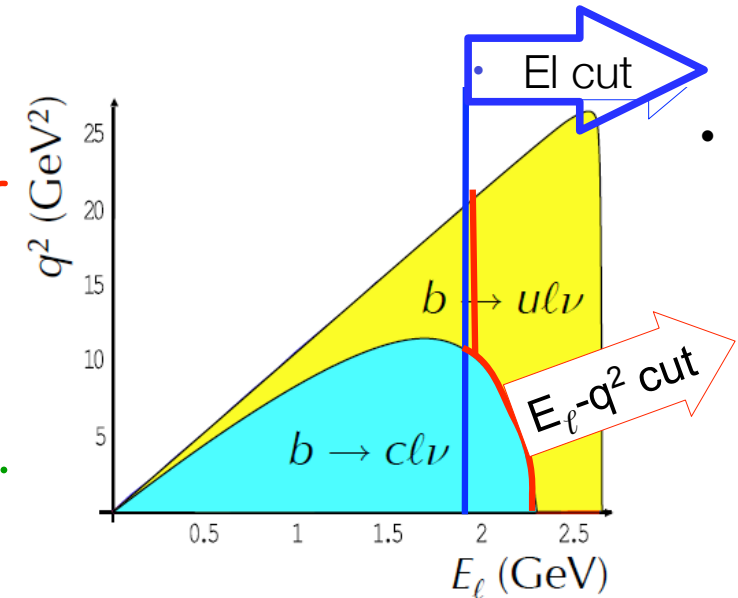
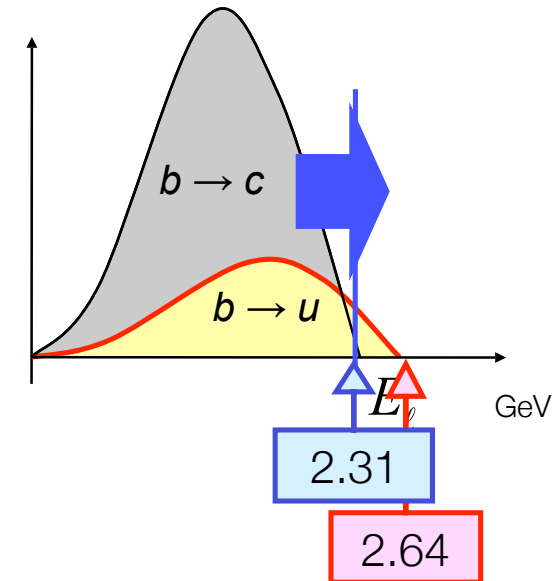
Use  $m_u \ll m_c$  difference in kinematics

- ★ Maximum lepton energy 2.64 vs. 2.31 GeV
- ★ First observations by CLEO (1990) used this technique

Only 6% of signal accessible

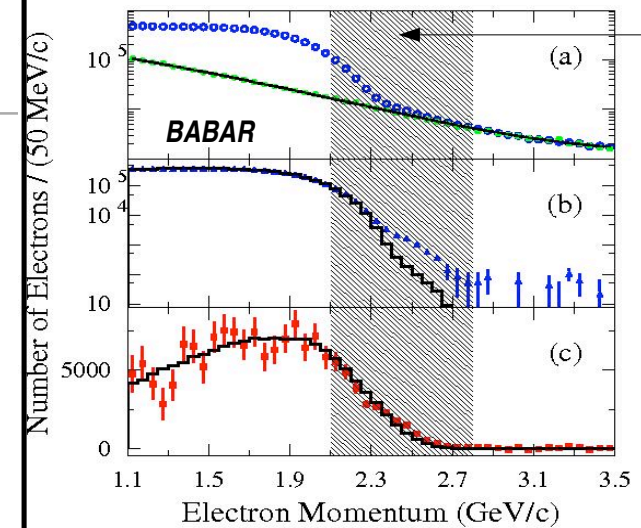
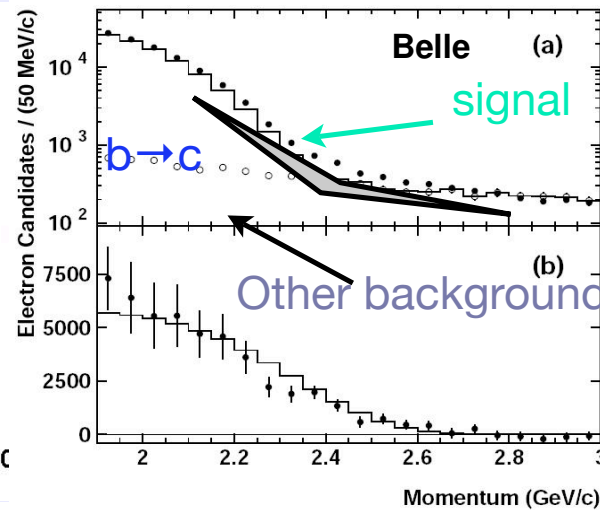
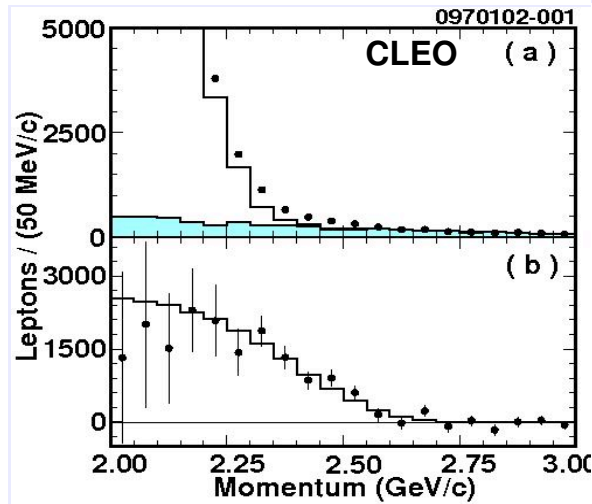
- ★ Small theory error requires low  $E_{\text{lepton}}$  cut
- ★ Small BG uncertainty requires high  $E_{\text{lepton}}$  cut

Measure partial BR in a region where S/N is acceptable and the width is reliably calculable.





# Endpoint method



Subtract off-peak data scaled to on-peak luminosity bin-by-bin.  
Fit MC to data in low energy region to constrain  $B \rightarrow X_c l \nu$  from data.

$B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu + B \rightarrow D^* l \nu$  (ratio fixed)  
 $B \rightarrow D^{**} l \nu$ ,  $B \rightarrow D^{(*)} \pi l \nu$  (Goity & Roberts)

$B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu + B \rightarrow D^* l \nu$  ( $D/D^*$  fixed)  
 $B \rightarrow D^{**} l \nu$   $D^{**}/D+D^*$  fitted ( $(D_1+D_2)/D^{**}$  fixed)

Simultaneous fit for non- $B\bar{B}$ ,  $B \rightarrow X_u l \nu$ ,  $B \rightarrow D l \nu$ ,

$B \rightarrow D^* l \nu$ ,  $B \rightarrow D^{**} l \nu$ ,  $B \rightarrow D^{(*)} \pi l \nu$ , other BG

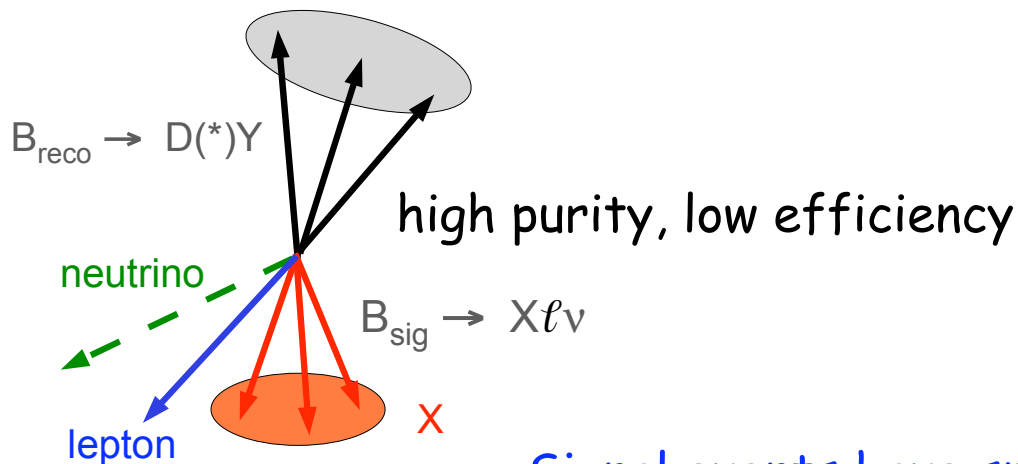
		$\Delta BR \times 10^{-4}$	$ V_{ub}  \times 10^{-3}$ % $\Delta$ ( exp -the. + the.)
PRL 88, 231803, 2002	Belle $27\text{fb}^{-1}: E_e > 1.9 \text{ GeV}$	$8.47 \pm 0.37_{\text{stat}} \pm 1.53_{\text{sys}}$	$4.61(1 \pm 9.3 + 5.0 - 6.7)$
PLB621, 28, 2005	BaBar: $E_e > 2.0 \text{ GeV}$	$5.72 \pm 0.41_{(\text{stat} \pm 0.65_{(\text{syst})})}$	$4.13(1 \pm 5.6 + 5.6 - 8.2)$
PRD 73, 012006, 2006	CLEO: $E_e > 2.0 \text{ GeV}$	$4.22 \pm 0.33_{\text{stat}} \pm 1.78_{\text{sys}}$	$3.77(1 \pm 12. + 6.9 - 10.)$

# Inclusive: fully reconstructed sample

Best access to kinematics with B meson "beam"

tag: charge and momentum

$m_u \ll m_c \rightarrow$  difference in kinematics



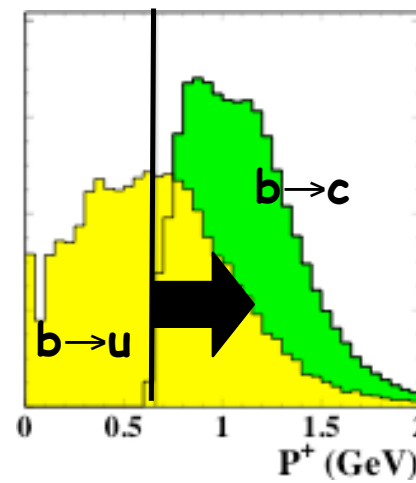
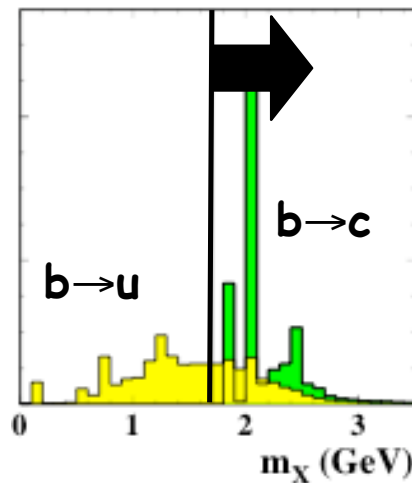
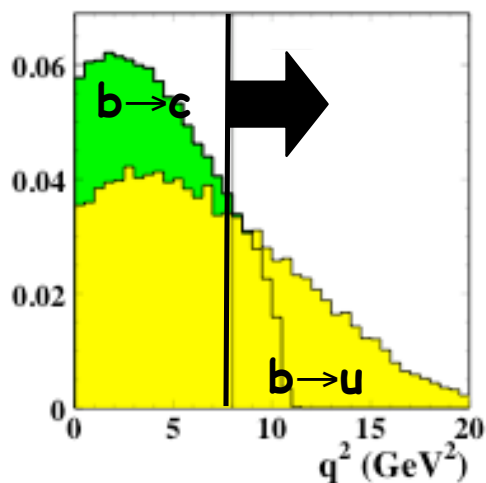
$q^2 =$  lepton-neutrino mass squared

$m_X =$  hadron system mass

$P^+ = E_X - |P_X|$

Signal events have smaller  $M_X$  and  $P_+$   $\rightarrow$  Larger  $E_l$  and  $q^2$

Not to scale!



# Relative branching fractions

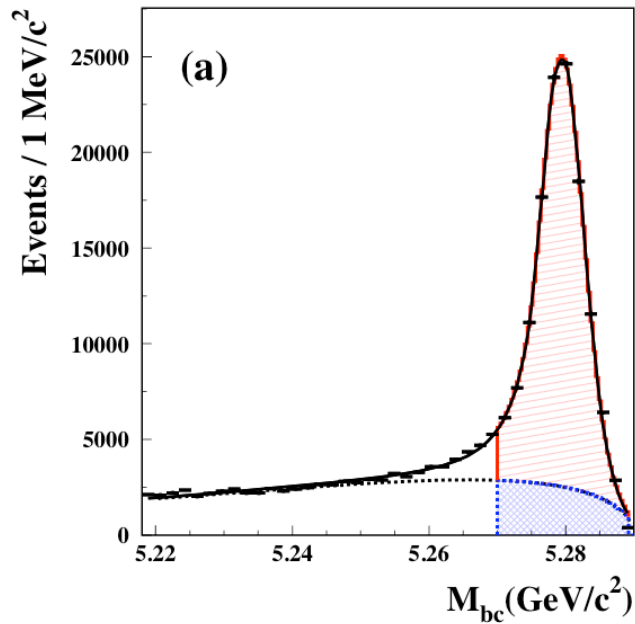
Signal side:

Reconstruct high momentum lepton

No strangeness

Event Level:

$\Sigma Q=0$ ,  $Q_{B_{reco}} \times Q_l = -1$ ,  $M_{miss}$ , etc.



★ Fit  $M_{bc}$  in bins of  $P_+$ ,  $M_x$ ,  $q^2$

★ Fit to  $P_+$ ,  $M_x$ ,  $q^2$  with various background and signal floated to determine background yield.

★ Relative BR to control systematics



Number of excess events

Unfolding factor F

PDG

$$\frac{\Delta\text{Br}(X_u l \nu)}{\text{Br}(X l \nu)} = \frac{N_{b \rightarrow u}}{N_{sl}} \cdot \boxed{F} \cdot \frac{\mathcal{E}_{\text{frec}}^{b \rightarrow u} \mathcal{E}_l^{b \rightarrow u}}{\mathcal{E}_{\text{frec}}^{sl} \mathcal{E}_l^{sl}}$$

Number of semileptonic events

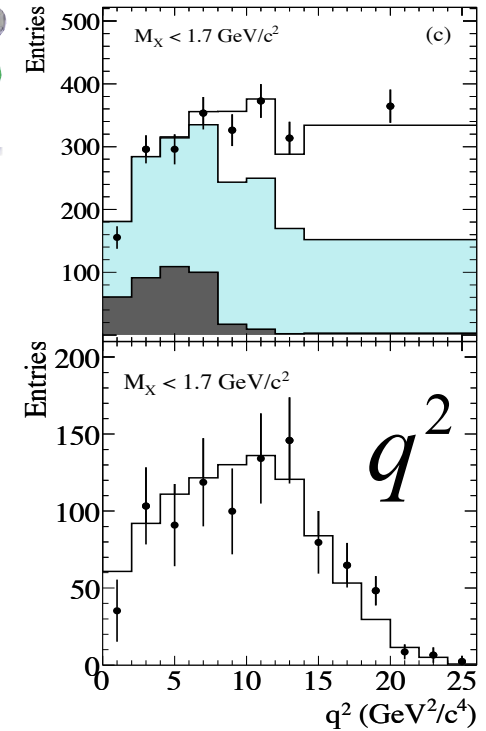
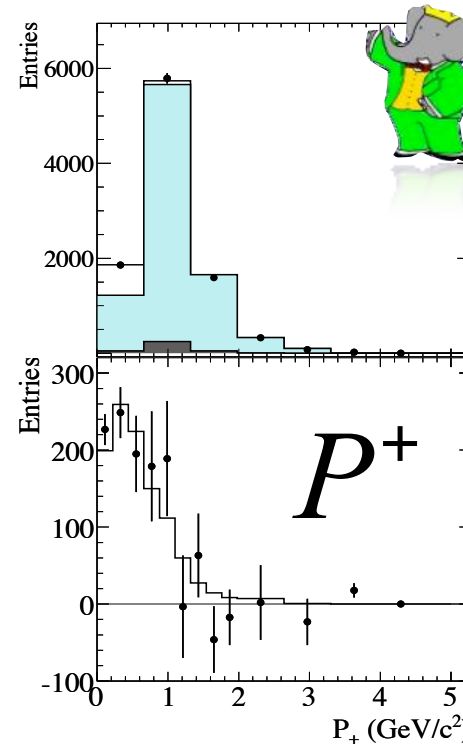
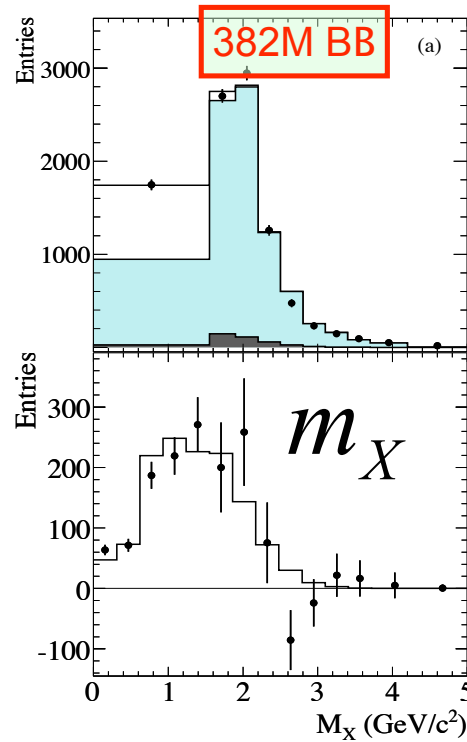
Ratio of efficiencies for  $b \rightarrow u$  and  $sl$

selection efficiency

# Inclusive

## $V_{ub}$

- $B \rightarrow X_u | n$
- $B \rightarrow X_c | n$
- $B \rightarrow X_u | n$  outside selected region



Kinematic Region	$B(B \rightarrow X_u   \nu) \times 10^{-3}$			$ V_{ub}  (10^{-3})$			Theory
	$\Delta$ (stat.)	sys.	th.)	$\Delta$ (stat.)	sys.	th.)	
$M_X < 1.55 \text{ GeV}$	$1.18 \pm 0.09 \pm 0.07 \pm 0.01$	$4.27 \pm 0.16 \pm 0.13 \pm 0.30$			BLNP		
		$4.56 \pm 0.17 \pm 0.14 \pm 0.32$			DGE		
$P_+ < 0.66 \text{ GeV}$	$0.95 \pm 0.10 \pm 0.08 \pm 0.01$	$3.88 \pm 0.19 \pm 0.16 \pm 0.28$			BLNP		
		$3.99 \pm 0.20 \pm 0.16 \pm 0.24$			DGE		
$M_X < 1.7 \text{ GeV} \ \& \ q^2 > 8.0 \text{ GeV}^2$	$0.76 \pm 0.08 \pm 0.07 \pm 0.02$	$4.48 \pm 0.22 \pm 0.19 \pm 0.30$			BLNP		
		$4.53 \pm 0.22 \pm 0.19 \pm 0.25$			DGE		
		$4.81 \pm 0.23 \pm 0.20 \pm 0.36$			BLL		

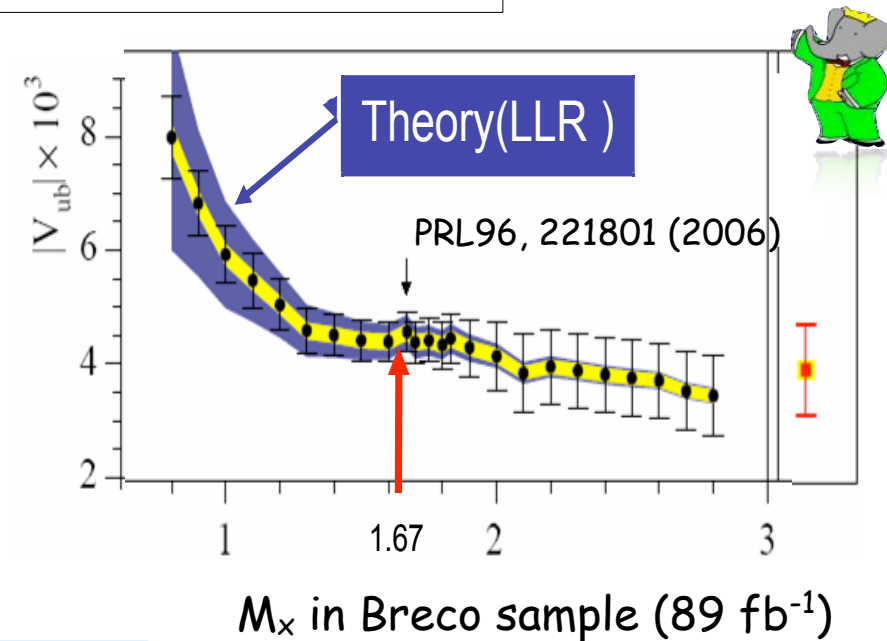
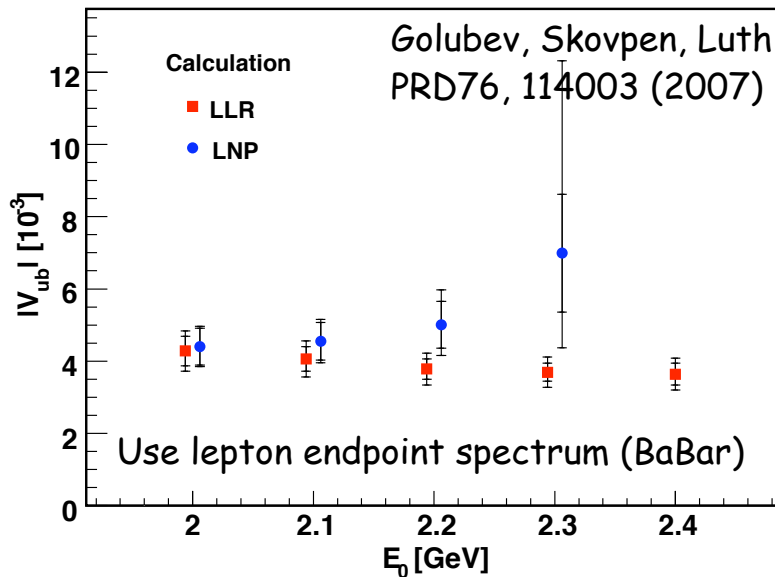
# SF independent analyses

Exploit the assumption that the leading shape functions are the same for all  $b \rightarrow q$  transitions.

LNP Lange, Neuber, Paz  
JHEP0510:084 (2005)

LLR Leibovich, Low, Rothstein  
PRD61,053006(2000)  
PLB513,83 (2001)

$$\int_0^{m_{max}} \frac{d\Gamma(b \rightarrow u)}{dm_X} dm_X \longleftrightarrow \int_{E_{min}}^{m_B/2} \frac{d\Gamma(b \rightarrow s\gamma)}{dE_\gamma} W(E_\gamma, E_{min}) dE_\gamma$$



Uncertainties due to non-perturbative power corrections increase rapidly near the kinematic end point (not included in LLR).

Results agree with analyses that rely on assumed shape function form.

E. Barberio

$$|V_{ub}| = (4.43 \pm 0.30_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.29_{\text{theo}}) \times 10^{-3}$$

# Inclusive $|V_{ub}|$

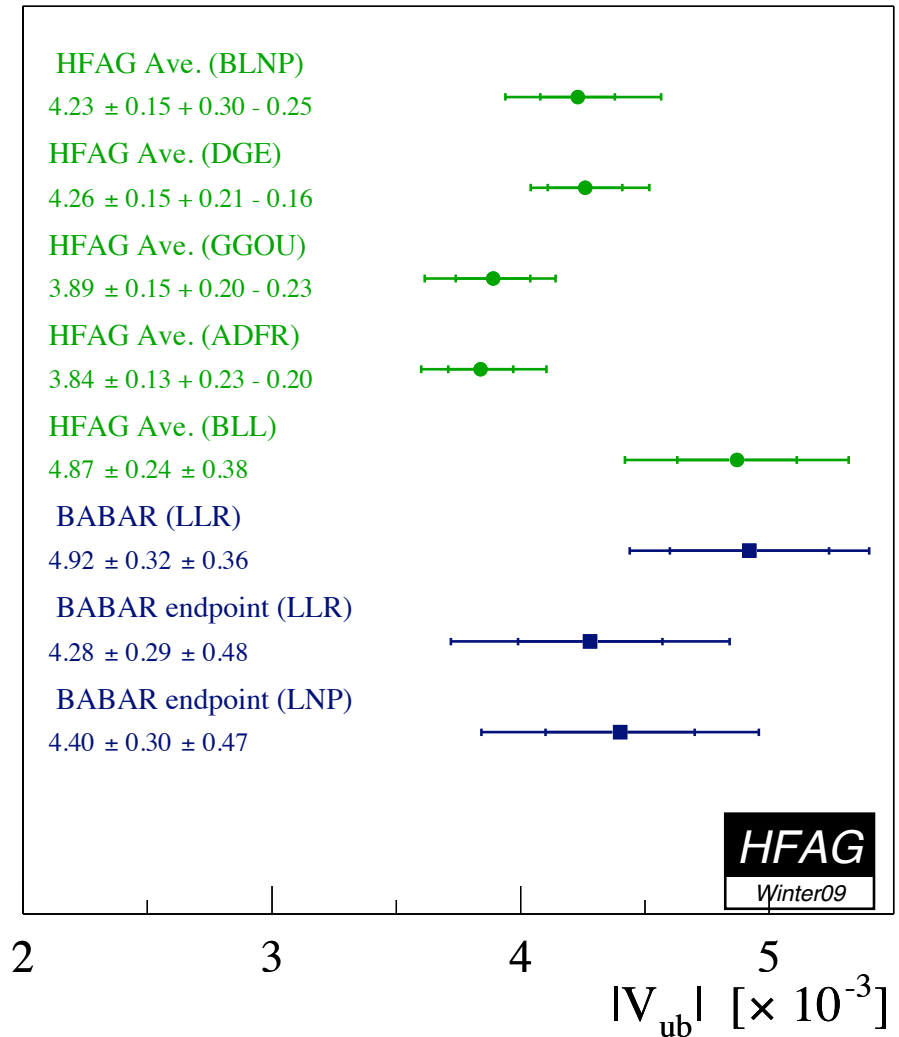
Different  $|V_{ub}|$  values  
from different calculations

OPE based + SF

- ▶ BLNP: HQE with systematic introduction of SF (PRD71:073006 (2005))
- ▶ BLL: phase space in  $m_x - q^2$  with reduced SF (PRD64:113004 (2001))
- ▶ GGOU: kinetic scheme (JHEP 10(2007) 058)
- ▶ LNP (LLR):  $b \rightarrow s \gamma$  directly related to  $b \rightarrow u l \nu$  (JHEP 0510:084 ( PLB 486:86 ))

No SF introduced  $\Rightarrow$  model non-perturbative QCD

- ▶ DGE: Dressed Gluon Exponentiation (JHEP 0601:097 (2006))
- ▶ ADFR: Analytic coupling (PRD74:03400(4,5,6) (2006))



all determinations moved of  $\sim -1 \times 10^{-4}$  from ICHEP08 due to  $m_b$  change

# Theory errors

BLNP as example

Weak annihilation 1.5% error

Expected  $<2\%$  of the total rate: potential problem for all inclusive determinations which include large  $q^2$  region (M. Voloshin, hep-ph/0106040)  
Reduce the effect by rejecting the high- $q^2$  region

Subleading Shape Function 1.8% error

Higher order non-perturbative corrections, matching

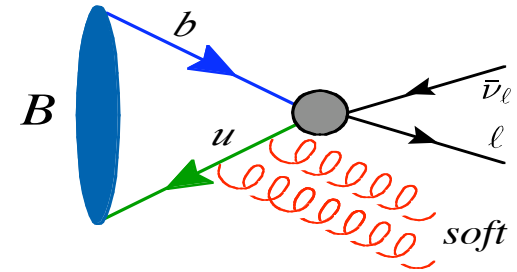
HQ parameters 5.3% error

Mainly  $m_b$ : kinematic cuts which give a higher  $m_b$  dependence e.g. for end-point  $\sim m_b^{11}$

No uncertainty is assigned for possible duality violations.

# Weak annihilation

Different BR between  $B^+$  and  $B^0$ : compare  $B^0$  partial rate to  $B$  averaged rate to enhance the WA contribution  
(PRD73, 012006 (2006))

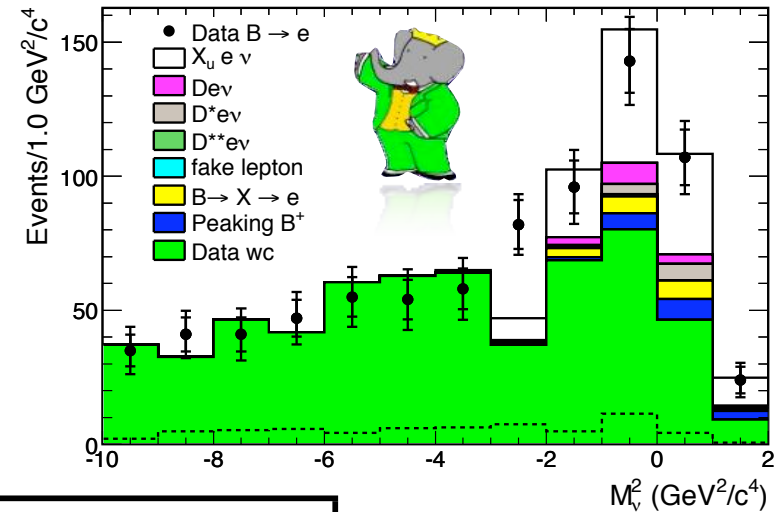


$$\frac{|\Gamma_{WA}|}{\Gamma_u} < 7.4\% \text{ at } 90\%C.L. \quad \text{CLEO, studing the } q^2 \text{ spectra} \\ \text{PRL96,121801 (2006)}$$

$$\frac{|\Gamma_{WA}|}{\Gamma_u} < \frac{3.8\%}{f_{WA}(2.3 - 2.6)} \text{ at } 90\%C.L.$$



arXiv: 0708.1753 383 M BB



$\Delta p$ (GeV)	$\Delta B(B) 10^4$	$\Delta B(B^0) 10^4$	$A^{+0}$
2.3-2.6	$2.31 \pm 0.10 \pm 0.18$	$1.30 \pm 0.21 \pm 0.07$	$0.08 \pm 0.15 \pm 0.08$
2.4-2.6	$0.75 \pm 0.04 \pm 0.06$	$0.76 \pm 0.15 \pm 0.05$	$-0.05 \pm 0.20 \pm 0.10$



# Inclusive $|V_{ub}|$

604 fb<sup>-1</sup>



Belle 09  
submitted  
to PRL

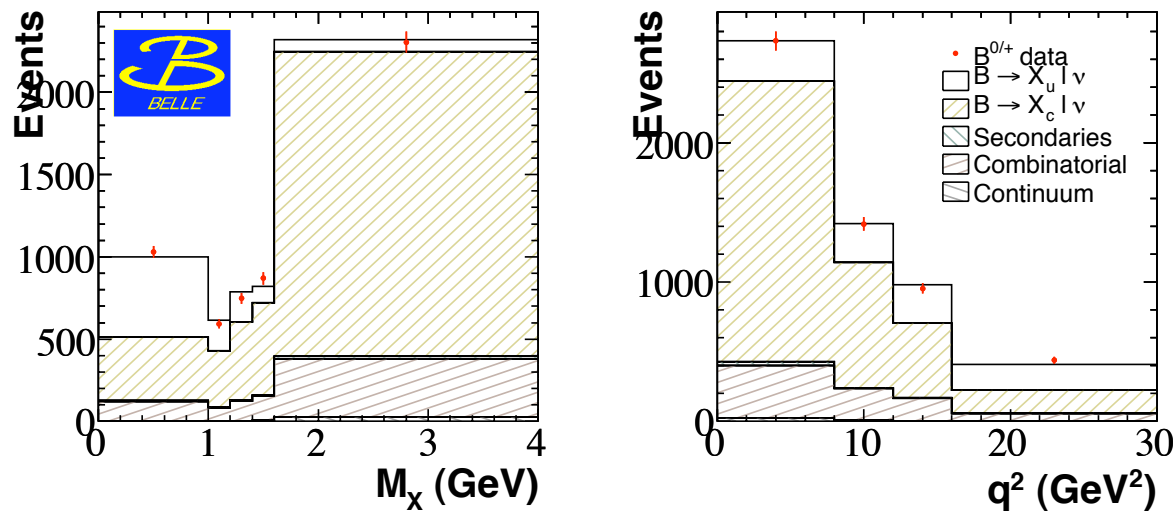
Irreducible uncertainties for  $V_{ub}$  are related to limited phase space.

Fully reconstructed sample signal and lepton with  $p_{lep}^{B^*} > 1 \text{ GeV}$

Boosted Decision Tree with many input:  $M_{miss}^2$ ,  $Q_{total}$ ,  $Q_{lepton}$ ,  $N_{lepton}$ ,  $D^*$  partial reco etc.....

Combinatorial estimated from MC, normalisation from sideband region.

2D fit to  $M_x, q^2$  with backgrounds and signal floated to get background yield.



$p_{lep}^{B^*} > 1 \text{ GeV}$

Main systematics: inclusive  $V_{ub}$  model 3.9% and exclusive  $V_{ub}$  model 3.9% (5.5% total)

# Inclusive $|V_{ub}|$

New Belle analysis

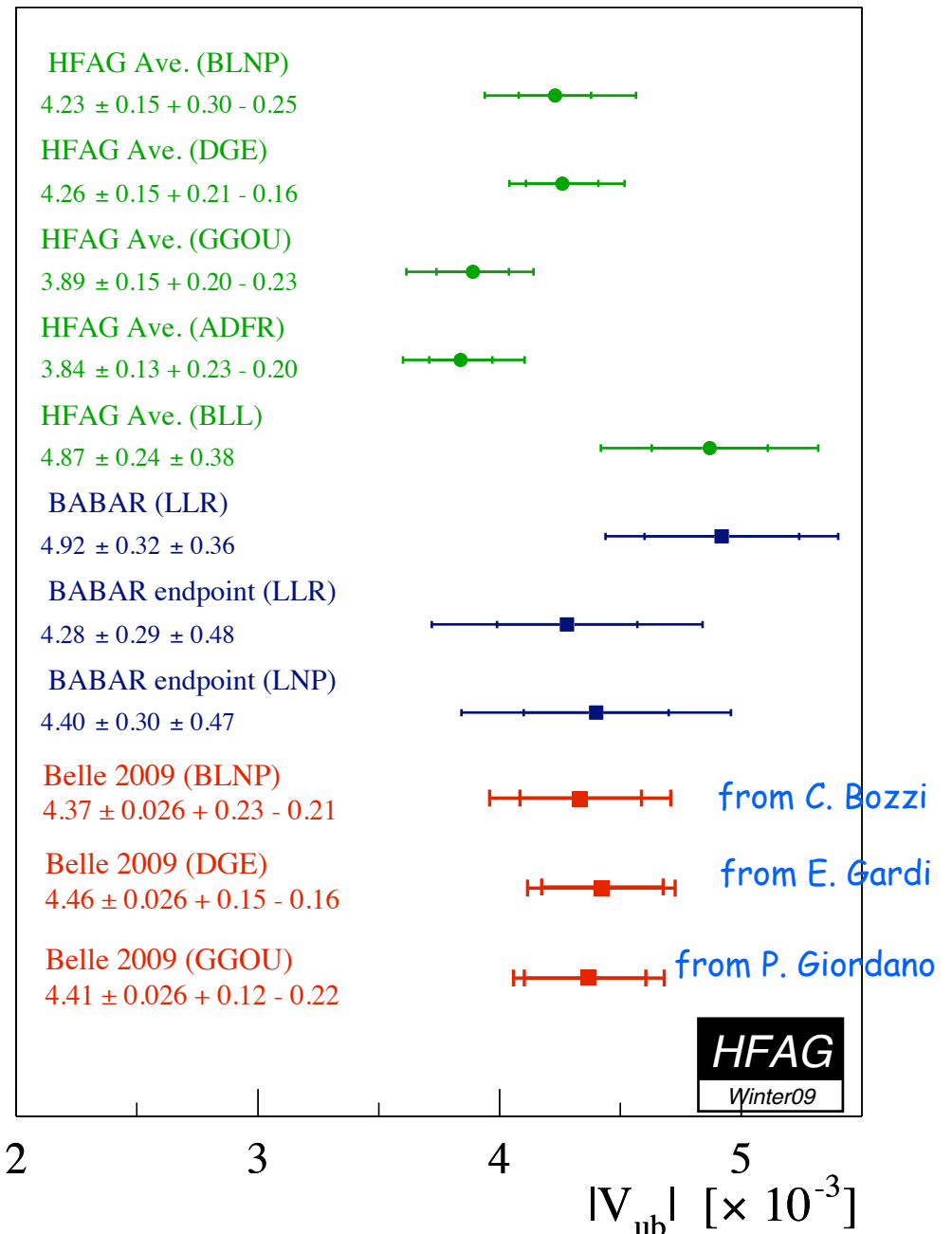
Measure absolute rate:



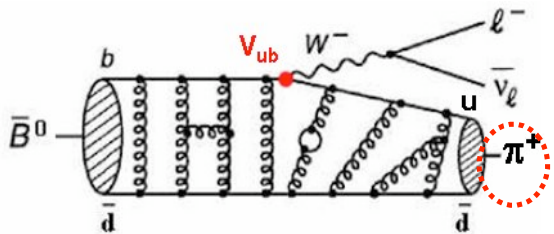
$$\Delta B(B \rightarrow Xu | \nu : p^{*B} > 1.0 \text{ GeV}/c) = 1.963(1 \pm 0.088 \text{ (stat.)} \pm 0.081 \text{ (sys.)}) \cdot 10^{-3}$$

Theoretical error smaller than the world average mainly due to a small power dependence on  $m_b$

~90% total phase space, thus theory error less correlated to other  $V_{ub}$  determinations



# Exclusive $|V_{ub}|$



Experimentally clean

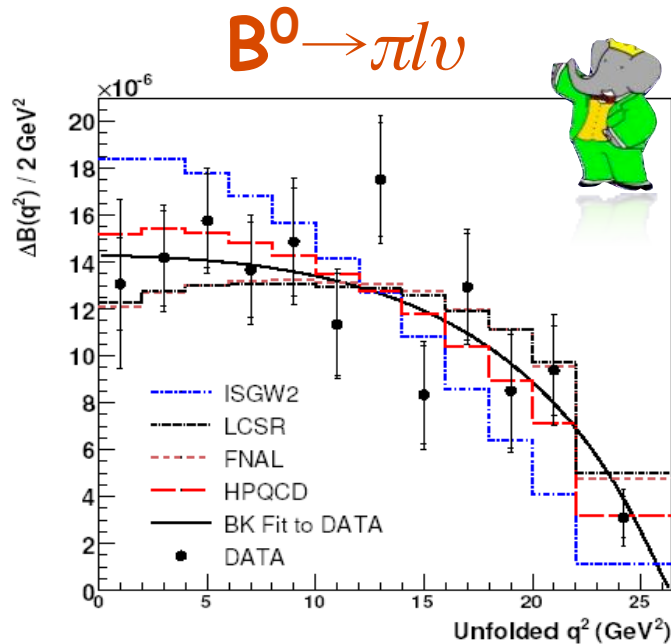
Currently only  $B \rightarrow \pi \ell \nu$  for  $|V_{ub}|$

$$\frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 P_\pi^3 |f_+(q^2)|^2$$

form factor:  $q^2$  shape and normalization needed

Form factor calculations from various methods:

- "unquenched" lattice QCD (HPQCD, Fermilab, ...)
- Light-Cone Sum Rules (Ball & Zwicky, ...)
- quark models (ISGW2, ...)



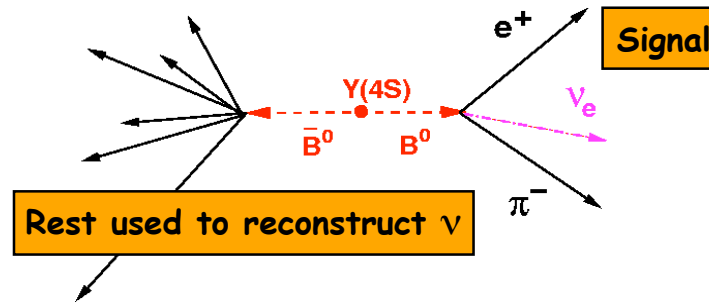
Phys.Rev.Lett. 98 (2007) 091801

LQCD and LCSR compatible with data ISGW2 quark-model incompatible (Prob<0.06%).

# Approaches to Measuring $B(B \rightarrow X_u | \nu)$ Exclusive

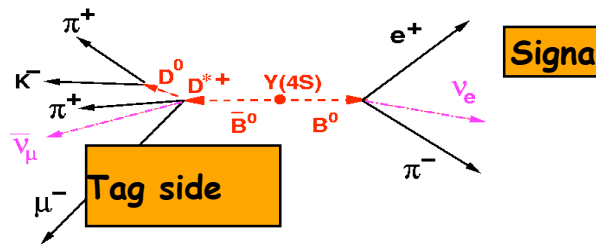
## Untagged

- ▶ Initial 4-momentum known.
- ▶ Missing 4-momentum =  $\nu$ .
- ▶ Reconstruct  $B \rightarrow X_u | \nu$  using  $m_B$  (beam-constrained) and  $\Delta E = E_B - E_{\text{beam}}$ .



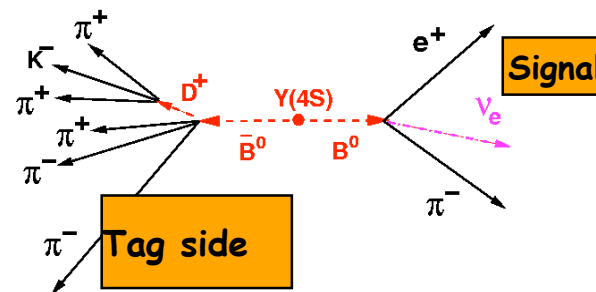
## Semileptonic Tag

- ▶ One B reconstructed in a selection of  $D^{(*)} | \nu$  modes.
- ▶ Two missing  $\nu$ s in event.
- ▶ Use kinematic constraints.



## Full Reconstruction Tag

- ▶ One B reconstructed completely in known  $b \rightarrow c$  mode.

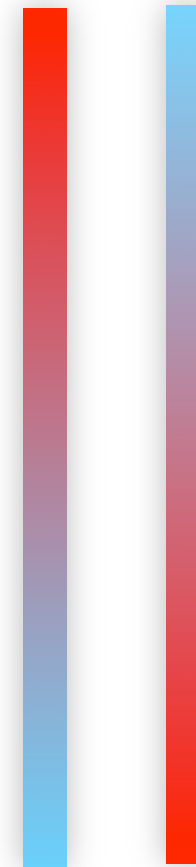


E. Barberio

Effi. Purity

High Low

Lumi.



$< 0.5 \text{ ab}^{-1}$

$< 1 \text{ ab}^{-1}$

$> 1 \text{ ab}^{-1}$

Low High

# Exclusive $V_{ub}$

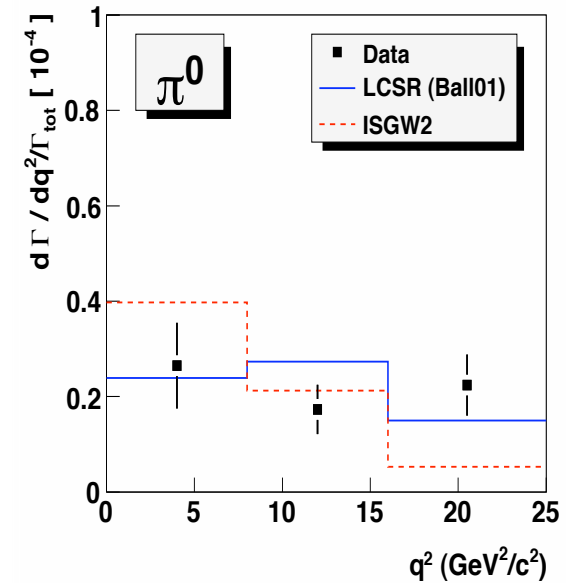
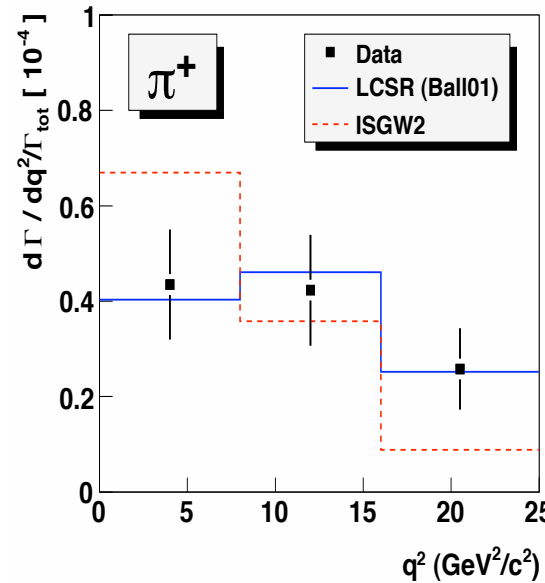
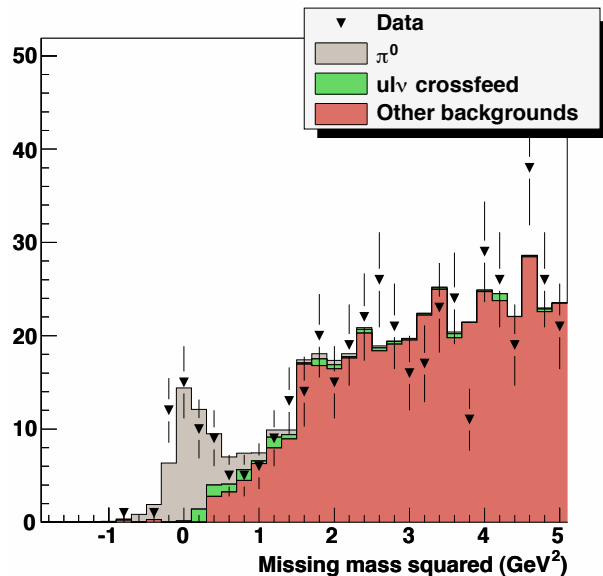
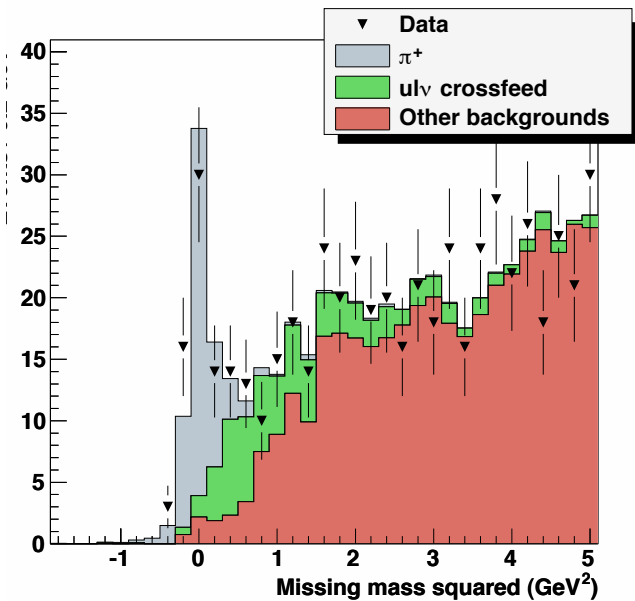
hep-ex 0812.12214



604 M BB

Fully reconstruct the  $B_{reco}$ : Know kinematics and flavor of signal B

Signal yields from  $M_{miss}^2 \approx M_{\nu}^2$  distribution in 3  $q^2$  bins

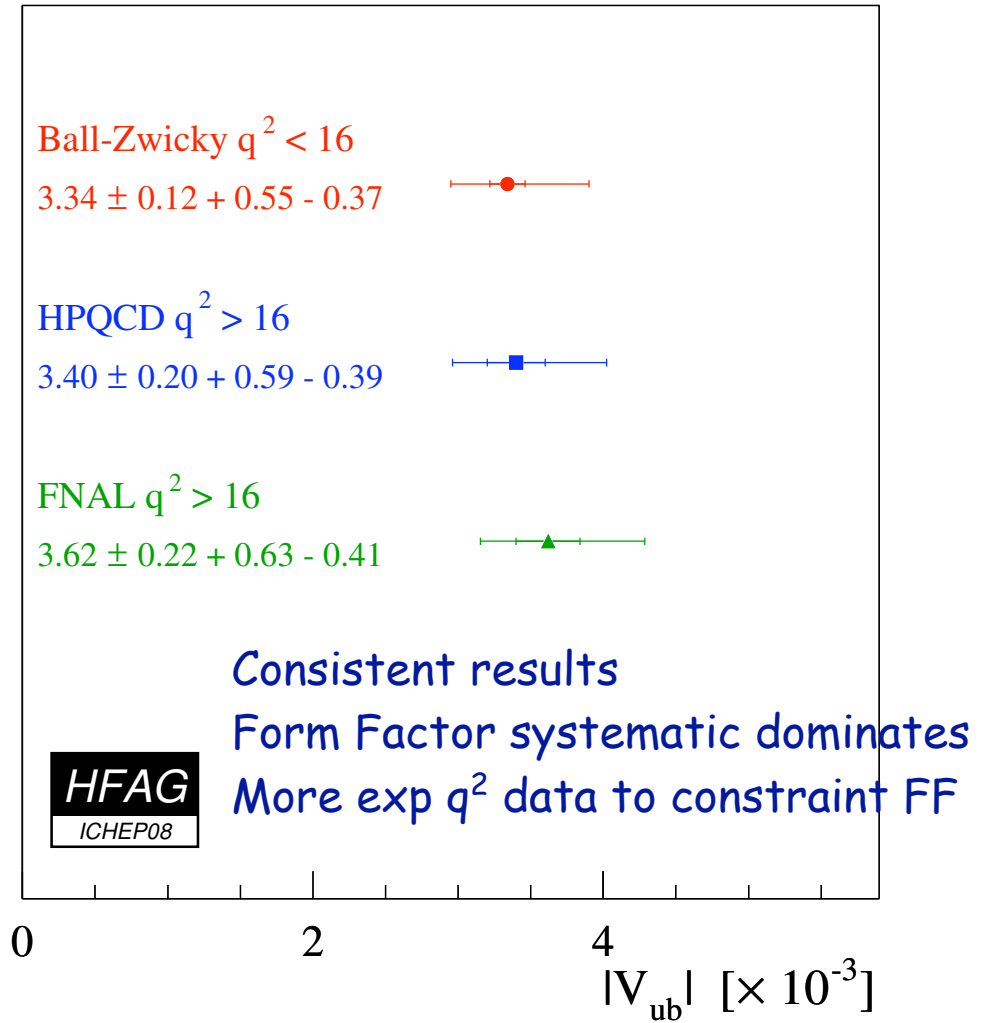
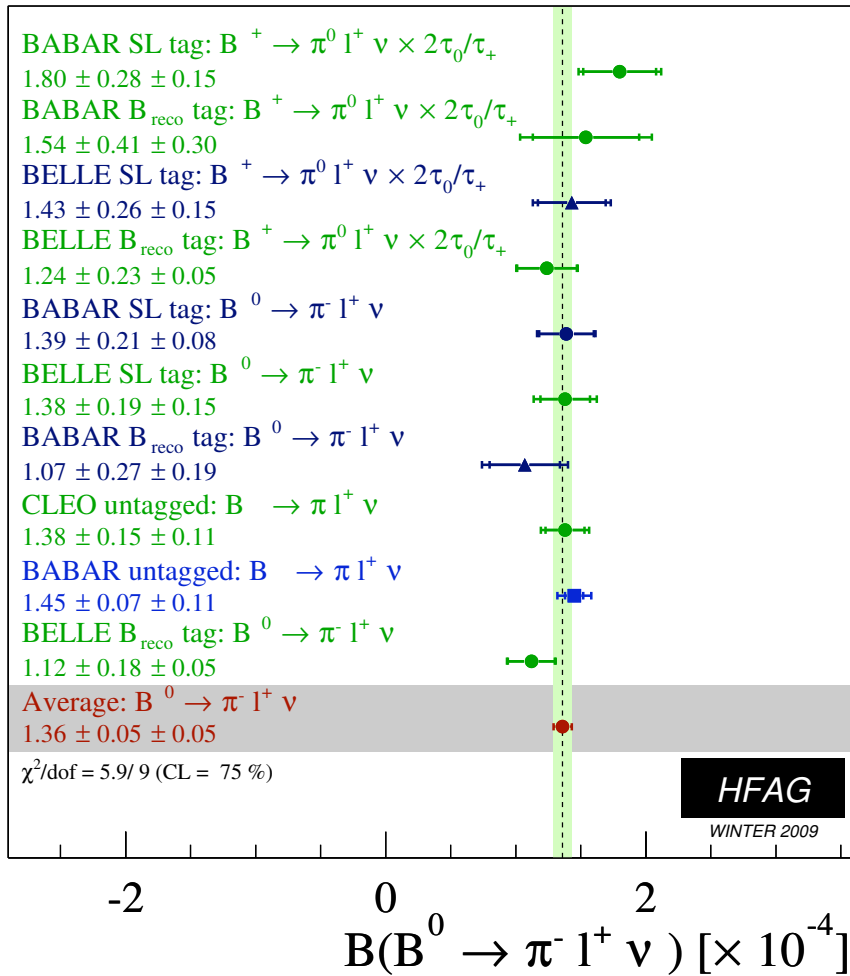


$$\Delta\mathcal{B}(B \rightarrow \pi\ell\nu)_{q^2 < 16 \text{ GeV}^2} = (0.82 \pm 0.12 \pm 0.04) \times 10^{-4}$$

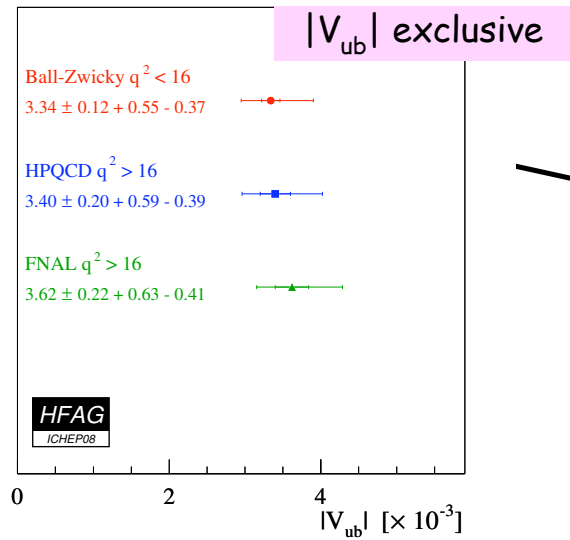
$$\Delta\mathcal{B}(B \rightarrow \pi\ell\nu)_{q^2 > 16 \text{ GeV}^2} = (0.31 \pm 0.07 \pm 0.02) \times 10^{-4}$$

E. Barberio

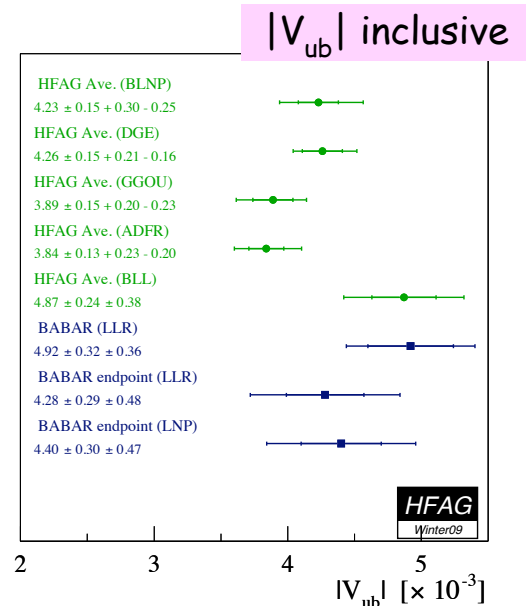
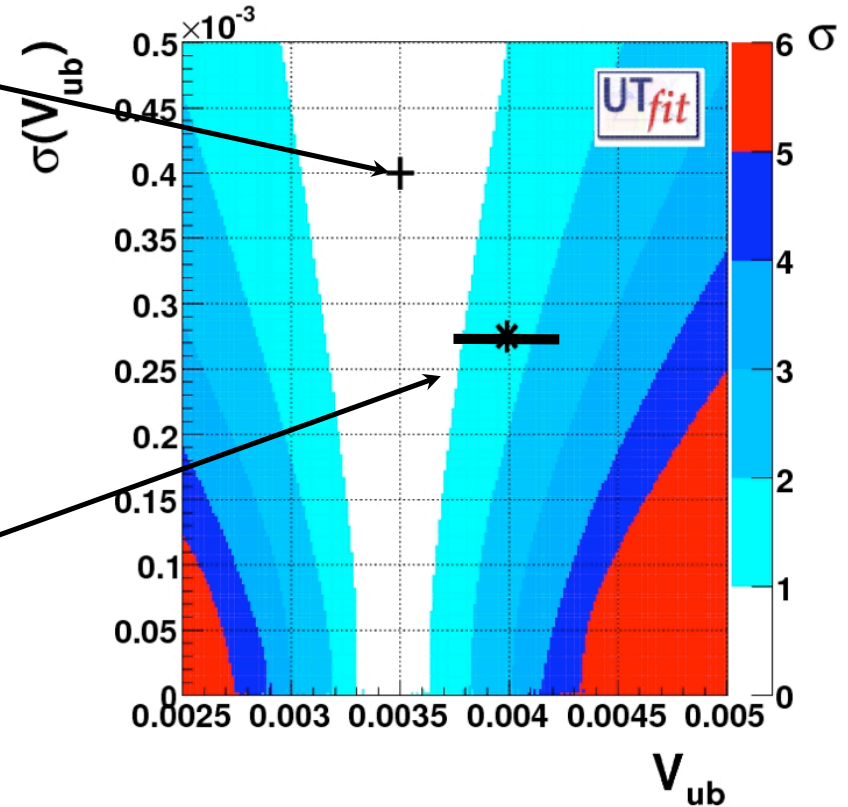
# Current status of $\text{Br}(B^0 \rightarrow \pi^- l^+ \nu)$ & $|V_{ub}|$



# $|V_{ub}|$ : inclusive vs exclusive



Most probable value of  $V_{ub}$  from measurements from other CKM parameters



# CONCLUSION AND OUTLOOK

Inclusive  $V_{ub}$   $\sim 6-7\%$  with the theoretical error is dominated by the extrapolation to the unmeasured phase space

The new Belle measurement alone has  $\sim 7\%$  error and covers 90% of phase space : all the extraction frameworks used are in good agreement

Exclusive: the error is coming down, it is the way to go in the future (super b-factories)

The inclusive vs exclusive: the gap is decreasing mainly due to the change in the value of  $m_b$



# $m_b$ more in details

comparison from  
A. Hoang  
Vxb workshop 2007  
Heidelberg

1S	$m_b$ (GeV)
	$4.70 \pm 0.03$
no $b \rightarrow s \gamma$	$4.751 \pm 0.058$

Kinetic	$m_b$ (GeV)
	$4.613 \pm 0.022 + 0.027$
no $b \rightarrow s \gamma$	$4.677 \pm 0.053$

$m^{1S} = 4.76$

$m^{1S} = 4.83$

1S fit

Kin fit

	$b \rightarrow s \gamma$	no $b \rightarrow s \gamma$	$b \rightarrow s \gamma$	no $b \rightarrow s \gamma$
$m^{1S}$	4.70	4.75	4.76	4.83
$\overline{m}(\overline{m})$	4.18	4.22	4.23	4.30

$\overline{m}(\overline{m})_{PDG} = 4.20 \pm 0.07 \text{ GeV}$

⋮

HFAG Winter09

1S	$m_b$ (GeV)
	$4.70 \pm 0.03$

Kinetic	$m_b$ (GeV)
	$4.620 \pm 0.035$
no $b \rightarrow s \gamma$	$4.678 \pm 0.051$

# New Belle Multivariate analysis

Measure absolute rate:

$$\Delta B (B \rightarrow Xu | \nu : p^{*B_1} > 1.0 \text{ GeV}/c) = 1.963 \cdot (1 \pm 0.088 (\text{stat.}) \pm 0.081 (\text{sys.})) \cdot 10^{-3}$$

Scheme	HQE par.	$ V_{ub} $
BLNP	$m_b(\text{SF}) = 4.650^{+0.043}_{-0.037} \text{ GeV}$ $\mu_{\pi^2}(\text{kin})(\text{SF}) = 0.256^{+0.057}_{-0.077} \text{ GeV}^2$	$4.37 \cdot (1 \pm 0.043(\text{stat.}) \pm 0.040(\text{sys.}))^{+0.031}_{-0.027} (m_b)^{+0.043}_{-0.040} (\text{th.}) \cdot 10^{-3}$
DGE	$m_b(\text{MSbar}) = 4.248 \pm 0.051 \text{ GeV}$	$4.46 \cdot (1 \pm 0.043(\text{stat.}) \pm 0.040(\text{sys.}))^{+0.032}_{-0.033} (m_b)^{+0.010}_{-0.015} (\text{th.}) \cdot 10^{-3}$
GGOU	$m_b(\text{kin}) = 4.620 \pm 0.035 \text{ GeV}$ $\mu_{\pi^2}(\text{kin}) = 0.424 \pm 0.042 \text{ GeV}^2$	$4.41 \cdot (1 \pm 0.043(\text{stat.}) \pm 0.040(\text{sys.}) \pm 0.019(m_b)^{+0.021}_{-0.045} (\text{th.})) \cdot 10^{-3}$

# Exclusive $V_{ub}$

arXiv:0805.2408 [hep-ex]

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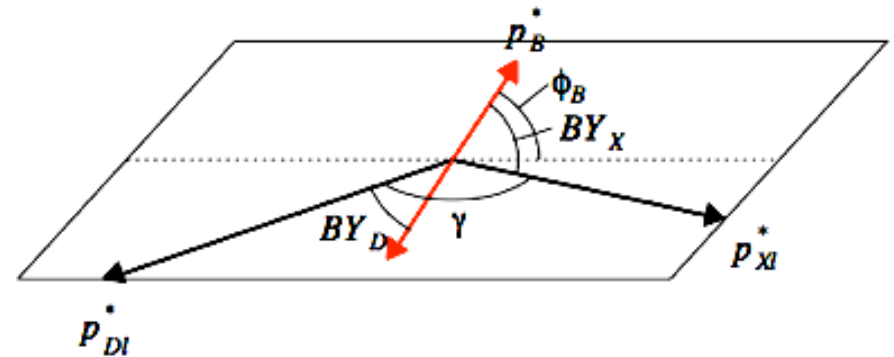
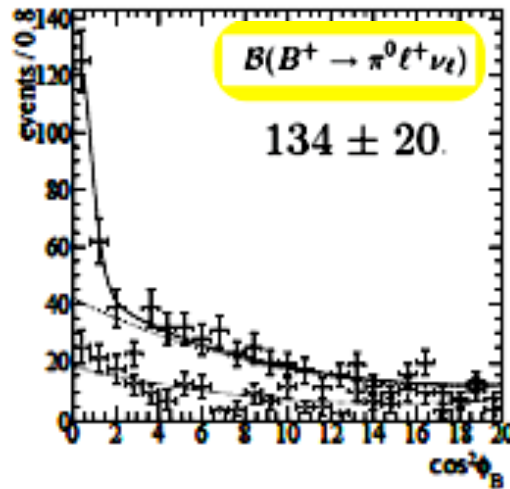
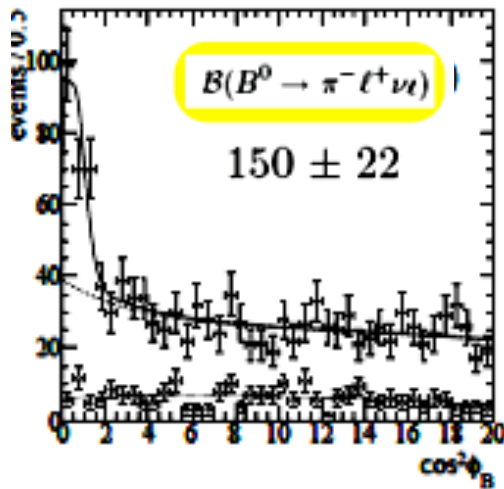


Semileptonic Tag

Tag one B in  $D^{(*)} | \nu$  and look for signal in the recoil;

Identify a signal lepton and search for a  $\pi^+$ ,  $\pi^-$ ,  $\pi^0$

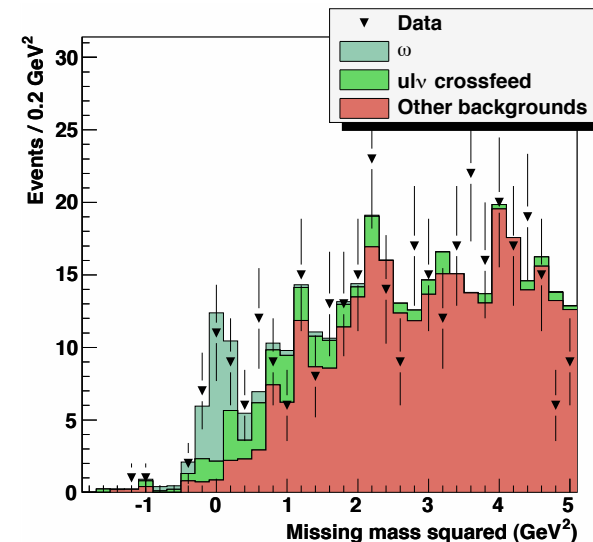
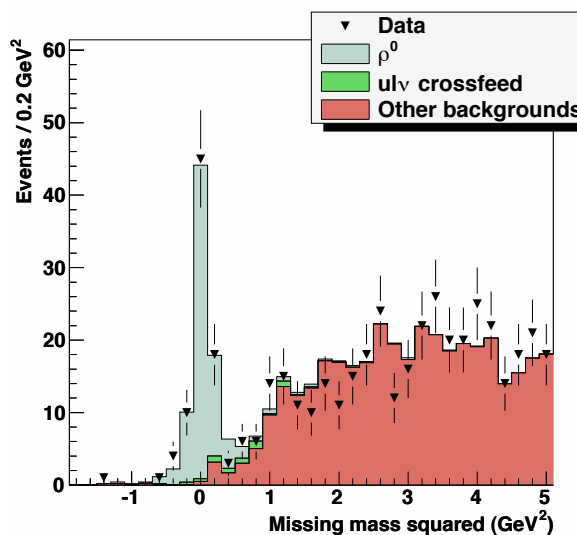
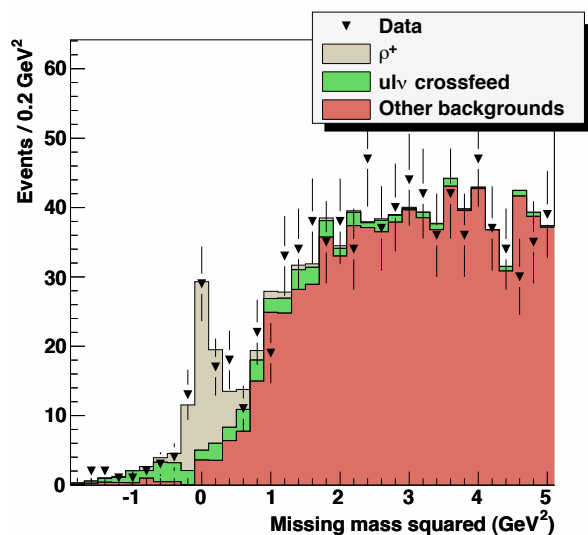
Signal yields from unbinned maximum likelihood fit to  $\cos^2 \varphi_B$ .



$$B(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = (1.38 \pm 0.21 \pm 0.07) \times 10^{-4}$$

$$B(B^+ \rightarrow \pi^0 \ell^+ \nu_\ell) = (0.96 \pm 0.15 \pm 0.07) \times 10^{-4}$$

# Full reconstruction



Mode	$\Delta\mathcal{B} [10^{-4}]$			$\mathcal{B} [10^{-4}]$
	$0 < q^2 < 8$ (GeV <sup>2</sup> )	$8 < q^2 < 16$ (GeV <sup>2</sup> )	$q^2 > 16$ (GeV <sup>2</sup> )	Sum (GeV <sup>2</sup> )
$B \rightarrow \rho^+ l\nu$	$0.74 \pm 0.29 \pm 0.04$	$1.01 \pm 0.28 \pm 0.05$	$0.81 \pm 0.21 \pm 0.04$	$2.56 \pm 0.46 \pm 0.12$
$B \rightarrow \rho^0 l\nu$	$0.72 \pm 0.15 \pm 0.03$	$0.70 \pm 0.13 \pm 0.03$	$0.39 \pm 0.11 \pm 0.02$	$1.80 \pm 0.23 \pm 0.07$
$B \rightarrow \omega l\nu$	$0.23 \pm 0.17 \pm 0.01$	$0.64 \pm 0.21 \pm 0.03$	$0.32 \pm 0.17 \pm 0.01$	$1.19 \pm 0.32 \pm 0.05$