

$|V_{ub}|$ and $|V_{cb}|$ from Inclusive B Decays

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Outline

- 1 Introduction
- 2 Inclusive $|V_{cb}|$
 - Global Fits
- 3 Inclusive $|V_{ub}|$
 - Current Status
 - New Developments

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1 Introduction

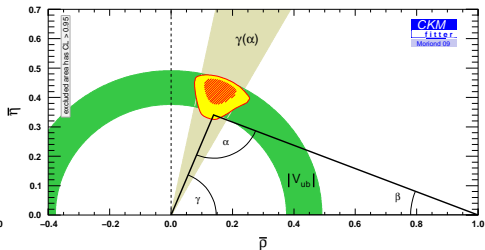
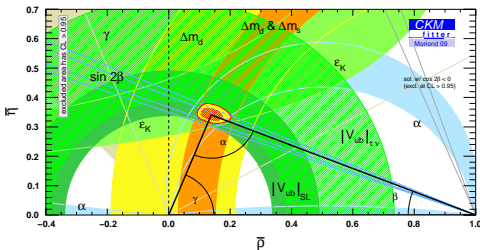
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3 Inclusive $|V_{ub}|$

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Why Care About $|V_{ub}|$ and $|V_{cb}|$?



$|V_{ub}|$ and $|V_{cb}|$ are important ingredients in Unitarity Triangle

- $|V_{ub}|$ dominant uncertainty in side opposite β
 - ▶ $\sin 2\beta$ favors small $|V_{ub}| \Rightarrow > 2\sigma$ tension (amplified by $B \rightarrow \tau\nu$)
- Constraint from ϵ_K depends on $|V_{cb}|^4$
- Crucial to obtain SM reference UT to compare tree and loop processes

To turn small discrepancies into hints of New Physics, model independent predictions are mandatory

Inclusive vs. Exclusive vs. Leptonic $|V_{ub}|$

Small but persistent systematic difference between inclusive and exclusive

$$\text{Leptonic:} \quad 10^3 |V_{ub}|_{B \rightarrow \tau \nu} = 5.2 \pm 0.5_{[\text{exp}]} \pm 0.4_{[f_B]}$$

$$\text{Inclusive OPE:} \quad 10^3 |V_{ub}|_{\text{BLL}} = 4.87 \pm 0.24_{[\text{exp}]} \pm 0.38_{[\text{theory}]}$$

$$\text{Inclusive SCET:} \quad 10^3 |V_{ub}|_{\text{BLNP}} = 4.32 \pm 0.16_{[\text{exp}]} \begin{matrix} +0.32 \\ -0.27 \end{matrix}_{[\text{theory}]}$$

$$\text{Exclusive:} \quad 10^3 |V_{ub}|_{B \rightarrow \pi \ell \nu} = 3.38 \pm 0.36_{[\text{exp}+\text{lattice}]}$$

- Uncertainties in inclusive determinations are underestimated
- Exclusive almost at **10%** from improved lattice calculation combined with model independent treatment of $B \rightarrow \pi \ell \nu$ form factor [Fermilab/MILC (2008)]
[→ see talk by Ruth Van de Water]

Before starting to get excited about a charged Higgs in $B \rightarrow \tau \nu$

- Inclusive and exclusive $|V_{ub}|$ have to converge
- Uncertainty on f_B should go down

Inclusive vs. Exclusive $|V_{cb}|$

Looks like $|V_{cb}|$ wants to get some attention as well

$$\text{Inclusive OPE: } 10^3 |V_{cb}|_{\text{kinetic}} = 41.48 \pm 0.48_{[\text{exp}]} \pm 0.58_{[\text{theory}]}$$

$$\text{Exclusive: } 10^3 |V_{cb}|_{B \rightarrow D \ell \nu} = 39.1 \pm 1.4_{[\text{exp}]} \pm 0.9_{[\text{lattice}]}$$

$$\text{Exclusive: } 10^3 |V_{cb}|_{B \rightarrow D^* \ell \nu} = 38.3 \pm 0.5_{[\text{exp}]} \pm 1.0_{[\text{lattice}]}$$

[→ see talk by Christoph Schwanda]

Recent first unquenched lattice calculation of $B \rightarrow D^* \ell \nu$ form factor

[Fermilab/MILC (2008)]

[→ see talk by Ruth Van de Water]

- Exclusive $|V_{cb}|$ is 8% lower than inclusive ($> 2\sigma$ discrepancy)
- Hard to imagine how inclusive $|V_{cb}|$ could go down more than $\sim 0.5\%$

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OPE for Inclusive Decays

Dependence on final state X drops out when summing over all X

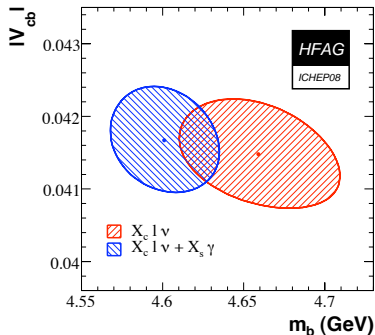
$$\begin{aligned}
 d\Gamma &= \sum_X \left| \text{Diagram 1} \right|^2 = \text{Im} \left[\text{Diagram 2} \right] \\
 &= \sum_n C_n(p) \times \text{Diagram 3} \\
 &= \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[C_0(\alpha_s) \mathbf{1} + \frac{0}{m_b} + C_1(\alpha_s) \frac{\lambda_1}{m_b^2} + C_2(\alpha_s) \frac{\lambda_2}{m_b^2} + \dots \right]
 \end{aligned}$$

- LD properties of B meson parametrized by MEs of local operators
 - ▶ $\mu_\pi^2 \sim -\lambda_1 \sim \langle k^2 \rangle$, $\mu_G^2 \sim 3\lambda_2 \sim \langle \sigma_{\mu\nu} G^{\mu\nu} \rangle \sim m_{B^*}^2 - m_B^2$
- SD physics contained in perturbative coefficients $C_n(p)$
 - ▶ $C_0(\alpha_s)$ given by perturbative quark decay
 - ▶ To get well-behaved α_s series need a SD mass $m_b^{1S}, m_b^{\text{kin}}, \dots$

Global $|V_{cb}|$ Fits

$|V_{cb}|$ is determined from a combined fit

- $B \rightarrow X_c l \nu$ partial rates (with cut on E_ℓ)
 - ▶ Normalization determines $|V_{cb}|$
- $B \rightarrow X_c l \nu$ lepton energy and hadronic mass moments
 - ▶ Shapes of distributions (moments) determine quark masses $m_{b,c}$ and nonperturbative parameters $\lambda_{1,2}, \dots$



Two schemes

- 1S [Bauer et al. (2002, 2004)], kinetic [Benson et al. (2003); Gambino, Uraltsev (2004)]

Current HFAG result in kinetic scheme

[→ see talk by Christoph Schwanda]

$$|V_{cb}| = 41.48 \cdot 10^{-3} \times (1 \pm 1.1\%_{\text{fit}} \pm 0.2\%_{\text{TB}} \pm 1.4\%_{\text{theory}})$$

⇒ Limited by theory uncertainty

$B \rightarrow X_s \gamma$ Moments and $|V_{cb}|$

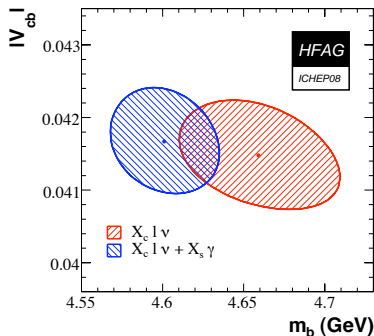
By default fit includes $B \rightarrow X_s \gamma$ photon energy moments

Input	$ V_{cb} $ [10^{-3}]	m_b^{kin} [GeV]
$B \rightarrow X_c l \nu + X_s \gamma$	$41.67 \pm 0.43_{\text{[fit]}} \pm \dots$	4.601 ± 0.034
$B \rightarrow X_c l \nu$ only	$41.48 \pm 0.47_{\text{[fit]}} \pm \dots$	4.659 ± 0.049

- Application of local OPE to $B \rightarrow X_s \gamma$ moments is on much less solid ground than for $B \rightarrow X_c l \nu$ moments

If goal is

- to determine $|V_{cb}|$
 - $B \rightarrow X_c l \nu$ alone is as precise, so no reason to include $B \rightarrow X_s \gamma$ here
- to obtain precise m_b as input for $|V_{ub}|$ determination
 - There is a better way to include $B \rightarrow X_s \gamma$ data (as I will show)



Theory Status

Rate is a double expansion in α_s and $(\Lambda_{\text{QCD}}/m_b)^n$

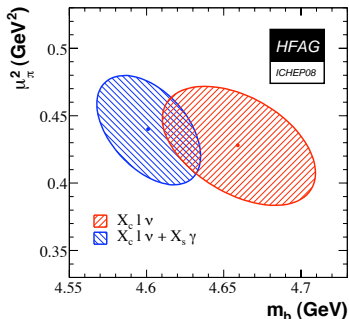
$$d\Gamma = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[C_0(\alpha_s) \mathbf{1} + \frac{0}{m_b} + C_1(\alpha_s) \frac{\lambda_1}{m_b^2} + C_2(\alpha_s) \frac{\lambda_2}{m_b^2} + \dots \right]$$

n		α_s^0	α_s^1	α_s^2/β_0	full α_s^2	
0		✓	✓	✓	(✓)	
2	$\lambda_1 \sim \mu_\pi^2$	✓	(✓)		(×)	✓ included in fits
2	$\lambda_2 \sim \mu_G^2$	✓	(×)		×	(✓) known, not yet included
3	$\sim 1/m_b^3$	✓	×		×	(×) being calculated
4	$\sim 1/m_b^4$	(✓)	×		×	×
4	$\sim 1/m_c^2 m_b^3$	(×)	×		×	×

- Probably most important missing pieces are $\alpha_s \mu_G^2$ corrections

Existing Theory Improvements

- $\alpha_s \mu_\pi^2$: Expect $\sim 20\%$ shift in value of μ_π^2
[Becher, Boos, Lunghi (2007)]
 - ▶ Likely no effect on $|V_{cb}|$, but will be interesting to see effect on m_b
- full α_s^2 vs. $\alpha_s^2 \beta_0$: Mostly affect total rate
[Melnikov; Czarnecki, Pak (2008)]
 - ▶ Shift $|V_{cb}|$ by -0.5% (kinetic scheme) or -0.3% (1S scheme)
- $1/m_b^4$: Small effect ($\sim 0.25\%$ on total rate) [Dassinger, Turczyk, Mannel (2007)]



Limiting 1.4% theory uncertainty in kinetic scheme fit comes from first fitting for total $\mathcal{B}(B \rightarrow X_c \ell \nu)$ and then converting to $|V_{cb}|$

- Was obtained from estimates of now (mostly) known contributions
- With total and differential rates known at same level in expansion should avoid additional step and directly fit for $|V_{cb}|$

Wishlist for Future Fits

$$|V_{cb}| = 41.48 \cdot 10^{-3} \times (1 \pm 1.1\%_{[\text{fit}]} \pm 0.2\%_{[\tau_B]} \pm 1.4\%_{[\text{theory}]})$$

Eagerly awaiting updated HFAG fit in the 1S scheme ...

How to gain confidence in $\mathcal{O}(1\%)$ (theory) uncertainties

- Please provide the theory expressions that are actually going into fits
- Compare separate fits at $\mathcal{O}(1, \alpha_s, \alpha_s^2)$
- Keep α_s as free fit parameter?
- How do you feel about adding theory errors in quadrature at **1%** level?
 - ▶ Separate out fit uncertainties into experimental and theoretical parts
 - ▶ Should think carefully about theory correlations
 - ▶ If feasible, should also try RFit [CKM Fitter] treatment of theory uncertainties

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$|V_{ub}|$ from Inclusive $B \rightarrow X_u \ell \nu$

Removing huge charm background requires stringent phase space cuts

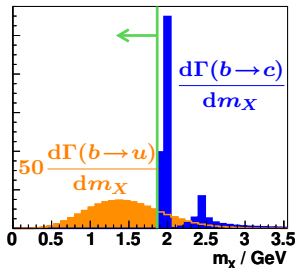
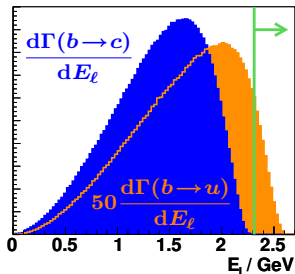
$$\mathcal{B}(B \rightarrow X_c \ell \nu) / \mathcal{B}(B \rightarrow X_u \ell \nu) \simeq 50$$

- Cuts can drastically enhance perturbative and nonperturbative corrections

Rates become sensitive to b -quark PDFs in B meson

- Determine shape of spectra
- Leading order: Universal shape function (SF) [Neubert (1993); Bigi et al. (1993)]
- $\mathcal{O}(\Lambda_{\text{QCD}}/m_b)$: Several more subleading shape functions [Bauer, Luke, Mannel (2001)]
- Need to be extracted from data (like any PDF)

[→ see talk by Elisabetta Barberio for recent measurements and averages]



Regions of Phase Space

Kinematic variables: $p_X^\pm = E_X \mp |\vec{p}_X|$

Shape function region (SCET region): $p_X^+ \ll p_X^-$

- Leading order in $1/m_b$ requires nonperturbative shape function $S(\omega)$

[Korchemsky, Sterman (1994); Bauer et al. (2001)]

$$d\Gamma = H(E_\ell, p_X^\pm) \int d\omega J[p_X^- (p_X^+ - \omega)] S(\omega)$$

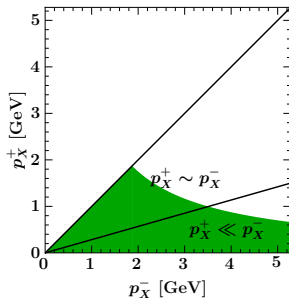
- $\mathcal{O}(\alpha_s^2)$ corrections recently completed

[Becher, Neubert (2005, 2006); Bonciani, Ferroglia; Asatrian et al.; Beneke et al.; Bell (2008)]

Local OPE region: $p_X^+ \sim p_X^-$ (large q^2 , small E_ℓ)

- Leading order in $1/m_b$ given by quark decay (as in $B \rightarrow X_c \ell \nu$) known to $\mathcal{O}(\alpha_s, \alpha_s^2 \beta_0)$ [De Fazio, Neubert (1999); Gardi, Ridolfi, Gambino (2006)]

Cut on $m_X < m_D$ does not imply $p_X^+ \ll p_X^- \Rightarrow$ depends on both regions



Current Approaches

Current approaches are essentially based on theory for one region and are extrapolated/modeled into other region

	BLNP [Bosch et al. (2004, 2005)]	GGOU [Gambino et al. (2007)]
based on	SCET region	local OPE region
SCET region	$\mathcal{O}(\alpha_s)$ NLL resummation	$\mathcal{O}(\alpha_s, \alpha_s^2 \beta_0)$ no resummation
local OPE region m_b scheme	partly $\mathcal{O}(\alpha_s)$, partly model tied to SF scheme m_b^{SF}	$\mathcal{O}(\alpha_s, \alpha_s^2 \beta_0)$ uses kinetic scheme m_b^{kin}
nonpert. input	LO: universal SF $S(\omega)$ $1/m_b$: 3 subleading SFs	3 LO distribution functions $F_i(k_+, q^2)$

- BLL: local OPE at large q^2 (consistency important cross check)
[Bauer, Ligeti, Luke (2000, 2001)]
- DGE: Fixed perturbative model for SF (from renormalon resummation)
[Andersen, Gardi (2006, 2008)]

Non-Experimental Uncertainties

Theoretical uncertainties

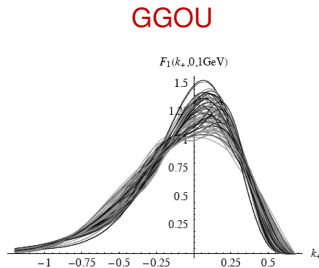
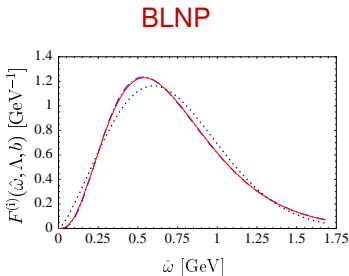
- Unknown higher orders in α_s , $1/m_b$ expansions
- Weak annihilation (open question \Rightarrow separate data into B^+ and B^0)

Uncertainties from input parameters

- m_b : Total rate $\sim |V_{ub}|^2 m_b^5$, partial rates with cuts $\sim |V_{ub}|^2 m_b^{\mathcal{O}(10)}$
 - ▶ Need precise m_b to get precise $|V_{ub}|$
 - ▶ Like to avoid scheme changes ($m_b^{1S} \leftrightarrow m_b^{SF} \leftrightarrow m_b^{\text{kin}}$)
 - ▶ Currently taken from $B \rightarrow X_c \ell \nu + X_s \gamma$ global $|V_{cb}|$ fits
- Leading shape/distribution function(s): Observables can depend on
 - only 1st moment $\simeq m_b$: total rate, q^2 spectrum
 - all moments, i.e. the full shape: p_X^+ , large E_ℓ
 - something in between: m_X
 - ▶ Ideally: Extract from data (e.g. $B \rightarrow X_s \gamma$ spectrum)
 - ▶ Currently: Modeled with 1st+2nd moment fixed by m_b, μ_π^2

Shape Function Models and Uncertainties

LO SF models
for fixed m_b, μ_π^2



$\delta|V_{ub}|$ (models)

2.2%

?

1.0%

$\delta|V_{ub}|(m_b, \mu_\pi^2)$

5.1%



3.9%

These model/SF uncertainties are an underestimate

- Use precise m_b, μ_π^2 from $|V_{cb}|$ fits but otherwise fixed model functions
- Shape variation should reflect the actual information we have

⇒ Currently, we do not know inclusive $|V_{ub}|$ to better than 10%

Caveats in Measurements

Monte Carlo signal model depends on the shape function

- Corresponding systematic uncertainty is correlated with m_b and SF uncertainty in the theory
- Can become dominant systematic uncertainty if signal shape is needed for background subtraction, e.g. Babar lepton endpoint [PRD 73, 012006 (2006)]

\mathcal{B} with E_ℓ^{cut} [GeV]	2.0	2.1	2.2	2.3
other sys unc. [%]	8.8	8.6	7.9	6.6
SF sys unc. [%]	6.0 – 13.3	3.5 – 8.6	1.6 – 4.0	0.3 – 0.8

Caveats in Measurements

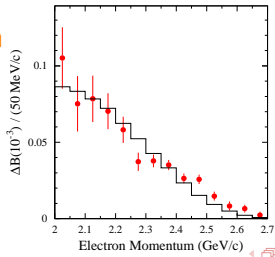
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Lepton-endpoint measurements define signal region with an explicit upper cut $E_\ell^{\Upsilon} < 2.6$ GeV

- Kinematic endpoint in $\Upsilon(4S)$ frame is $E_\ell^{\Upsilon} < 2.81$ GeV
- Rate is clearly nonzero for $E_\ell^{\Upsilon} > 2.6$ GeV
- Impossible to calculate on theory side



Strategy Towards Precision $|V_{ub}|$

Precision of inclusive $|V_{ub}|$ depends on

- How well we know m_b and SF and correlation between them
- Ability to (consistently) combine many different measurements
 - ▶ Different kinematic cuts: E_ℓ , m_X , q^2 , p_X^+
 - ▶ Different analysis techniques: hadronic tag, untagged

First, reduce SF uncertainties by incorporating all available information on it

- Perturbative constraints (perturbative tail and RGE)
- Moment constraints (m_b , λ_1 from $B \rightarrow X_{cl\nu}$)
- Shape information from $B \rightarrow X_s\gamma$ and $B \rightarrow X_{ul\nu}$ spectra

Then repeat success strategy of inclusive $|V_{cb}|$

- Perform global fit to all available data
- Simultaneously determine $|V_{ub}|$ and inputs (m_b , SF)

[Bernlochner, Lacker, Ligeti, Stewart, FT, K. Tackmann (work in progress)]



New and Improved Approach to Shape Function

[Ligeti, Stewart, FT (2008)]

Start with perturbative constraints on shape function. Derive factorized form

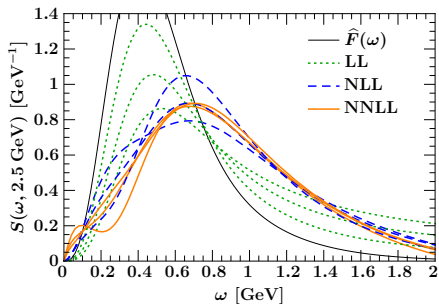
$$S(\omega, \mu_\Lambda) = \int dk \hat{C}_0(\omega - k, \mu_\Lambda) \hat{F}(k)$$

$\hat{F}(k)$ purely nonperturbative part

- Determines peak region

$\hat{C}_0(\omega, \mu_\Lambda)$ perturbative (partonic SF)

- Determines tail consistent with RGE
- Known to $\mathcal{O}(\alpha_s, \alpha_s^2)$
[Bauer, Manohar (2003); Becher, Neubert (2005)]
- For given $\hat{F}(k)$ can calculate $S(\omega)$ order by order in α_s , vary μ_Λ to estimate perturbative uncertainty

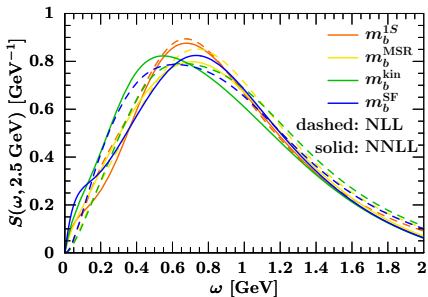


$\mu_\Lambda = (1.0, 1.3, 1.8) \text{ GeV}$
+ RGE to $\mu = 2.5 \text{ GeV}$

Different Short Distance Schemes

\widehat{C} and \widehat{F} defined in generic short distance scheme, can use any m_b scheme!

$$\begin{aligned}
 S(\omega) &= \int dk C_0^{\text{pole}}(\omega - k) F^{\text{pole}}(k) \\
 &= \int dk C_0^{1S}(\omega - k) F^{1S}(k) \\
 &= \int dk C_0^{\text{kin}}(\omega - k) F^{\text{kin}}(k) \\
 &= \int dk C_0^{\text{SF}}(\omega - k) F^{\text{SF}}(k) = \dots
 \end{aligned}$$



Moments of $\widehat{F}(k)$ given by corresponding SD HQE parameters $\widehat{m}_b, \widehat{\lambda}_1, \dots$ (at any order in α_s), e.g.

$$\int dk k^n F^{1Si}(k) = M_n = \begin{cases} 1 & (n = 0) \\ m_B - m_b^{1S} & (n = 1) \\ -\lambda_1^i/3 + (m_B - m_b^{1S})^2 & (n = 2) \end{cases}$$

⇒ Can avoid having to switch from different m_b scheme used for $B \rightarrow X_c \ell \nu$

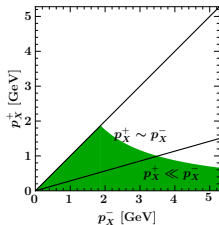
Master Formula

[Ligeti, Stewart, FT (to appear)]

Separation $S = \hat{C} \otimes \hat{F}$ allows to consistently connect

SCET region: $p_X^+ \ll p_X^-$

Local OPE region: $p_X^+ \sim p_X^-$

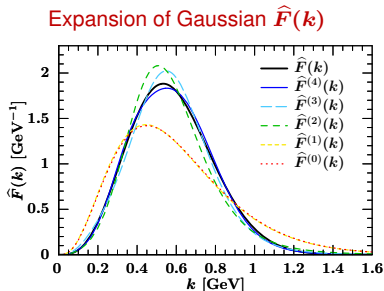
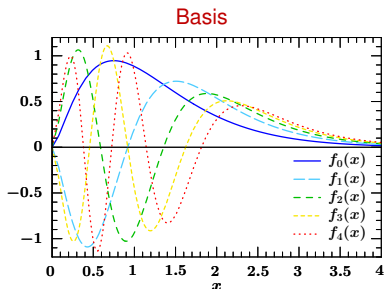


$$d\Gamma = |V_{ub}|^2 K(E_\ell, p_X^-, p_X^+) \int dk \widehat{W}_{\text{pert}}(p_X^-, p_X^+, k) \widehat{F}(k)$$

- Combines optimal descriptions for different phase space regions
- Smooth transition between correct fixed-order result in local OPE region and RGE improved result in SCET region
- Not the case in any previous approach!

Next: Determine nonperturbative input function $\widehat{F}(k)$

Designer Orthonormal Basis Functions

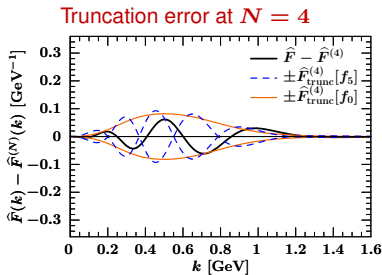
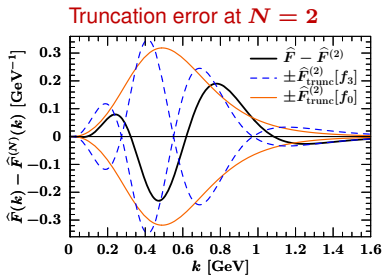


Design suitable orthonormal basis for $\widehat{F}(k)$ (formally model independent)

$$\widehat{F}(\lambda x) = \frac{1}{\lambda} \left[\sum_{n=0}^{\infty} c_n f_n(x) \right]^2 \quad \text{with} \quad \int dk \widehat{F}(k) = \sum_{n=0}^{\infty} c_n^2 = 1$$

- Builds an orthonormal basis on top of any given model function
- Keep terms up to $n \leq N$ as required by precision of data
- Experimental uncertainties and correlations can be properly captured by uncertainties and correlations in basis coefficients c_n

Estimating Residual Model Dependence



Truncating series at $n \leq N$ introduces residual dependence on basis model

- Overall size of truncation error scales with $1 - \sum_{n=0}^N c_n^2$
- Can test expansion by varying N and underlying basis model
- Choose final N so that truncation error is small compared to experimental uncertainties in coefficients

⇒ Allows for systematic, fully data driven SF uncertainties

Setup for Global $|V_{ub}|$ Fit

Use expansion $\widehat{F}(k) = [\sum_n c_n f_n(k)]^2$ in master formula and moments

$$d\Gamma = |V_{ub}|^2 \sum_{n,m} c_n c_m K(E_\ell, p_X^\pm) \int dk \widehat{W}_{\text{pert}}(p_X^\pm, k) f_n(k) f_m(k)$$

$$M_j(m_b^{1S}, \lambda_1^i) = \sum_{n,m} c_n c_m \int dk k^j f_n(k) f_m(k)$$

Perform combined fit (similar to $|V_{cb}|$)

- $B \rightarrow X_u \ell \nu$ partial rates
 - ▶ Normalization determines $|V_{ub}|$
- $B \rightarrow X_s \gamma$ and $B \rightarrow X_u \ell \nu$ spectra
 - ▶ Shapes of distributions constrain $\widehat{F}(k)$ through basis coefficients c_n
- Known moments of $\widehat{F}(k)$
 - ▶ Consistently combines existing constraints on m_b^{1S}, λ_1^i (from $B \rightarrow X_c \ell \nu$ or anywhere else) with $B \rightarrow X_u \ell \nu$ and $B \rightarrow X_s \gamma$ data

Proof-of-Concept

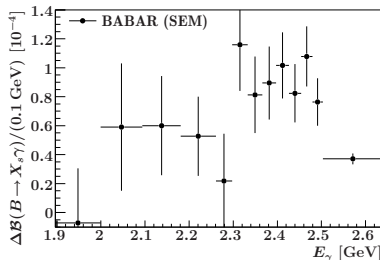
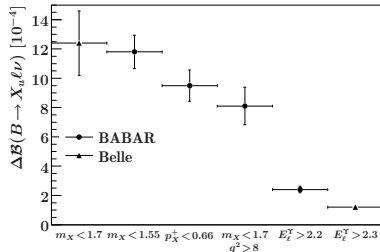
[Bernlochner, Lacker, Ligeti, Stewart, FT, K. Tackmann (work in progress)]

As proof-of-concept, fit to

- $B \rightarrow X_u \ell \nu$ hadronic tag
 - ▶ BABAR: $m_X, m_X - q^2, p_X^+$
 - ▶ Belle: m_X
- $B \rightarrow X_u \ell \nu$ lepton endpoint
 - ▶ BABAR: $E_\ell^{\Upsilon} > 2.2 \text{ GeV}$
 - ▶ Belle: $E_\ell^{\Upsilon} > 2.3 \text{ GeV}$
- $B \rightarrow X_s \gamma$ spectra
 - ▶ Babar sum over exclusive modes
 - ▶ Babar hadronic tag (not shown)
- m_b^{1S}, λ_1 from $B \rightarrow X_c \ell \nu$
 - ▶ Belle fit in 1S scheme

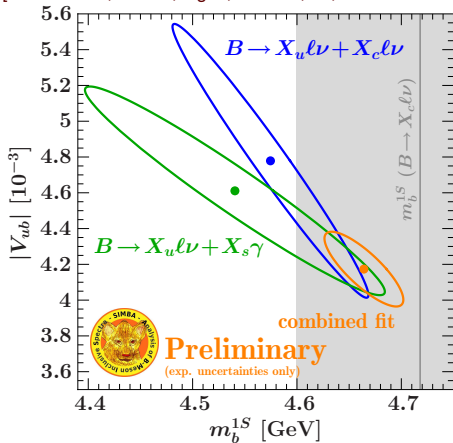
$$m_b^{1S} = (4.72 \pm 0.12) \text{ GeV}$$

$$\lambda_1 = (-0.31 \pm 0.09) \text{ GeV}^2$$

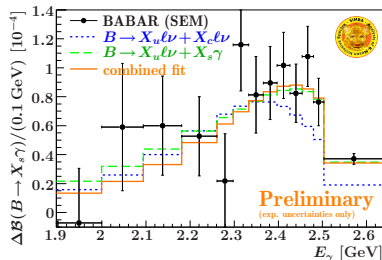
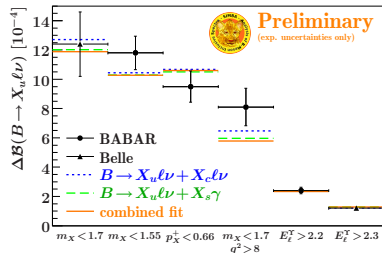


Proof-of-Concept

[Bernlochner, Lacker, Ligeti, Stewart, FT, K. Tackmann (work in progress)]



- Wrong E_γ spectrum without $B \rightarrow X_s \gamma$
- Significant improvement from combining $B \rightarrow X_s \gamma$ and $B \rightarrow X_c l \nu$



(Caution: Fits do not include theory uncertainties yet) ▶

Conclusions

Tensions between inclusive and exclusive determinations

- Somewhat disturbing for $|V_{cb}|$
- For $|V_{ub}|$ remains to be seen with improved analyses

Inclusive $|V_{cb}|$ from global fits between 1% – 2%

- Already theory limited, pushing theory below 1% very hard
 - Need to be careful how to treat theory uncertainties at $\mathcal{O}(1\%)$ level
- ⇒ Some theory improvements still possible, will increase confidence

Inclusive $|V_{ub}|$

- Improved treatment of SF and multiple phase space regions
- Work in progress towards combining all information into global fit similar to $|V_{cb}|$
- Will provide more rigorous uncertainties and test of theory
- Key to precision inclusive $|V_{ub}|$ from Super Flavor Factory

