Neutrinos vs Quarks
MNS vs CKM

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Based on work done in collaboration with K.T. Mahanthappa
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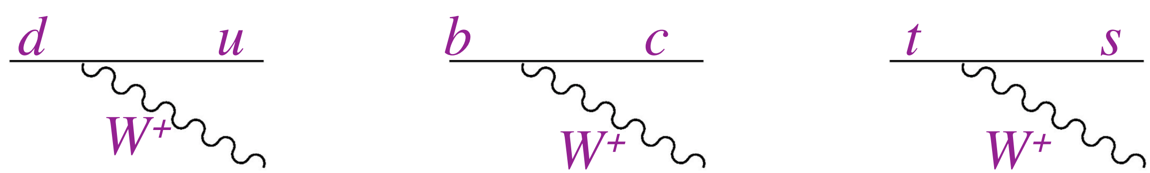
Mysteries of Masses and Mixing in SM

- Mixing among quarks



$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{cd} & V_{td} \\ V_{us} & V_{cs} & V_{ts} \\ V_{ub} & V_{cb} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

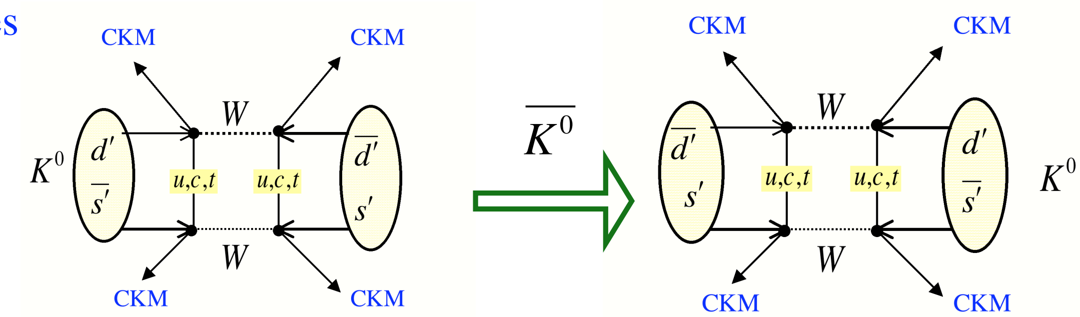
mass eigenstates — CKM matrix — weak eigenstates

Mass states: *d', s', b'*

Charged current weak states: *d, s, b*

CKM matrix → 3 mixin angles, 1 phase

Mass eigenstates = mixture of weak eigenstates



Compelling Neutrino Oscillation Evidences

Atmospheric Neutrinos:

SuperKamiokande (up-down asymmetry, L/E, θz dependence of μ-like events)

dominant channel: $\nu_\mu \to \nu_\tau$

next: K2K, MINOS, CNGS (OPERA)

Solar Neutrinos:

Homestake, Kamiokande, SAGE, GALLEX/GNO, SK, SNO, BOREXINO, KamLAND

dominant channel: $\nu_e \to \nu_{\mu,\tau}$

next: BOREXINO, KamLAND, ...

LSND:

dominant channel: $\overline{\nu}_\mu \to \overline{\nu}_e$

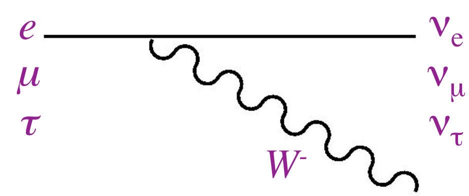
MiniBOONE -- negative result (2007)

Leptonic Mixing Parameters

- three neutrino mixing $\nu_{\ell L} = \sum_{j=1}^{3} U_{\ell j} \nu_{jL} \quad \ell = e,\ \mu,\ \tau$

- mismatch between weak and mass eigenstates $\nu_{1,\,2,\,3} \rightarrow m_{1,\,2,\,3}$

$$\mathcal{L}_{cc} = (\ \overline{\nu}_1, \quad \overline{\nu}_2, \quad \overline{\nu}_3\ )\gamma^\mu U^\dagger \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} W^+_\mu$$



- PMNS matrix

$$U = V \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix} \qquad V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

atm reactor solar

- Dirac CP-violating phase: $\delta = [0, 2\pi]$
- Majorana CP-violating phases: $\alpha_{21},\ \alpha_{31}$

CKM Matrix ⟺ PMNS Matrix

- Quark mixings are small

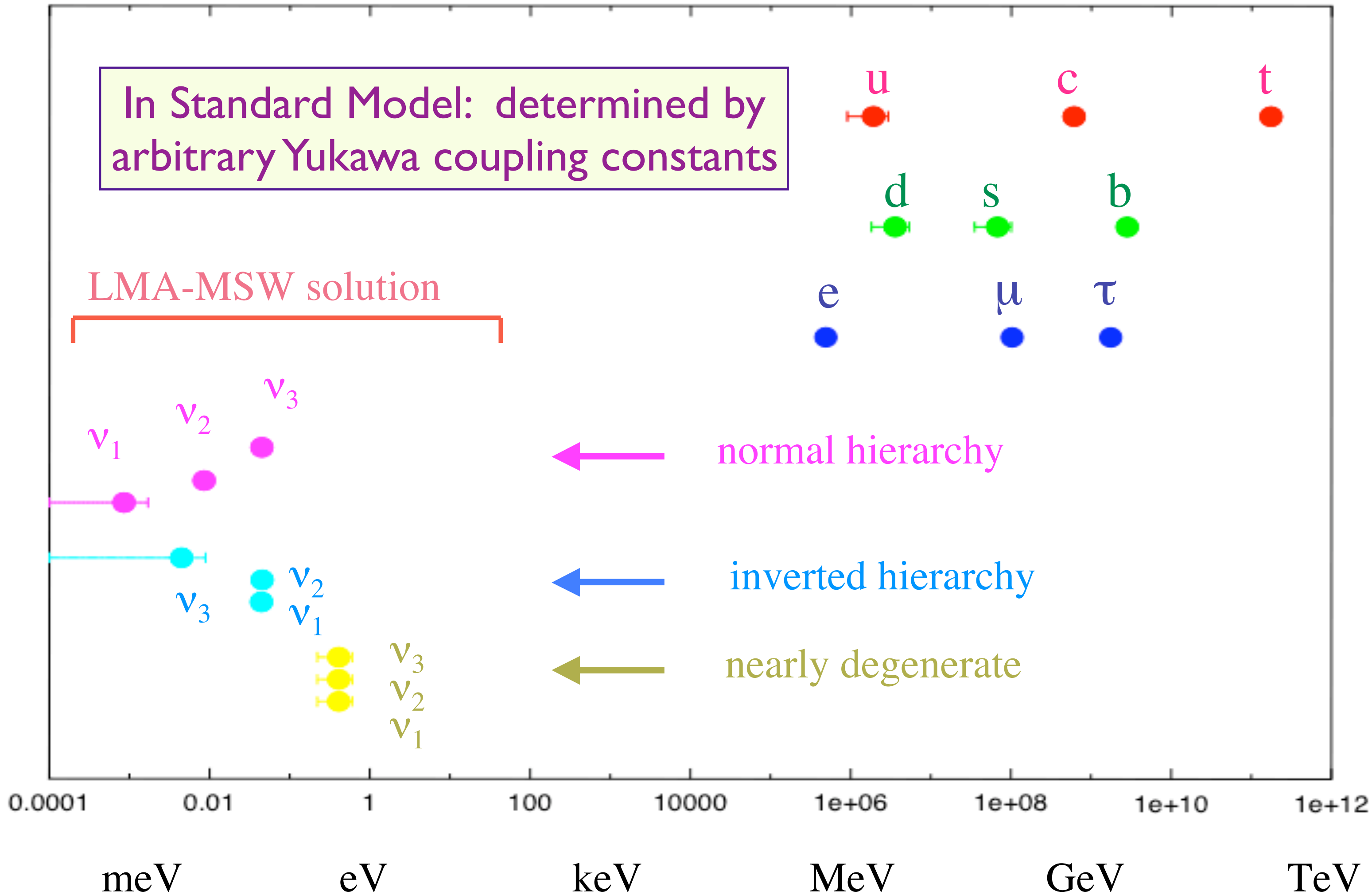
$$V_{CKM} \sim \begin{pmatrix} 0.9745-0.9757 & 0.219-0.224 & 0.002-0.005 \\ 0.218-0.224 & 0.9736-0.9750 & 0.036-0.046 \\ 0.004-0.014 & 0.034-0.046 & 0.9989-0.9993 \end{pmatrix}$$

- Lepton mixings are large

$$U_{MNS} \sim \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.0-0.16 \\ 0.24-0.52 & 0.44-0.69 & 0.63-0.79 \\ 0.26-0.52 & 0.47-0.71 & 0.60-0.77 \end{pmatrix}$$

In Standard Model: determined by arbitrary Yukawa coupling constants

Mass spectrum of elementary particles



Origin of Mass Hierarchy & Flavor Mixing

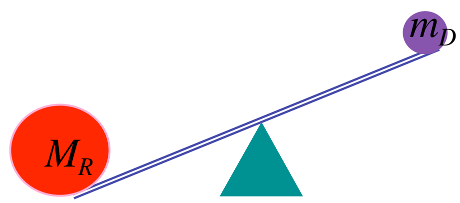
- no fundamental origin of fermion mass hierarchy and flavor mixing has been found or suggested
- less ambitious aim: reduce the number of parameters in the Yukawa sector
- parameter reductions by imposing symmetries
  - grand unified gauge symmetry
    - allowed relations between up, down, charged lepton and neutrino masses ⇒ connections between quark and lepton sectors
  - family symmetry
    - allow relations among three families ⇒ further reduction of parameters
  - supersymmetry
    - required by data to get correct predictions

Seesaw Mechanism

Minkowski, 1977; Gell-mann, Ramond, Slansky, 1981;
Yanagida, 1979; Mohapatra, Senjanovic, 1981

- Introduce right-handed neutrinos, which are SM gauge singlets
  [predicted in many GUTs, e.g. SO(10)]
- integrating out RH neutrinos: effective mass matrix

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

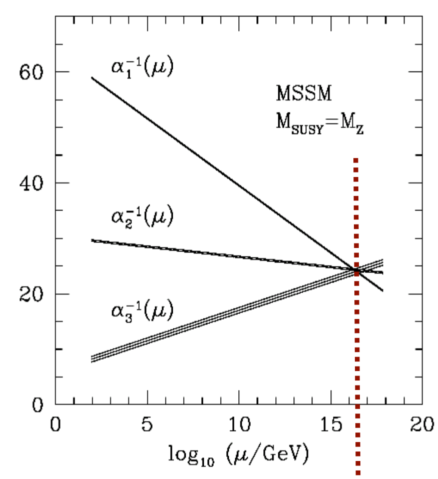


light neutrino mass: $m_\nu \sim \frac{m_D}{M_R} m_D \quad \gg m_D$

heavy neutrino mass: $M \sim M_R$

$$m_\nu \sim \sqrt{\Delta m^2_{atm}} \sim 0.05 \text{ eV}, \; m_D \sim m_t \sim 172 \text{ GeV}$$

$$\Rightarrow M_R \sim 10^{15} \text{GeV} \sim M_{GUT}$$



$M_{GUT} \sim 10^{16}$ GeV

SO(10) GUT

- RH neutrino accommodated in the model

$$16 = \overline{5} + 10 + 1$$

$$1 \rightarrow \nu_R$$

$$16 = (3, 2, 1/6) \sim \begin{bmatrix} u & u & u \\ d & d & d \end{bmatrix}$$

$$+ (3^*, 1, -2/3) \sim (u^c \; u^c \; u^c)$$

$$+ (3^*, 1, 1/3) \sim (d^c \; d^c \; d^c)$$

$$+ (1, 2, -1/2) \sim \begin{bmatrix} \nu \\ e \end{bmatrix}$$

$$+ (1, 1, 1) \sim e^c$$

$$+ (1, 1, 0) \sim \nu^c$$

- Natural for seesaw: offer both ingredients, i.e. RH neutrino & heavy scale neutrino oscillation strongly support SO(10)!!
- Quark & Leptons reside in the same GUT multiplets
- One set of Yukawa coupling for a given GUT multiplet

➡ SO(10) relates quarks and leptons (intra-family relations)

➡ reduce # of parameters in Yukawa sector

Models Based on SUSY SO(10)

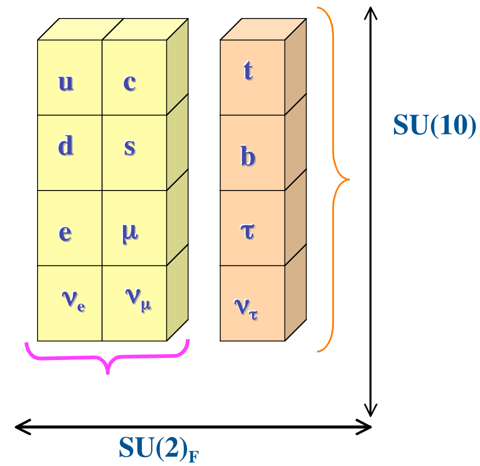
- large neutrino mixing from neutrino sector

$$U_{MNS} = U^{+}_{e,L} U_{\nu,L}$$

SO(10) GUT + SU(2) family symmetry

Barbieri, Hall, Raby, Romanino; ...

SO(10) → SU(4) x SU(2)$_L$ x SU(2)$_R$
→ SU(3) x SU(2)$_L$ x U(1)$_Y$



- symmetric mass matrices: M.-C.C & K.T. Mahanthappa, PRD 2000

Up-type quarks ⇔ Dirac neutrinos

Down-type quarks ⇔ charged leptons

$$\text{seesaw} \;\Rightarrow\; M_\nu \sim \begin{pmatrix} 0 & 0 & * \\ 0 & 1 & 1 \\ * & 1 & 1 \end{pmatrix}$$

12 parameters accommodate 22 fermion masses, mixing angles and CP phases in both quark and lepton sectors

- prediction for θ_{13}:

$$\sin\theta_{13} \sim \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2} \sim O(0.1) \Rightarrow \text{LMA}$$

Models Based on SUSY SO(10)

- large neutrino mixing from charged lepton sector — Albright & Barr

$$U_{MNS} = U^{\dagger}_{e,L} U_{\nu,L}$$

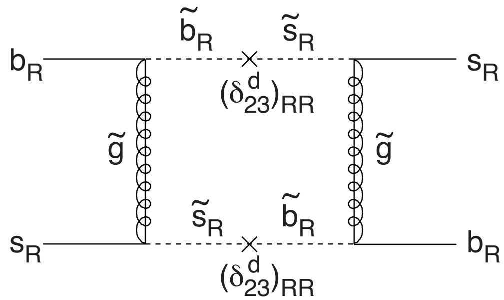
- lopsided mass matrices:

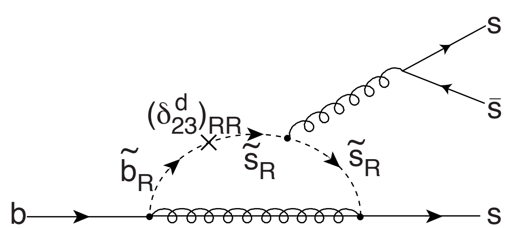
$$M_d^T = M_e \sim \begin{pmatrix} * & * & * \\ * & * & 1 \\ * & * & 1 \end{pmatrix}$$

SO(10) → SU(5)
→ SU(3) x $SU(2)_L$ x $U(1)_Y$

down-type quarks ⇔ charged leptons

- large mixing in $U_{e,L}$
  - large mixing in $U_{d,R}$ (effects in B physics) — D. Chang, A. Masiero, H. Murayama, 2002



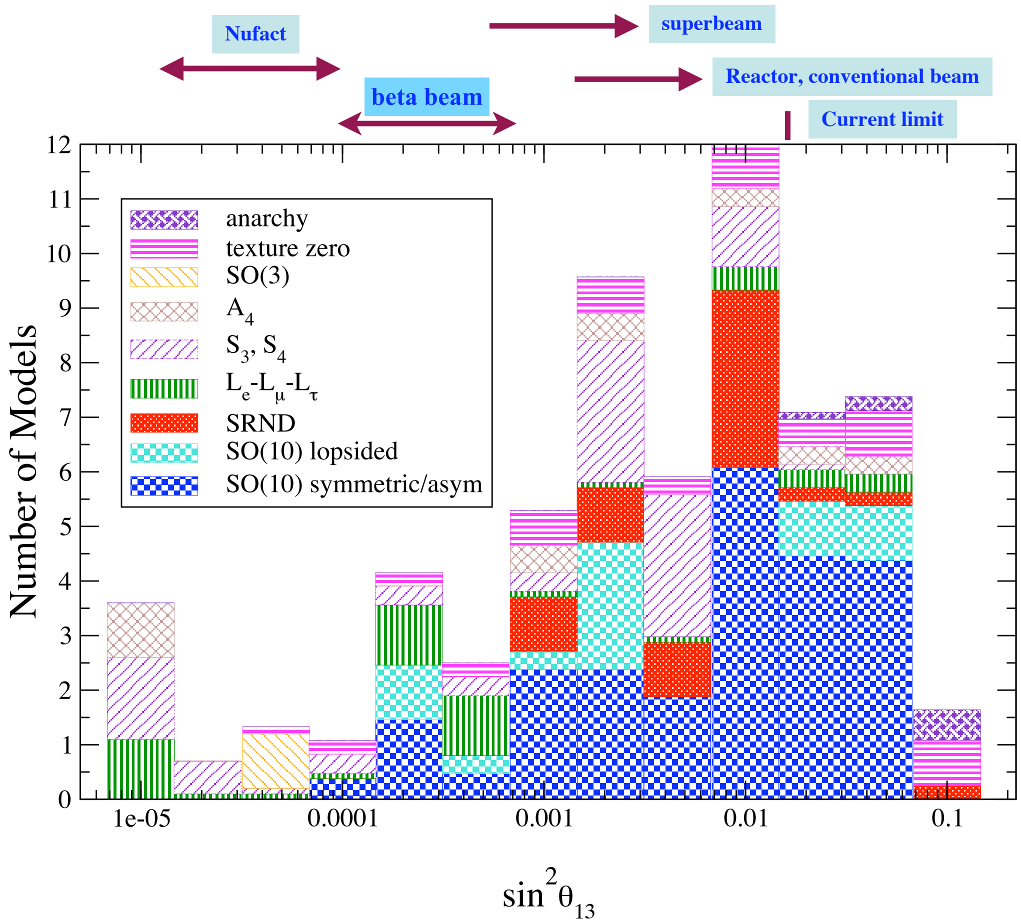


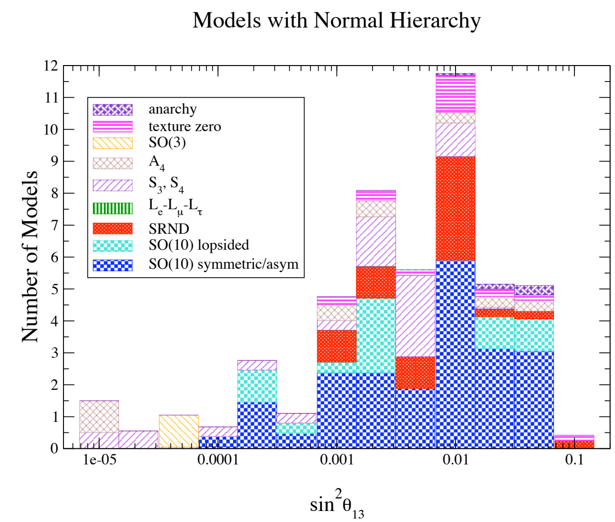
  - large $\mu \to e + \gamma$ rate

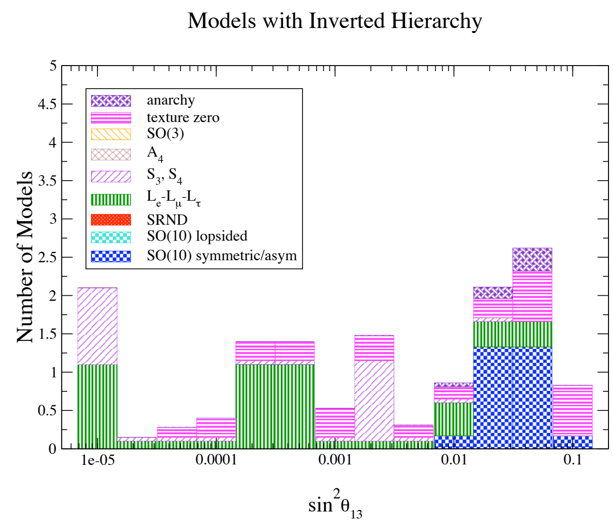
- prediction for θ_{13}: can be small; $\sin\theta_{13} \sim 0.05$

Distinguishing Models

C. Albright & M.-C.C, PRD 2006







	$\sin^2 2\theta_{13}$	$\sin\theta_{13}$
current limit	10^{-1}	0.16
reactor	10^{-2}	0.05
Conventional beam	10^{-2}	0.05
superbeam	3×10^{-3}	2.7×10^{-2}
Neutrino factory	$(5\text{-}50) \times 10^{-5}$	$(3.5\text{-}11) \times 10^{-3}$

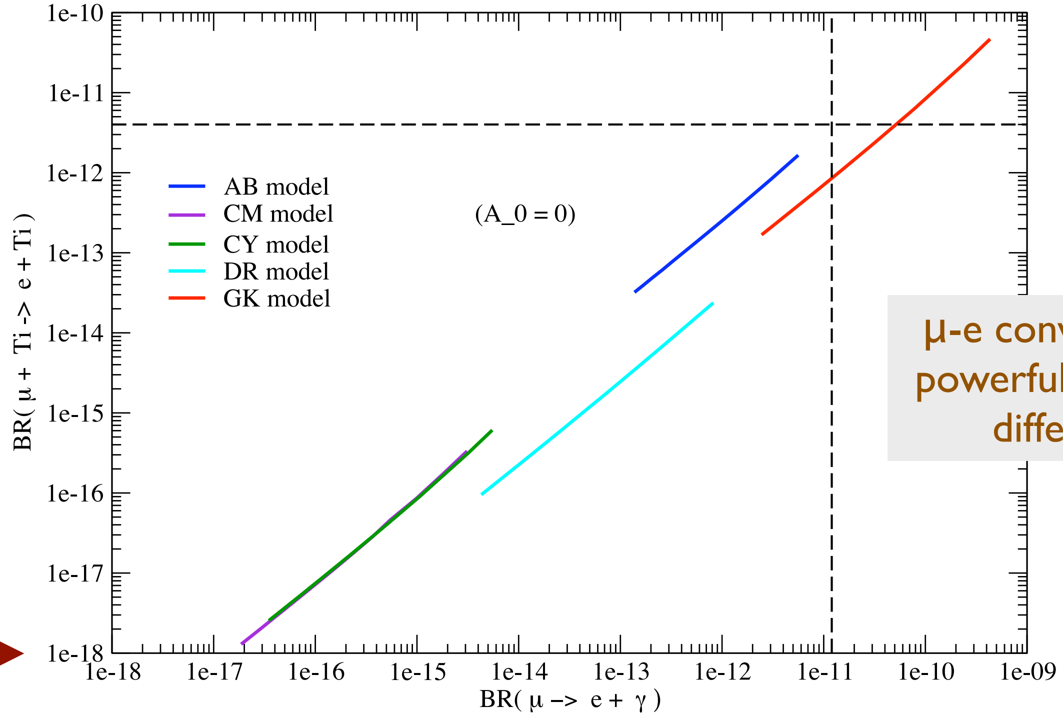
LFV Rare Processes

C. Albright & M.-C.C, PRD 2008

predictions for LFV processes in five viable SUSY SO(10) models:

-- assuming MSUGRA boundary conditions

-- including Dark Matter constraints from WMAP



μ-e conversion could be powerful in distinguishing different models

sensitivity of proposed MECO-type exp

reach at MEG

Tri-bimaximal Neutrino Mixing

- Neutrino Oscillation Parameters (2σ)

Schwetz, Tortola, Valle (Aug 2008)

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{pmatrix}$$

$$\sin^2\theta_{23} = 0.5^{+0.14}_{-0.12}, \quad \sin^2\theta_{12} = 0.304^{+0.044}_{-0.032}$$

- indication for non-zero θ_{13}:

Bari group, June 2008

$$\sin^2\theta_{13} = 0.01^{+0.016}_{-0.011}\ (1\sigma)$$

consistent with $\theta_{13} = 0$

- Tri-bimaximal neutrino mixing:

Harrison, Perkins, Scott, 1999

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\sin^2\theta_{\rm atm,\,TBM} = 1/2 \qquad \sin\theta_{13,\rm TBM} = 0.$$

$$\sin^2\theta_{\odot,\rm TBM} = 1/3 \qquad \tan^2\theta_{\odot,\rm TBM} = 1/2$$

$$\tan^2\theta_{\odot,\rm exp} = 0.429$$

new KamLAND result: $\tan\theta^2_{\odot,exp} = 0.47^{+0.06}_{-0.05}$

Tri-bimaximal Neutrino Mixing

- Neutrino mass matrices:

$$M = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} \longrightarrow \sin^2 2\theta_{23} = 1 \qquad \theta_{13} = 0$$

solar mixing angle NOT fixed

  - S3 Mohapatra, Nasri, Yu, 2006; ...
  - D4 Grimus, Lavoura, 2003; ...
  - μ-τ symmetry Fukuyama, Nishiura, '97; Mohapatra, Nussinov, '99; Ma, Raidal, '01; ...

- if A+B = C + D $\longrightarrow \tan^2\theta_{12} = 1/2$ TBM pattern
  - A4 Ma, '04; Altarelli, Feruglio, '06; .....
  - Z3 × Z7 Luhn, Nasri, Ramond, 2007

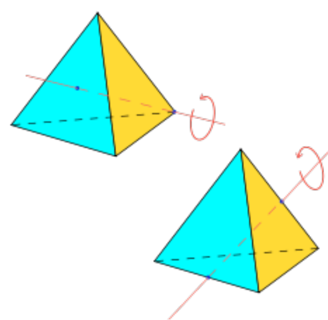
Non-abelian Finite Family Symmetry

- TBM mixing matrix: can be realized in finite group family symmetry based on A4 Ma & Rajasekaran, '01
- even permutations of 4 objects

  S: (1234) → (4321)

  T: (1234) → (2314)
- invariance group of Tetrahedron
- orbifold compactification:

  6D → 4D on T2/Z2



Altarelli, Feruglio, '06

- Deficiencies:
  - does NOT give rise to CKM mixing: $V_{ckm} = 1$
  - does NOT explain mass hierarchy
  - all CG coefficients real

Group Theory of T′

Frampton & Kephart, IJMPA (1995)

- Double covering of tetrahedral group A4:
- in-equivalent representations of T':

A4: $1,\ 1',\ 1'',\ 3$ ⟶ TBM for neutrinos

other: $2,\ 2',\ 2''$ ⟶ 2 +1 assignments for quarks

- generators:

$$S^2 = R,\ T^3 = 1,\ (ST)^3 = 1,\ R^2 = 1$$

R=1: $1,\ 1',\ 1'',\ 3$ (vector)

R= -1: $2,\ 2',\ 2''$ (spinorial)

- generators: in 3-dim representations, T-diagonal basis

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

Group Theory of T′

- product rules:

$$1^0 \equiv 1,\ 1^1 \equiv 1',\ 1^{-1} \equiv 1''$$

$$1^a \otimes r^b = r^b \otimes 1^a = r^{a+b} \qquad \text{for } r = 1,2 \qquad a,b = 0,\pm 1$$

$$1^a \otimes 3 = 3 \otimes 1^a = 3$$

$$2^a \otimes 2^b = 3 \oplus 1^{a+b}$$

$$2^a \otimes 3 = 3 \otimes 2^a = 2 \oplus 2' \oplus 2''$$

$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

J. Q. Chen & P. D. Fan, J. Math Phys 39, 5519 (1998)

★ complex CG coefficients in T′

complexity cannot be avoided by different basis choice

- spinorial x spinorial ⊃ vector:

$$2 \otimes 2 = 2' \otimes 2'' = 2'' \otimes 2' = 3 \oplus 1 \qquad 3 = \begin{pmatrix} \left(\frac{1-i}{2}\right)(\alpha_1\beta_2 + \alpha_2\beta_1) \\ i\alpha_1\beta_1 \\ \alpha_2\beta_2 \end{pmatrix}$$

- spinorial x vector ⊃ spinorial:

$$2 \otimes 3 = 2 \oplus 2' \oplus 2'' \qquad 2 = \begin{pmatrix} (1+i)\alpha_2\beta_2 + \alpha_1\beta_1 \\ (1-i)\alpha_1\beta_3 - \alpha_2\beta_1 \end{pmatrix}$$

A Novel Origin of CP Violation

M.-C.C., K.T. Mahanthappa, arXiv:0904.1721

- Conventionally:
  - Explicit CP violation: complex Yukawa couplings
  - Spontaneous CP violation: complex Higgs VEVs

★ complex CG coefficients in T' $\Rightarrow$ explicit CP violation

  - real Yukawa couplings, real Higgs VEVs
  - CP violation in both quark and lepton sectors determined by complex CG coefficients
  - no additional parameters needed $\Rightarrow$ extremely predictive model!!

Tri-bimaximal Neutrino Mixing

- fermion charge assignments:

$$\begin{pmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \end{pmatrix}_L \sim 3, \quad e_R \sim 1, \quad \mu_R \sim 1'', \quad \tau_R \sim 1'$$

- SM Higgs ~ singlet under T′
- operator for neutrino masses: $\frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda} + \frac{\langle\eta\rangle}{\Lambda}\right)$
- two scalar (flavon) fields for neutrino sector: $\xi \sim 3, \quad \eta \sim 1$

$T' \to G_{TST^2}$: $\quad \langle\xi\rangle = \xi_0 \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\qquad T' - \text{invariant:} \quad \langle\eta\rangle = u\Lambda$

- product rules:

$$3 \otimes 3 = 3 \oplus 3 \oplus 1 \oplus 1' \oplus 1''$$

Tri-bimaximal Neutrino Mixing

- neutrino masses: triplet Higgs contribution

$$3_S = \frac{1}{3}\begin{pmatrix} 2\alpha_1\beta_1 - \alpha_2\beta_3 - \alpha_3\beta_2 \\ 2\alpha_3\beta_3 - \alpha_1\beta_2 - \alpha_2\beta_1 \\ 2\alpha_2\beta_2 - \alpha_1\beta_3 - \alpha_3\beta_1 \end{pmatrix} \qquad 1 = \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$$

- neutrino masses: singlet contribution $1 = \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2$
- Resulting mass matrix:

$$M_\nu = \frac{\lambda v^2}{M_x}\begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

$$V_\nu^{\mathrm{T}} M_\nu V_\nu = \mathrm{diag}(u + 3\xi_0,\, u,\, -u + 3\xi_0)\frac{v_u^2}{M_x} \qquad U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

General conditions for Form Diagonalizablility in seesaw: M.-C. Chen, S. F. King, arXiv:0903.0125

Tri-bimaximal Neutrino Mixing

- charged lepton sector -- non-GUT models
  - operators for charged fermion masses:

$$(\ell\phi)_1 e_R(1) + (\ell\phi)_{1'}\mu_R(1'') + (\ell\phi)_{1''}\tau_R(1')$$

  - scalar sector: flavon triplet for charged lepton sector $\phi \sim 3$

$$\begin{aligned} 1 &= \alpha_1\beta_1 + \alpha_2\beta_3 + \alpha_3\beta_2 \\ 1' &= \alpha_3\beta_3 + \alpha_1\beta_2 + \alpha_2\beta_1 \\ 1'' &= \alpha_2\beta_2 + \alpha_1\beta_3 + \alpha_3\beta_1 \end{aligned}$$

$$T' \to G_T : \qquad \langle\phi\rangle = \phi_0\Lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- resulting charged lepton mass matrix: diagonal
- leptonic mixing matrix = tri-bimaximal

$$V_{MNS} = V_{e,L}^{\dagger} V_\nu = \mathcal{I} \cdot U_{TBM} = U_{TBM}$$

- in our model: SU(5) GUT ⇒ corrections from charged lepton sector

The Model

M.-C.C., K.T. Mahanthappa, Phys. Lett. B652 (2007) 34; arXiv:0904.1721

- Symmetry: SU(5) x T′
- Particle Content $10(Q,\, u^c,\, e^c)_L \qquad \overline{5}(d^c, \ell)_L$

	T_3	T_a	$\overline{F}$	H_5	$H'_{\overline{5}}$	Δ_{45}	ϕ	ϕ'	ψ	ψ'	ζ	N	ξ	η
SU(5)	10	10	$\overline{5}$	5	$\overline{5}$	45	1	1	1	1	1	1	1	1
T'	1	2	3	1	1	$1'$	3	3	$2'$	2	$1''$	$1'$	3	1
Z_{12}	ω^5	ω^2	ω^5	ω^2	ω^2	ω^5	ω^3	ω^2	ω^6	ω^9	ω^9	ω^3	ω^{10}	ω^{10}
Z'_{12}	ω	ω^4	ω^8	ω^{10}	ω^{10}	ω^3	ω^3	ω^6	ω^7	ω^8	ω^2	ω^{11}	1	1

$\omega = e^{i\pi/6}$.

- additional $Z_{12} \times Z'_{12}$ symmetry:
  - ★ predictive model: only 9 operators allowed up to at least dim-7
  - ★ vacuum misalignment: neutrino sector vs charged fermion sector
  - ★ mass hierarchy: lighter generation masses allowed only at higher dim

The Model

- Lagrangian: only 9 operators allowed!!

$$
\begin{aligned}
\mathcal{L}_{\rm Yuk} &= \mathcal{L}_{\rm TT} + \mathcal{L}_{\rm TF} + \mathcal{L}_{\rm FF} \\
\mathcal{L}_{\rm TT} &= y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi \zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3 \\
\mathcal{L}_{\rm TF} &= \frac{1}{\Lambda^2} y_b H'_{\overline{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3} \left[y_s \Delta_{45} \overline{F} T_a \phi \psi N + y_d H'_{\overline{5}} \overline{F} T_a \phi^2 \psi' \right] \\
\mathcal{L}_{\rm FF} &= \frac{1}{M_x \Lambda} \left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F} \xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F} \eta \right] ,
\end{aligned}
$$

Λ: cutoff scale above which the family symmetry T' is exact
M_x: scale at which the lepton number violating operator is generated

Neutrino Sector

- Operators: $\mathcal{L}_{\rm FF} = \frac{1}{M_x \Lambda}\left[\lambda_1 H_5 H_5 \overline{F}\,\overline{F}\xi + \lambda_2 H_5 H_5 \overline{F}\,\overline{F}\eta\right] \longrightarrow \frac{HHLL}{M}\left(\frac{\langle\xi\rangle}{\Lambda} + \frac{\langle\eta\rangle}{\Lambda}\right)$
- Symmetry breaking:

$$T' \to G_{TST^2}: \quad \langle\xi\rangle = \xi_0\,\Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad T' - \text{invariant:} \quad \langle\eta\rangle = u_0 \Lambda$$

- Resulting mass matrix:

$$M_\nu = \begin{pmatrix} 2\xi_0 + u_0 & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & -\xi_0 + u_0 \\ -\xi_0 & -\xi_0 + u_0 & 2\xi_0 \end{pmatrix} \frac{\lambda v^2}{M_x}$$

only vector representations involved
⇒ all CG are real

⇒ Majorana phases either 0 or π

$$U_{\rm TBM}^T M_\nu U_{\rm TBM} = \text{diag}(u_0 + 3\xi_0, u_0, -u_0 + 3\xi_0)\frac{v_u^2}{M_X}$$

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Form diagonalizable:
-- no adjustable parameters
-- neutrino mixing from CG coefficients!

Up Quark Sector

- Operators: $$\mathcal{L}_{\rm TT} = y_t H_5 T_3 T_3 + \frac{1}{\Lambda^2} y_{ts} H_5 T_3 T_a \psi\zeta + \frac{1}{\Lambda^2} y_c H_5 T_a T_a \phi^2 + \frac{1}{\Lambda^3} y_u H_5 T_a T_a \phi'^3$$
- top mass: allowed by T′
- lighter family acquire masses thru operators with higher dimensionality

➡ dynamical origin of mass hierarchy

- symmetry breaking:

$$T' \to G_T : \qquad \langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \ , \ \langle\psi\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi_0' \Lambda$$

dim-6

$$T' \to G_{TST^2} : \qquad \langle\phi'\rangle = \phi_0' \Lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

dim-7

no contributions to elements involving 1st family; true to all levels

- Mass matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u$$

both vertor and spinorial reps involved
⇒ complex CG

Down Quark Sector

- operators: $\mathcal{L}_{\rm TF} = \frac{1}{\Lambda^2} y_b H'_{\bar{5}} \overline{F} T_3 \phi \zeta + \frac{1}{\Lambda^3}\left[y_s \Delta_{45} \overline{F} T_a \phi \psi N + y_d H'_{\bar{5}} \overline{F} T_a \phi^2 \psi' \right]$

- generation of b-quark mass ⇒ breaking of T′ ⇒ dynamical origin for hierarchy between m_b and m_t

- symmetry breaking:

$$T' \to G_T: \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \phi_0 \Lambda \, , \; \langle \psi \rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \psi'_0 \Lambda \qquad T' \to \text{nothing:} \quad \langle \psi' \rangle = \psi'_0 \Lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

- mass matrix:

SU(5) CG for GJ relations

$$M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi'_0 & 0 \\ -(1-i)\phi_0\psi'_0 & \psi_0 N_0 & 0 \\ \phi_0\psi'_0 & \phi_0\psi'_0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0, \quad \leftrightarrow \quad M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi'_0 & \phi_0\psi'_0 \\ (1+i)\phi_0\psi'_0 & -3\psi_0 N_0 & \phi_0\psi'_0 \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0$$

complex T' CG

- consider 2nd, 3rd families only: TBM exact

- at M_{GUT}: Georgi-Jarlskog relations

$$m_d \simeq 3 m_e \qquad m_\mu \simeq 3 m_s \qquad m_b \simeq m_\tau$$

→ corrections to TBM related to θc

Quark and Lepton Mixing Matrices

- CKM mixing matrix:

$$M_u = \begin{pmatrix} i\phi_0'^3 & \frac{1-i}{2}\phi_0'^3 & 0 \\ \frac{1-i}{2}\phi_0'^3 & \phi_0'^3 + (1-\frac{i}{2})\phi_0^2 & y'\psi_0\zeta_0 \\ 0 & y'\psi_0\zeta_0 & 1 \end{pmatrix} y_t v_u \qquad M_d = \begin{pmatrix} 0 & (1+i)\phi_0\psi_0' & 0 \\ -(1-i)\phi_0\psi_0' & \psi_0 N_0 & 0 \\ \phi_0\psi_0' & \phi_0\psi_0' & \zeta_0 \end{pmatrix} y_b v_d \phi_0,$$

V_{cb} $\quad$ V_{ub}

$$\theta_c \simeq \left|\sqrt{m_d/m_s} - e^{i\alpha}\sqrt{m_u/m_c}\right| \sim \sqrt{m_d/m_s},$$

Georgi-Jarlskog relations ⇒ $V_{d,L} \neq I$

SU(5) ⇒ $M_d = (M_e)^T$

⇒ corrections to TBM related to θ_c

- MNS matrix:

$$M_e = \begin{pmatrix} 0 & -(1-i)\phi_0\psi_0' & \phi_0\psi_0' \\ (1+i)\phi_0\psi_0' & -3\psi_0 N_0 & \phi_0\psi_0' \\ 0 & 0 & \zeta_0 \end{pmatrix} y_b v_d \phi_0 \quad \Longrightarrow \quad \theta_{12}^e \simeq \sqrt{\frac{m_e}{m_\mu}} \simeq \frac{1}{3}\sqrt{\frac{m_d}{m_s}} \sim \frac{1}{3}\theta_c$$

$$U_{\rm MNS} = V_{e,L}^\dagger U_{\rm TBM} = \begin{pmatrix} 1 & -\theta_c/3 & * \\ \theta_c/3 & 1 & * \\ * & * & 1 \end{pmatrix} \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2}$$

new QLC relation!

leptonic Dirac CP phase ⇐ complex CG

"Usual" Quark-Lepton Complementarity

lepton mixing

parameter	Best-fit value	3σ range
θ_{12}	33.2^o	$28.7^o - 38.1^o$
θ_{23}	45^o	$35.7^o - 55.6^o$
θ_{13}	2.6^o	$0 - 12.5^o$

quark mixing

parameter	Best-fit value	3σ range
θ_c	12.88^o	$12.75^o - 13.01^o$
θ^q_{23}	2.36^o	$2.25^o - 2.48^o$
θ^q_{13}	0.21^o	$0.17^o - 0.25^o$

$$\theta_{12} + \theta_c = 45^o$$

Raidal, '04; Smirnov & Minakata, '04

quark-lepton complementarity relation

↕

quark-lepton unification?

more generally:

$$\theta_{12} + \theta_C \left(\frac{1}{\sqrt{2}} + \frac{\theta_C}{4} \right) \simeq \frac{\pi}{4}$$

k

Plentinger, Seidl, Winter, 08; Frampton, Matsuzaki, 08; King 05; King Antusch, 05

RG effects: $\Delta\theta_c \sim \theta_c^4$

MSSM: normal hierarchy $\Delta\theta_{12} < 0.1°$ Schmidt & Smirnov, '06

Motivate measurements of neutrino mixing angles to at least the accuracy of the measured quark mixing angles

Numerical Results

- Experimentally:

$$m_u : m_c : m_t = \theta_c^{7.5} : \theta_c^{3.7} : 1 \qquad m_d : m_s : m_b = \theta_c^{4.6} : \theta_c^{2.7} : 1$$

- Model Parameters: (RG corrections included)

$$M_u = \begin{pmatrix} ig & \frac{1-i}{2}g & 0 \\ \frac{1-i}{2}g & g+(1-\frac{i}{2})h & k \\ 0 & k & 1 \end{pmatrix} y_t v_u$$

$$\frac{M_d}{y_b v_d \phi_0 \zeta_0} = \begin{pmatrix} 0 & (1+i)b & 0 \\ -(1-i)b & c & 0 \\ b & b & 1 \end{pmatrix}$$

$$\begin{aligned} b &\equiv \phi_0 \psi_0'/\zeta_0 = 0.00304 \\ c &\equiv \psi_0 N_0/\zeta_0 = -0.0172 \\ k &\equiv y' \psi_0 \zeta_0 = -0.0266 \\ h &\equiv \phi_0^2 = 0.00426 \\ g &\equiv \phi_0'^3 = 0.0000145 \end{aligned}$$

$$y_t = 1 \qquad y_b \phi_0 \zeta_0 \simeq m_b/m_t \simeq 0.011$$

7 parameters in charged fermion sector

- RG corrections from M_{GUT} to M_z:
  - mass ratios not renormalized
  - mixing parameters:

$$\frac{Q(M_x)}{Q(M_Z)} = \xi\,, \quad Q = A,\ V_{ij},\ (ij) = (13, 31, 23, 32) \qquad \xi = (M_x/M_z)^{-h_t^2/(16\pi^2)} \simeq 0.811$$

$$\frac{J_{cp}(M_x)}{J_{cp}(M_z)} = \xi^2\,,$$

Numerical Results

- CKM Matrix:

$$V_{CKM} = \begin{pmatrix} 0.974 & 0.227 & 0.00412e^{-i45.6^o} \\ -0.227 - 0.000164e^{i45.6^o} & 0.974 - 0.0000384e^{i45.6^o} & 0.0411 \\ 0.00932 - 0.00401e^{i45.6^o} & -0.0400 - 0.000935e^{i45.6^o} & 1 \end{pmatrix}$$

$$|V_{CKM}| = \begin{pmatrix} 0.974 & 0.227 & 0.00412 \\ 0.227 & 0.973 & 0.0412 \\ 0.00718 & 0.0408 & 0.999 \end{pmatrix}$$

- CP violation measures

$$\beta \equiv \arg\left(\frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) = 23.6^o, \ \sin 2\beta = 0.734 \ ,$$

$$\alpha \equiv \arg\left(\frac{-V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) = 110^o \ ,$$

$$\gamma \equiv \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = \delta_q = 45.6^o \ ,$$

$$J \equiv \mathrm{Im}(V_{ud}V_{cb}V_{ub}^*V_{cs}^*) = 2.69 \times 10^{-5} \ ,$$

$$A = 0.798$$
$$\overline{\rho} = 0.299$$
$$\overline{\eta} = 0.306$$

predicting:
9 masses, 3 mixing angles, 1 CP Phase;
agree with exp within 3σ

Numerical Results

- diagonalization matrix for charged leptons:

$$\begin{pmatrix} 0.997e^{i177^o} & 0.0823e^{i131^o} & 1.31\times10^{-5}e^{-i45^o} \\ 0.0823e^{i41.8^o} & 0.997e^{i176^o} & 0.000149e^{-i3.58^o} \\ 1.14\times10^{-6} & 0.000149 & 1 \end{pmatrix}$$

- MNS Matrix:

$$|m_1| = 0.0156 \text{ eV}, \quad |m_2| = 0.0179 \text{ eV}, \quad |m_3| = 0.0514 \text{ eV}$$

$$\begin{pmatrix} 0.838 & 0.542 & 0.0583e^{-i227^o} \\ -0.385-0.0345e^{i227^o} & 0.594-0.0224e^{i227^o} & 0.705 \\ 0.384-0.0346e^{i227^o} & -0.592-0.0224e^{i227^o} & 0.707 \end{pmatrix} \rightarrow |U_{MNS}| = \begin{pmatrix} 0.838 & 0.542 & 0.0583 \\ 0.362 & 0.610 & 0.705 \\ 0.408 & 0.577 & 0.707 \end{pmatrix}$$

→ $$\sin^2 2\theta_{atm} = 1, \quad \tan^2\theta_{\odot} = 0.419, \quad |U_{e3}| = 0.0583$$

prediction for Dirac CP phase: δ = 227 degrees

$$J_\ell = -0.00967$$

Note that these predictions do NOT depend on u_0 and ξ_0

- neutrino masses: using best fit values for Δm^2

2 parameters in neutrino sector

$$u_0 = -0.0593, \quad \xi_0 = 0.0369, \quad M_X = 10^{14} \text{ GeV}$$

$$|m_1| = 0.0156 \text{ eV}, \quad |m_2| = 0.0179 \text{ eV}, \quad |m_3| = 0.0514 \text{ eV}$$

- Majorana phases $\alpha_{21} = \pi \qquad \alpha_{31} = 0.$

predicting: 3 masses,
3 mixing angles, 3 CP Phases;
both θ_{sol} & θ_{atm} agree with exp

Neutrino Mass Sum Rule

- sum rule among three neutrino masses:

$$m_1 - m_3 = 2m_2$$

- the mass eigenvalues:

$$\begin{aligned} m_1 &= u_0 + 3\xi_0 \\ m_2 &= u_0 \\ m_3 &= -u_0 + 3\xi_0 \end{aligned}$$

$$\begin{aligned} \Delta m^2_{atm} &\equiv |m_3|^2 - |m_2|^2 = -12 u_0 \xi_0 \\ \Delta m^2_{\odot} &\equiv |m_2|^2 - |m_1|^2 = -9\xi_0^2 - 6 u_0 \xi_0 \end{aligned}$$

- leads to sum rule

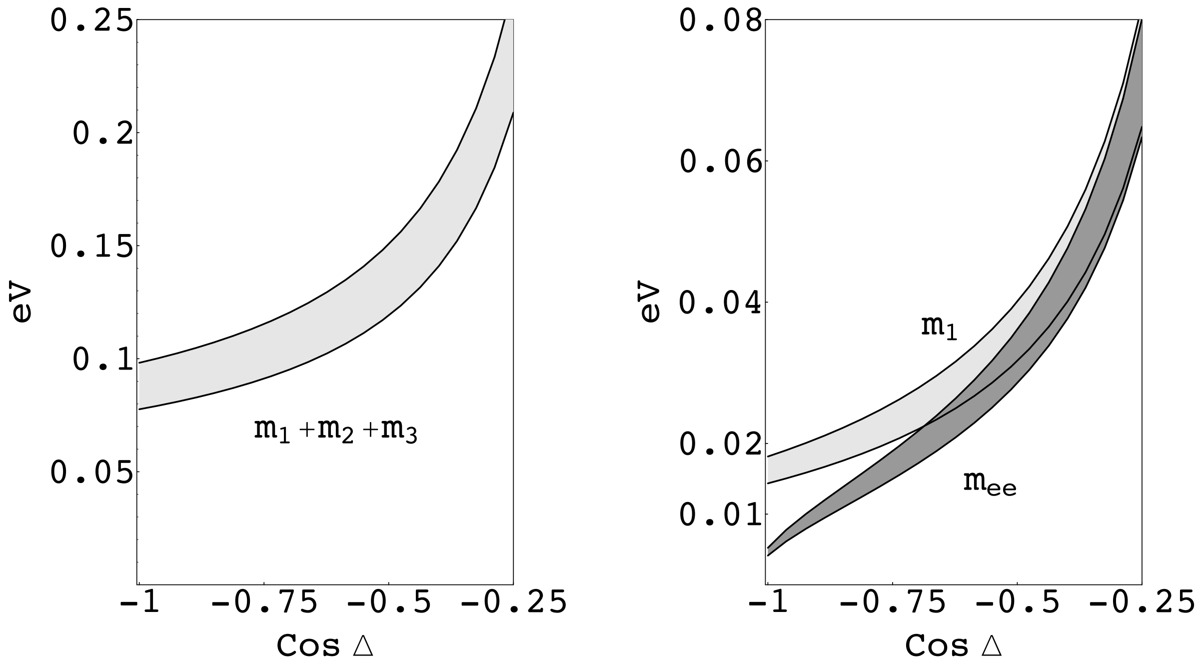
$$\Delta m^2_{\odot} = -9\xi_0^2 + \frac{1}{2}\Delta m^2_{atm} \longrightarrow \Delta m^2_{atm} > 0$$

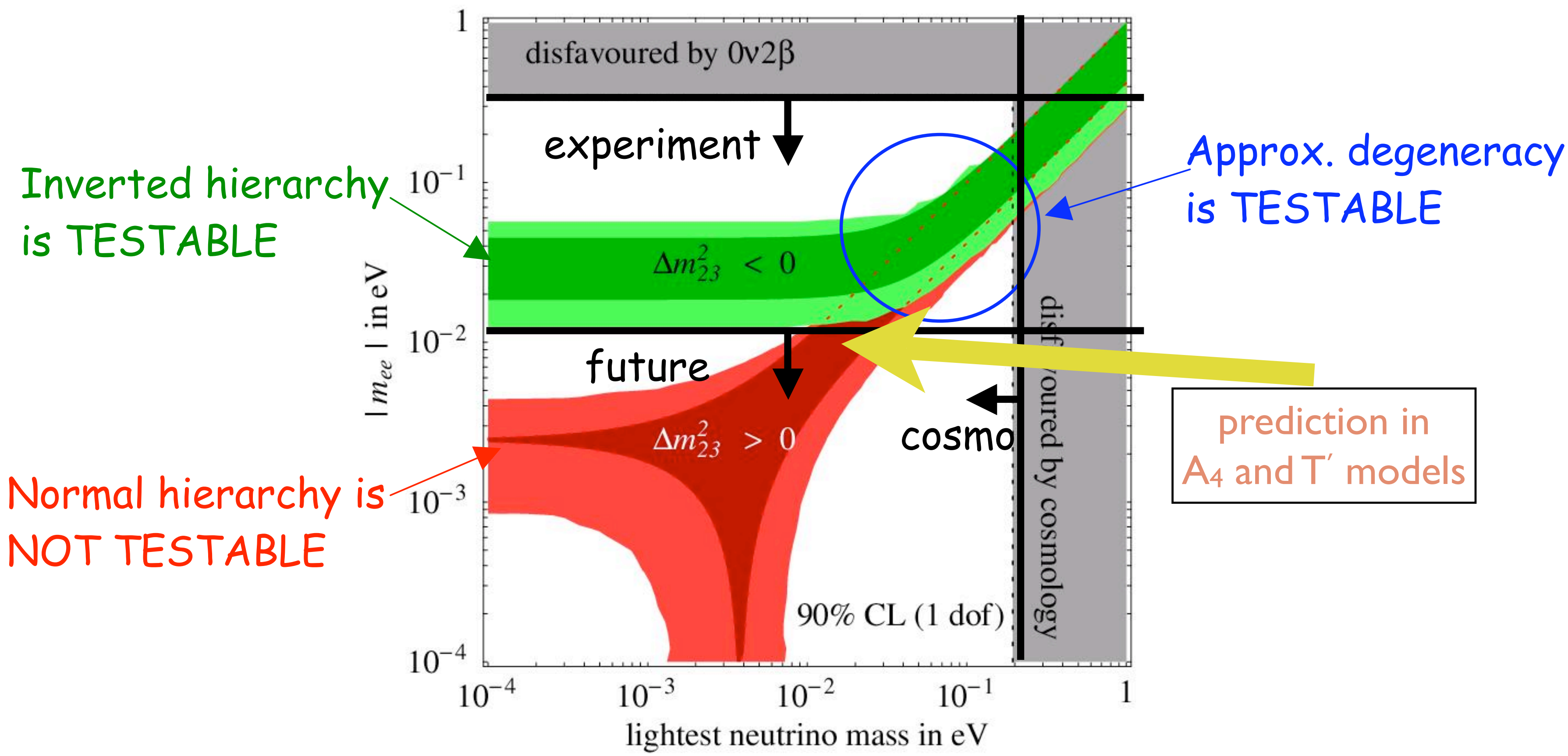
normal hierarchy predicted!!

M.-C.C., K.T. Mahanthappa, Phys. Lett. B652 (2007) 34

Neutrino Mass Sum Rule

For A4: Altarelli et al, 2006





from: F. Feruglio, A. Strumia, F. Vissani ('02)

Summary

- SU(5) x T′ symmetry:

  near tri-bimaximal lepton mixing ⇔ realistic CKM matrix

- complex CG coefficients in T′: origin of CPV both in quark and lepton sectors

- Z_{12} x Z_{12}': only 9 parameters in Yukawa sector
  - ★ dynamical origin of mass hierarchy (including m_b vs m_t)
  - ★ forbid Higgsino-mediated proton decay

- interesting sum rules:

leptonic Dirac CP phase: δ = 227 degrees

$$\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c\cos\delta$$

$$\theta_{13} \simeq \theta_c/3\sqrt{2} \sim 0.05$$

right amount to account for discrepancy between exp best fit value and TBM prediction