Electroweak Corrections to W/Z Production

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Contents

Introduction

M_{W} measurements @ Tevatron and LHC

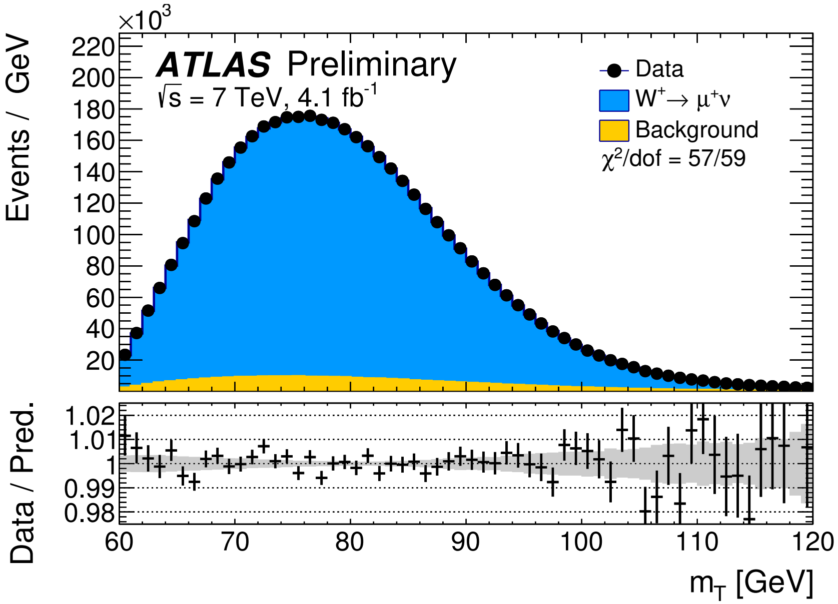
CDF: $80.387\,\mathrm{GeV} \pm 19\,\mathrm{MeV}$

D0: $80.375\,\mathrm{GeV} \pm 23\,\mathrm{MeV}$

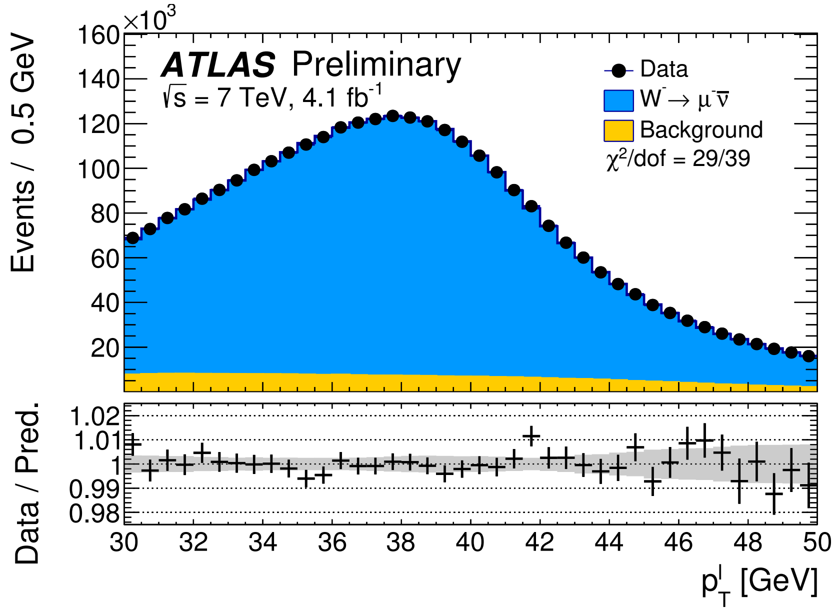
ATLAS: $80.370\,\mathrm{GeV} \pm 19\,\mathrm{MeV}$ from fits to distributions in

a) transverse W-boson mass

$$M_{\mathrm{T},l\nu} = \sqrt{2(E_{\mathrm{T},l}\not{E}_{\mathrm{T}} - \mathbf{p}_{\mathrm{T},l}\cdot\not{\mathbf{p}}_{\mathrm{T}})}$$



b) transverse lepton momentum $p_{\mathrm{T},l}$



Sensitivity to M_{W} via Jacobian peaks from W resonance at

$$M_{\mathrm{T},l\nu} \sim M_{\mathrm{W}} \qquad p_{\mathrm{T},l} \sim M_{\mathrm{W}}/2$$

$\Rightarrow$ Reduction of ΔM_{W} requires high theoretical precision in W resonance region !

Spirit of this talk

- review EW corrections to single-W/Z production processes
- particular focus to resonance regions

  EW corrections: $\Delta M_{\mathrm{W}}^{\mathrm{EW}} \sim 100(50)\,\mathrm{MeV}$ for bare (dressed) leptons
- emphasize recent developments
- set the stage for a discussion of EW issues / uncertainties

ATLAS-CONF-2016-113:

Decay channel	$W \to e\nu$		$W \to \mu\nu$	
Kinematic distribution	p_{T}^{ℓ}	m_{T}	p_{T}^{ℓ}	m_{T}
δm_W [MeV]				
FSR (real)	< 0.1	< 0.1	< 0.1	< 0.1
Pure weak and IFI corrections	3.3	2.5	3.5	2.5
FSR (pair production)	3.6	0.8	4.4	0.8
Total	4.9	2.6	5.6	2.6

Electroweak corrections

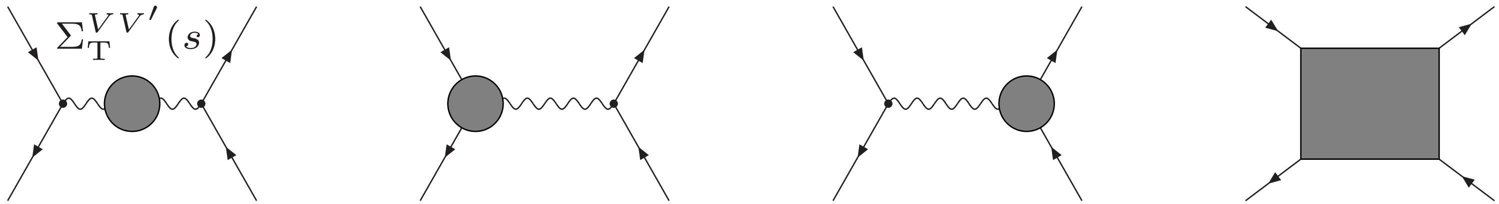
to W production

SM predictions for W/Z production:

- NNLO QCD (differential) — Melnikov, Petriello ’06; Catani et al. ’09; Gavin et al. ’10,’12
- QCD resummations / parton showers — Arnold, Kauffman ’91; Balazs et al. ’95; R.K.Ellis et al. ’97; Qiu, Zhang ’00; Kulesza et al. ’01,’02; ...
- NLO EW (+ h.o. improvements) — Baur et al. ’97–’04; Brein et al. ’99; S.D., Krämer ’01 Zykunov ’01,’05; Arbuzov et al. ’05,’06; Carloni Calame et al. ’06,’07; Brensing et al. ’07 S.D., Huber ’09
- NLO QCD/EW POWHEG matching — Bernaciak, Wackeroth ’12; Barze et al. ’12,’13; Mück, Oymanns ’16
- NNLO QCD + parton shower — Hoeche et al. ’14; Karlberg et al. ’14
- $\mathcal{O}(\alpha\alpha_\mathrm{s})$ corrs. near resonances — S.D., Huss, Schwinn ’14,’15
- Theoretical uncertainties in M_W determination — Carloni Calame et al. ’16

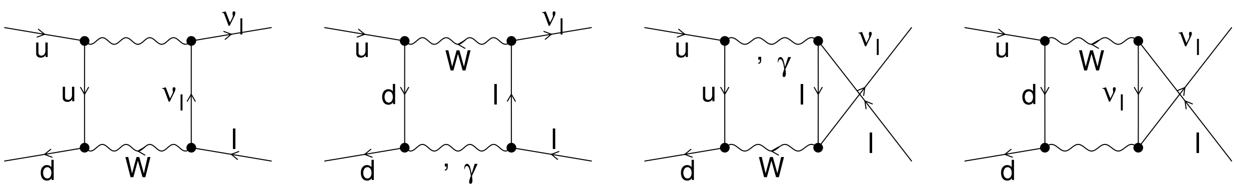
Ingredients of the NLO EW calculation

Loop corrections:



VV' self-energies $\quad$ $Vq\bar{q}'$ and Vll' vertex corrections $\quad$ box diagrams

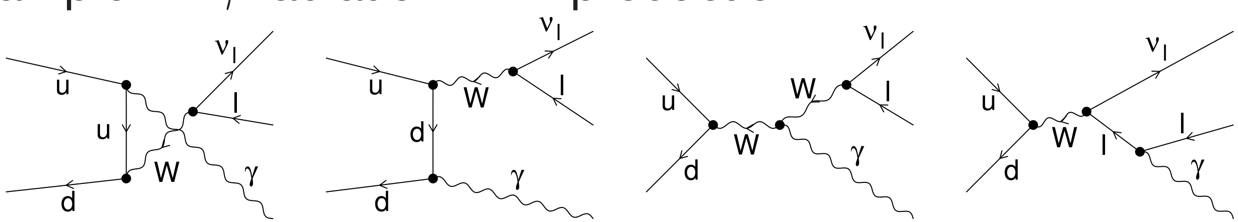
Example: box corrections to W production



Real-emission corrections:

QCD: g emission, $q\mathrm{g}$ channels; EW: γ emission, $q\gamma/\gamma\gamma$ channels

Example: γ radiation in W production

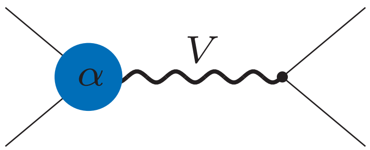


Field-theoretical requirement:

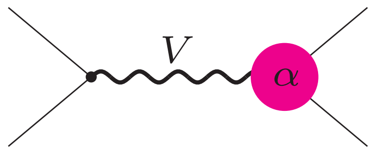
gauge-invariant description of resonance with higher-order corrections

Break-ups of the EW corrections:

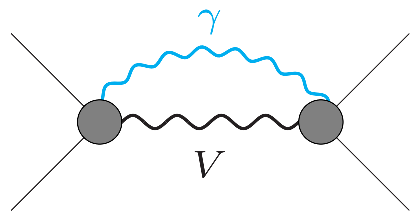
- Near resonance: pole expansion separates production and decay



Initial-State (IS) corr.



Final-State (FS) corr.



Non-fact. IS–FS corr.

Note:

◇ splitting possible both for W and Z production

◇ splitting only meaningful near resonance

- Separation of photonic corrections:

◇ not meaningful for W production

$\hookrightarrow$ only leading-log part of photonic corrections unique

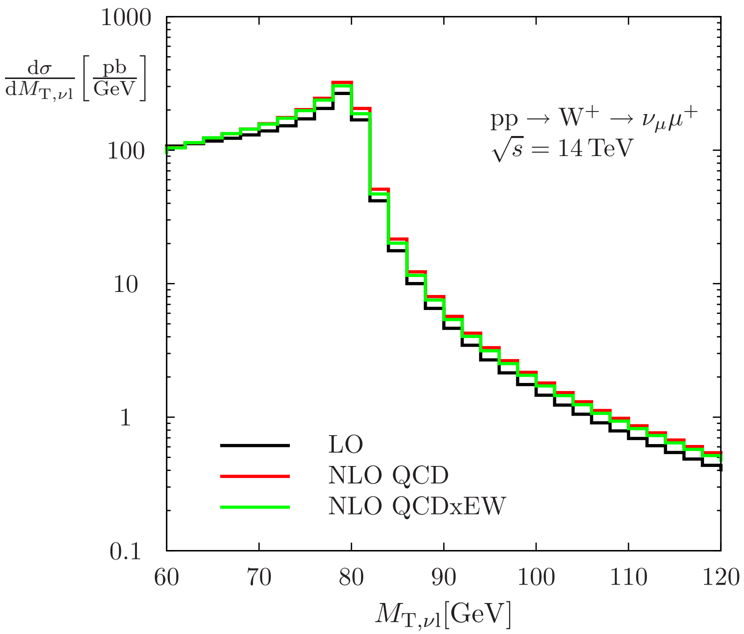
◇ meaningful and useful for Z production

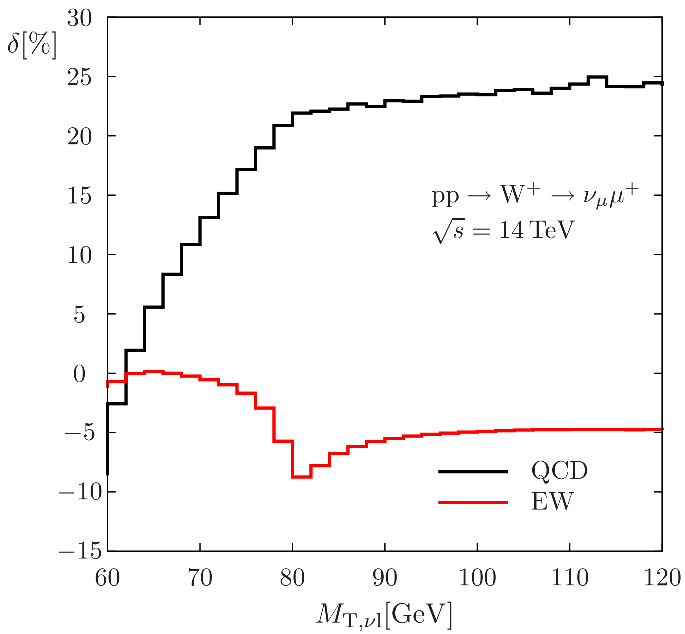
$\hookrightarrow$ 4 gauge-independent parts at NLO:

– photonic FS corrs. $\propto Q_\ell^2$, IS corrs. $\propto Q_q^2$, IS–FS corrs. $\propto Q_q\, Q_\ell$

– purely weak corrections

Transverse-mass distribution for W production





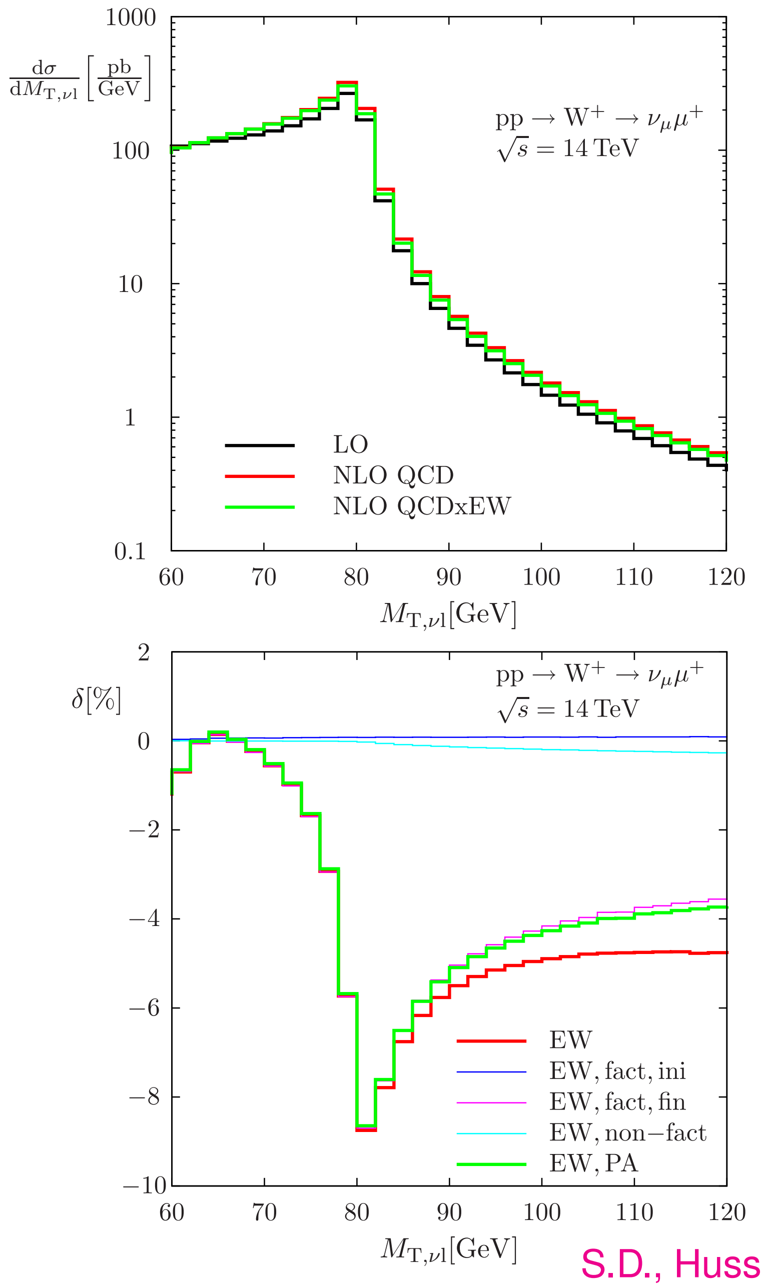
Features of $M_{T,\nu l}$:

- stability wrt QCD corrs./uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_T$

Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

Transverse-mass distribution for W production



Features of $M_{T,\nu l}$:

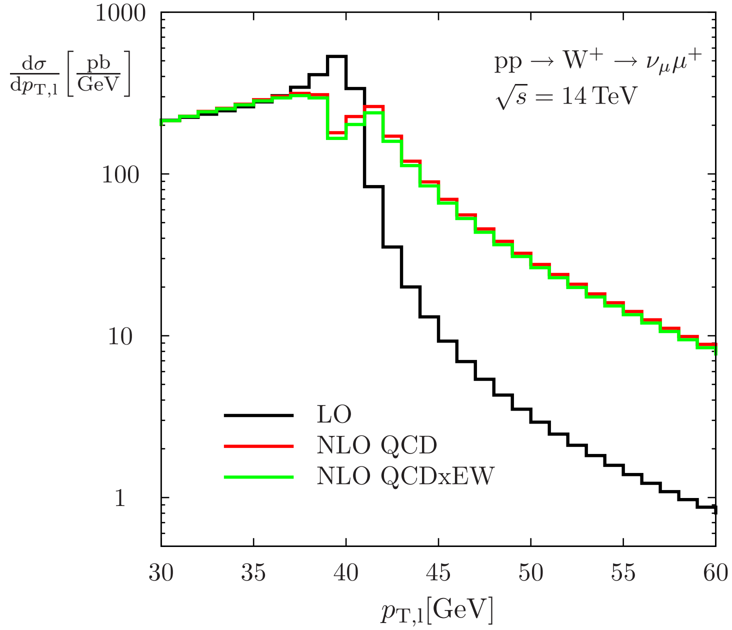
- stability wrt QCD corrs./uncertainties (insensitive to jet recoil)
- sensitive to detector effects via $\not{E}_T$

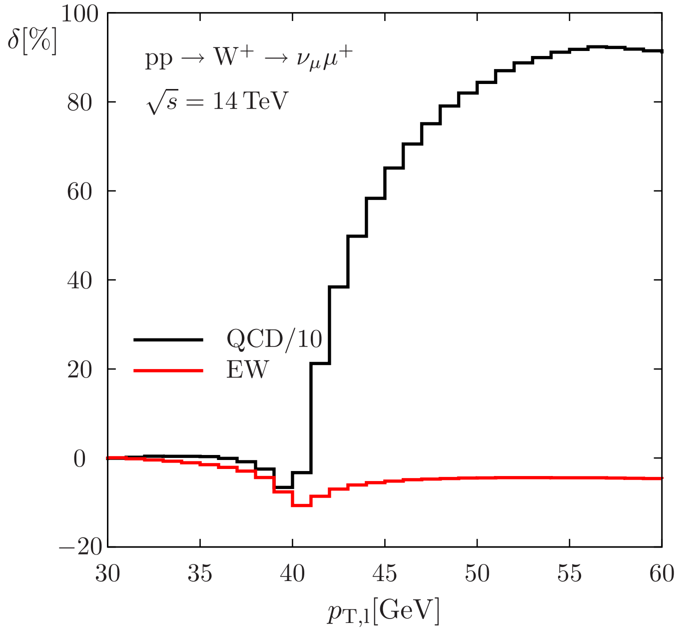
Break-up of the EW correction:

- Pole Approximation
  = IS + FS + non-fact. IS–FS
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

S.D., Huss, Schwinn '14

Transverse-momentum distribution for W production





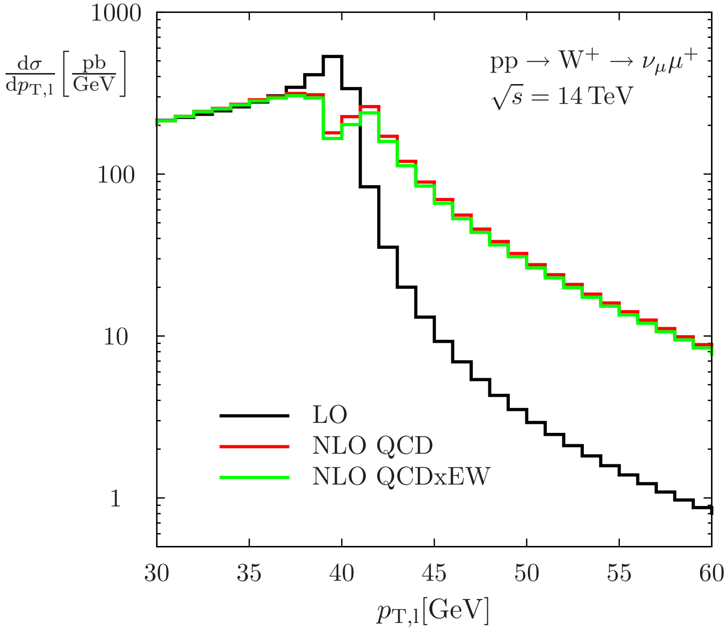
Features of $p_{T,l}$:

- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties

Corrections:

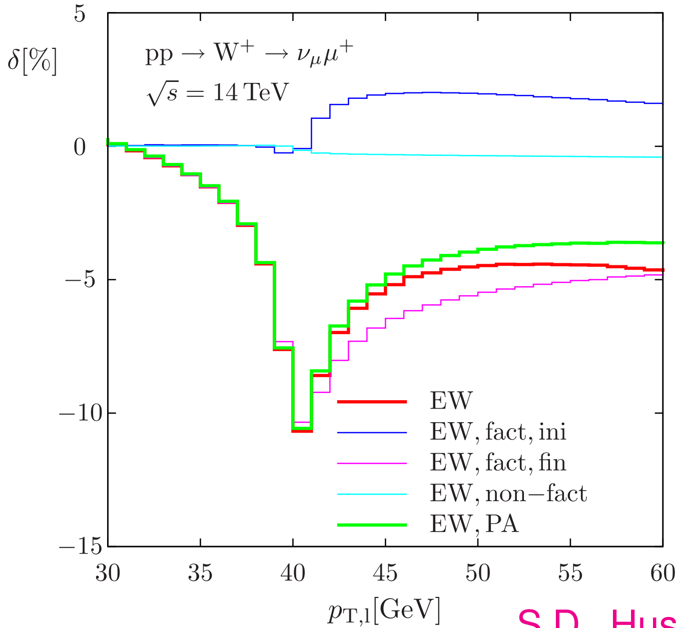
- QCD corrections huge above resonance (jet recoil)
- EW corrections distort resonance shape as well

Transverse-momentum distribution for W production



Features of $p_{T,l}$:

- stability wrt detector effects
- sensitive to QCD effects/modelling/uncertainties



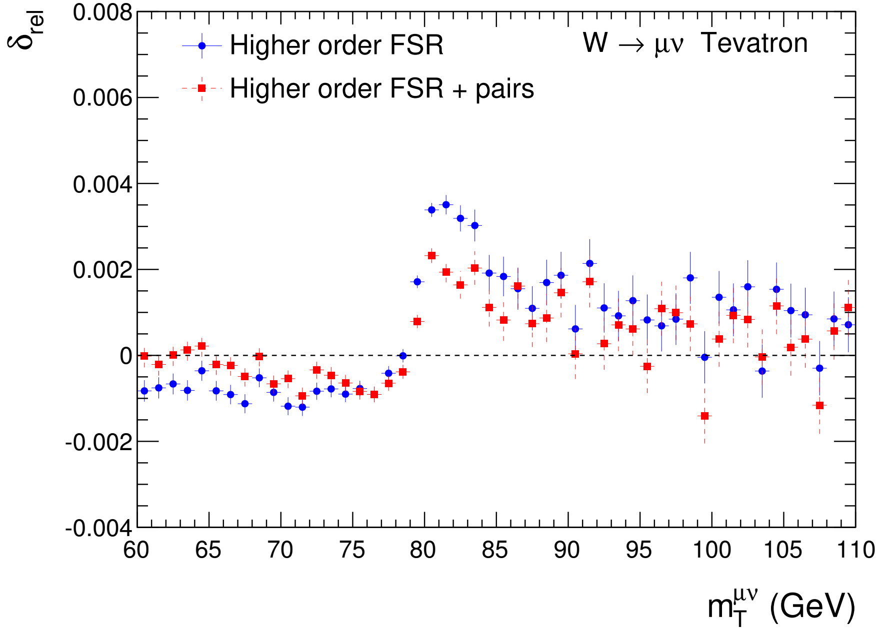
Break-up of the EW corrections:

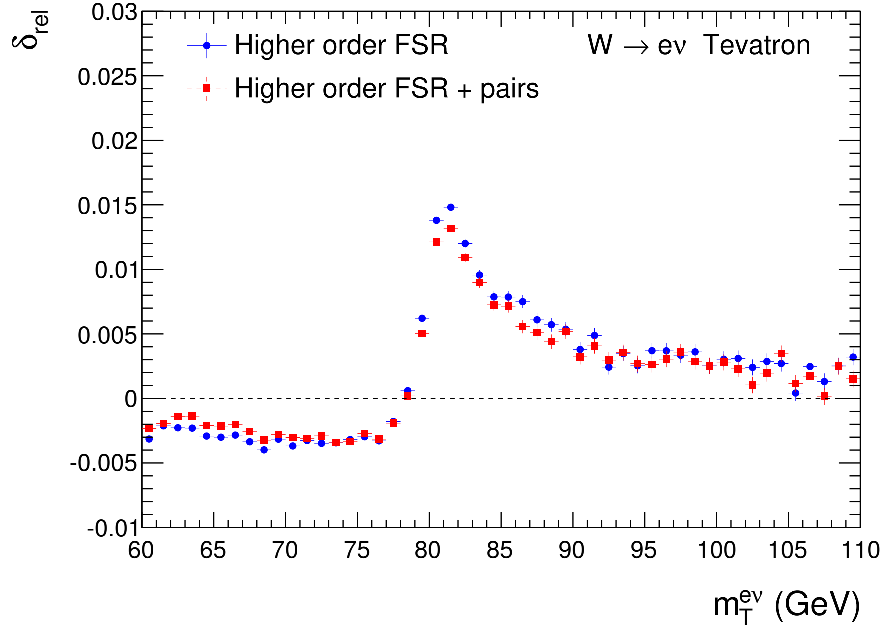
- PA works well near resonance
- factorizable FS corrs. distort resonance shape
- factorizable IS corrs. overwhelmed by QCD
- non-fact. corrs. flat and negligible

S.D., Huss, Schwinn '14

Multi-photon and lepton-pair corrections to W production:

Carloni Calame et al. '16





Estimated shifts on M_W: Carloni Calame et al. '16

$pp \to W^+$, $\sqrt{s} = 14$ TeV	M_W shifts (MeV)			
Templates accuracy: LO	$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$ (bare)	
Pseudo–data accuracy	M_T	p_T^ℓ	M_T	p_T^ℓ
HORACE only FSR-LL at $\mathcal{O}(\alpha)$	-94±1	-104±1	-204±1	-230±2
HORACE FSR-LL	-89±1	-97±1	-179±1	-195±1
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PHOTOS FSR-LL	-92±1	-100±2	-182±1	-199±2

Difference is multi-γ effect beyond NLL FSR

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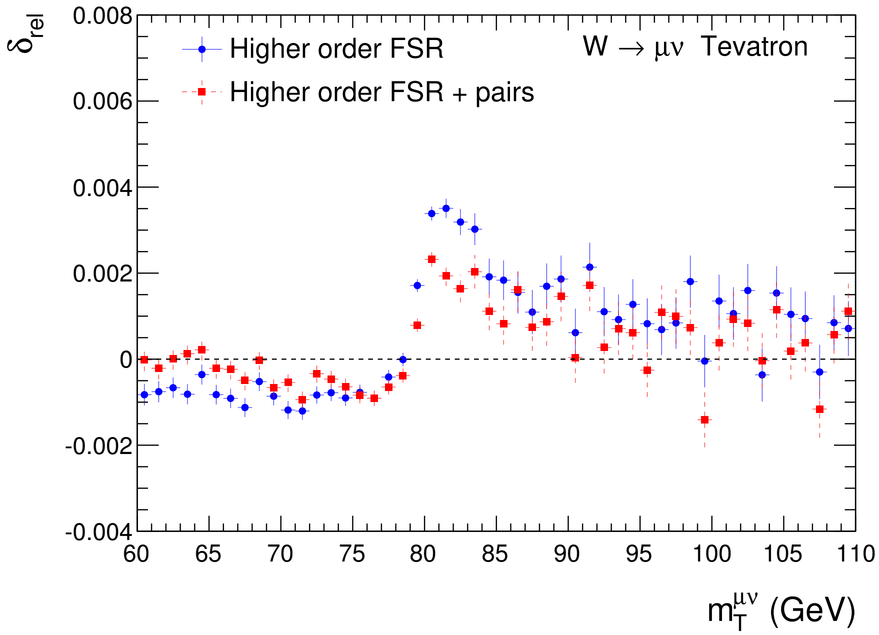

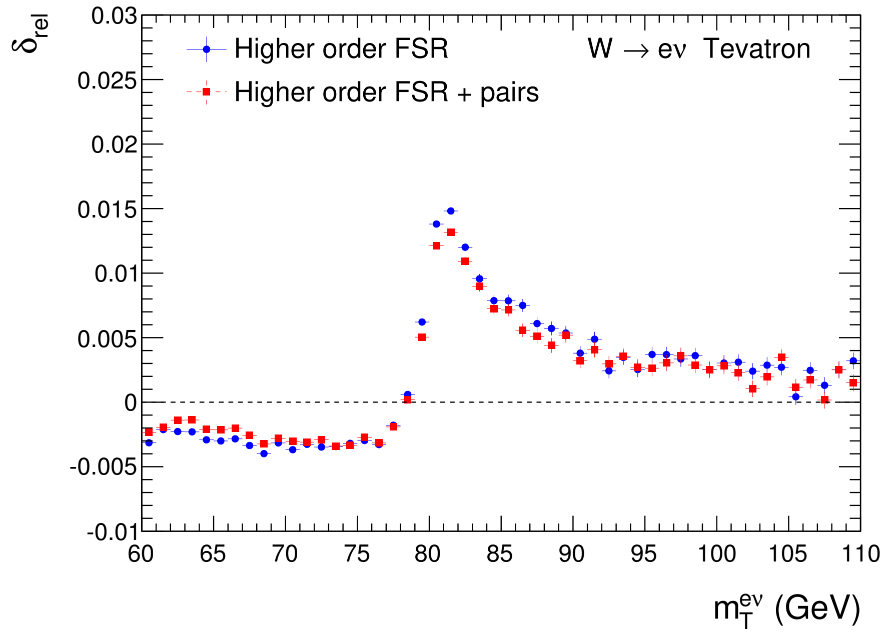

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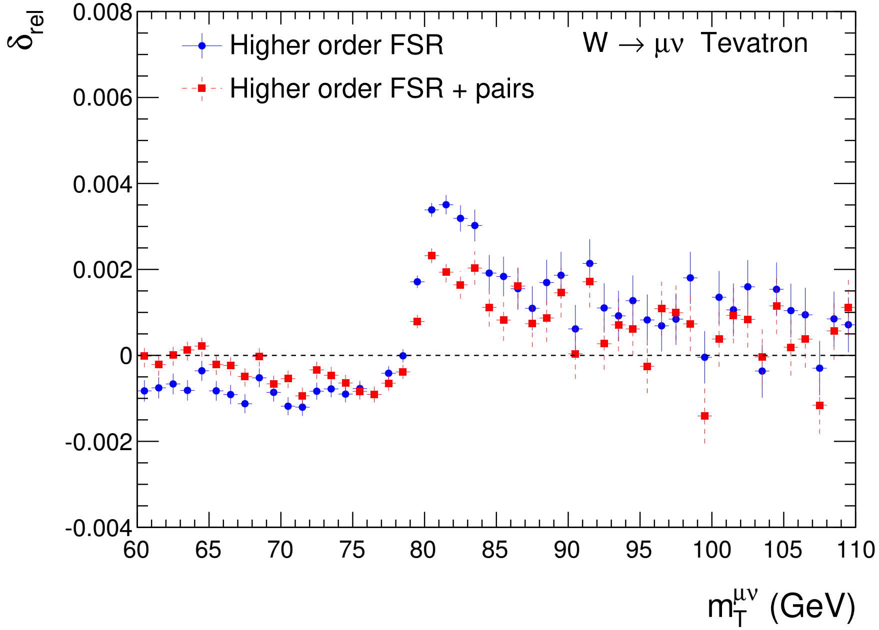
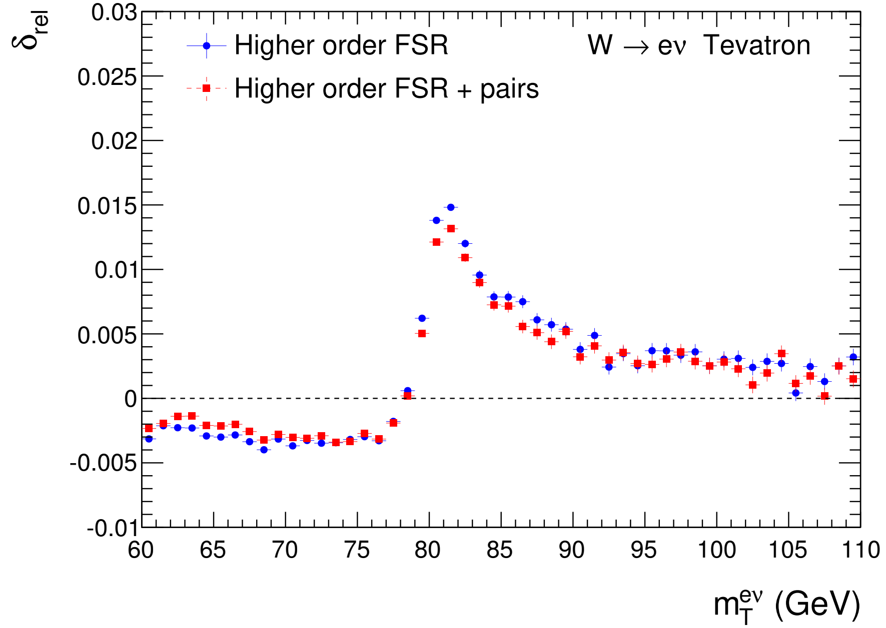
Difference is NLO EW — FSR LL

Multi-photon and lepton-pair corrections to W production:

Carloni Calame et al. '16





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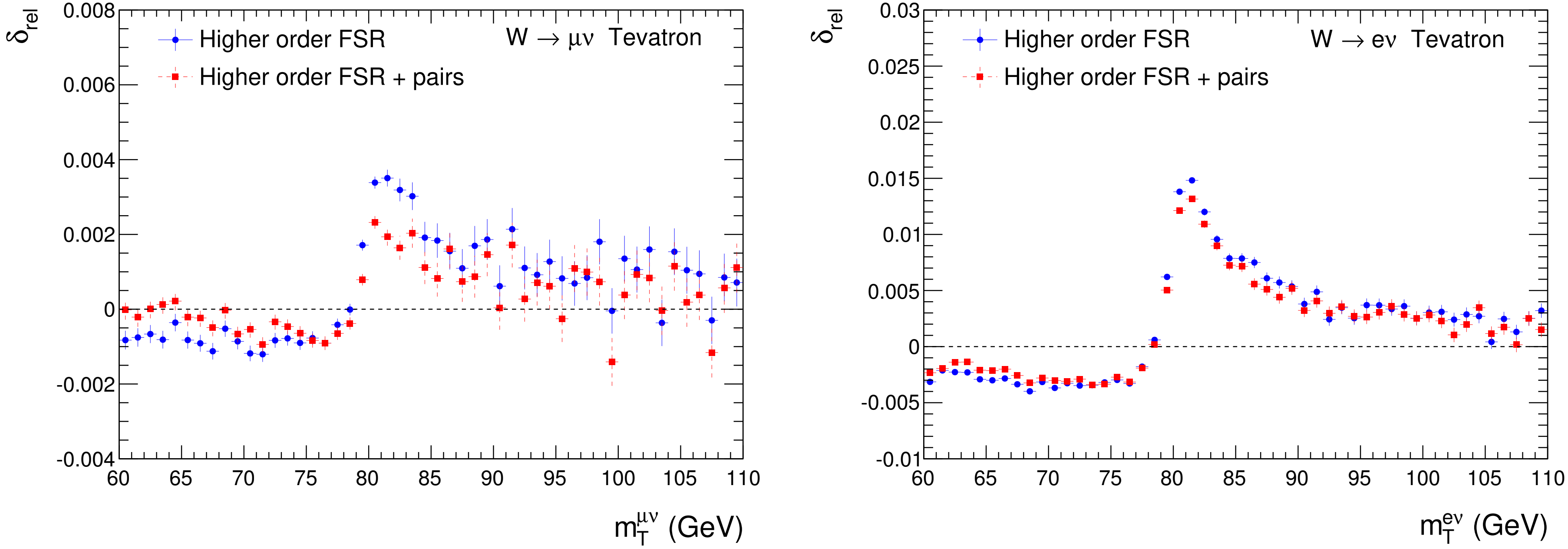
Difference is lepton-pair correction

Multi-photon and lepton-pair corrections to W production:

Carloni Calame et al. '16



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Difference is

$\sim$ FSR LL uncertainty $\sim 3\,\mathrm{MeV}$!

Combination of QCD and EW corrections

Combination of QCD and EW corrections to inclusive W/Z production

Issue unambiguously fixed only by calculating the 2-loop $\mathcal{O}(\alpha\alpha_s)$ corrections, until then rely on approximations and estimate the uncertainties:

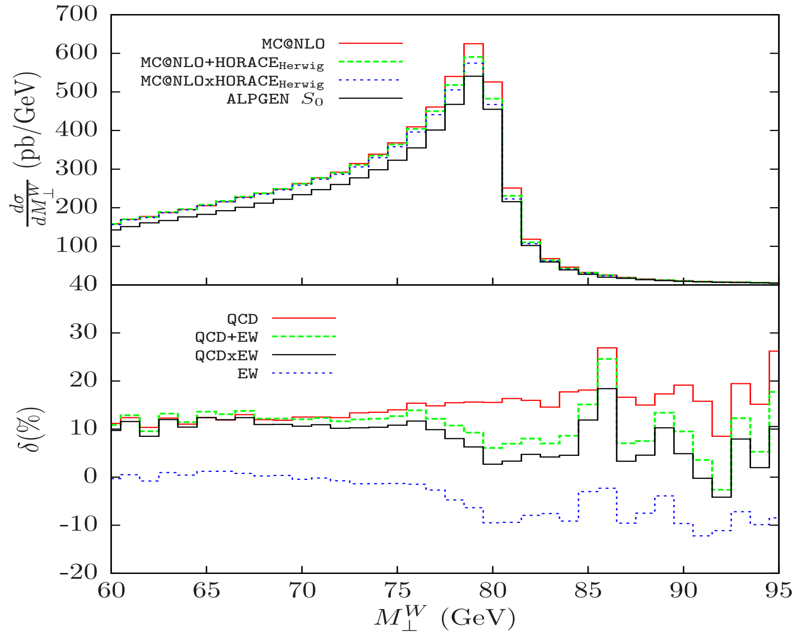
Comparison of two extreme alternatives:

$$(1 + \delta^{\rm NLO}_{\rm QCD} + \delta^{\rm NLO}_{\rm EW})$$

versus

$$(1 + \delta^{\rm NLO}_{\rm QCD}) \times (1 + \delta^{\rm NLO}_{\rm EW})$$

Balossini et al. '09 (HORACE)



Difference at %-level
with shape distortion
$\hookrightarrow$ limits precision in M_W measurement

$\Rightarrow$ Dominant $\mathcal{O}(\alpha\alpha_s)$ corrections calculated for resonance region

S.D., Huss, Schwinn '14,'15

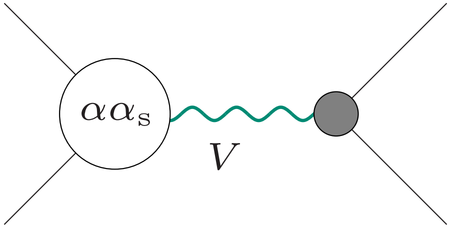
Full calculation in progress in various groups ...

$\mathcal{O}(\alpha\alpha_s)$ corrections in pole approximation S.D., Huss, Schwinn '14,'15

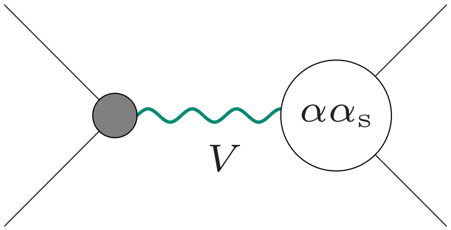
$\hookrightarrow$ take only leading (=resonant) contributions in expansion about resonance poles

Factorizable contributions:

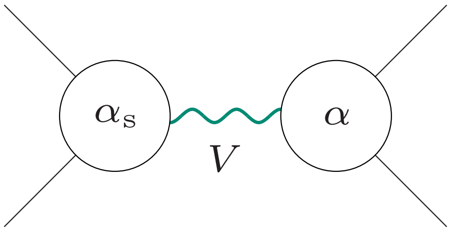
(only virtual contributions indicated)



- not yet known,
  but no significant resonance distortion expected
- no PDFs with $\mathcal{O}(\alpha\alpha_s)$ corrections



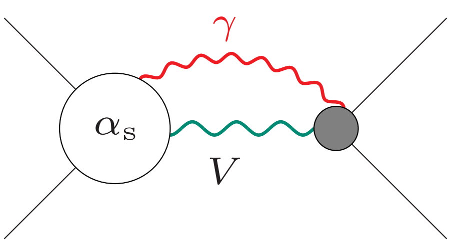
- only $Vl\bar{l}'$ counterterm contributions
- calculated $\rightarrow$ very small, uniform correction



- significant resonance distortions from FSR
- calculated and
  compared to leading-log parton shower approach

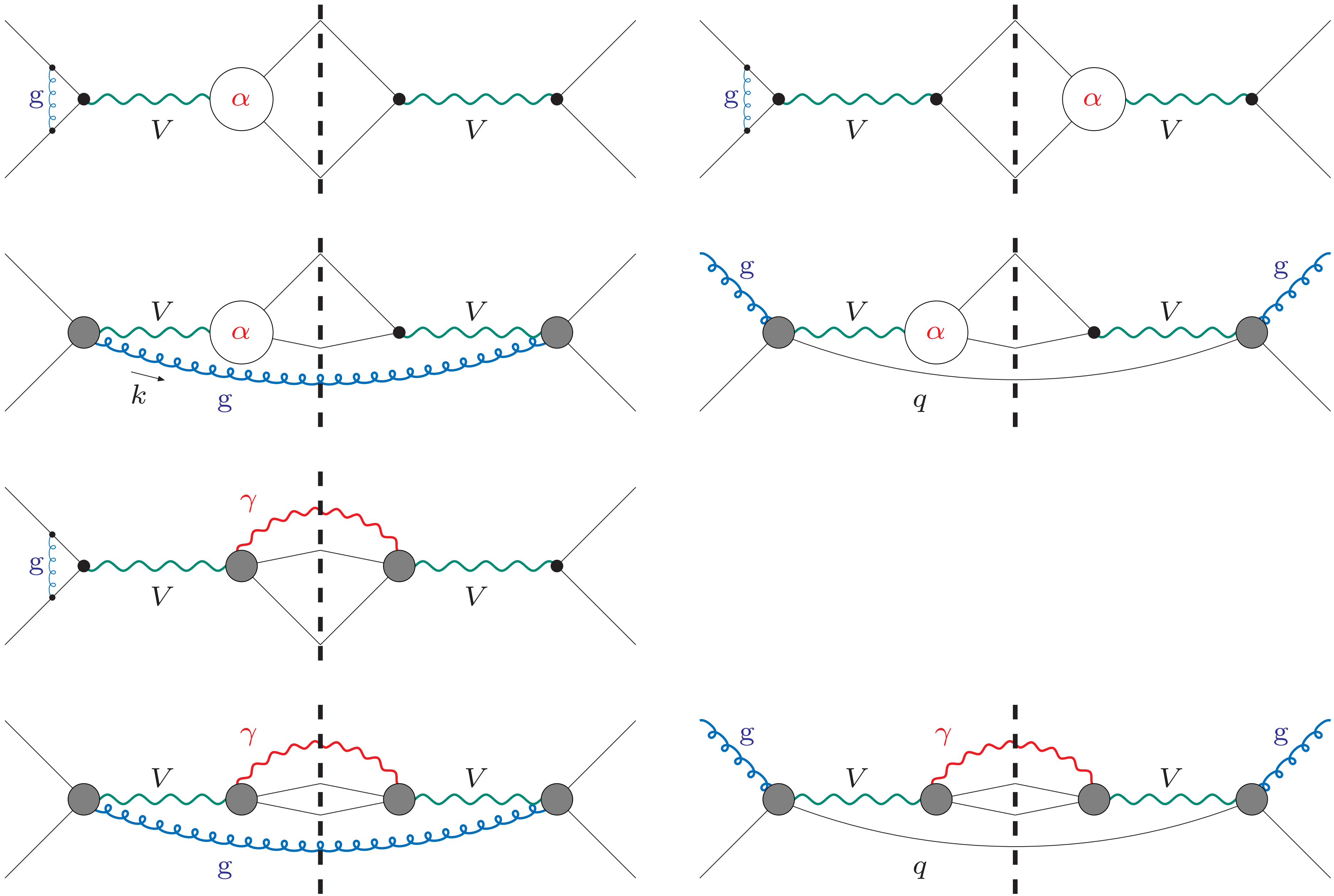
Non-factorizable contributions:

(only virtual contributions indicated)



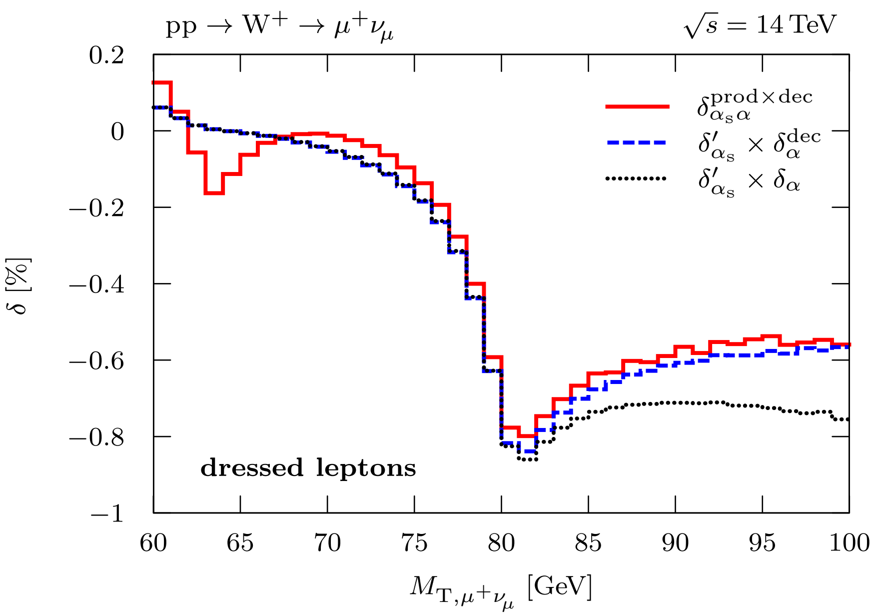
- could induce shape distortions
- calculated $\rightarrow$ phenomenologically negligible

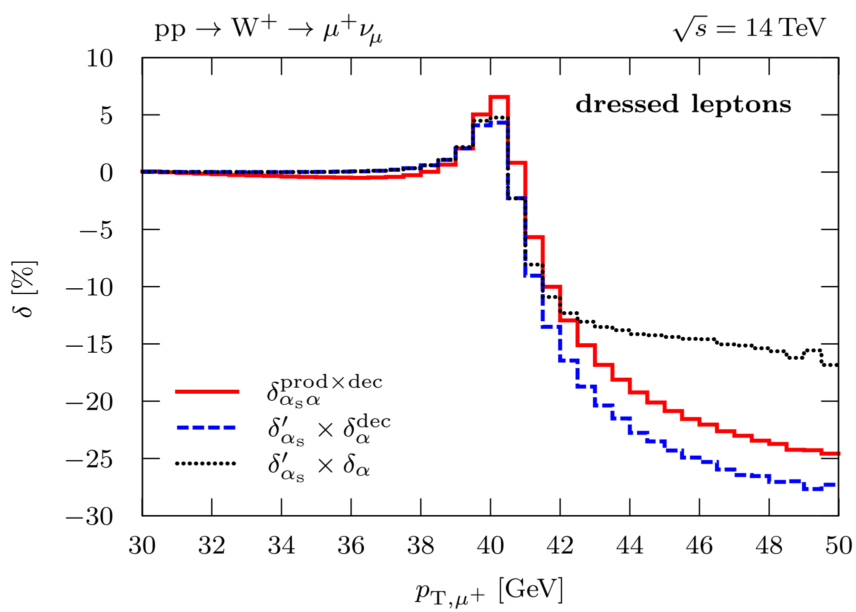
Initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections



Initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections S.D., Huss, Schwinn '15

W production: (γ recombination applied, "dressed leptons")





Naive factorization works!

Naive factorization deteriorates for $p_{T,\mu^+} \gtrsim M_W/2$

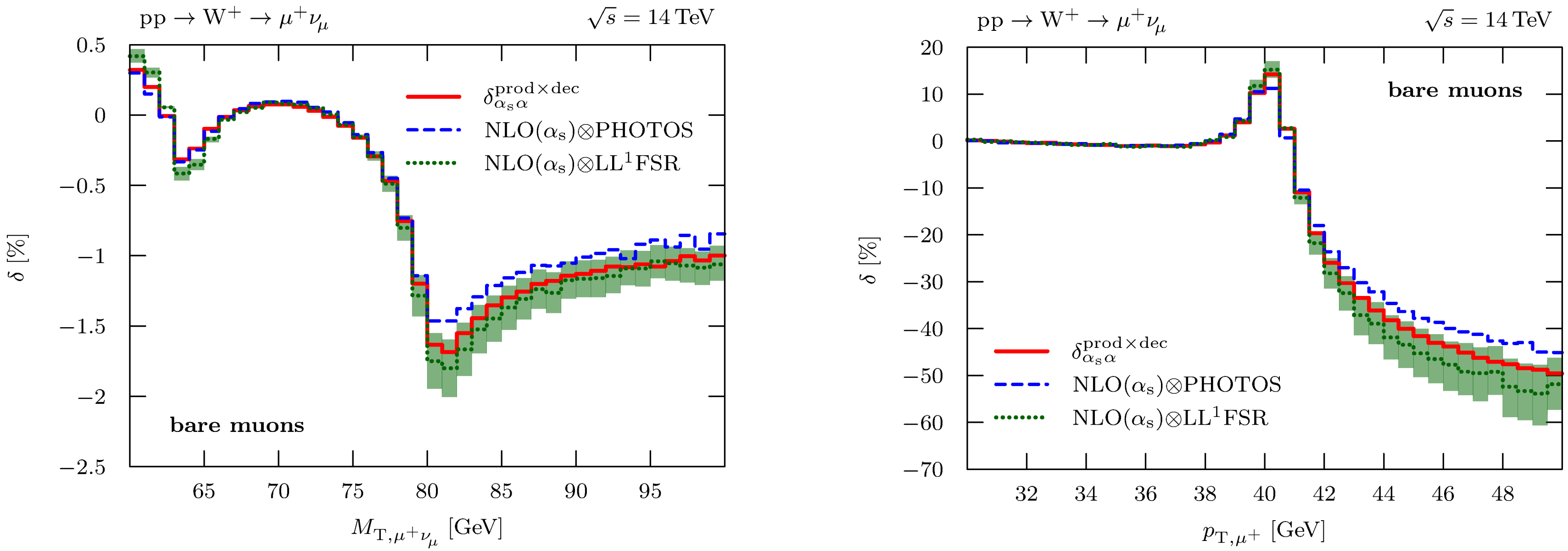
Important issues:

- comparison of $\delta^{\text{ini-fin}}_{\alpha_s\alpha}$ with MC approach $d\sigma_{\alpha_s} \otimes (\gamma \text{ shower})$
- estimate shifts in M_W by $\delta^{\text{ini-fin}}_{\alpha_s\alpha}$

Initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections

S.D., Huss, Schwinn '15

W production: (γ recombination applied, "dressed leptons")



Two QED FSR leading-log approaches: "Differential factorization" works!

- QED structure-function:
- QED parton-shower PHOTOS

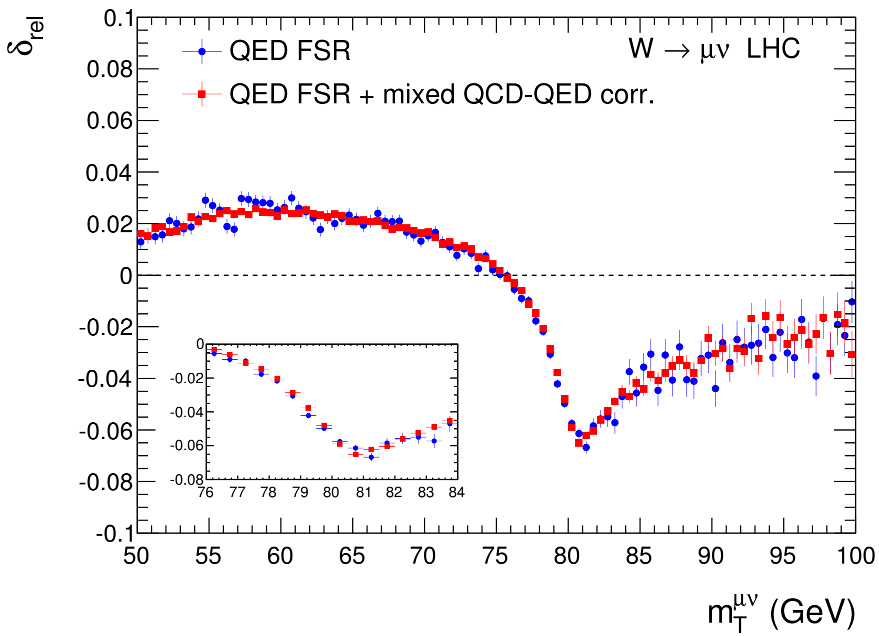
Barberio, van Eijk, Was

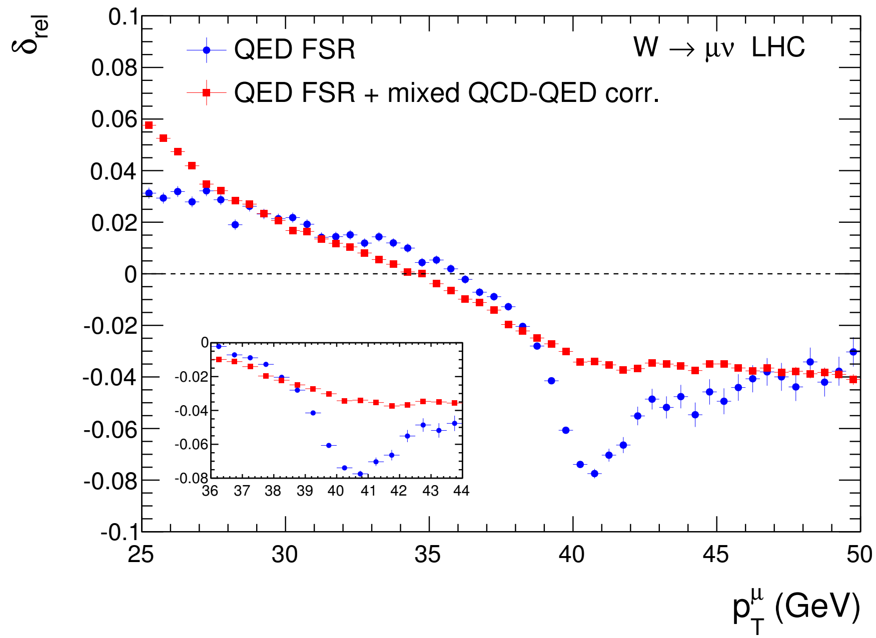
Estimated shift $\Delta M_W^{\alpha_s\alpha}$:	fixed order	POWHEG-V2two-rad (Carloni Calame et al. '16)	
bare μ	$-14\,\mathrm{MeV}$	$-16.0 \pm 3\,\mathrm{MeV}$	good agreement!
dressed leptons	$-4\,\mathrm{MeV}$	?	

Parton-shower-improved predictions

Impact of QCD corrections on QED parton showers:

Carloni Calame et al. '16





QED FSR = $\mathrm{QED_{PS}}$ / LO, HORACE

QED FSR + mixed QCD-QED = $\mathrm{QCD_{NLOPS}} \otimes \mathrm{QED_{PS}} / \mathrm{QCD_{NLOPS}}$, POWHEG-v2

Note: (NLO QCD + PS) $\otimes$ QED PS misses
(NLO QCD + PS) $\otimes$ (non-FSR NLO EW)

$\hookrightarrow$ improved by POWHEG-V2`two-rad` (see also Mück, Oymanns '16)

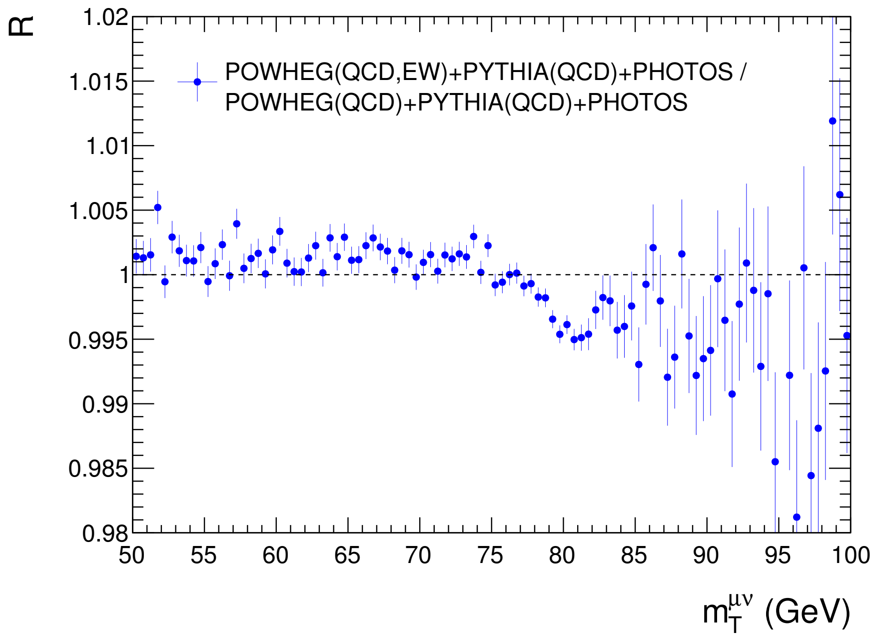
NLO EW PS matching – Resonance improvement necessary!

Carloni Calame et al. ’16; Mück, Oymanns ’16

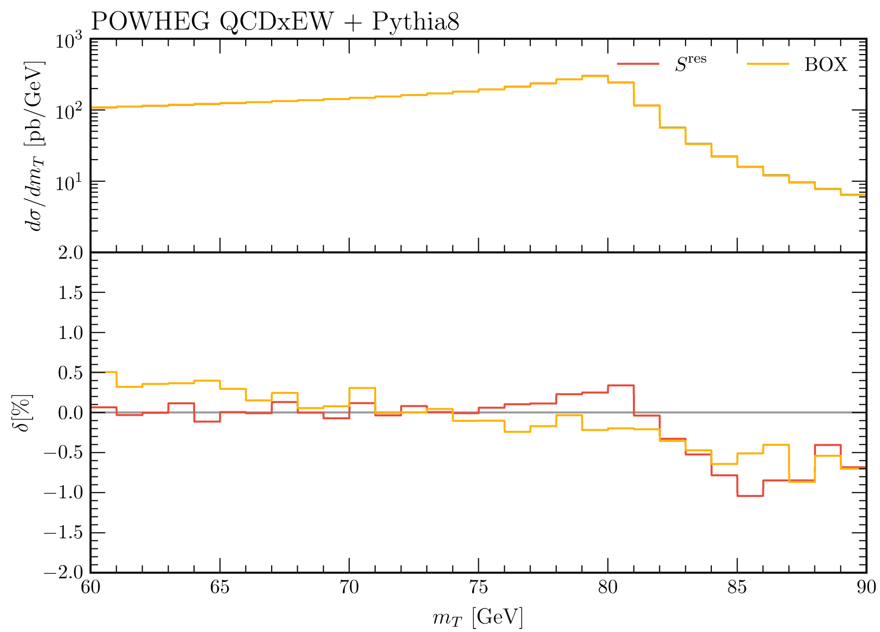
Impact of EW matching without resonance improvement:

Effect of resonance improvement:

WG report CERN-LPCC-2016-002 [1606.02330]



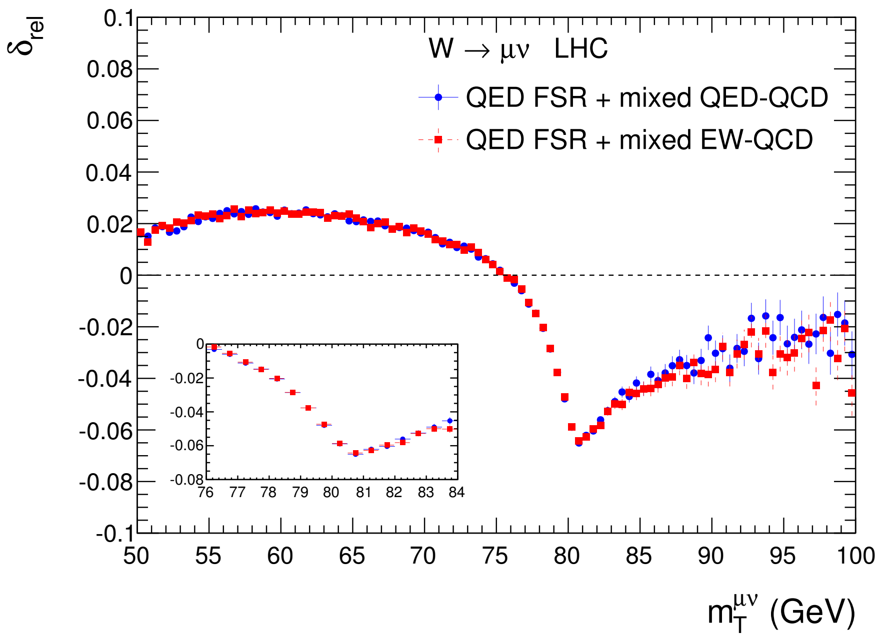
Mück, Oymanns ’16

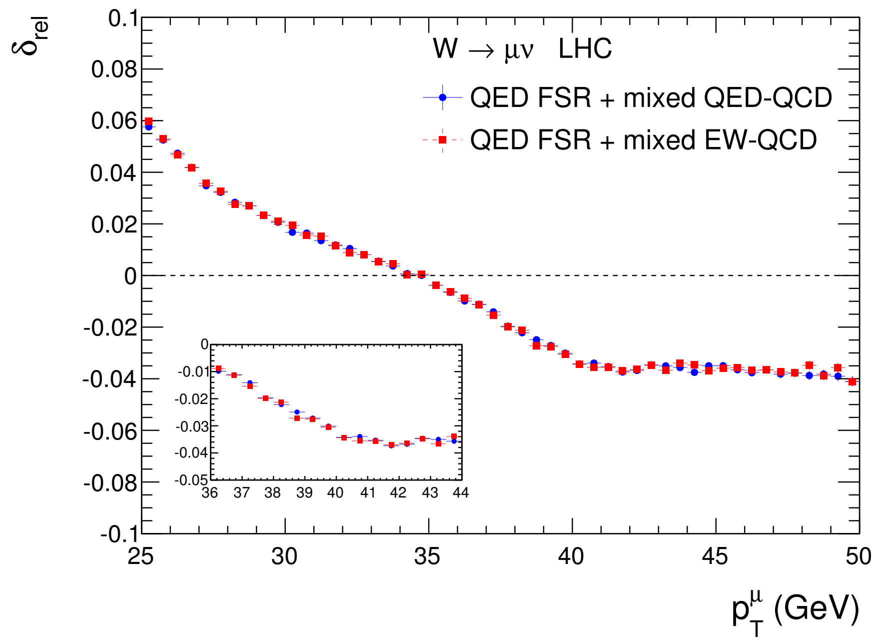


Missing resonance improvement leads to artefacts of $\sim 0.5{-}1\%$ in M_T distribution!

Improved results based on (NLO QCD + PS) $\otimes$ (NLO EW + PS)

Carloni Calame et al. '16





QED FSR + mixed QED-QCD = $\mathrm{QCD_{NLOPS}} \otimes \mathrm{QED_{PS}} / \mathrm{QCD_{NLOPS}}$

QED FSR + mixed EW-QCD = $\mathrm{QCD_{NLOPS}} \otimes \mathrm{EW_{NLOPS}} / \mathrm{QCD_{NLOPS}}$, POWHEG-V2two-rad

EW shifts on M_W by NLO-PS-matched corrections wrt LO predictions

Carloni Calame et al. '16

$pp \to W^+$, $\sqrt{s} = 14$ TeV		M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD$_\mathrm{PS}$		$W^+ \to \mu^+\nu$		$W^+ \to e^+\nu$(dres)	
Pseudodata accuracy	QED FSR	M_T	p_T^ℓ	M_T	p_T^ℓ
NLO-QCD+(QCD+QED)$_\mathrm{PS}$	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
NLO-QCD+(QCD+QED)$_\mathrm{PS}$	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
NLO-(QCD+EW)+(QCD+QED)$_\mathrm{PS}$two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
NLO-(QCD+EW)+(QCD+QED)$_\mathrm{PS}$two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2

- Impact of QCD corrs. on EW shift ΔM_W:
  $\sim 1\,\mathrm{MeV}$ in M_T, but some $100\,\mathrm{MeV}$ in p_T^ℓ
- PHOTOS shower closer to full NLO EW than PYTHIA shower
  $\hookrightarrow$ use PYTHIA shower only with full NLO EW
- updated version POWHEG-v2 two-rad should be used
  (changes in ΔM_W by ~ 5–$10\,\mathrm{MeV}$ in M_T)
  $\hookrightarrow$ estimated accuracy in ΔM_W: ~ 1–$2\,\mathrm{MeV}$

Note: Extraction of $\Delta M_\mathrm{W}^{\alpha_s\alpha} = -16\,\mathrm{MeV}$ induced by mixed QCD–EW corrs. requires several runs with different setups.

Conclusions

M_W determination @ LHC

- recent ATLAS measurement catches up with Tevatron result ($\Delta M_\mathrm{W} = 19\,\mathrm{MeV}$)
- promises M_W with accuracy $\Delta M_\mathrm{W} \lesssim 10\,\mathrm{MeV}$

Precision calculations for Drell–Yan physics

- NNLO QCD + NLO EW + QCD resummations etc. known
- dominant $\mathcal{O}(\alpha\alpha_\mathrm{s})$ correction near resonances
- POWHEG matching for NLO QCD+EW $\otimes$ QCD+QED PS
  public tool: POWHEG-v2 `two-rad`

Open issues / room for improvement

- general-purpose MC generator with state-of-the-art QCD and EW corrections
- full $\mathcal{O}(\alpha\alpha_\mathrm{s})$ correction

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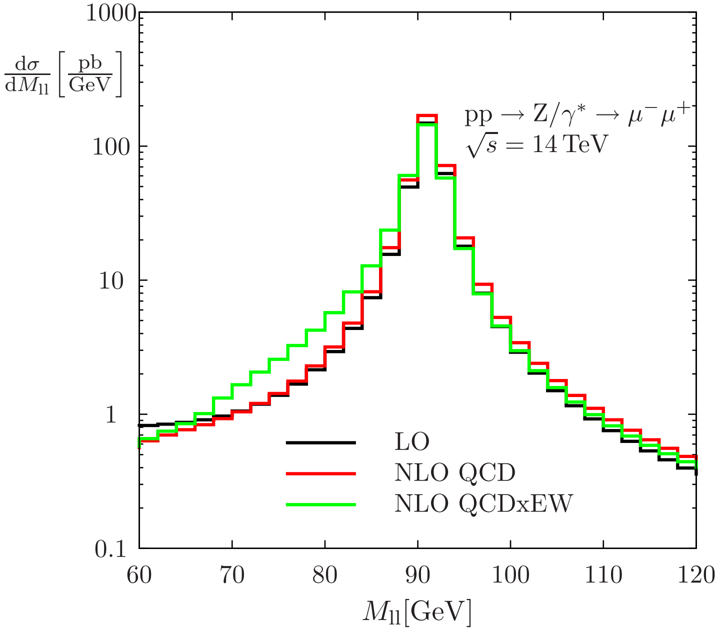
"In ~~football as in watchmaking~~ *particle theory*, talent and elegance mean nothing without rigour and precision."
[Lionel Messi]

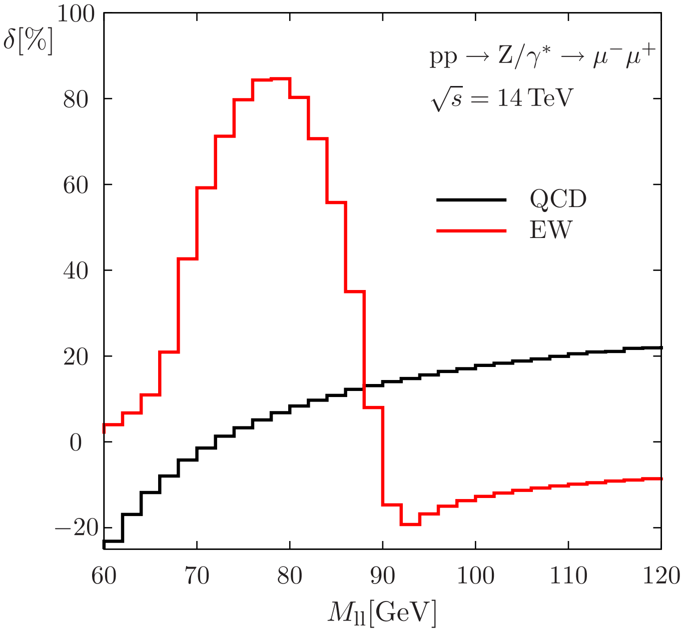
Backup slides

EW corrections to Z production

Invariant-mass distribution for Z production



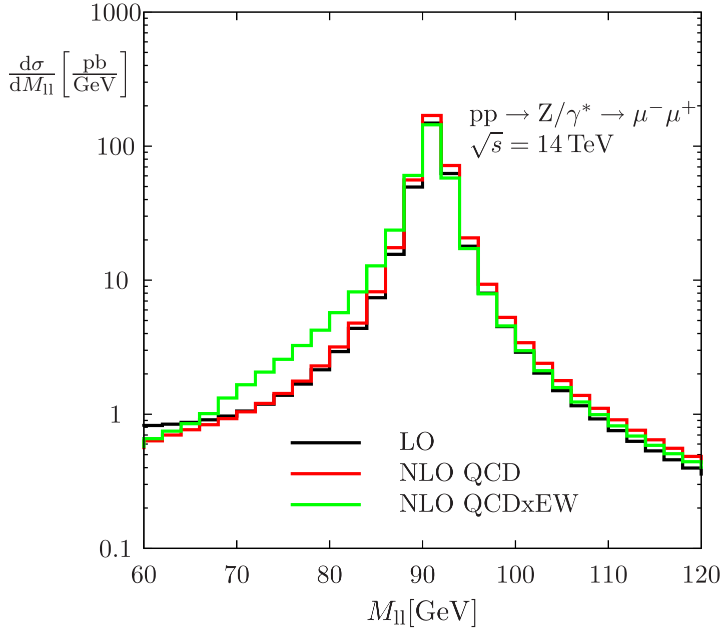
Reference process for M_W measurement

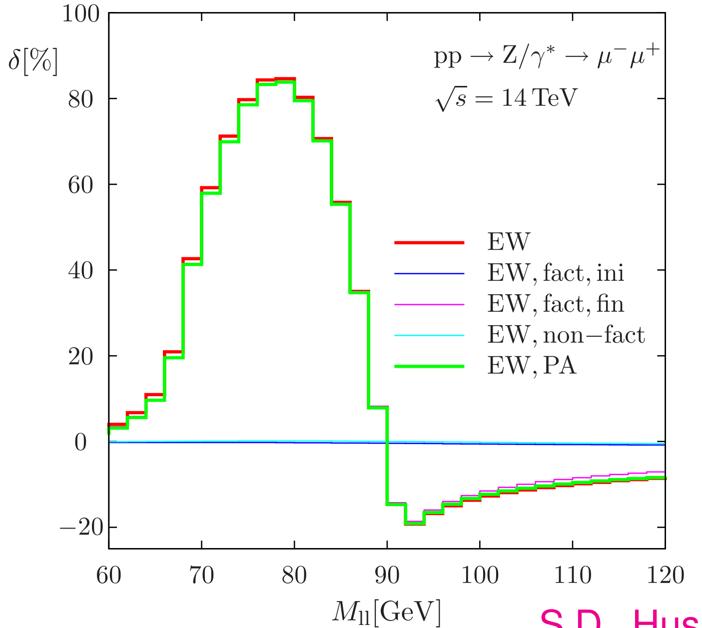


Corrections:

- QCD corrections quite flat near resonance
- EW corrections distort resonance shape

Invariant-mass distribution for Z production





S.D., Huss, Schwinn ’14

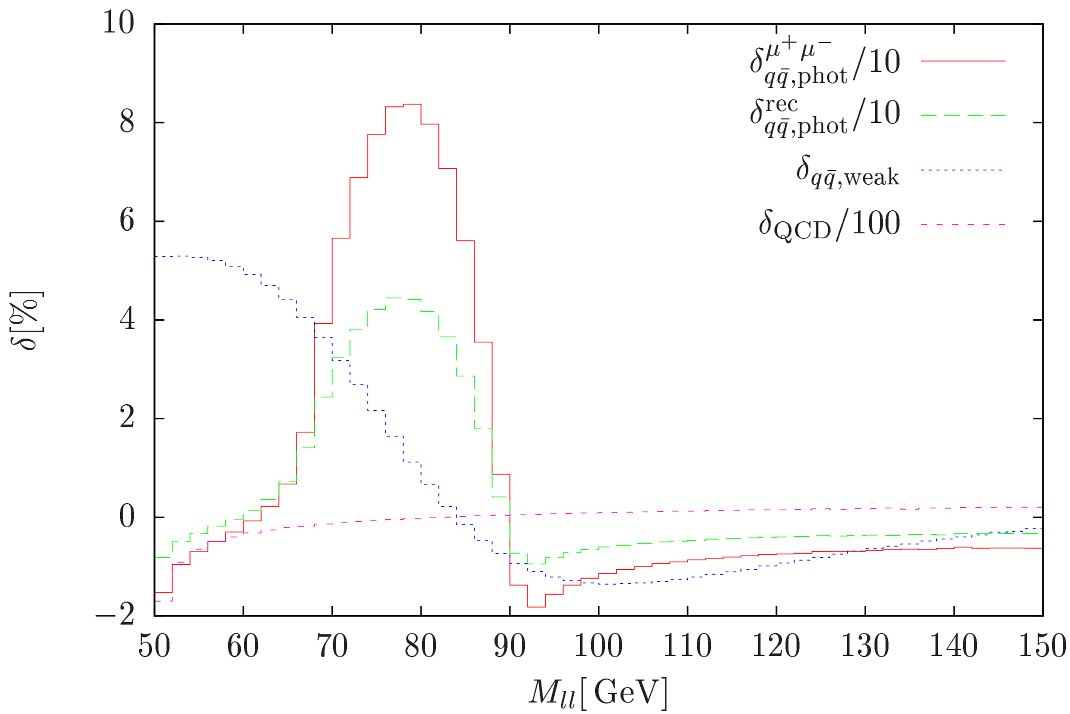
Reference process for M_W measurement

Break-up of the EW corrections:

- PA reproduces EW corr near resonance
- resonance distortion merely due to factorizable FS correction
- factorizable IS and non-fact. corrections flat (and even negligible)

Isolation of NLO weak corrections to M_{ll} distribution in Z production

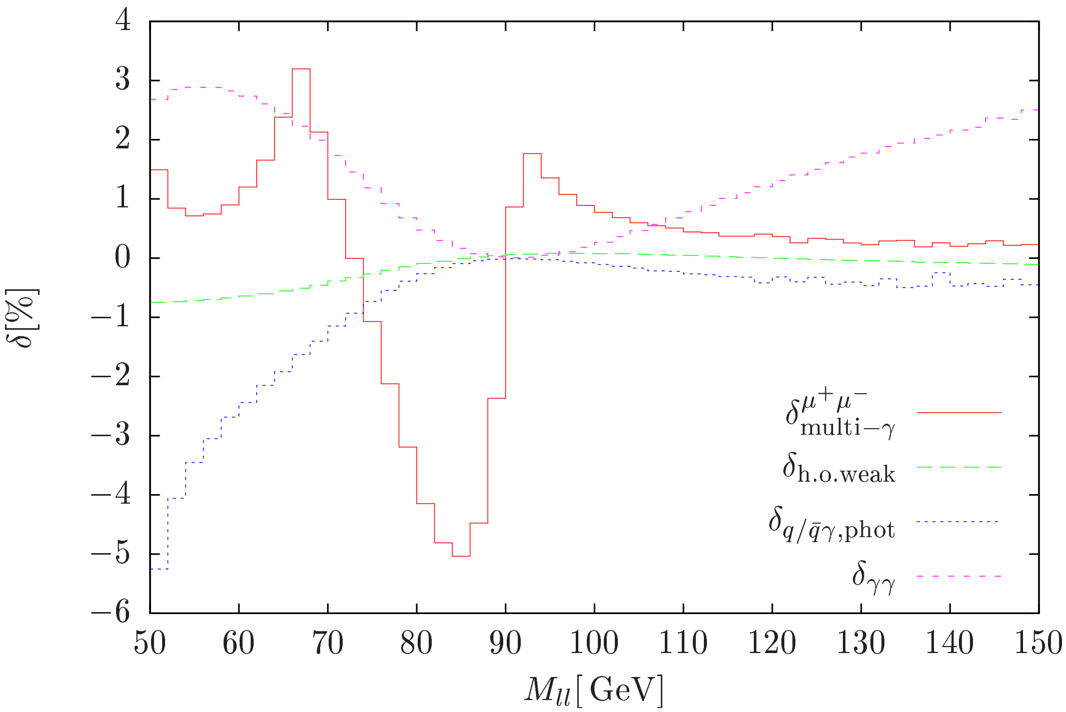
S.D., Huber '09



- photonic corrections deliver major effect!
  - large radiative tail for $M_{ll} \lesssim M_{\mathrm{Z}}$ from photonic final-state radiation
  - photon recombination (=dressed leptons) reduces FSR corrs. drastically (cancellation of large mass-singular corrections $\propto (\alpha \ln m_\ell)^n$ a la KLN)
- weak corrections only moderate $\sim \mathcal{O}(1\%)$ at $M_{ll} \sim M_{\mathrm{Z}}$

EW corrections beyond NLO to M_{ll} distribution in Z production

S.D., Huber '09

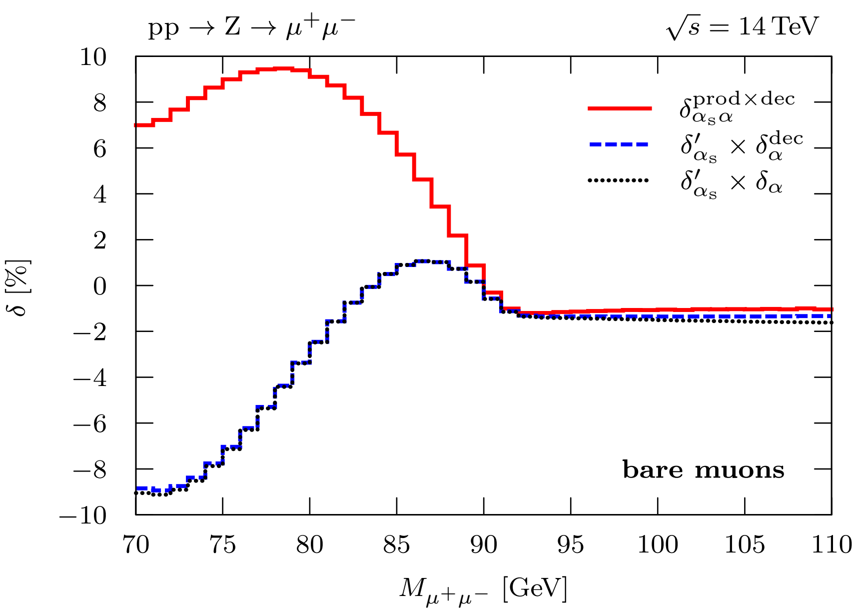


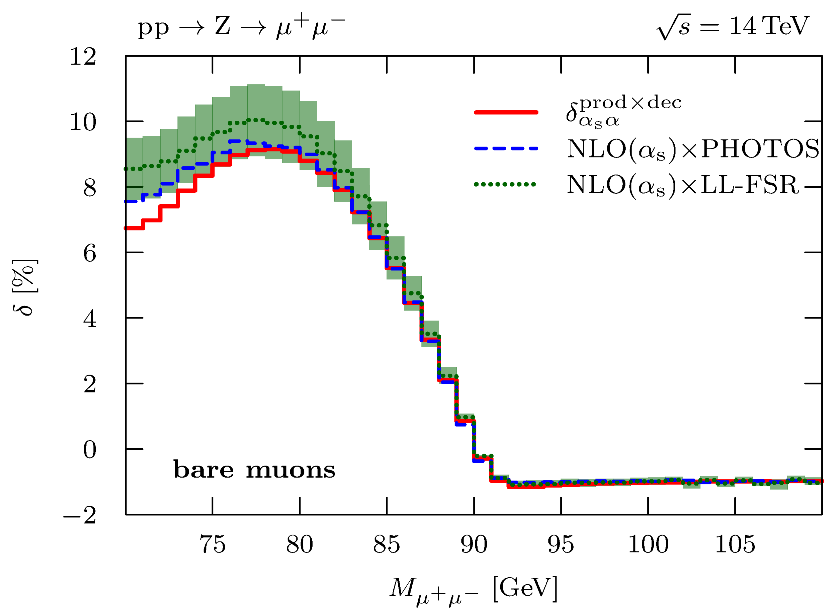
- multi-photon emission significant in resonance region
- higher-order weak corrs. ($\Delta\alpha$, $\Delta' rho$) negligible at $M_{ll} \sim M_Z$
- $q\gamma$/$\gamma\gamma$ channels negligible at $M_{ll} \sim M_Z$

Initial–final factorizable $\mathcal{O}(\alpha\alpha_s)$ corrections S.D., Huss, Schwinn '15

Z production: (no γ recombination applied, "bare leptons")

"naive factorization" versus "differential factorization"



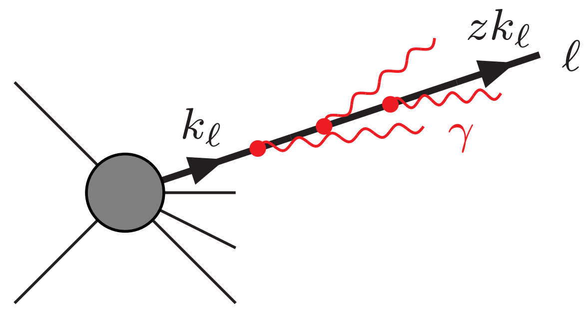


Naive factorization fails !

Differential factorization works!

FSR off leptons

Collinear final-state radiation (FSR) off leptons



Leading logarithmic effect is universal:

$$\sigma_{\mathrm{LL,FSR}} = \underbrace{\int \mathrm{d}\sigma^{\mathrm{LO}}(k_l)}_{\text{hard scattering}} \int_0^1 \mathrm{d}z \; \underbrace{\Gamma_{\ell\ell}^{\mathrm{LL}}(z,Q^2)}_{\text{leading-log structure function, } Q = \text{typ. scale}} \; \Theta_{\mathrm{cut}}(zk_l)$$

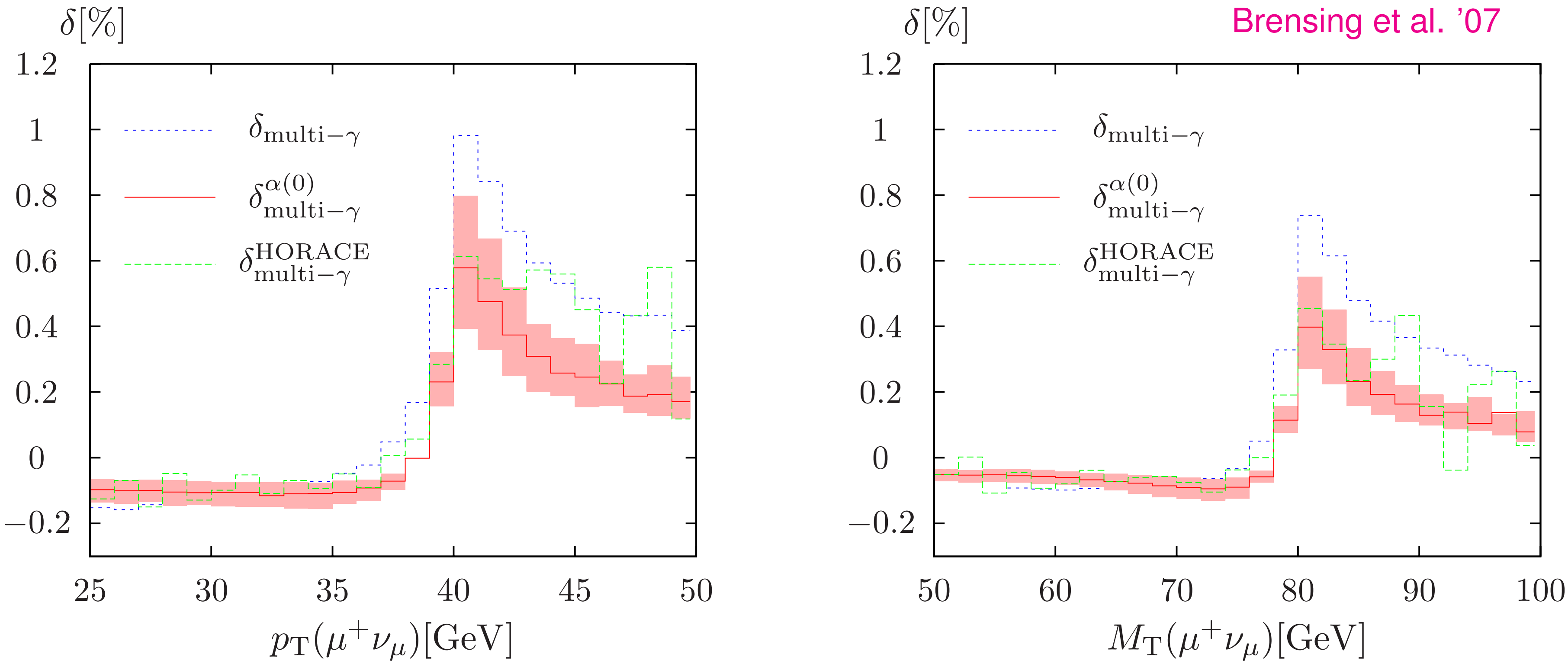
- $\Gamma_{\ell\ell}^{\mathrm{LL}}(z,Q^2)$ known to $\mathcal{O}(\alpha^5)$ + soft exponentiation, equivalent description by QED parton showers
- $\mathcal{O}(\alpha)$ approximation: $\Gamma_{\ell\ell}^{\mathrm{LL},1}(z,Q^2) = \frac{\alpha(0)}{2\pi}\left[\ln\left(\frac{Q^2}{m_\ell^2}\right) - 1\right]\left(\frac{1+z^2}{1-z}\right)_+$
- Alternative approach: QED parton shower
  $\hookrightarrow$ advantage: photons described with finite p_{T} and definite multiplicity

Impact on predictions:

- log-enhanced corrections for “bare” leptons (muons) $\rightarrow$ large radiative tails
- KLN theorem: mass-singular FSR effects cancel if $(\ell\gamma)$ system is inclusive
  (full integration over z)
- full FSR not universal, in general not even separable from other EW corrections

Higher-order photonic corrections:

Brensing et al. '07



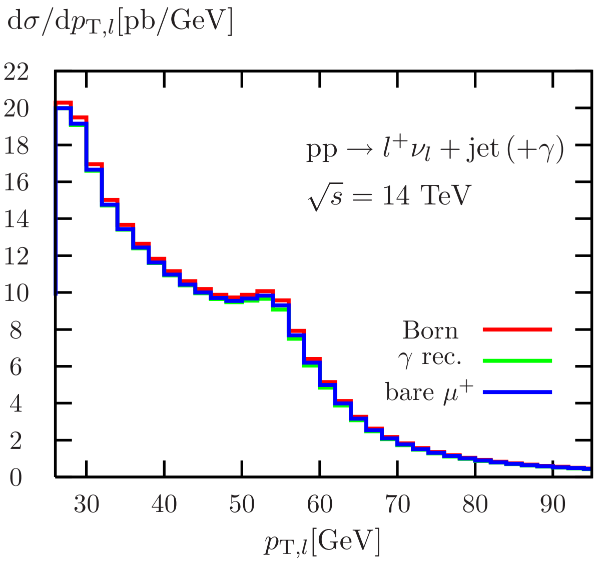
Multi-γ effects (beyond NLO!) relevant near resonance,
logarithmic approximations by structure-functions and parton showers
in good agreement

$\hookrightarrow$ uncertainties can be estimated by scale variations

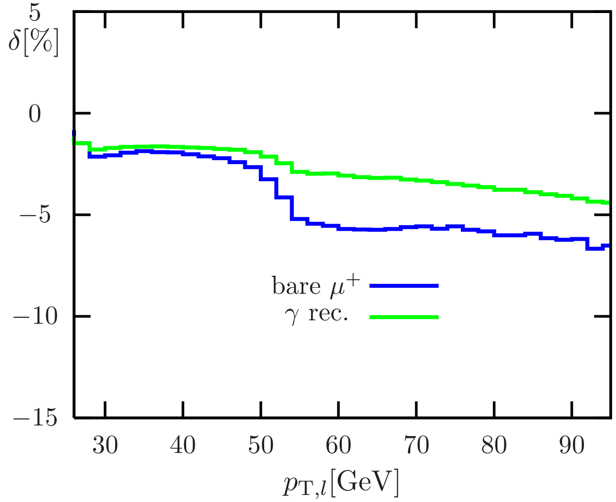
More on mixed QCD–EW corrections

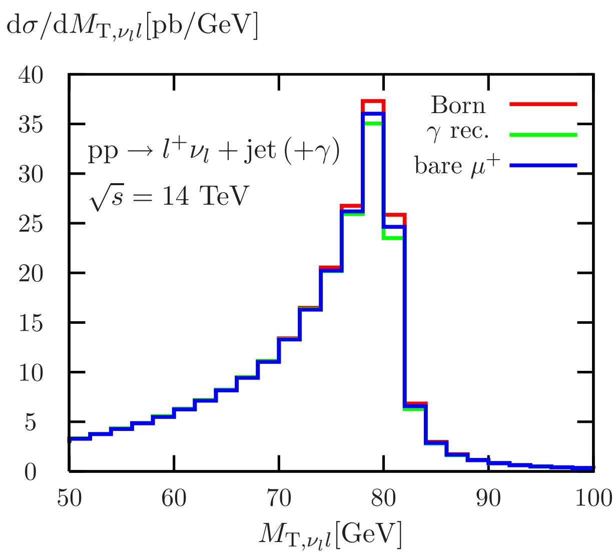
Comparison of EW corrections to W+jet and single (jet-inclusive) W production

$\hookrightarrow$ argument for factorization QCD×EW if EW corrections coincide

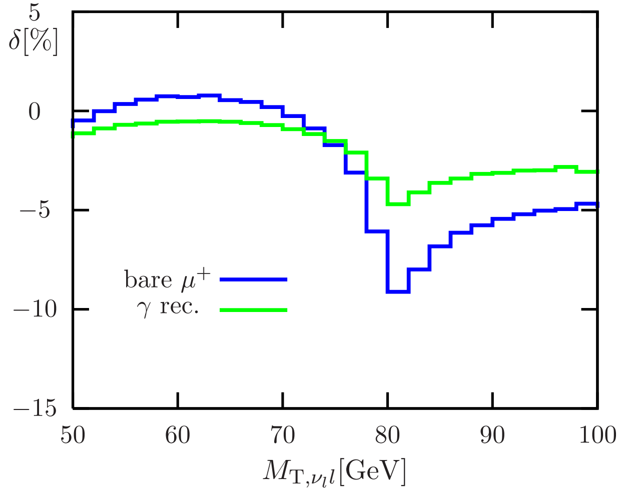


Denner et al. '09





Denner et al. '09



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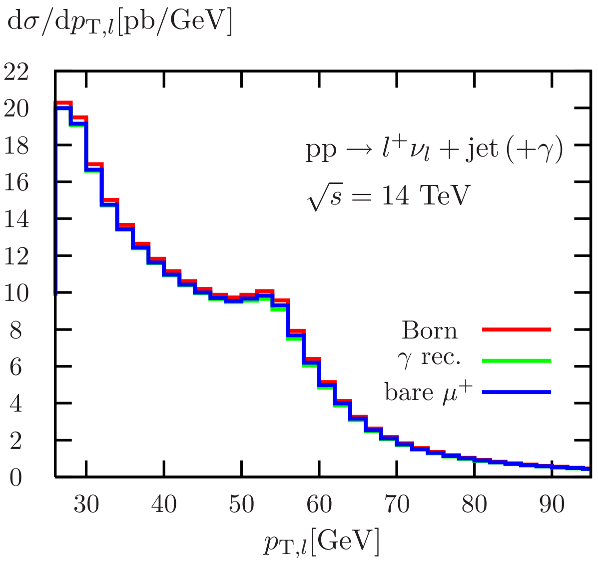
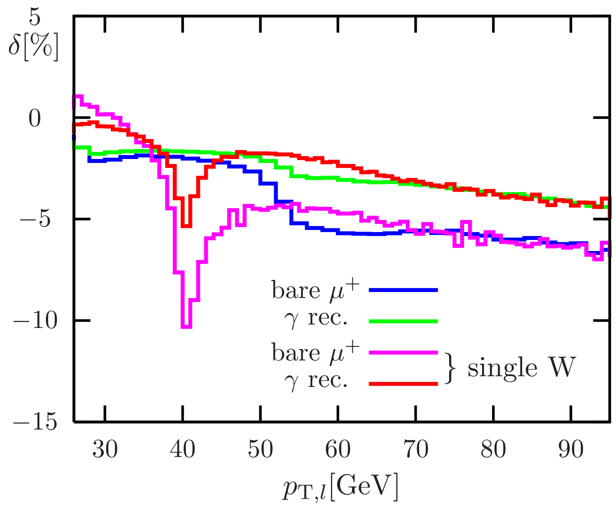
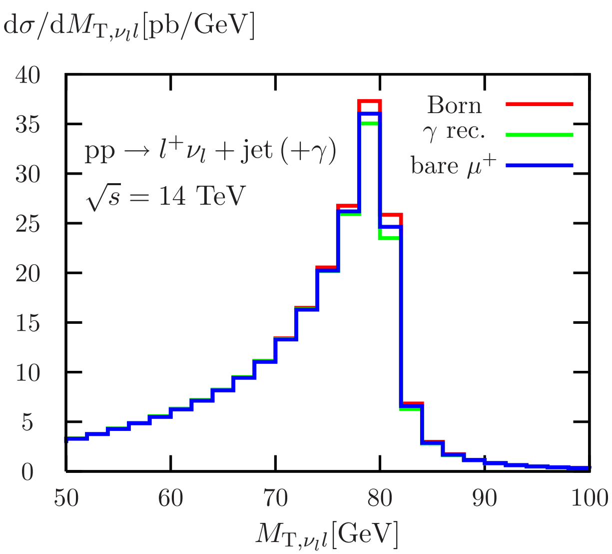


Denner et al. ’09

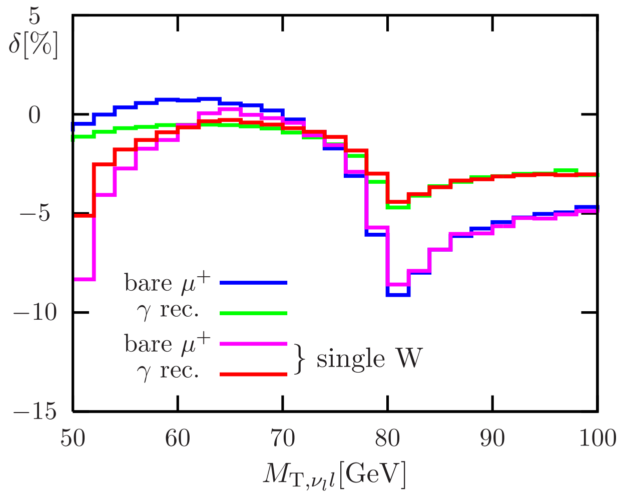


Jet recoil destroys simple factorization !

Single-W results from S.D./Krämer ’01; Brensing et al. ’07



Denner et al. ’09



EW corrections factorize from hard gluon emission near Jacobian peak !

Full $\mathcal{O}(\alpha\alpha_s)$ corrections versus naive factorization

NLO QCD and EW corrections:

$$\sigma^{\mathrm{NLO_s}} \equiv \sigma^{\mathrm{LO}} \underbrace{(1+\delta_{\alpha_s})}_{=K^{\mathrm{NLO}}_{\mathrm{QCD}}} = \sigma^0 + \sigma^{\mathrm{LO}} \underbrace{\left(\frac{\sigma^{\mathrm{LO}}-\sigma^0}{\sigma^{\mathrm{LO}}} + \delta_{\alpha_s}\right)}_{\equiv \delta'_{\alpha_s}},$$

$$\Delta\sigma^{\mathrm{NLO_{ew}}} = \sigma^0\,\delta_\alpha, \qquad \sigma^0 = \text{LO contribution with NLO PDFs}$$

$\mathcal{O}(\alpha\alpha_s)$-corrected cross section:

$$\sigma^{\mathrm{NNLO_{s\otimes ew}}} = \sigma^{\mathrm{NLO_s}} + \Delta\sigma^{\mathrm{NLO_{ew}}} + \underbrace{\Delta\sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{ini-fin}}}_{=\sigma^{\mathrm{LO}}\,\delta^{\mathrm{ini-fin}}_{\alpha_s\alpha}}$$

Naive factorization @ $\mathcal{O}(\alpha\alpha_s)$:

$$\sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{naive\,fact}} = \sigma^{\mathrm{NLO_s}}(1+\delta_\alpha) = \sigma^{\mathrm{LO}}(1+\delta_{\alpha_s})(1+\delta_\alpha)$$

$\Rightarrow$ Comparison of relative corrections:

$$\frac{\sigma^{\mathrm{NNLO_{s\otimes ew}}} - \sigma^{\mathrm{NNLO_{s\otimes ew}}}_{\mathrm{naive\,fact}}}{\sigma^{\mathrm{LO}}} = \delta^{\mathrm{ini-fin}}_{\alpha_s\alpha} - \delta'_{\alpha_s}\delta_\alpha$$

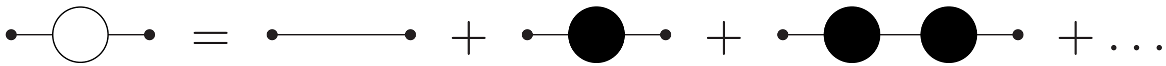
Unstable particles in QFT

Problem of unstable particles:

description of resonances requires resummation of propagator corrections

$\hookrightarrow$ mixing of perturbative orders potentially violates gauge invariance

Dyson series and propagator poles (scalar example)



$$G^{\phi\phi}(p) = \frac{\mathrm{i}}{p^2-m^2} + \frac{\mathrm{i}}{p^2-m^2}\,\mathrm{i}\Sigma(p^2)\,\frac{\mathrm{i}}{p^2-m^2} + \ldots = \frac{\mathrm{i}}{p^2-m^2+\Sigma(p^2)}$$

$\Sigma(p^2) =$ renormalized self-energy, $m =$ ren. mass

stable particle: $\mathrm{Im}\{\Sigma(p^2)\} = 0$ at $p^2 \sim m^2$

$\hookrightarrow$ propagator pole for real value of p^2,
renormalization condition for physical mass m: $\Sigma(m^2) = 0$

unstable particle: $\mathrm{Im}\{\Sigma(p^2)\} \neq 0$ at $p^2 \sim m^2$

$\hookrightarrow$ location μ^2 of propagator pole is complex,
possible definition of mass M and width Γ: $\mu^2 = M^2 - \mathrm{i}M\Gamma$

Commonly used mass/width definitions:

- "on-shell mass/width" $M_{\rm OS}/\Gamma_{\rm OS}$: $\quad M_{\rm OS}^2 - m^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \stackrel{!}{=} 0$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to M_{\rm OS}^2}{\widetilde{\quad}} \frac{1}{(p^2 - M_{\rm OS}^2)(1 + {\rm Re}\{\Sigma'(M_{\rm OS}^2)\}) + {\rm i}\,{\rm Im}\{\Sigma(p^2)\}}$$

comparison with form of Breit–Wigner resonance $\dfrac{R_{\rm OS}}{p^2 - m^2 + {\rm i}m\Gamma}$

yields: $\quad M_{\rm OS}\Gamma_{\rm OS} \equiv {\rm Im}\{\Sigma(M_{\rm OS}^2)\} \,/\, (1 + {\rm Re}\{\Sigma'(M_{\rm OS}^2)\}), \qquad \Sigma'(p^2) \equiv \dfrac{\partial\Sigma(p^2)}{\partial p^2}$

- "pole mass/width" M/Γ: $\quad \mu^2 - m^2 + \Sigma(\mu^2) \stackrel{!}{=} 0$

complex pole position: $\quad \mu^2 \equiv M^2 - {\rm i}M\Gamma$

$$\hookrightarrow \quad G^{\phi\phi}(p) \underset{p^2\to \mu^2}{\widetilde{\quad}} \frac{1}{(p^2 - \mu^2)[1 + \Sigma'(\mu^2)]} = \frac{R}{p^2 - M^2 + {\rm i}M\Gamma}$$

Note: $\quad \mu =$ gauge independent for any particle (pole location is property of S-matrix)

$M_{\rm OS} =$ gauge dependent at 2-loop order $\quad$ Sirlin '91; Stuart '91; Gambino, Grassi '99; Grassi, Kniehl, Sirlin '01

Relation between "on-shell" and "pole" definitions:

Subtraction of defining equations yields:

$$M_{\rm OS}^2 + {\rm Re}\{\Sigma(M_{\rm OS}^2)\} \;=\; M^2 - {\rm i}M\Gamma + \Sigma(M^2 - {\rm i}M\Gamma)$$

Equation can be uniquely solved via recursion in powers of coupling α:

ansatz: $M_{\rm OS}^2 = M^2 + c_1\alpha^1 + c_2\alpha^2 + \dots$

$M_{\rm OS}\Gamma_{\rm OS} = M\Gamma + d_2\alpha^2 + d_3\alpha^3 + \dots,$ $\quad c_i, d_i =$ real

counting in α: $\quad M_{\rm OS}, M = \mathcal{O}(\alpha^0), \quad \Gamma_{\rm OS}, \Gamma, \Sigma(p^2) = \mathcal{O}(\alpha^1)$

Result:

$$\begin{aligned} M_{\rm OS}^2 &= M^2 + {\rm Im}\{\Sigma(M^2)\}\,{\rm Im}\{\Sigma'(M^2)\} \;+\; \mathcal{O}(\alpha^3) \\ M_{\rm OS}\Gamma_{\rm OS} &= M\Gamma + {\rm Im}\{\Sigma(M^2)\}\,{\rm Im}\{\Sigma'(M^2)\}^2 \\ &\quad + \tfrac{1}{2}\,{\rm Im}\{\Sigma(M^2)\}^2\,{\rm Im}\{\Sigma''(M^2)\} \;+\; \mathcal{O}(\alpha^4) \end{aligned}$$

i.e. $\{M_{\rm OS}, \Gamma_{\rm OS}\} \;=\; \{M, \Gamma\} \;+$ gauge-dependent 2-loop corrections

Important examples: W and Z bosons

In good approximation: $\mathrm{W} \to f\bar{f}'$, $\mathrm{Z} \to f\bar{f}$ with masses fermions f, f'

so that: $$\mathrm{Im}\{\Sigma_{\mathrm{T}}^{\mathrm{V}}(p^2)\} = p^2 \times \frac{\Gamma_{\mathrm{V}}}{M_{\mathrm{V}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

$$\hookrightarrow \; M_{\mathrm{OS}}^2 = M^2 + \Gamma^2 + \mathcal{O}(\alpha^3) \qquad M_{\mathrm{OS}}\Gamma_{\mathrm{OS}} = M\Gamma + \frac{\Gamma^3}{M} + \mathcal{O}(\alpha^4)$$

In terms of measured numbers:

W boson: $M_{\mathrm{W}} \approx 80\,\mathrm{GeV}, \quad \Gamma_{\mathrm{W}} \approx 2.1\,\mathrm{GeV}$

$\hookrightarrow \; M_{\mathrm{W,OS}} - M_{\mathrm{W,pole}} \approx 28\,\mathrm{MeV}$

Z boson: $M_{\mathrm{Z}} \approx 91\,\mathrm{GeV}, \quad \Gamma_{\mathrm{Z}} \approx 2.5\,\mathrm{GeV}$

$\hookrightarrow \; M_{\mathrm{Z,OS}} - M_{\mathrm{Z,pole}} \approx 34\,\mathrm{MeV}$

Exp. accuracy: $\Delta M_{\mathrm{W,exp}} = 29\,\mathrm{MeV}, \quad \Delta M_{\mathrm{Z,exp}} = 2.1\,\mathrm{MeV}$

$\hookrightarrow$ Difference in definitions phenomenologically important !

Example of W and Z bosons continued:

Approximation of massless decay fermions:

$$\Gamma_{\mathrm{V,OS}}(p^2) \;=\; \Gamma_{\mathrm{V,OS}} \times \frac{p^2}{M^2_{\mathrm{V,OS}}}\,\theta(p^2), \qquad \mathrm{V} = \mathrm{W}, \mathrm{Z}$$

Fit of W/Z resonance shapes to experimental data:

- ansatz $\left|\frac{R'}{p^2 - m'^2 + \mathrm{i}\gamma' p^2/m'}\right|^2$ yields: $m' = M_{\mathrm{V,OS}}, \quad \gamma' = \Gamma_{\mathrm{V,OS}}$
- ansatz $\left|\frac{R}{p^2 - m^2 + \mathrm{i}\gamma m}\right|^2$ yields: $m = M_{\mathrm{V,pole}}, \quad \gamma = \Gamma_{\mathrm{V,pole}}$

Note: the two forms are equivalent:

$$R = \frac{R'}{1 + \mathrm{i}\gamma'/m'}, \quad m^2 = \frac{m'^2}{1 + \gamma'^2/m'^2}, \quad m\gamma = \frac{m'\gamma'}{1 + \gamma'^2/m'^2}$$

$\hookrightarrow$ consistent with relation between “on-shell” and “pole” definitions !