Theory Calculations of/for the W Boson Mass

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- 1. Introduction: Electroweak Precision Observables
- 2. Status and future of M_W in the SM
- **3**. M_W in the MSSM
- 4. What precision do we want?
- 5. Conclusions

Experimental situation:

LHC/ILC/FCC-ee/CEPC/... will provide (high!) accuracy measurements!

Theory situation:

- Measurements are performed using theory predictions
- measured observables have to be compared with theoretical predictions (in various models: SM, MSSM, ...)

Full uncertainty is given by the (linear) sum of experimental and theoretical uncertainties!

Theoretical uncertainties for electroweak and Higgs-boson precision measurements at the FCC-ee

Conveners: A. Freitas¹, S. Heinemeyer², Contributors: M. Beneke³, A. Blondel⁴, A. Hoang⁵, P. Janot⁶, J. Reuter⁷, C. Schwinn⁸, and S. Weinzierl⁹

 \Rightarrow Write-up as CDR contribution (only SM covered so far)

 \Rightarrow Many "theory numbers" are collider independent . . .

⇒ Here: SM numbers are based on this write-up additional MSSM numbers as show-case

Where we need theory prediction:

1. Prediction of the measured quantity Example: M_W

 \rightarrow at the same level or better as the experimental precision

2. Prediction of the measured process to extract the quantity Example: $pp \rightarrow W^+W^- \rightarrow 4f \Rightarrow$ see Stefan's talk \rightarrow better than then "pure" experimental precision

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Two types of theory uncertainties:

- 1. intrinsic: missing higher orders
- 2. parametric: uncertainty due to exp. uncertainty in SM input parameters Example: m_t , m_b , α_s , $\Delta \alpha_{had}$, ...

Combining uncertainties:

- 1. LHC/ILC/FCC-ee/CEPC/... (pure) experimental (anticipated) precision
- 2. Intrinsic uncertainties
- 3. Parametric uncertainties
 - \rightarrow taking into account the improved precision of SM paramters from LHC measurements

Total uncertainty:

total =
$$\sqrt{\text{experimental}^2 + \text{parametric}^2 + \text{intrinsic}}$$

Options for the evaluation of intrinsic uncertainties:

- 1. Determine all prefactors of a certain diagram class (couplings, group factors, multiplicities, mass ratios) and assume the loop is $\mathcal{O}(1)$
- 2. Take the known contribution at *n*-loop and (n-1)-loop and thus estimate the n + 1-loop contribution:

$$\frac{(n+1)(\text{estimated})}{n(\text{known})} \approx \frac{n(\text{known})}{(n-1)(\text{known})}$$

⇒ simplified example! Has to be done "coupling constant by coupling constant"

3. Variation of $\mu^{\overline{MS}}$ (QCD!, EW?)

- 4. Compare different renormalizations
- \Rightarrow Mostly used here: 1 & 2

Electroweak Precision Observables

Comparison of observables with theory:

Precision data:
$$M_W, \sin^2 \theta_{\rm eff}, a_{\mu}, M_h$$
Theory:
 ${\rm SM, MSSM}, \ldots$ \downarrow

Test of theory at quantum level: Sensitivity to loop corrections, e.g. \boldsymbol{X}



SM: limits on M_H , BSM: limits on M_X

Very high accuracy of measurements and theoretical predictions needed \Rightarrow only models "ready" so far: SM, MSSM

$$\begin{split} \mathbf{M}_{W} & \text{(best from } e^{+}e^{-} \text{ threshold scan)} \\ \sigma_{\mathsf{had}}^{0} &= \sum_{q} \sigma_{q}(M_{Z}^{2}), \\ \Gamma_{Z} &= \sum_{f} \Gamma[Z \to f\bar{f}], \quad (\text{from a fit to } \sigma_{f}(s) \text{ at various values of } s) \\ R_{\ell} &= \left[\sum_{q} \sigma_{q}(M_{Z}^{2})\right] / \sigma_{\ell}(M_{Z}^{2}), \quad (\ell = e, \mu, \tau) \\ R_{q} &= \sigma_{q}(M_{Z}^{2}) / \left[\sum_{q} \sigma_{q}(M_{Z}^{2})\right], \quad (q = b, c) \\ A_{\mathsf{FB}}^{f} &= \frac{\sigma_{f}(\theta < \frac{\pi}{2}) - \sigma_{f}(\theta > \frac{\pi}{2})}{\sigma_{f}(\theta < \frac{\pi}{2}) + \sigma_{f}(\theta > \frac{\pi}{2})} \equiv \frac{3}{4} \mathcal{A}_{e} \mathcal{A}_{f}, \\ A_{\mathsf{FB}}^{f} &= \frac{\sigma_{f}(P_{e} < 0) - \sigma_{f}(P_{e} > 0)}{\sigma_{f}(P_{e} < 0) + \sigma_{f}(P_{e} > 0)} \equiv \mathcal{A}_{e} |P_{e}| \\ \mathcal{A}_{f} &= 2 \frac{g_{V_{f}}/g_{A_{f}}}{1 + (g_{V_{f}}/g_{A_{f}})^{2}} = \frac{1 - 4|Q_{f}|\sin^{2}\theta_{\mathsf{eff}}^{f}}{1 - 4|Q_{f}|\sin^{2}\theta_{\mathsf{eff}}^{f}} + 8(|Q_{f}|\sin^{2}\theta_{\mathsf{eff}}^{f})^{2}} (f = \ell, b, \ldots) \end{split}$$

2. Status and future of M_W in the SM

Existing higher-order corrections to the EWPO [taken from A. Freitas '16]



Current intrinsic uncertainties:

Quantity	current experimental unc.	curr	ent intrinsic unc.
M_W [MeV]	15	4	$(\alpha^3, \alpha^2 \alpha_s)$
$\sin^2 heta_{ m eff}^\ell$ $[10^{-5}]$	16	4.5	$(\alpha^3, \alpha^2 \alpha_s)$
Γ_Z [MeV]	2.3	0.5	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s, \alpha \alpha_s^2)$
$R_b \ [10^{-5}]$	66	15	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s)$
R_l [10 ⁻³]	25	5	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s)$

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Current parametric uncertainties:

Quantity	$\delta m_t = 0.9 \text{ GeV}$	$\delta(\Delta \alpha_{\rm had}) = 10^{-4}$	$\delta M_Z = 2.1 \text{ MeV}$
$\delta M_W^{\sf para}$ [MeV]	5.5	2	2.5
$\delta \sin^2 \theta_{\rm eff}^{\ell, \rm para}$ [10 ⁻⁵]	3.0	3.6	1.4

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⇒ Current intrinsic/parametric uncertainties are substantially smaller than current experimental uncertainties :-)

- assume to go substantially beyond what is known now
- assume that many theorists will put many² hours of work into it in the next $\mathcal{O}(20)$ years (motivation?)
- do not assume that magically new calculational methods are invented
- are overall optimistic

 \Rightarrow they should be taken seriously!

Saying "Ah, theorists will have to work a bit harder and solve this" is not a realistic option!

Future intrinsic uncertainties:

Quantity	Current intrinsic unc.		Projected unc.
M_W [MeV]	4	$(\alpha^3, \alpha^2 \alpha_s)$	1
$\sin^2 \theta_{\rm eff}^{\ell}$ [10 ⁻⁵]	4.5	$(\alpha^3, \alpha^2 \alpha_s)$	1.5
Γ_Z [MeV]	0.5	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s, \alpha \alpha_s^2)$	0.2
$R_b \ [10^{-5}]$	15	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s)$	7
R_l [10 ⁻³]	5	$(\alpha_{bos}^2, \alpha^3, \alpha^2 \alpha_s)$	1.5

These calculations are required for the projection:

- complete $\mathcal{O}\left(\alpha\alpha_s^2\right)$ corrections
- fermionic $\mathcal{O}\left(\alpha^2 \alpha_s\right)$ corrections
- double-fermionic $\mathcal{O}\left(\alpha^{3}\right)$ corrections
- leading four-loop corrections enhanced by the top Yukawa coupling
- the $\mathcal{O}(\alpha_{bos}^2)$ corrections are not the leading uncertainties now

For these calculations, qualitatively new developments of existing loop integration techniques will be required, but no conceptual paradigm shift.

- 1. M_H : ~ 100 MeV \Rightarrow negligible
- 2. M_Z : ~ 2.1 MeV $\Leftarrow e^+e^-$ improvement important?!
- 3. $\alpha_s(M_Z)$: better than ~ 0.001 \Rightarrow negligible?? $\Leftarrow e^+e^-$ improvement important?!
- 4. m_t : maybe down to ~ 0.5 GeV at the LHC $\Leftarrow e^+e^-$ improvement important?!
- 5. m_b : from lattice calculations \Rightarrow negligible $\delta m_b \sim 10$ MeV (still under discussion, too optimistic?)
- 6. $\Delta \alpha_{had}$: BES III and Belle II: $\delta(\Delta \alpha_{had}) \sim 5 \times 10^{-5}$

 $\delta m_t = 0.5 \text{ GeV}, \ \delta(\Delta \alpha_{had}) = 5 \times 10^{-5}, \ \delta M_Z = 2.1 \text{ MeV}$

Quantity	δm_t	$\delta(\Delta \alpha_{\sf had})$	δM_Z	total parametric
$\delta M_W^{\sf para}$ [MeV]	3.0	1	2.5	4
$\delta \sin^2 \theta_{\text{eff}}^{\ell,\text{para}}$ [10 ⁻⁵]	1.7	1.8	1.4	2.8

 $total = \sqrt{experimental^2 + parametric^2} + intrinsic$

 \Rightarrow parametric uncertainties of the same order as experimental accuracy \Rightarrow not negligible

The above numbers have all been obtained assuming the SM as calculational framework.

The SM constitutes the model in which highest theoretical precision for the predictions of EWPO can be obtained.

We know that BSM physics must exist! (DM, gravity, ...)

As soon as BSM physics will be discovered, an evaluation of the EWPO in any preferred BSM model will be necessary.

The corresponding theory uncertainties, both intrinsic and parametric, can then be larger (as known for the MSSM).

A dedicated theory effort (beyond the SM) would be needed in this case.

3. M_W in the MSSM

Superpartners for Standard Model particles

Standard particles

SUSY particles



1.) New contributions from SUSY particles:



2.) CPV effects via new complex phases

3.) large Yukawa corrections: $\sim m_t^4 \log\left(\frac{m_{\tilde{t}_1}m_{\tilde{t}_2}}{m_t^2}\right)$

4.) large corrections from the b/\tilde{b} sector for large tan β

5.) non-decoupling SUSY effects: $\sim \log \frac{M_{\text{SUSY}}}{M_W}$

Corrections to M_W , $\sin^2 \theta_{\text{eff}} \rightarrow \text{approximation via the } \rho$ -parameter:

 ρ measures the relative strength between neutral current interaction and charged current interaction

$$\rho = \frac{1}{1 - \Delta \rho} \qquad \Delta \rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}$$

(leading, process independent terms)

 $\Delta \rho$ gives the main contribution to EW observables:



$$\Delta \rho^{\text{SUSY}}$$
 from \tilde{t}/\tilde{b} loops > 0 $\Rightarrow M_W^{\text{SUSY}} \gtrsim M_W^{\text{SM}}$

 $\Delta \rho^{\text{SUSY}}$ from \tilde{t}/\tilde{b} loops $> 0 \implies M_W^{\text{SUSY}} \gtrsim M_W^{\text{SM}}$

SM result for M_W :

- full one-loop
- full two-loop
- leading 3-loop via $\Delta\rho$
- leading 4-loop via $\Delta\rho$

Best MSSM result for M_W :

[S.H., W. Hollik, G. Weiglein, L. Zeune '13]

- full SM result (via fit formel)
- full MSSM one-loop (incl. complex phases)
- all existing two-loop $\Delta \rho$ contributions
- \Rightarrow non- $\Delta \rho$ one-loop and $\Delta \rho$ two-loop contributions sometimes non-negligible!

<u>Example</u>: Prediction for M_W in the SM and the MSSM : [S.H., W. Hollik, D. Stockinger, G. Weiglein, L. Zeune '13]



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MSSM band: scan over SUSY masses

overlap: SM is MSSM-like MSSM is SM-like

SM band: variation of M_H^{SM}

4. Which M_W precision do we want?



 \Rightarrow ILC precision becomes interesting . . .

\Rightarrow more interesting: see effects of new physics!

Example MSSM scenario (I):

[S.H., G. Weiglein, L. Zeune '13]

\Rightarrow extensive parameter scan:

Parameter	Minimum	Maximum
μ	-2000	2000
$M_{\tilde{E}_{1,2,3}} = M_{\tilde{L}_{1,2,3}}$	100	2000
$M_{\tilde{Q}_{1,2}} = M_{\tilde{U}_{1,2}} = M_{\tilde{D}_{1,2}}$	500	2000
$M_{ ilde{Q}_3}$	100	2000
$M_{ ilde{U}_{3}}$	100	2000
$M_{ ilde{D}_3}$	100	2000
$A_e = A_\mu = A_\tau$	-3 $M_{ ilde{E}}$	З $M_{ ilde{E}}$
$A_u = A_d = A_c = A_s$	-3 $M_{ ilde{Q}_{12}}$	З $M_{ ilde{Q}_{12}}$
A_b	-3 max $(M_{ ilde{Q}_3}, M_{ ilde{D}_3})$	$\operatorname{3max}(M_{ ilde Q_3},M_{ ilde D_3})$
A_t	-3 max $(M_{ ilde{Q}_3}, M_{ ilde{U}_3})$	$\operatorname{3max}(M_{ ilde Q_3},M_{ ilde U_3})$
aneta	1	60
M_3	500	2000
M_A	90	1000
M_2	100	1000

Example MSSM scenario (I): effects of EW particles



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Example MSSM scenario (I): effects of EW particles



Effects of stops:



All points HiggsBounds allowed

Effects of stops:



 $\ldots \oplus \, m_{{\widetilde q}_{1,2}}, m_{{\widetilde g}} >$ 1200 GeV





 $\ldots \oplus m_{\tilde{l}}, m_{\tilde{\chi}_i^\pm}, m_{\tilde{\chi}_j^0} > 500 \,\, {
m GeV}$

Example MSSM scenario (II):

[S.H., G. Weiglein, L. Zeune '15]

 $m_{\tilde{t}_1} = 400 \pm 40$ GeV, Other masses $\gtrsim 500$ GeV $M_W^{exp} = 80.375 \pm 0.005$ GeV, 80.385 ± 0.005 GeV, 80.395 ± 0.005 GeV



\Rightarrow precision below 5 MeV required, the better the better!

Example MSSM cenario (II):

[S.H., G. Weiglein, L. Zeune '15]

 $m_{\tilde{t}_1} = 400 \pm 40$ GeV, Other masses $\gtrsim 500$ GeV $M_W^{exp} = 80.375 \pm 0.005$ GeV, 80.385 ± 0.005 GeV, 80.395 ± 0.005 GeV



\Rightarrow precision below 5 MeV required, the better the better!

5. Conclusions

- High anticipated experimental precision for M_W at the LHC
- Important: theory uncertainties: intrinsic and parametric

$$total = \sqrt{experimental^2 + parametric^2} + intrinsic$$

- Realistic/optimistic future estimates:
 - future intrinsic uncertainties: $\sim 1~\text{MeV}$
 - − future parametric uncertainties: ~ 4 MeV ⇒ improvement only with e^+e^- colliders?
- MSSM calculation: large effects possible also from EW particles!
- Desired precision? O(5 MeV) total unc. interesting!

Further Questions?

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\Rightarrow improvement in α_s crucial

 e^+e^- collider: precision measurement:

$$R_l := \frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to l^+ l^-)}$$

Improvement down to $\delta^{\exp} \alpha_s \sim 0.001 - 0.0001$ possible?!

Note: TH uncertainty (assuming fermionic 3-loop corrections): $\delta R_l^{\text{theo}} \sim 0.0015 \Rightarrow \delta \alpha_s^{\text{theo}} \sim 0.00015 \Rightarrow hard to beat \dots$ SM input: $\Delta \alpha_{had} \Rightarrow$ could be limiting factor!

From $e^+e^- \rightarrow$ had. using dispersion relation

today: $\delta(\Delta \alpha_{had}) \sim 10^{-4}$ possible improvement in the future: $\delta(\Delta \alpha_{had}) \sim 5 \times 10^{-5}$

Direct determination at FCC-ee from $e^+e^- \rightarrow f\bar{f}$ off the Z peak [P. Janot '15] possible improvement in the future: $\delta(\Delta \alpha_{had}) \sim 2 \times 10^{-5} \Rightarrow TU$ neglected

Calculation of $e^+e^- \rightarrow f\bar{f}$ needed at 3-loop and beyond: [A. Freitas '16] current techniques (2L/3L): corrections of ~ 10⁻³ new calculation methods (2L/3L): corrections of ~ 10⁻⁴ unknown methods 3L: $\leq 10^{-5}$ unknown methods 4L: ~ 10⁻⁵ (+ higher-orders in real photon emission) \Rightarrow improvement unclear $\Rightarrow \delta(\Delta \alpha_{had}) \sim 3 \times 10^{-5}$ Additional uncertainty for M_W from threshold scan:

Not only $e^+e^- \rightarrow W^{(*)}W^{(*)}$, but $e^+e^- \rightarrow WW \rightarrow 4f$ needed

Current status:

full one-loop for $2 \rightarrow 4$ process

[A. Denner, S. Dittmaier, M. Roth, D. Wackeroth '99-'02]

 \Rightarrow extraction of M_W at the level of $\sim 6 \text{ MeV}$

Most recent improvement:

leading 2L corrections from EFT

[Actis, Beneke, Falgari, Schwinn '08]

 \Rightarrow impact on M_W at the level of $\sim 3 \text{ MeV}$

 \Rightarrow full 2L for 2 \rightarrow 4 process not foreseeable

Potentially possible:

2L resummed higher-order terms for $e^+e^- \rightarrow WW$ and $W \rightarrow ff'$

 \Rightarrow extraction of M_W at $\sim 1 \text{ MeV}$? \oplus pure exp. uncertainty of $\sim 0.5 \text{ MeV}$

Overview about all EWPO:



Surprisingly good agreement: χ^2 /d.o.f. = 18.1/14 (p = 20%)

Most quantities measured with 1%–0.1% precision

GFitter coll. '14

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Most quantities measured with 1%–0.1% precision

A few interesting deviations:

M_{W}	$(\sim 1.4\sigma)$
σ_{had}^{0}	$(\sim 1.5\sigma)$
$A_{\ell}(SLD)$	$(\sim 2\sigma)$
A^b_{FB}	$(\sim 2.5\sigma)$
$(g_{\mu} - 2)$	$(\sim 3\sigma)$

GFitter coll. '14

Current fit to M_H :

[GFitter '14]



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Precise M_H test with the ILC precision: [*GFitter '13*] [*LEPEWWG '13*]



 $\Rightarrow \delta M_H^{\text{ind}} \lesssim 6 \text{ GeV}$ ← only ILC analysis available so far \Rightarrow extremely sensitive test of SM (and BSM) possible