Tutorial 1

OMA School, Pavia,

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1 Acceleration

1. Why does not a magnetic field contribute to the longitudinal acceleration of a charged particle with initial speed v?
2. If we assume a electric field and the acceleration to be along the z direction, calculate the kinetic energy gained from the field.

2 Local radius, rigidity

We wish to design an electron ring with a radius of 200 m. Let us assume that only 50% of the circumference is occupied by bending magnets:

- What will be the local radius of bend ρ in these magnets if they all have the same strength?
- If the momentum of the electrons is 12 GeV/c, calculate the rigidity $B\rho$ and the field in the dipoles.

3 Phase space

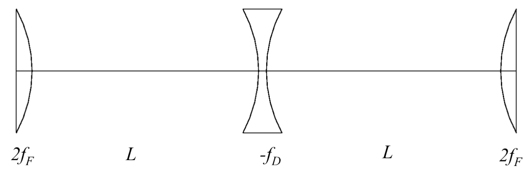
1. Sketch the emittance ellipse of a particle beam in horizontal x-x' phase space (I) at the position of a transverse waist, (II) when the beam is divergent and (III) when the beam is convergent.
2. Phase Space Representation of a Particle Source:
   - Consider a source at position s_0 with radius w emitting particles. Make a drawing of this setup in configuration space and in phase space. Which part of phase space can be occupied by the emitted particles?
   - Any real beam emerging from a source like the one above will be clipped by aperture limitations of the vacuum chamber. This can be modeled by assuming that a distance d away from the source there is an iris with an opening with radius $R = w$. Make a drawing of this setup in configuration and phase space. Which part of phase space is occupied by the beam at a location after the iris?

4 FODO lattice

A quadrupole doublet consists of two lenses of focal length f_1 and f_2 separated by a drift length L. Assume that the lenses are thin and show that the transport matrix of this system is

$$M = \begin{pmatrix} 1 - L/f_1 & L \\ -1/f^* & 1 - L/f_2 \end{pmatrix} \text{ where } \frac{1}{f^*} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

A FODO cell can be considered as the simplest block of the magnetic structure of modern accelerators and storage rings. It consists of a magnet structure of focusing (F) and defocusing (D) quadrupole lenses in alternating order (see schematic below). Its transfer matrix can be calculated using the matrix of the quadrupole doublet (above) with $f_1 = +2f$ and $f_2 = -2f$ followed (and multiplied) by another quadrupole doublet matrix with $f_1 = -2f$ and $f_2 = +2f$.



Show that the transfer matrix of a FODO system in thin lens approximation is as follows:

$$M_{FODO} = \begin{pmatrix} 1 - \frac{L^2}{2f^2} & 2L + \frac{L^2}{f} \\ -\frac{L}{2f^2}(1 - \frac{L}{2f}) & 1 - \frac{L^2}{2f^2} \end{pmatrix}$$

5 Cyclotron

Let us to consider a cyclotron with a magnetic field strength of 1 T. If a proton beam is injected:

- What is the maximum velocity of the protons for a maximum radius of 50 cm?
- What is the corresponding maximum kinetic energy?
- If the maximum voltage across the gap is 50 kV, how many revolutions does a proton make before it reaches its maximum energy?
- How much time does a proton spend in this accelerator?

6 OTR

1. Optical Transition Radiation (OTR) is a workhorse for diagnosing a beam's spatial and angular properties. OTR is generated as the beam intercepts a metallic foil. Imaging the beam with OTR provides a high resolution measurement of the spatial distribution of the beam; imaging the beam on two screens separated in a free drift region allows a measure of the beam's rms divergence. Under the proper conditions OTR can thus be used to measure the beam's rms emittance.

   - Assume that the beam is axially symmetric and has equal rms horizontal (x) and vertical (y) divergences. Show how the divergence can be measured by writing a simple equation for divergence in terms of the rms size of the beam as it travels in a free drift space between two screens separated by a distance L. Provide a diagram to illustrate your calculations.

2. The angular distribution of OTR produced from a single foil can also be used to measure the beam divergence. This is due to the fact that the angular distribution of OTR for a single charged particle in the beam has the form:

   $$I(\theta) \sim \frac{\theta^2}{(\gamma^{-2} + \theta^2)^2}$$

   where $I(\theta)$ is the intensity of the radiation, θ is the angle measured between the velocity vector of the particle and the direction of observation and γ is the Lorentz factor of the beam. In the absence of beam divergence this pattern has a null in the center. Beam divergence increases the observed minimum value and widens the distribution.

   - Can you qualitatively explain why divergence has this effect?
   - How would you go about computing the effect of divergence if e.g. the beam particles had a Gaussian distribution of trajectory angles?

3. Practically speaking, single foil OTR AD pattern can be used to extract the beam divergence down to a value of about 10% of the angle of peak intensity. Once the rms (x, y) beam sizes and divergences are known, the rms (x, y) geometrical emittances can be calculated; e.g., for x, the x rms geometric emittance is given by

   $$\epsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle}$$

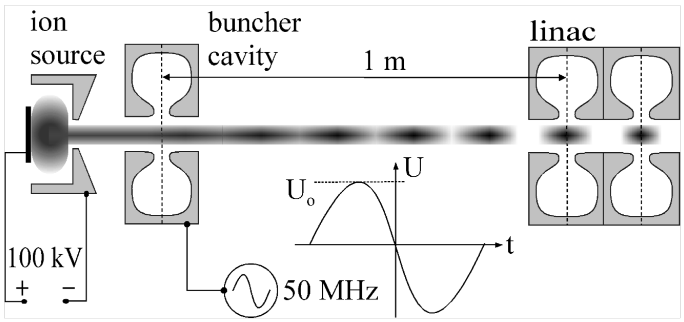
   If the beam has an energy $E = 100$ MeV:

   - What is minimum value of the rms x divergence that can be measured with single foil OTR?
   - If the beam is focussed to an x waist condition and a value of rms $x = 0.1$ mm, what is the corresponding normalized rms x emittance of the beam?

Bonus!

7 Ion source and bunching

Assume a continuous proton beam from a 100 kV ion source. A buncher cavity operating at a frequency of 50 MHz modulates the energy of the protons. The corresponding modulation of velocities leads to different times of flight to a location further downstream, and thus to a formation of beam bunches from the initially continuous current. This process is called velocity bunching or ballistic bunching. At a distance $L = 1$ m after the buncher the bunching should be optimum in order to inject the protons into a linac there.



Question: What's the required voltage amplitude U_0 of the buncher cavity?
Hints:

- What's the proton beam velocity?
- Is it allowed to do a non-relativistic calculation?
- Assume that the buncher is "short", i.e. any variation of fields inside the buncher may be neglected.
- Calculate time of flight for a proton at time δt with respect to a reference particle.
- Introduce approximations:

  a) linearize the time dependance ($\sin \omega t$) of the electric field in the buncher.

  b) Assume the energy change due to the buncher is small compared to the initial energy.