

# Top Mass at Threshold

## - Update on Recent Studies -

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CERN

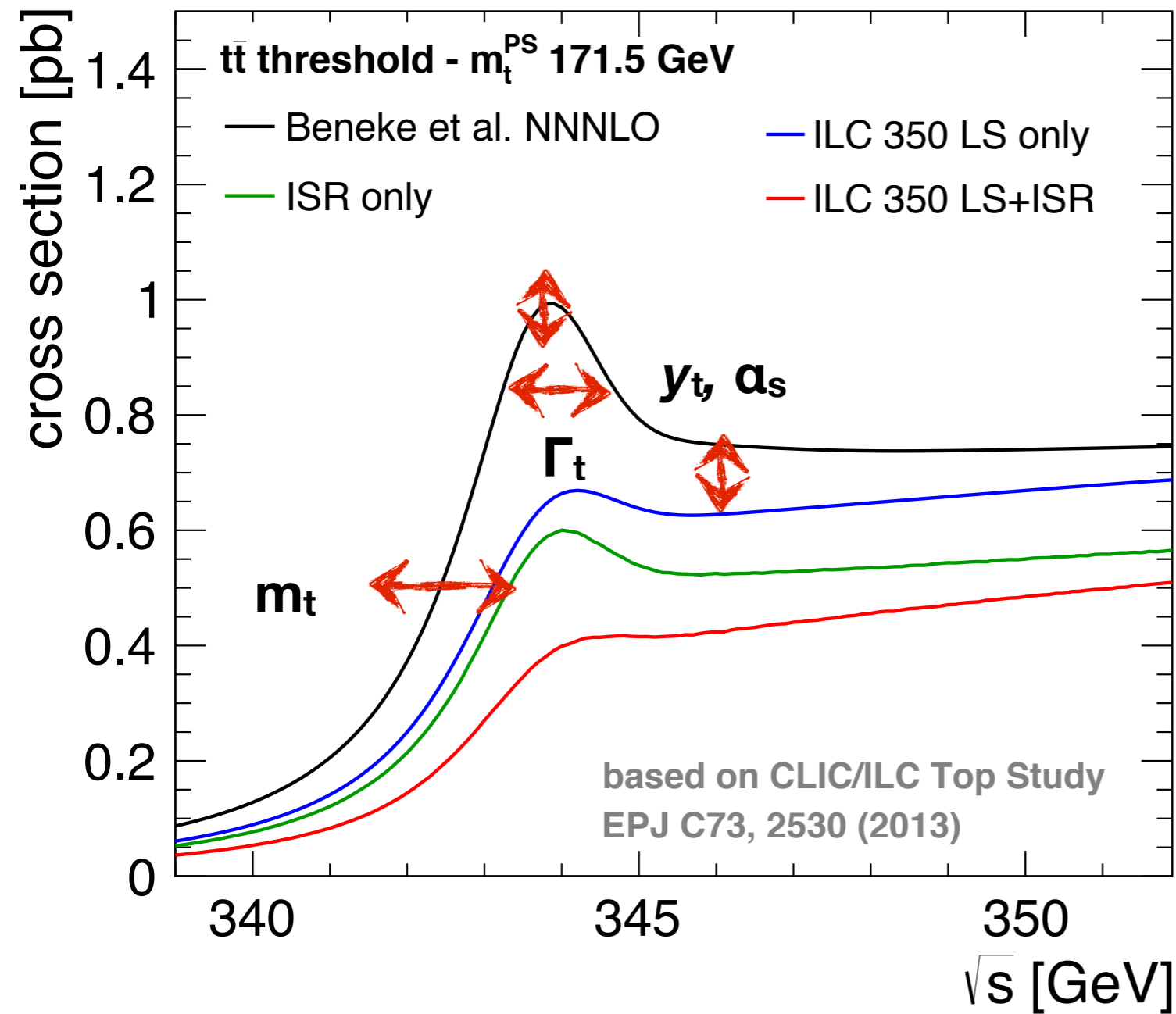


# Outline

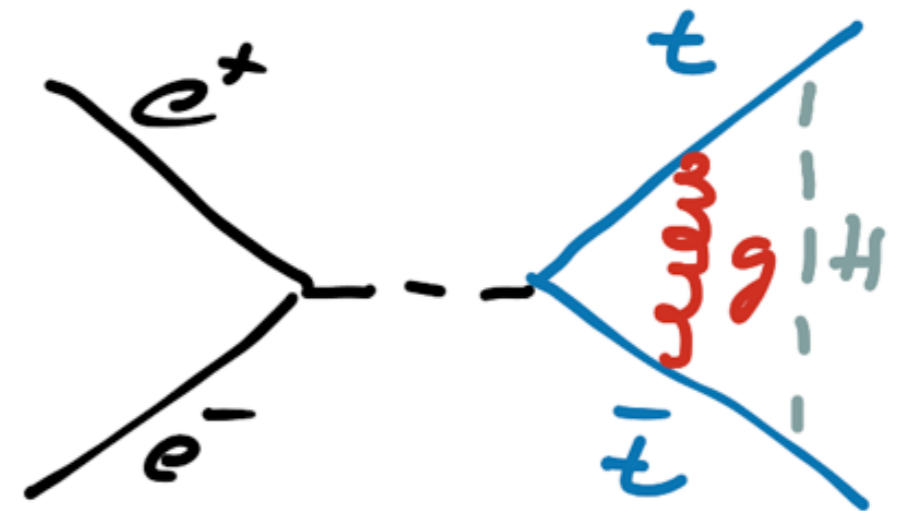
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- Motivation & Systematics Status
- Exploring multi-parameter extractions:
  - Mass & Width
  - Mass & Yukawa coupling
- Comparison of different  $e^+e^-$  colliders
- The effect of the choice of mass scheme
- Summary

# Threshold Scans: The Motivation

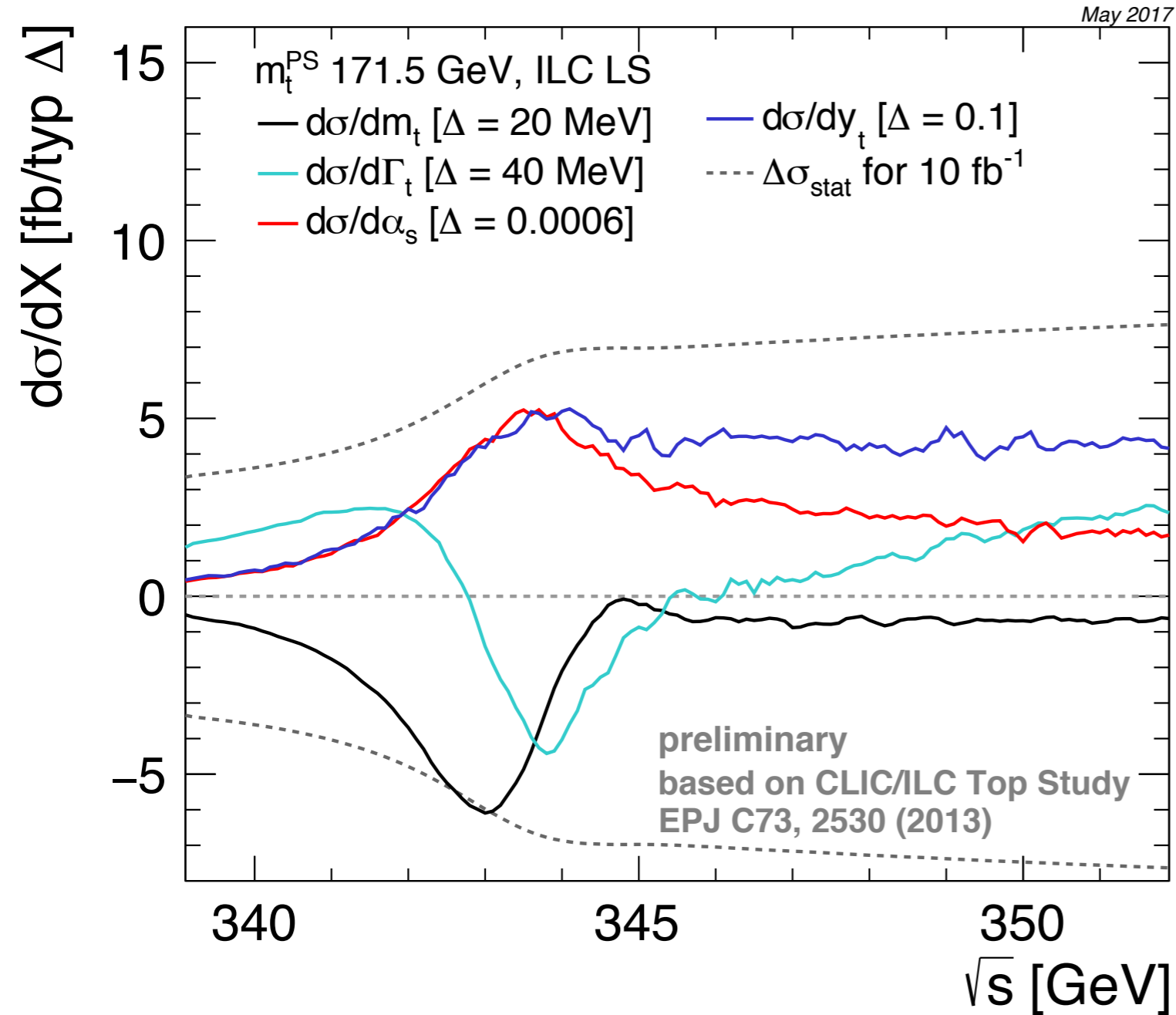


- The cross-section around the threshold is affected by several properties of the top quark and by QCD
  - Top mass, width, Yukawa coupling
  - Strong coupling constant

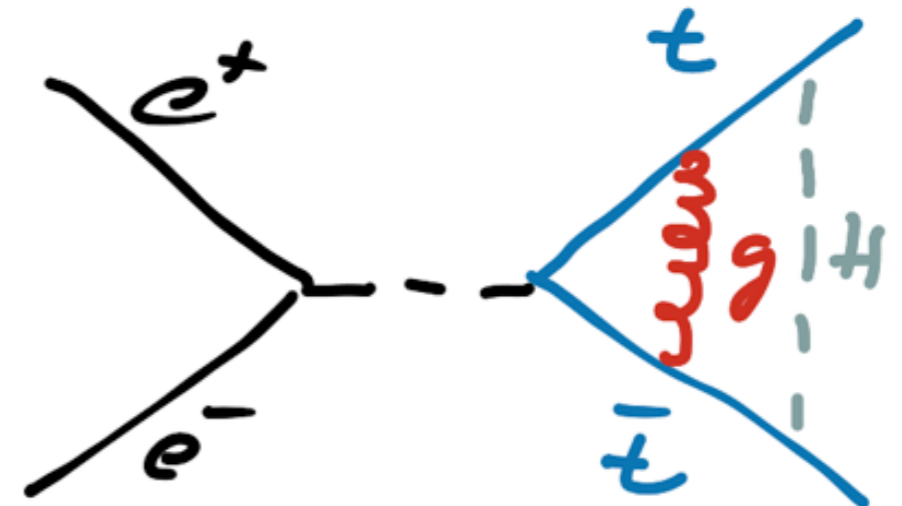


- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external  $\alpha_s$  helps

# Threshold Scans: The Motivation



- The cross-section around the threshold is affected by several properties of the top quark and by QCD
  - Top mass, width, Yukawa coupling
  - Strong coupling constant



- Effects of some parameters are correlated; dependence on Yukawa coupling rather weak - precise external  $\alpha_s$  helps



# Top Mass Uncertainties - Status

- A number of studies in Tesla, ILC, CLIC contexts: Expected statistical uncertainty **20 - 30 MeV** (for  $100 \text{ fb}^{-1}$ )

error source	$\Delta m_t^{\text{PS}}$ [MeV]
stat. error ( $200 \text{ fb}^{-1}$ )	13
theory (NNNLO scale variations, PS scheme)	40
parametric ( $\alpha_s$ , current WA)	35
non-resonant contributions (such as single top)	$< 40$
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	$< 10$
beam energy uncertainty	$< 17$
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

- Summary of status for ILC “New Particles” Report arXiv:1702.05333

# The Basics for all Studies presented today

- Experimental details:
  - Based on CLIC / ILC top threshold study ( EPJ C73, 2530 (2013) ):
    - CLIC\_ILD Detector model
    - Threshold simulated using efficiency & backgrounds from full simulations, signal scaled according to theory input
    - Assuming ILC TDR luminosity spectrum
- Theory input:
  - NNNLO QCD Theory calculations, using QQbar\_threshold (arXiv:1605.03010)
    - M. Beneke, Y. Kiyo, P. Marquard, A. Penin, J. Piclum, M. Steinhauser, Phys. Rev. Lett. 115, 192001 (2015)
    - ▶ Including NNNLO Higgs effects, NLO non-resonant EW contributions, NLO QED
      - M. Beneke, A. Maier, J. Piclum, T. Rauh, Nucl. Phys. B899, 180 (2015)
  - Using the ***PS Mass Scheme*** as the “native” scheme of the calculation, also using MSbar and 1S schemes to explore scheme dependence

Thanks to Martin Beneke, Andreas Meyer, Jan Piclum for help and fruitful discussions!

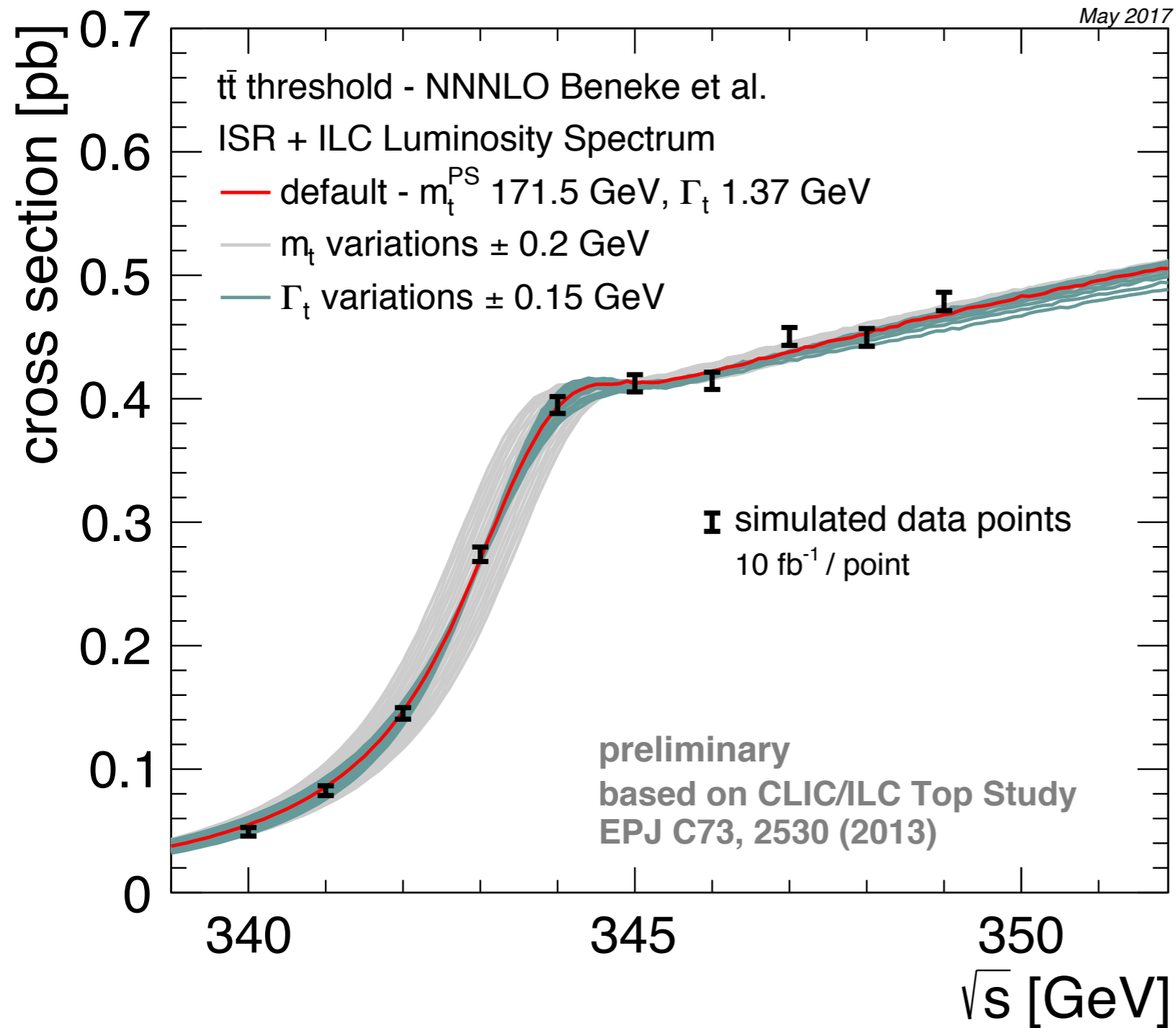


# Multi-Parameter Fits @ ILC

- Mass & Width
- Mass & Yukawa Coupling

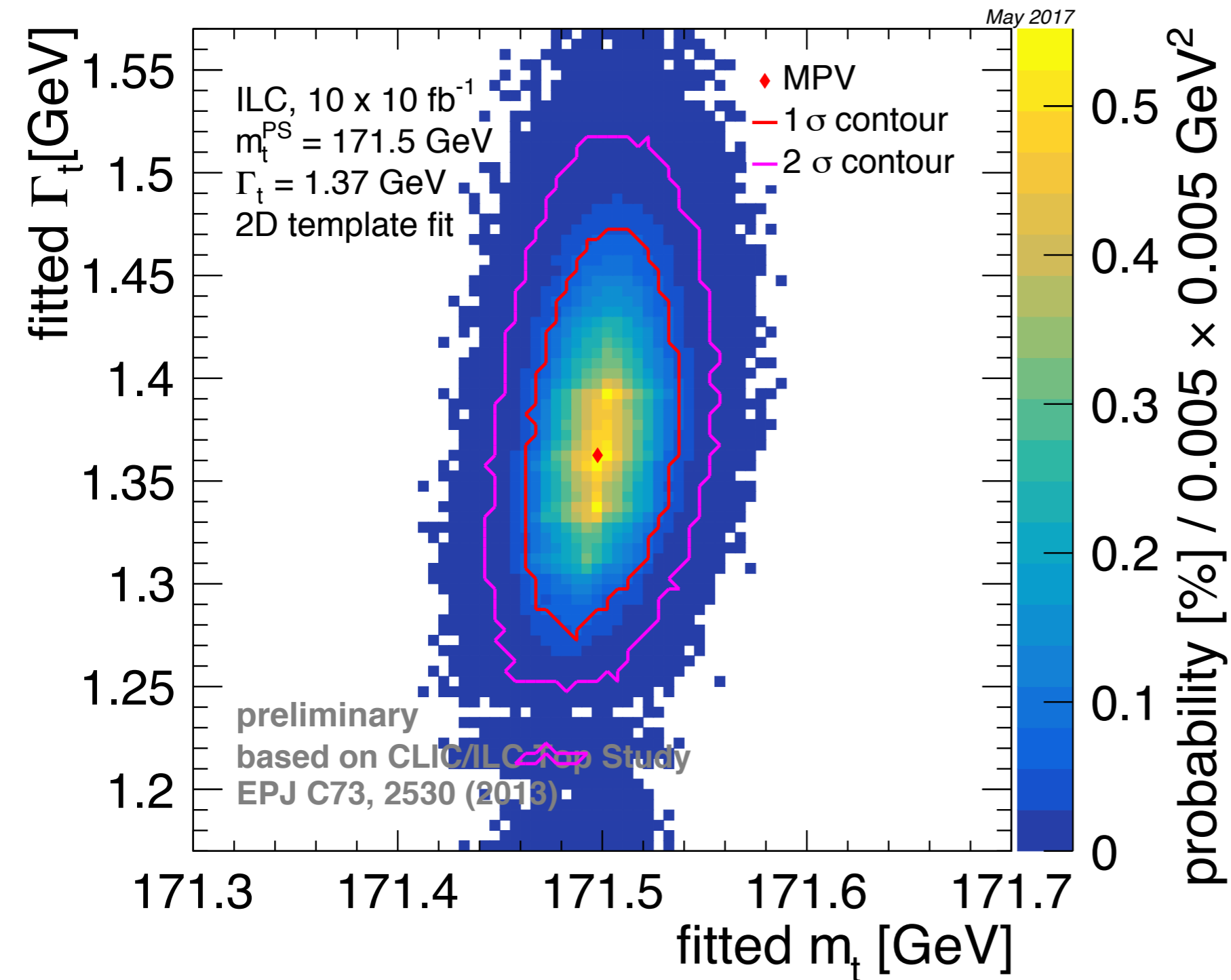
Note: Hot of the press - but fully QA'ed yet...

# Mass & Width - Illustrating the Sensitivity



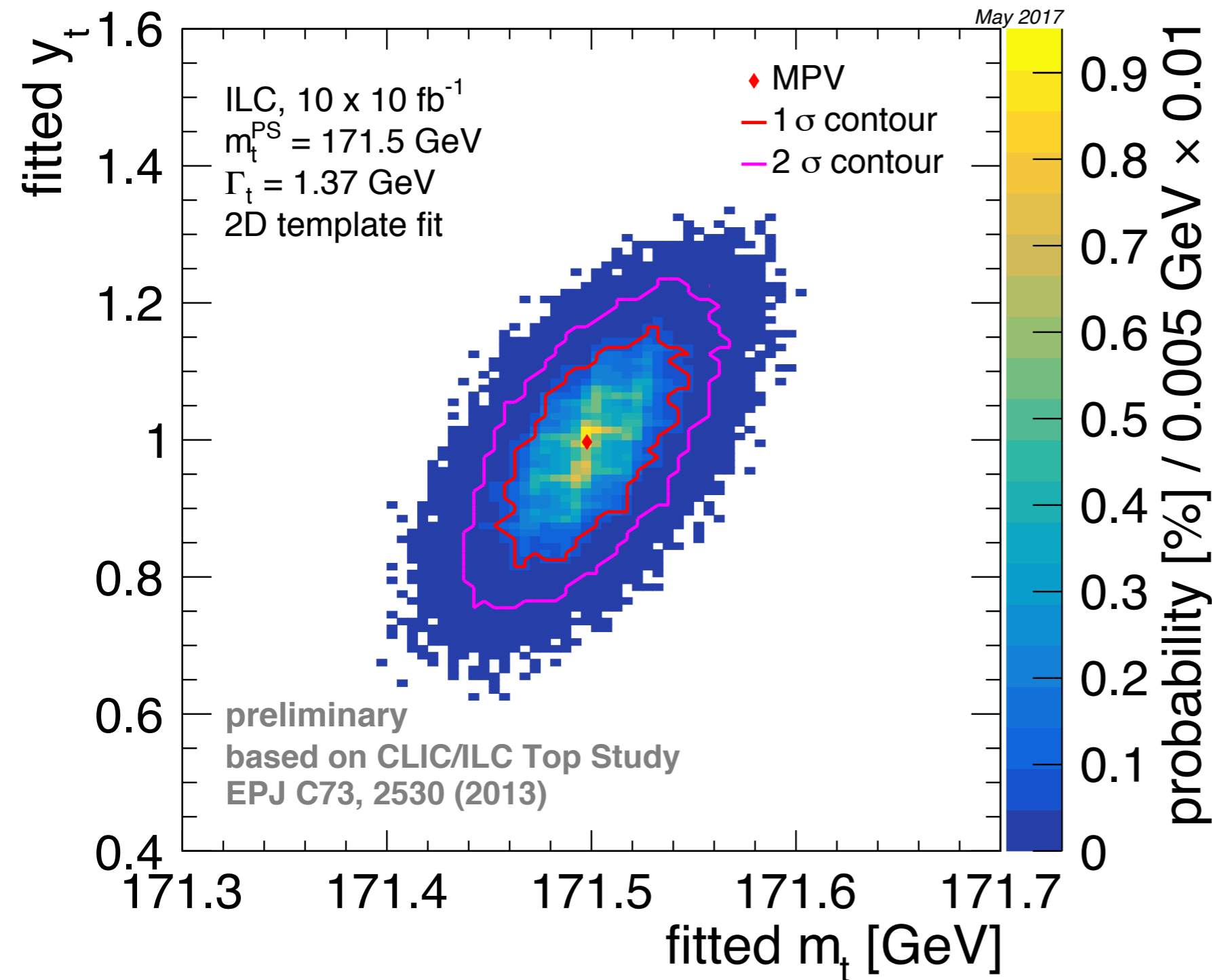


# Mass & Width: 2D Template Fit



- 1D mass resolution (assuming def.  $\Gamma_t$ )  
**18 MeV**
- 1D width resolution (assuming def.  $m_t$ )  
**43 MeV**
- Extension of 2D  $1 \sigma$  contour:
  - $m_t +39 -35 \text{ MeV}$
  - $\Gamma_t +109 - 90 \text{ MeV}$
  - correlation 0.26

# Mass & Yukawa Coupling: 2D Fit



- 1D mass resolution (assuming def.  $y_t$ )  
**18 MeV**
- 1D width resolution (assuming def.  $m_t$ )  
**0.067**
- Extension of 2D  $1 \sigma$  contour:  
 **$m_t +49 -45 \text{ MeV}$**   
 **$y_t +0.17 -0.18 \text{ MeV}$** 
  - correlation 0.61

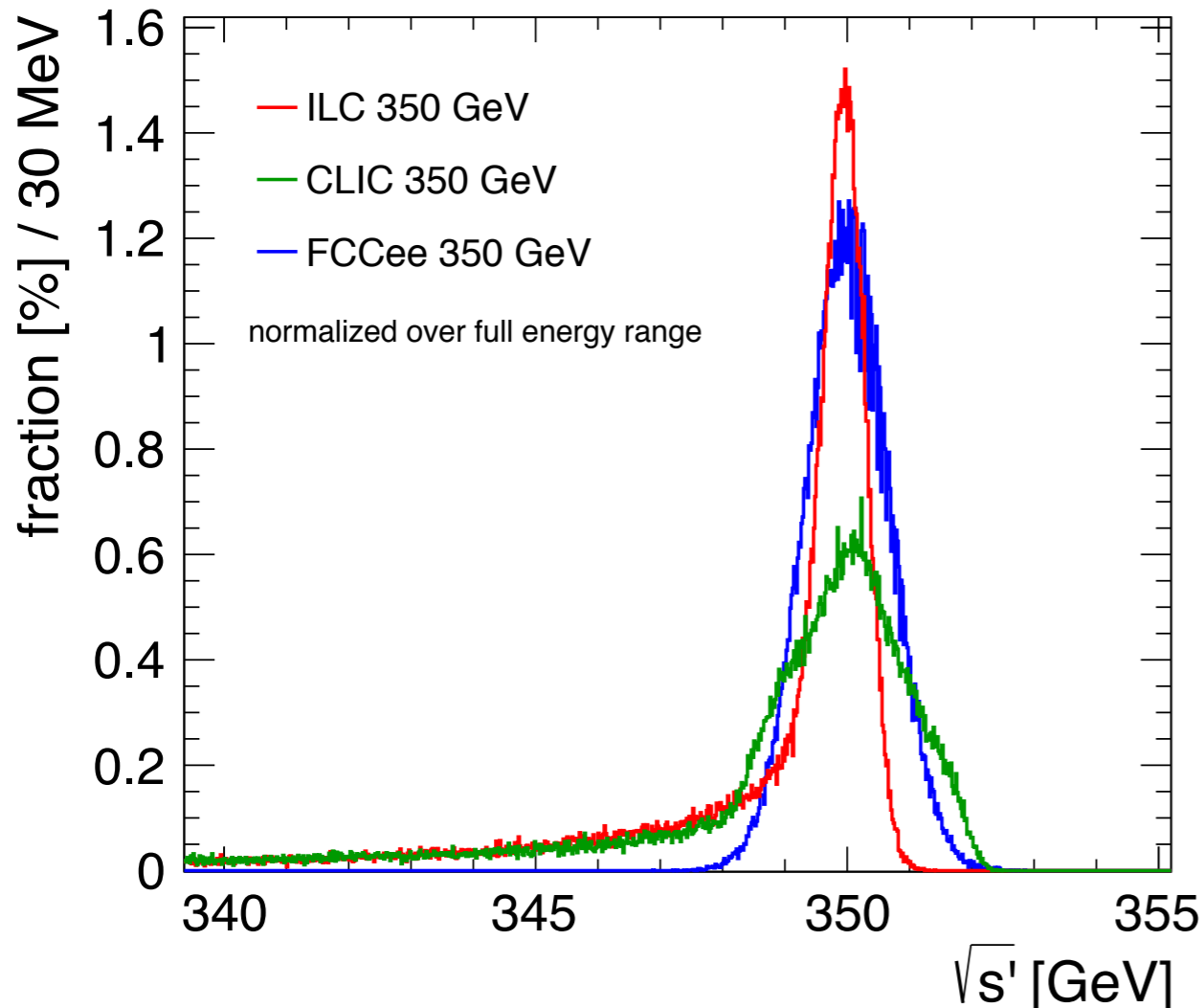
# Multi-Parameter Fits: The Impact of the Lumi Spectrum

- Mass & Width at
  - ILC
  - CLIC
  - FCCee

Yukawa coupling coming soon ...

# The Top Threshold at different $e^+e^-$ Colliders

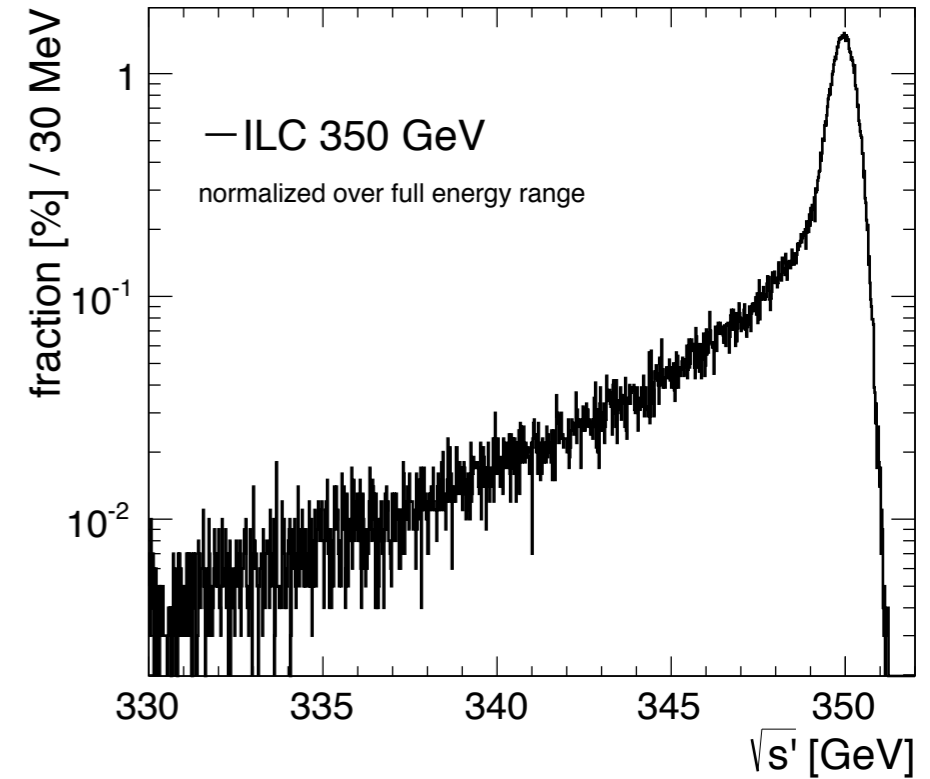
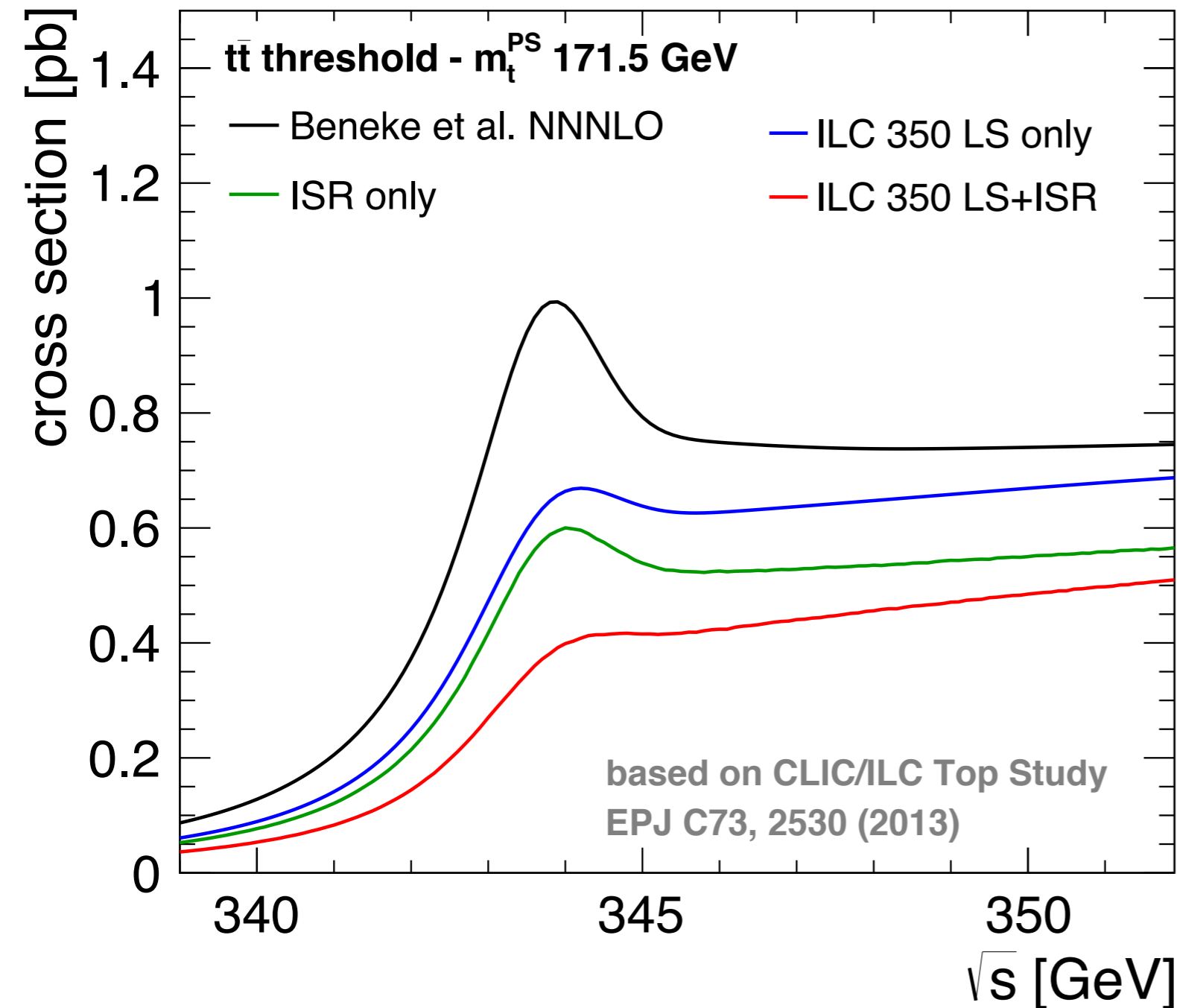
- Three  $e^+e^-$  colliders currently in discussion are capable or reaching the top threshold: ILC, CLIC, FCCee
  - In terms of a threshold scan, the most relevant difference is the lumi spectrum
    - NB: Assumptions on integrated luminosity are also very different - ignored here: Always assume  $100 \text{ fb}^{-1}$  spread over 10 points



- ILC and CLIC luminosity spectra from full machine simulations
- FCCee assumed to be gaussian, with a spread of 0.19% (realistically, there probably is a small beamstrahlungs-tail which would make the distribution asymmetric, but the spectrum will be closer to a gaussian than the LCs in any case)

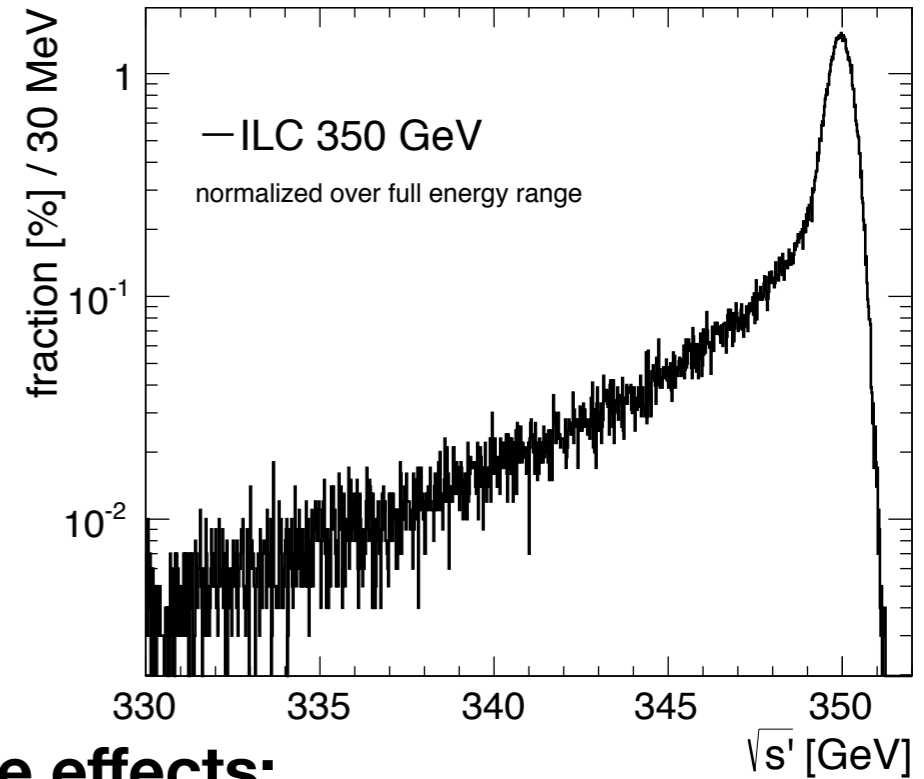
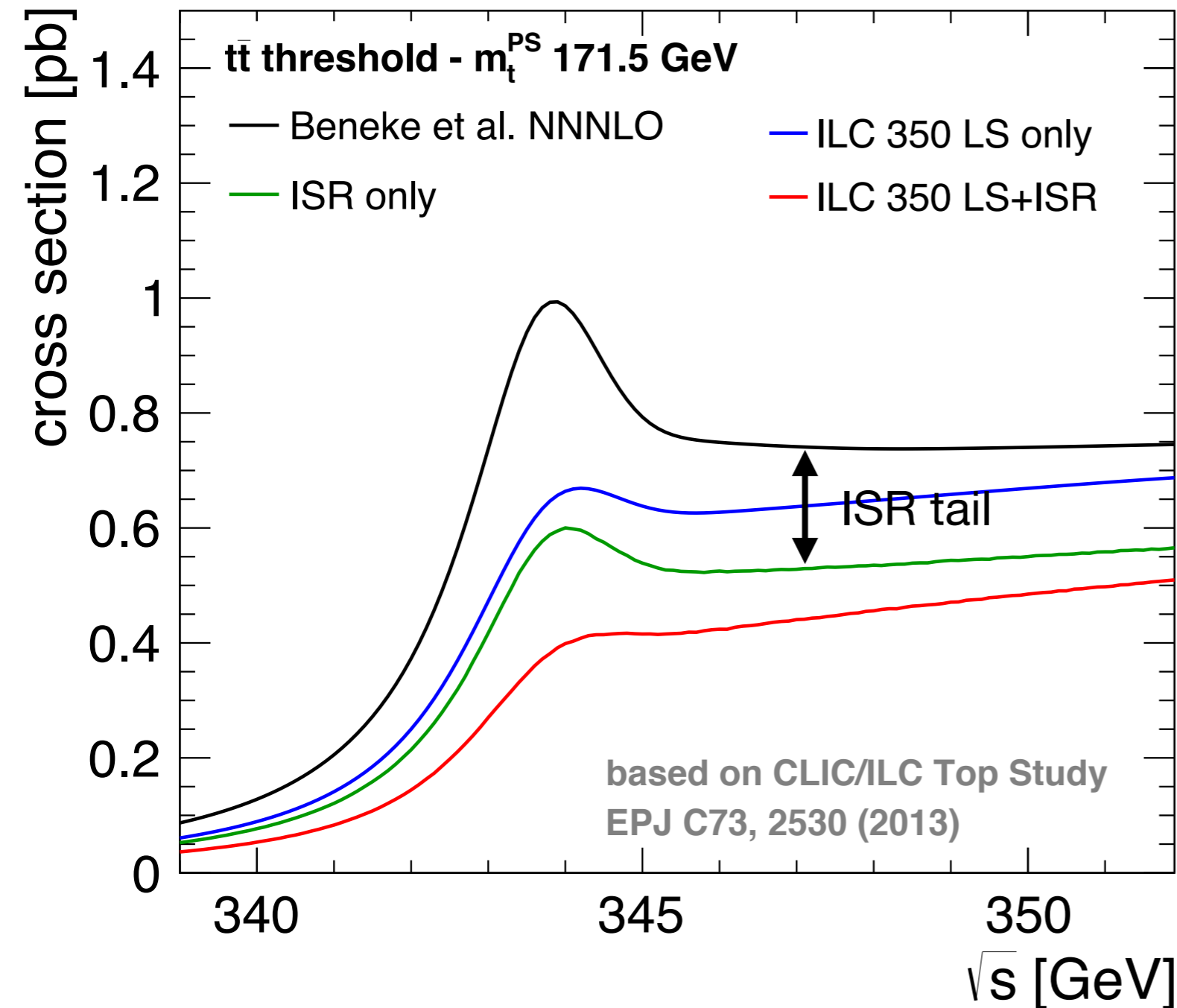
# The Top Threshold at different $e^+e^-$ Colliders

- What the luminosity spectrum does to the threshold:



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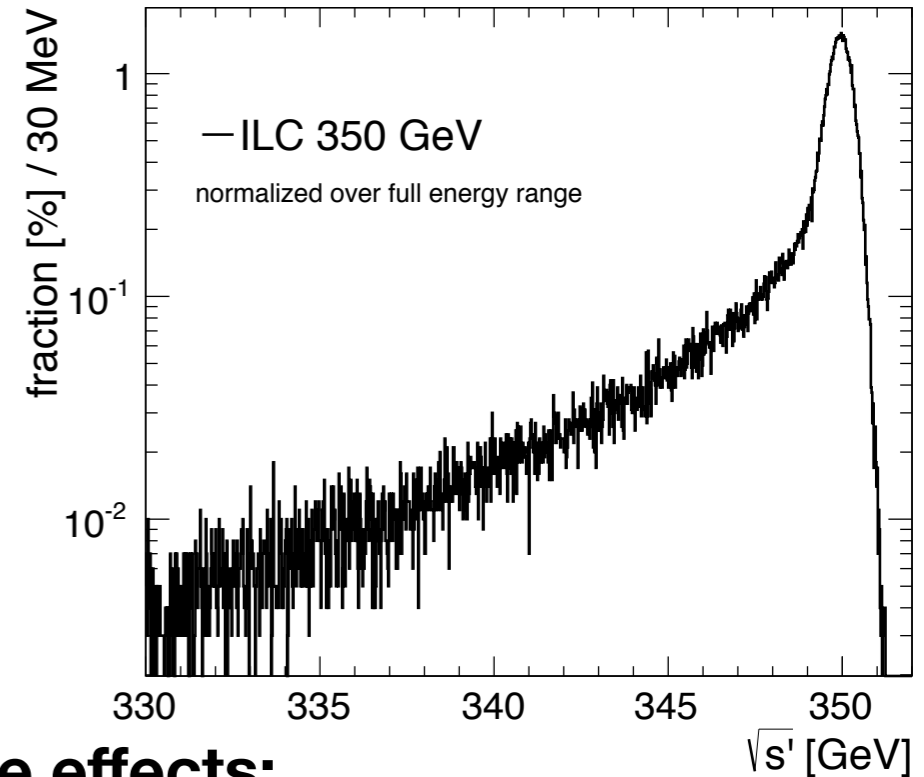
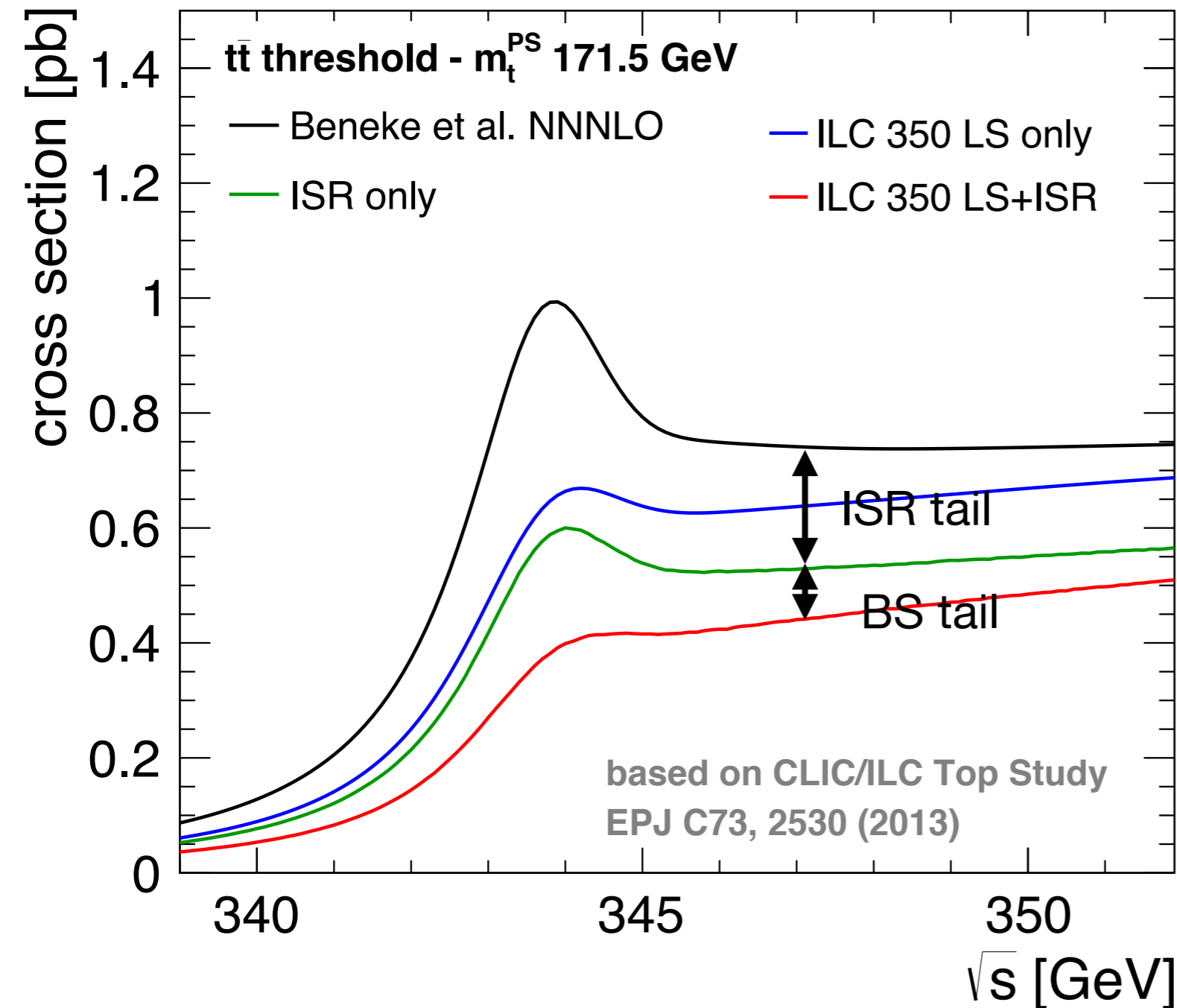


## The effects:

- ISR tail: lowering of effective L at top energy

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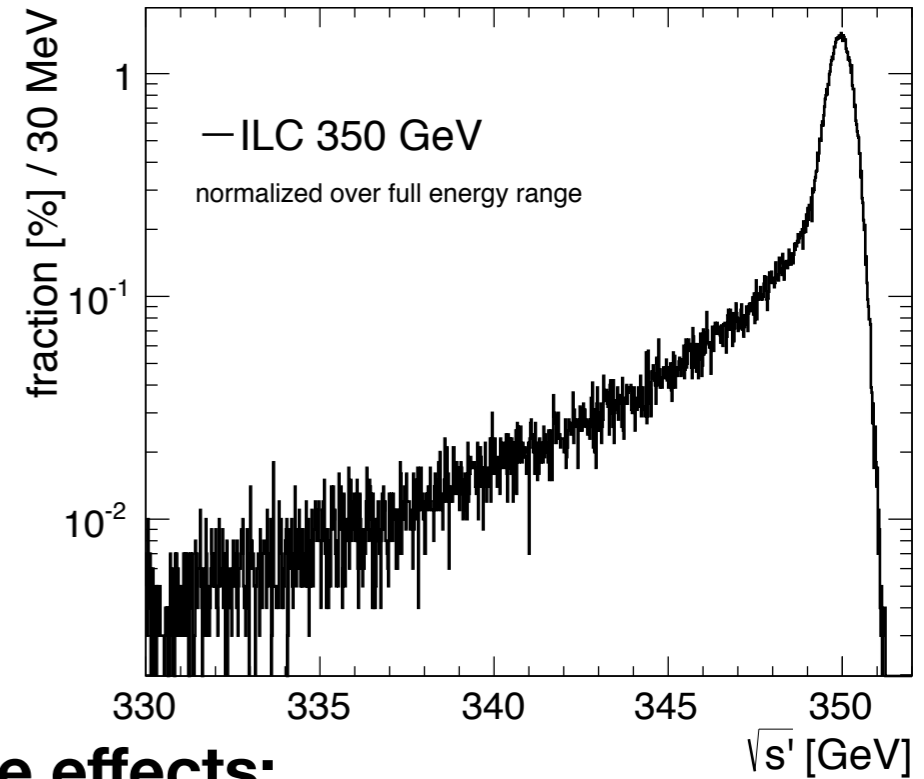
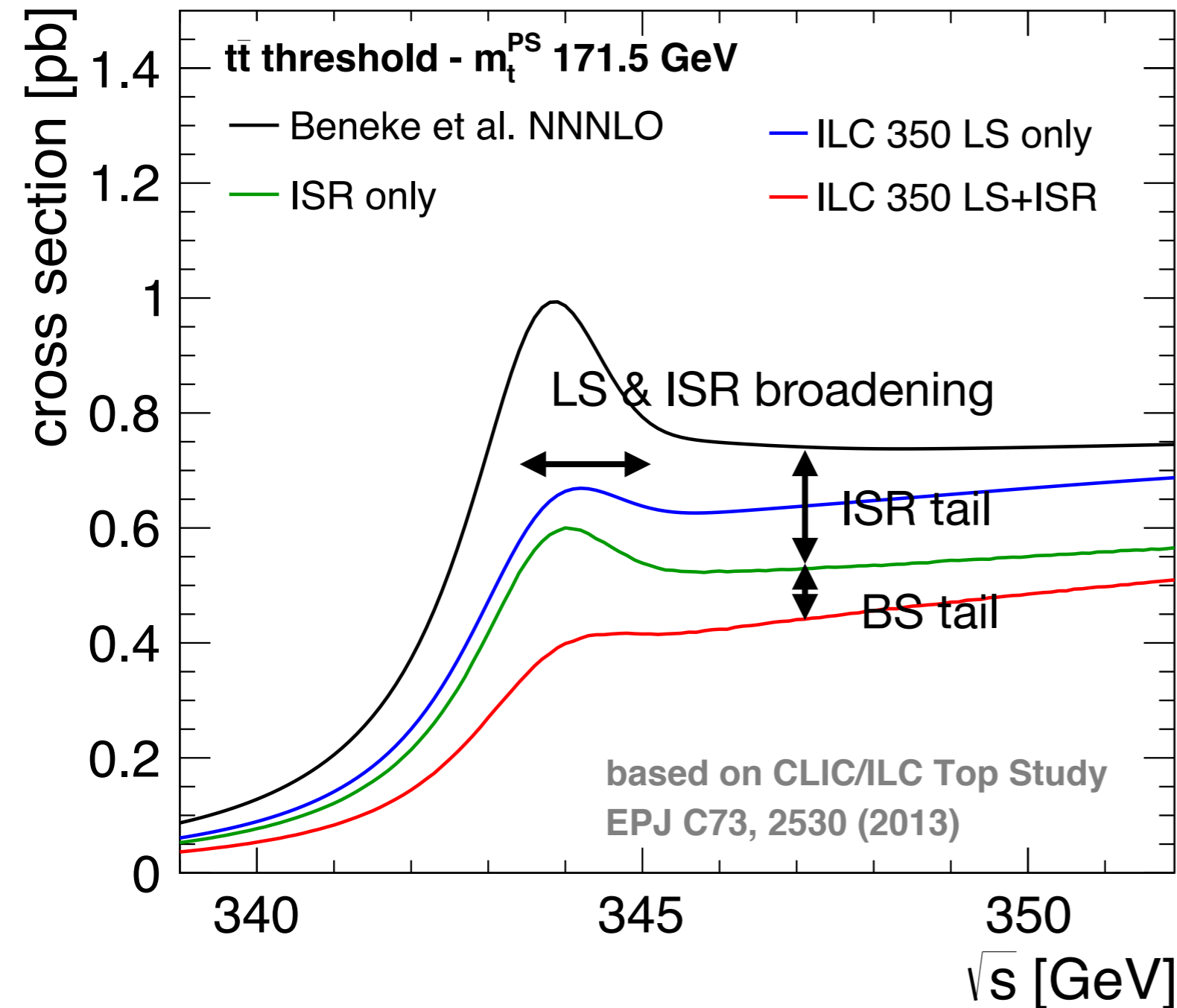


## The effects:

- ISR tail: lowering of effective L at top energy
- BS tail: lowering of effective L at top energy

# The Top Threshold at different $e^+e^-$ Colliders

- What the luminosity spectrum does to the threshold:

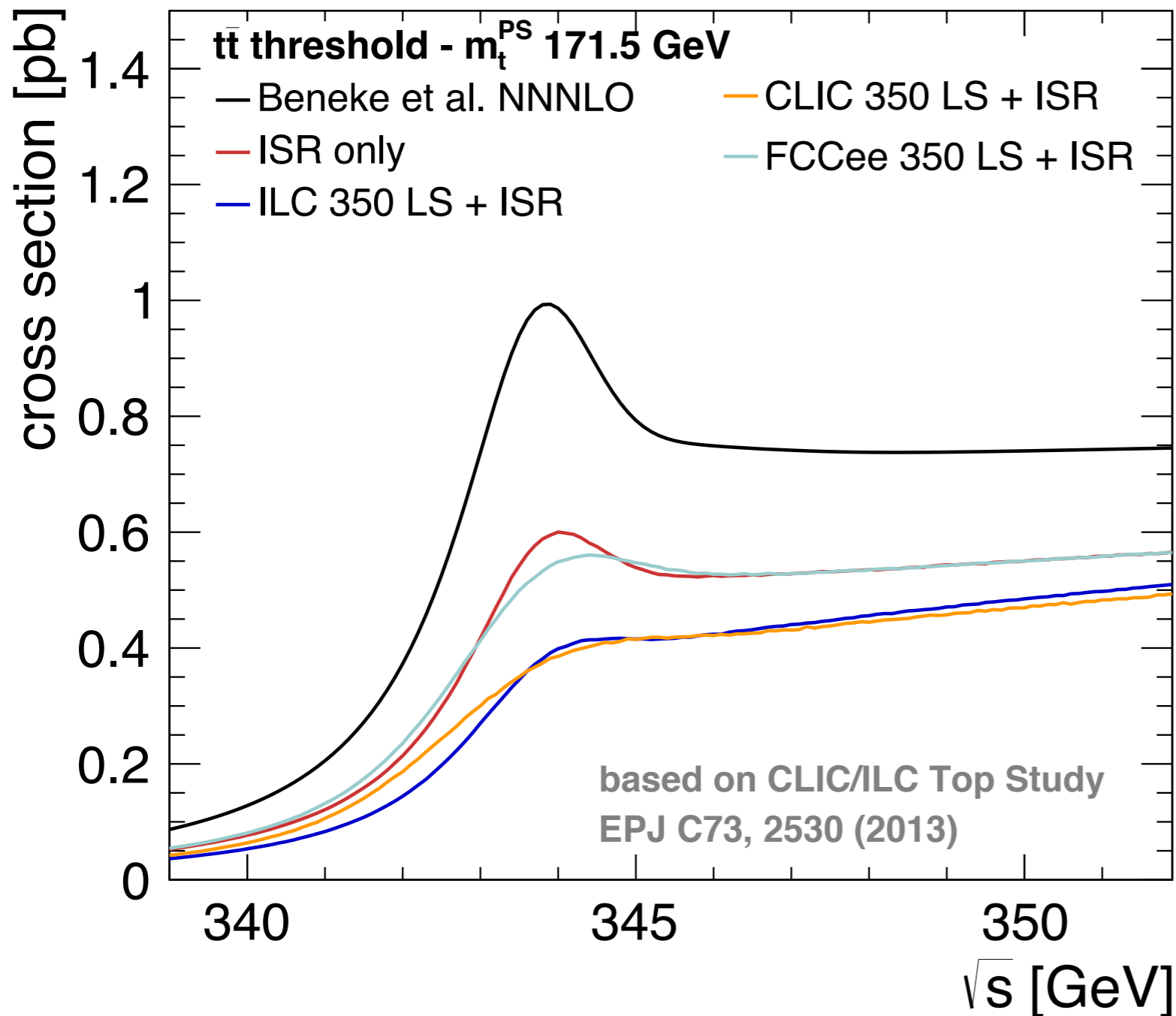


## The effects:

- ISR tail: lowering of effective L at top energy
- BS tail: lowering of effective L at top energy
- LS & ISR broadening: smearing of Xsection due to beam energy spread, BS tail and ISR

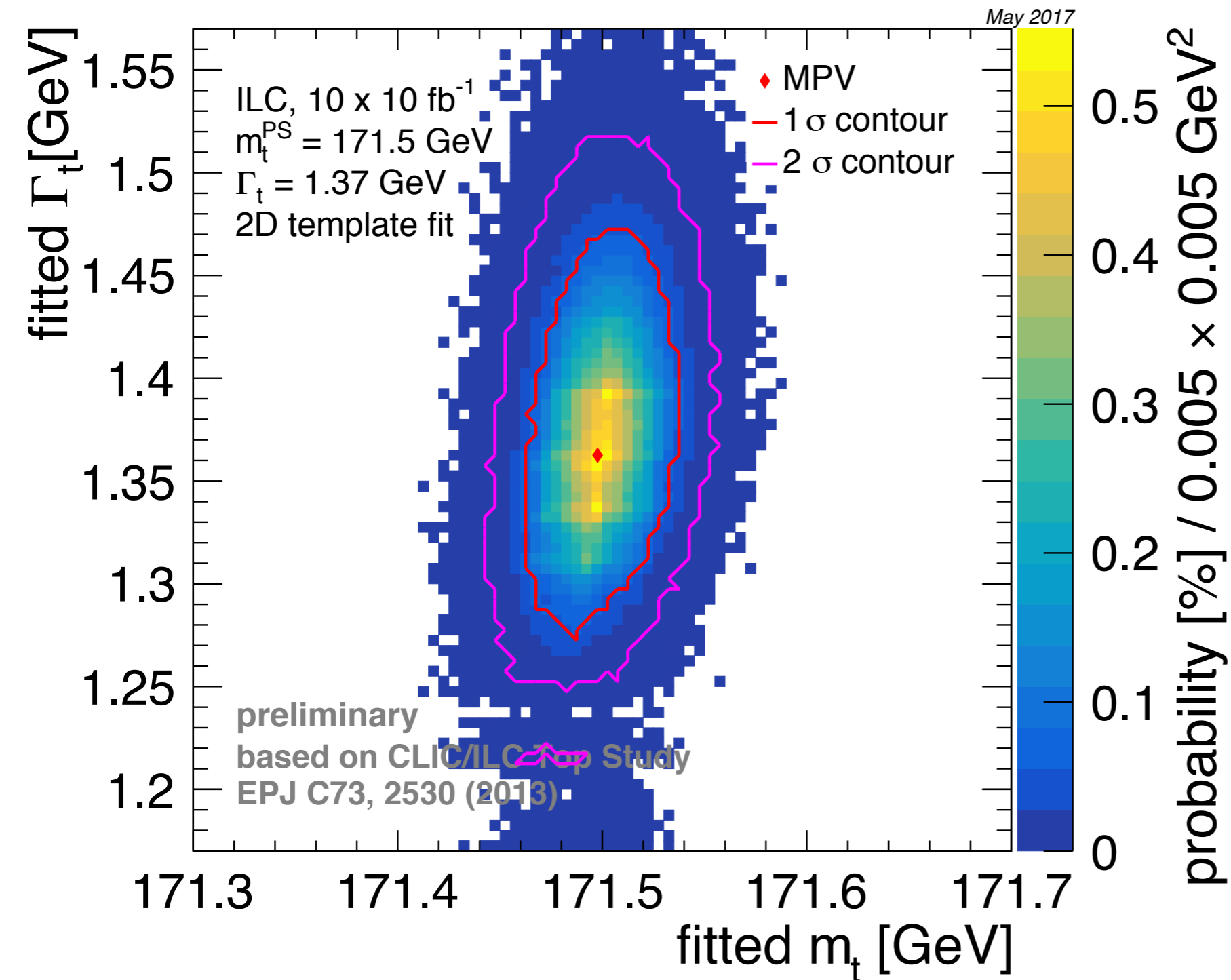


# The Top Threshold at different $e^+e^-$ Colliders

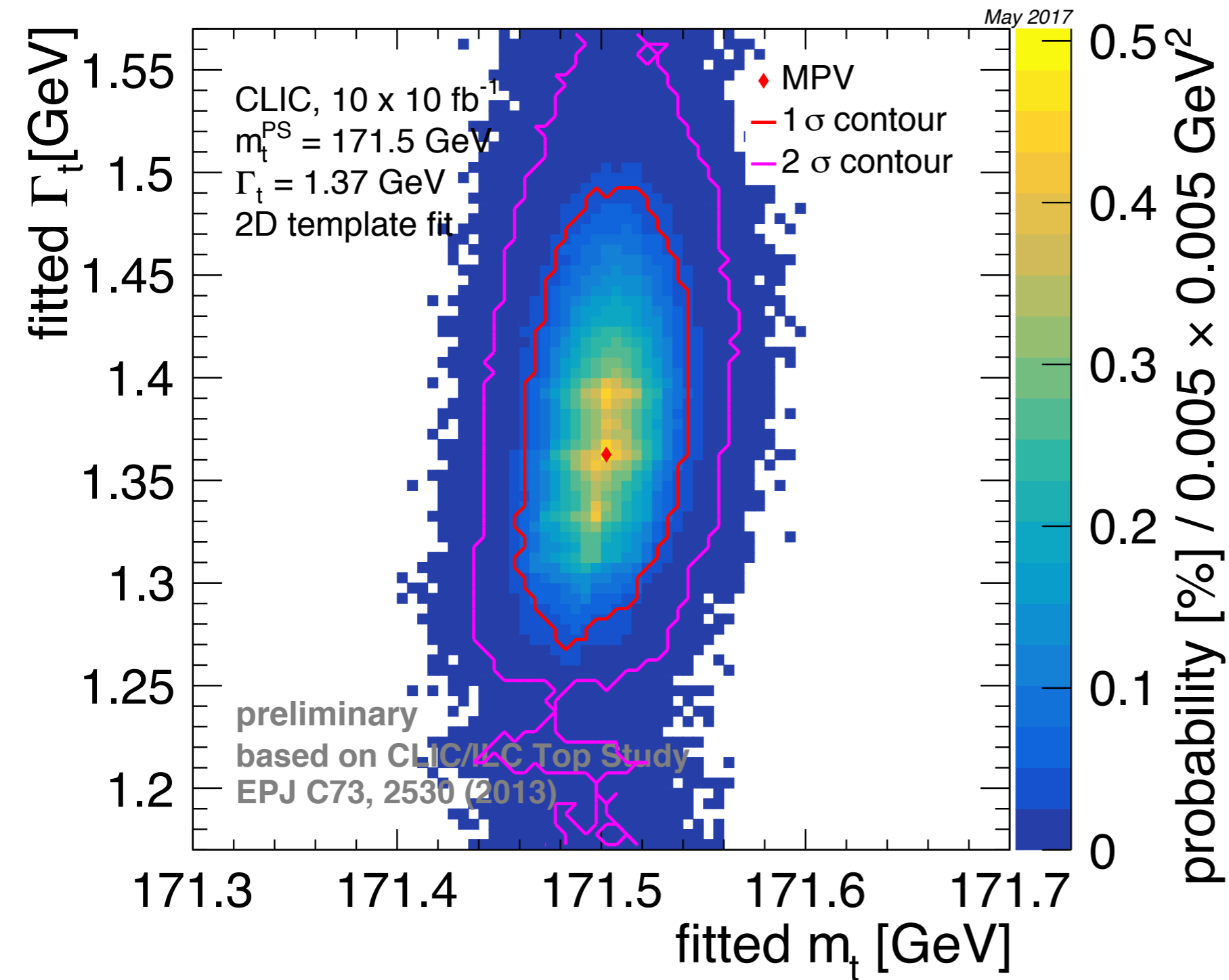


- The observable cross section for ILC, CLIC, FCCee
  - ~30% larger for FCCee than LCs: expect smaller statistical uncertainty
  - broader for CLIC than for ILC: expect somewhat larger statistical uncertainty at CLIC

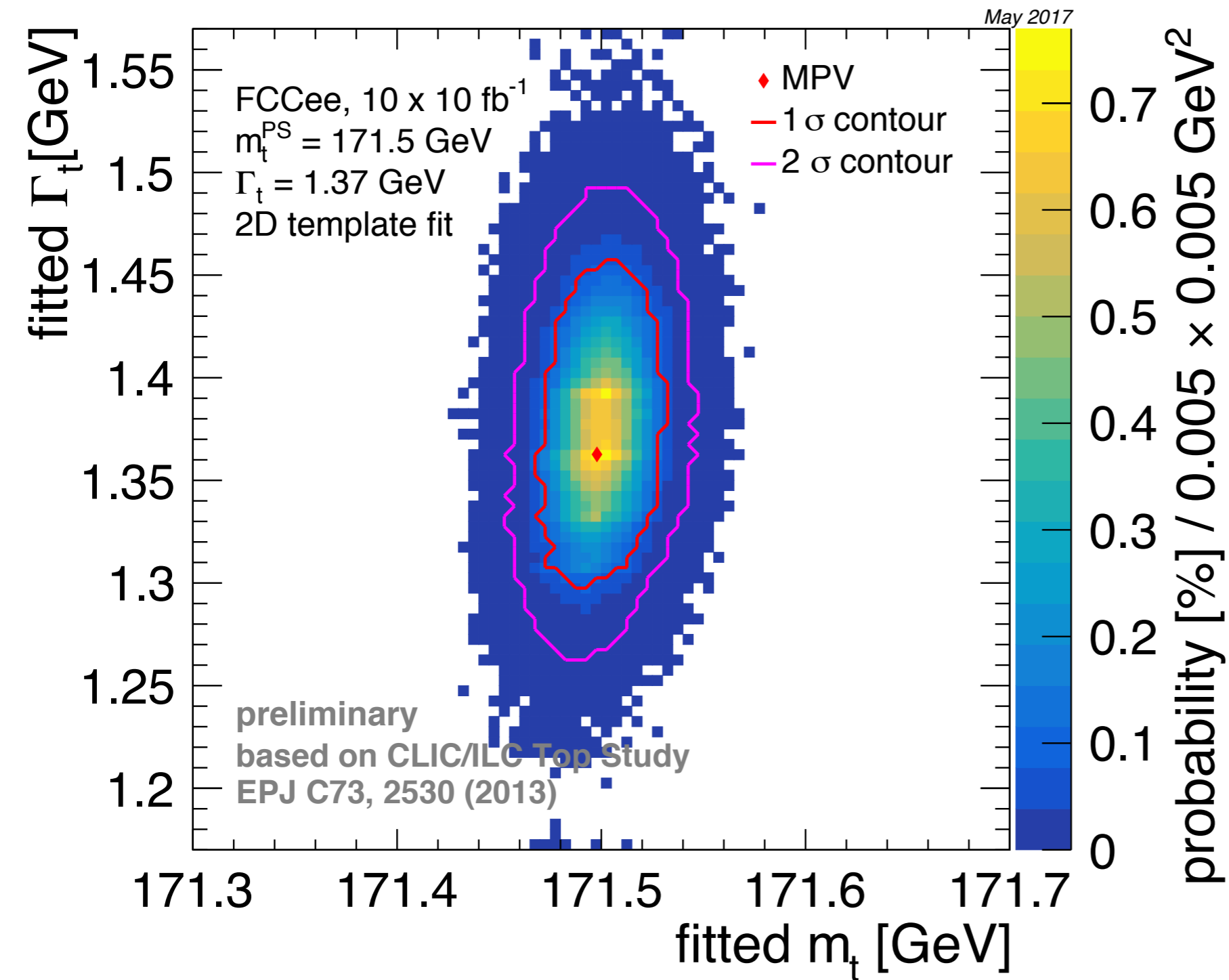
# Mass & Width: 2D Template Fit ILC, CLIC, FCCee



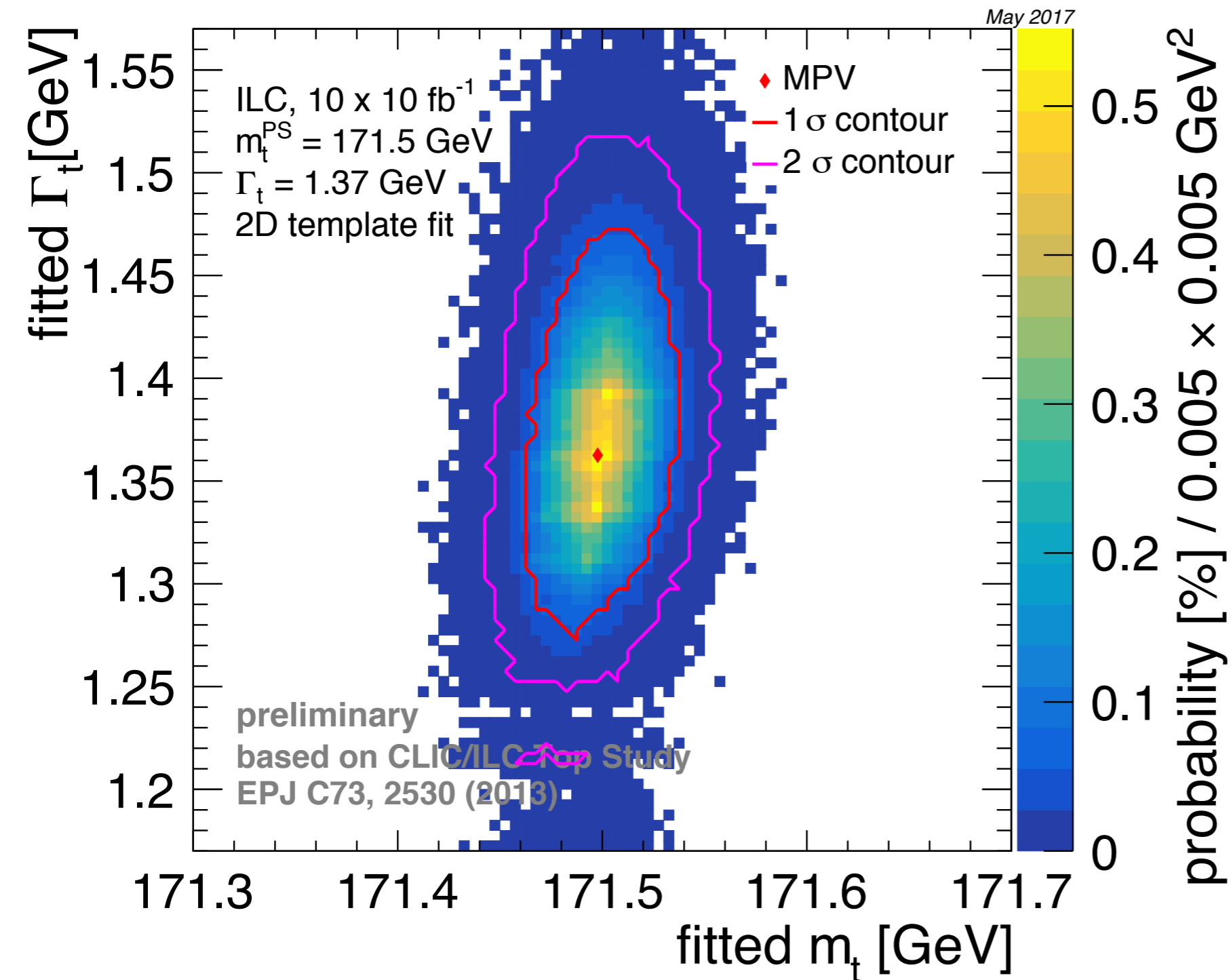
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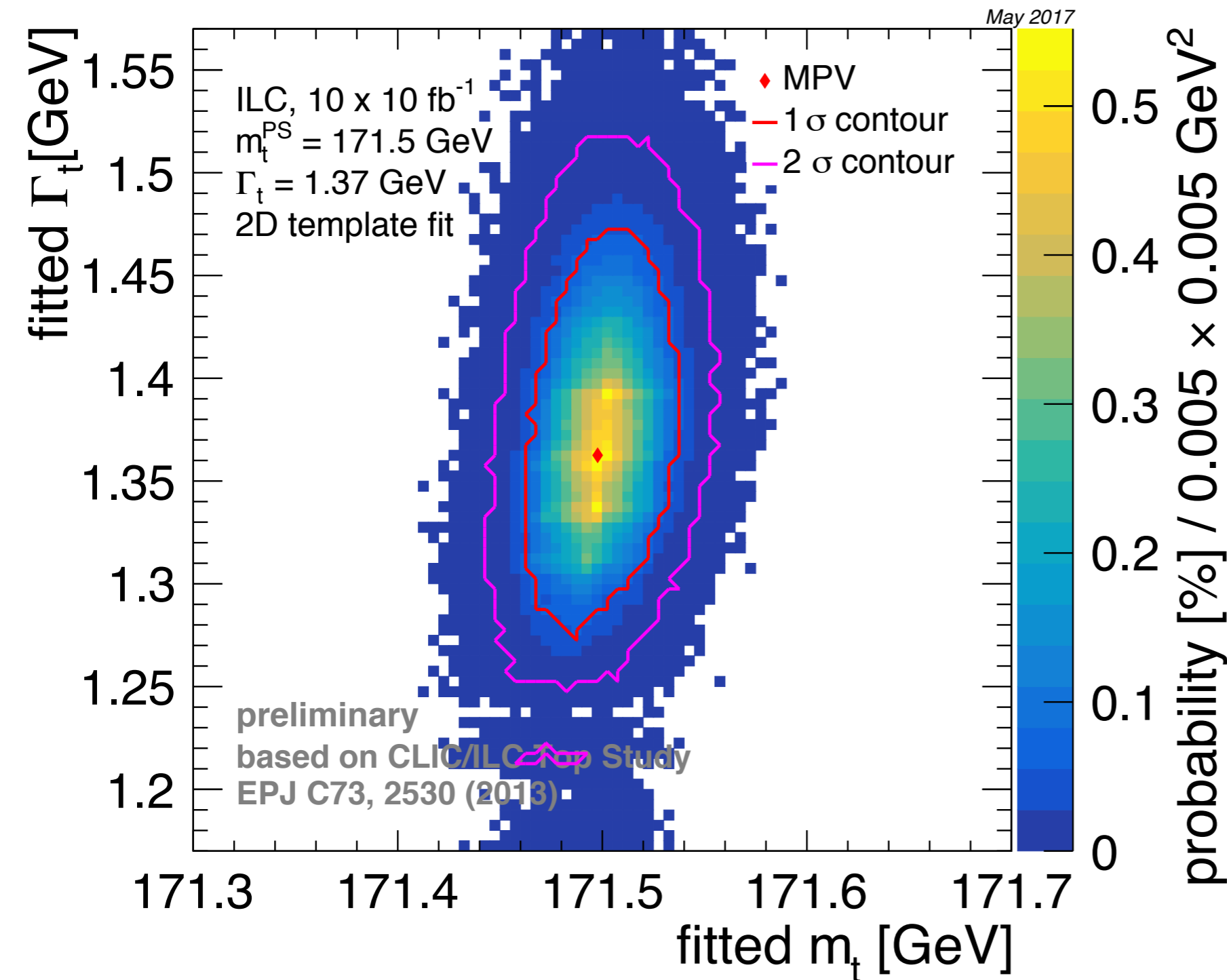
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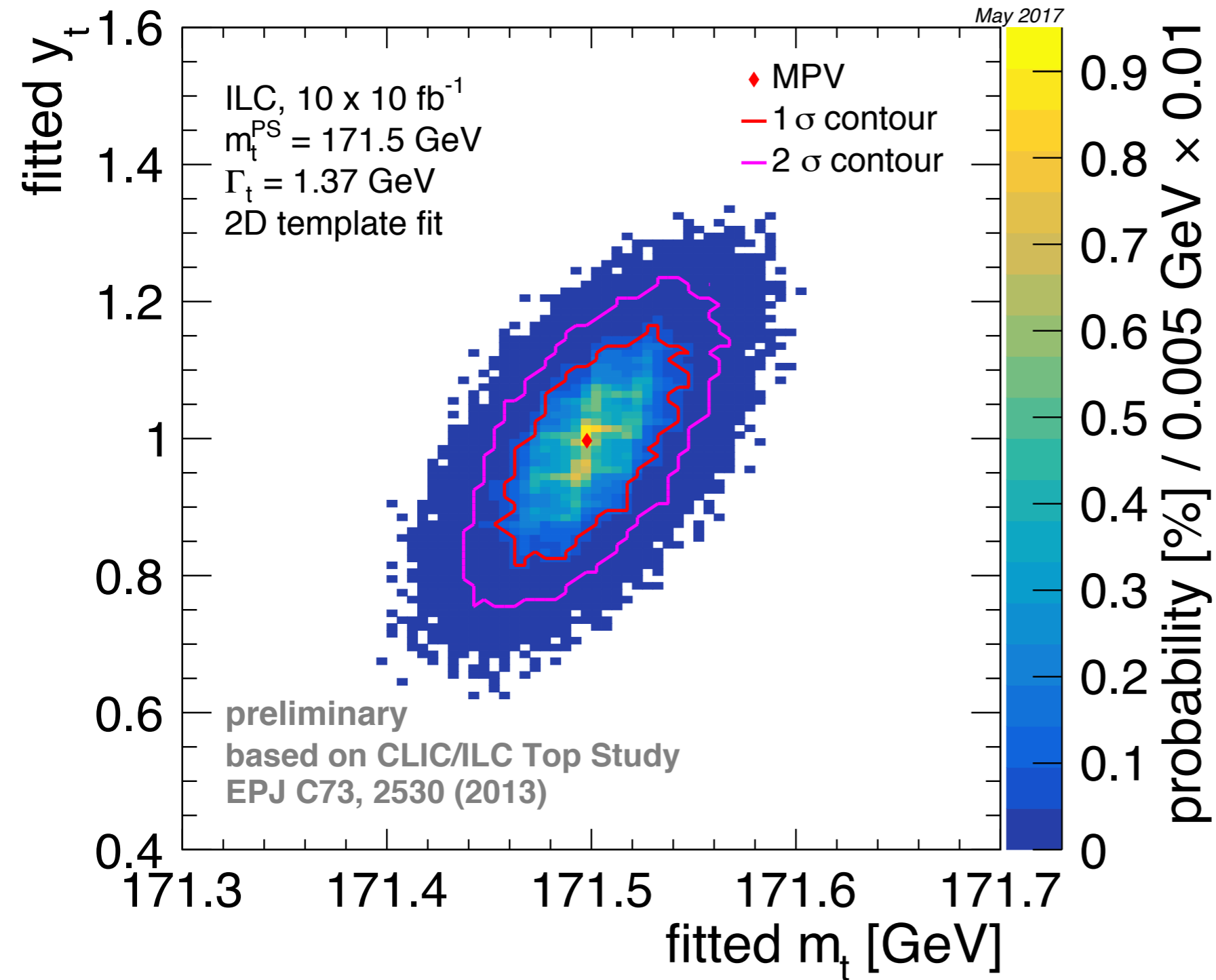
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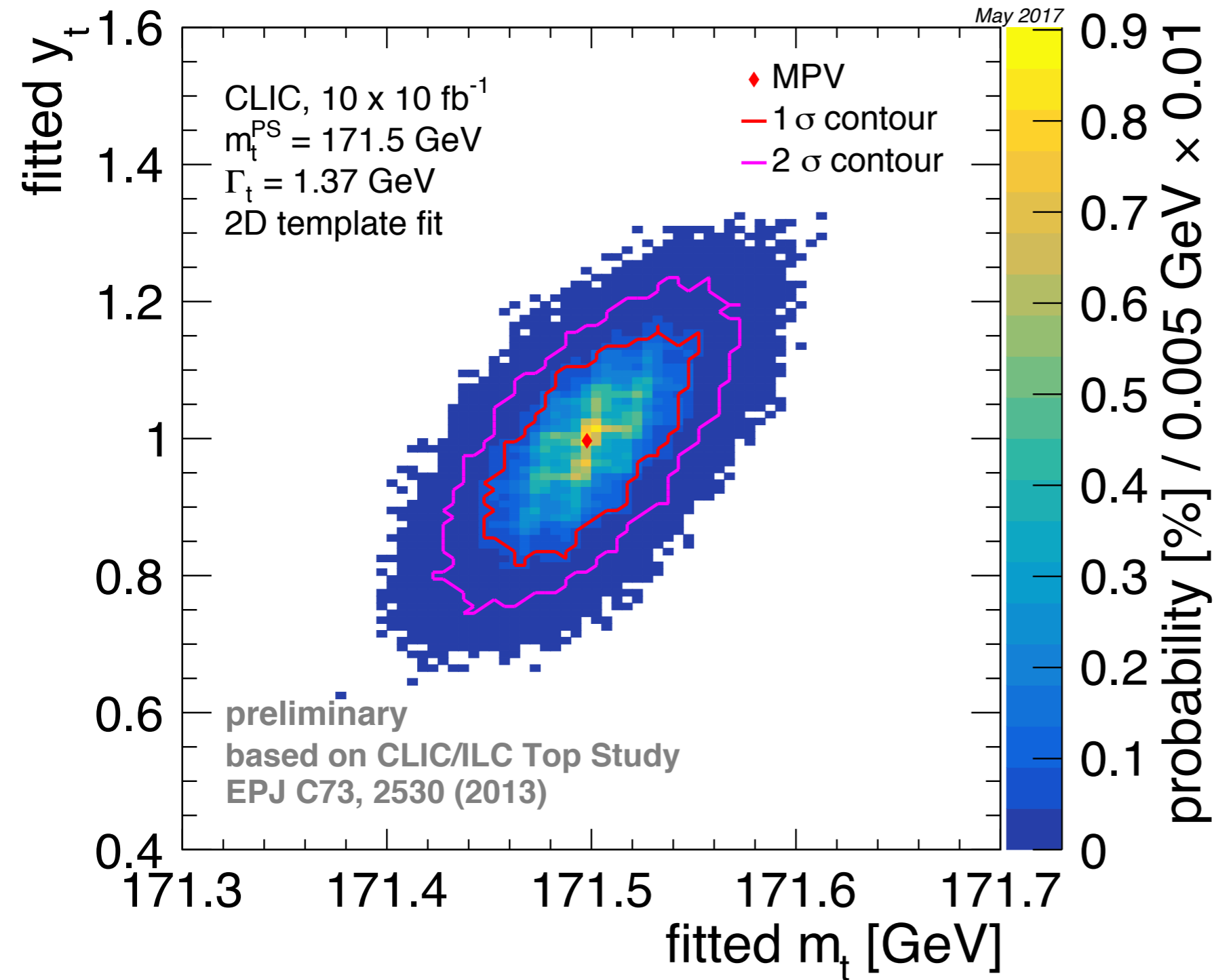
- 1D mass resolution (assuming def.  $\Gamma_t$ )
  - 18 MeV (ILC)**
  - 21 MeV (CLIC)**
  - 16 MeV (FCCee)**
- 1D width resolution (assuming def.  $m_t$ )
  - 43 MeV (ILC)**
  - 51 MeV (CLIC)**
  - 37 MeV (FCCee)**
- Extension of 1  $\sigma$  contour:
 

$m_t$ +39 -35 MeV	(ILC)
$\Gamma_t$ +90 -45 MeV	
$m_t$ +40 -45 MeV	(CLIC)
$\Gamma_t$ +130 -95 MeV	
$m_t$ +35 -30 MeV	(FCCee)
$\Gamma_t$ +95 -65 MeV	

# Mass & Yukawa: 2D Template Fit ILC, CLIC, FCCee

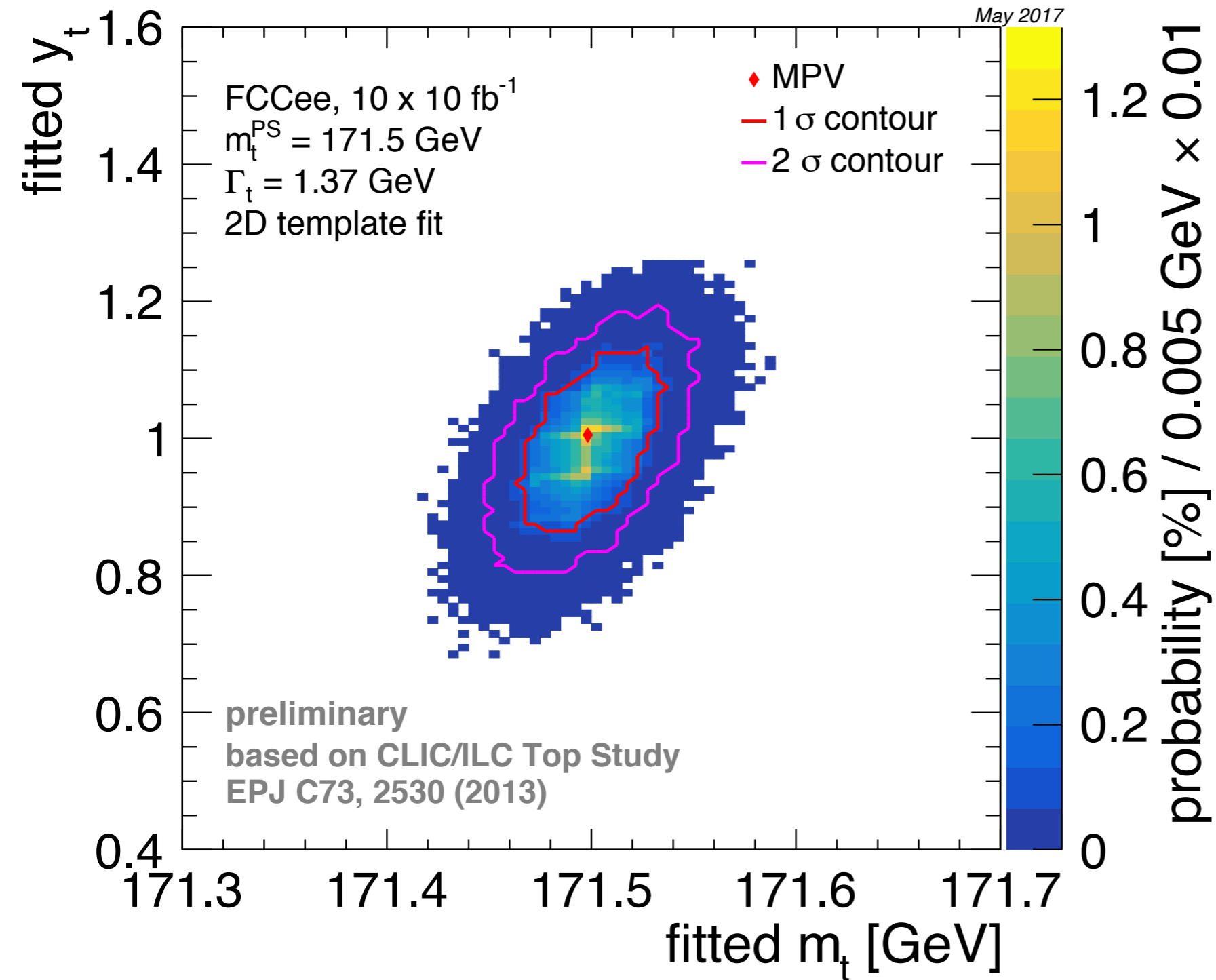


# Mass & Yukawa: 2D Template Fit ILC, CLIC, FCCee

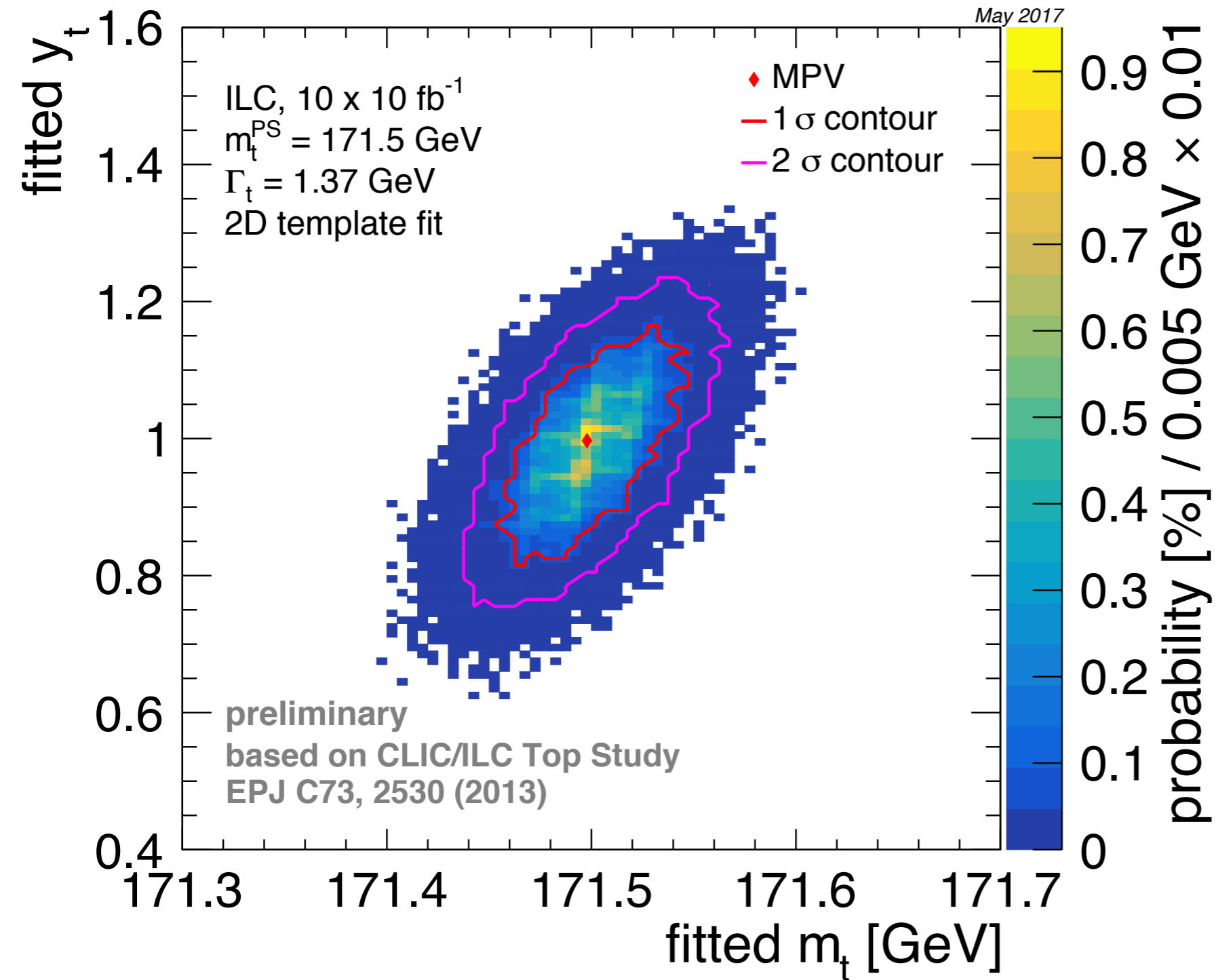




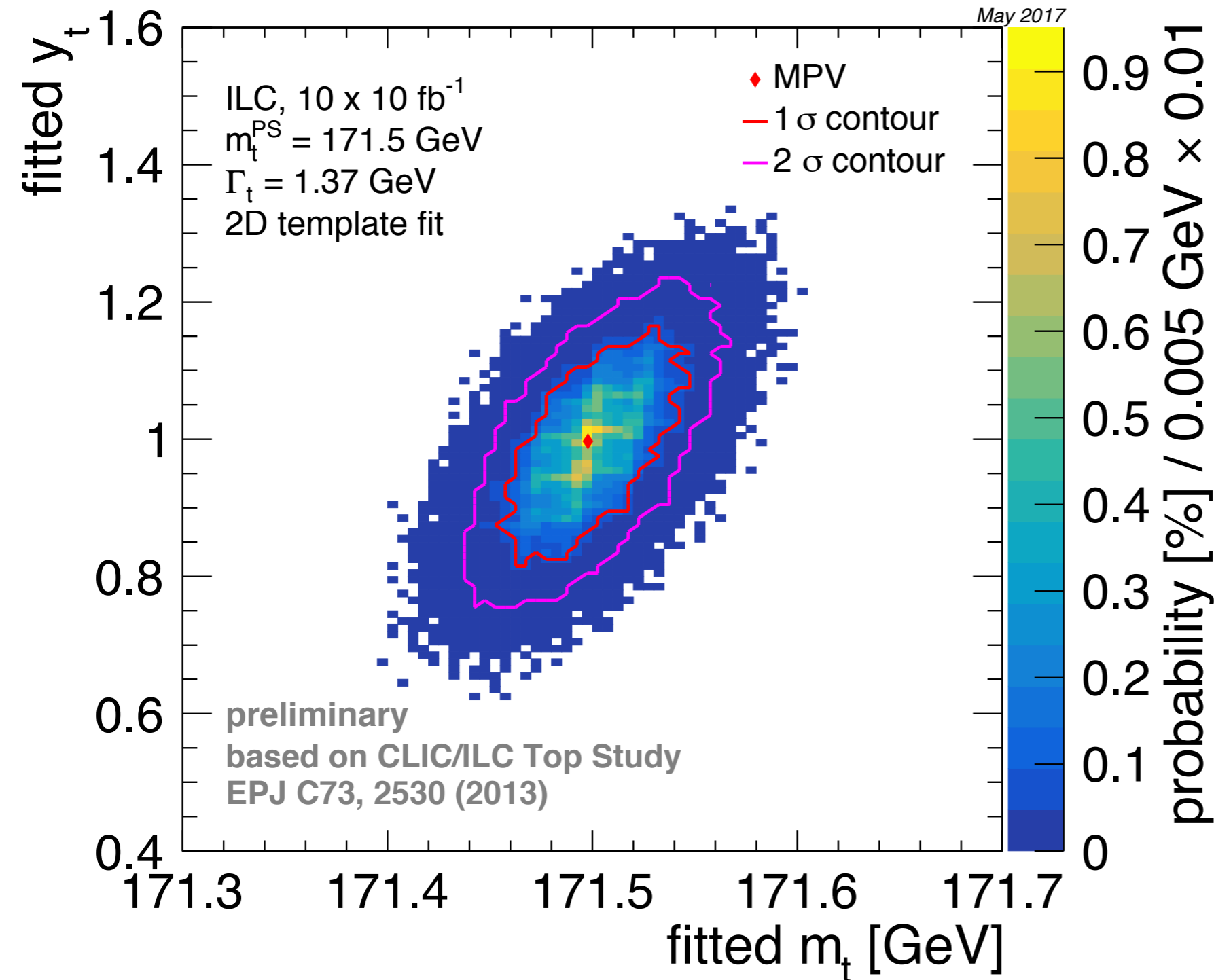
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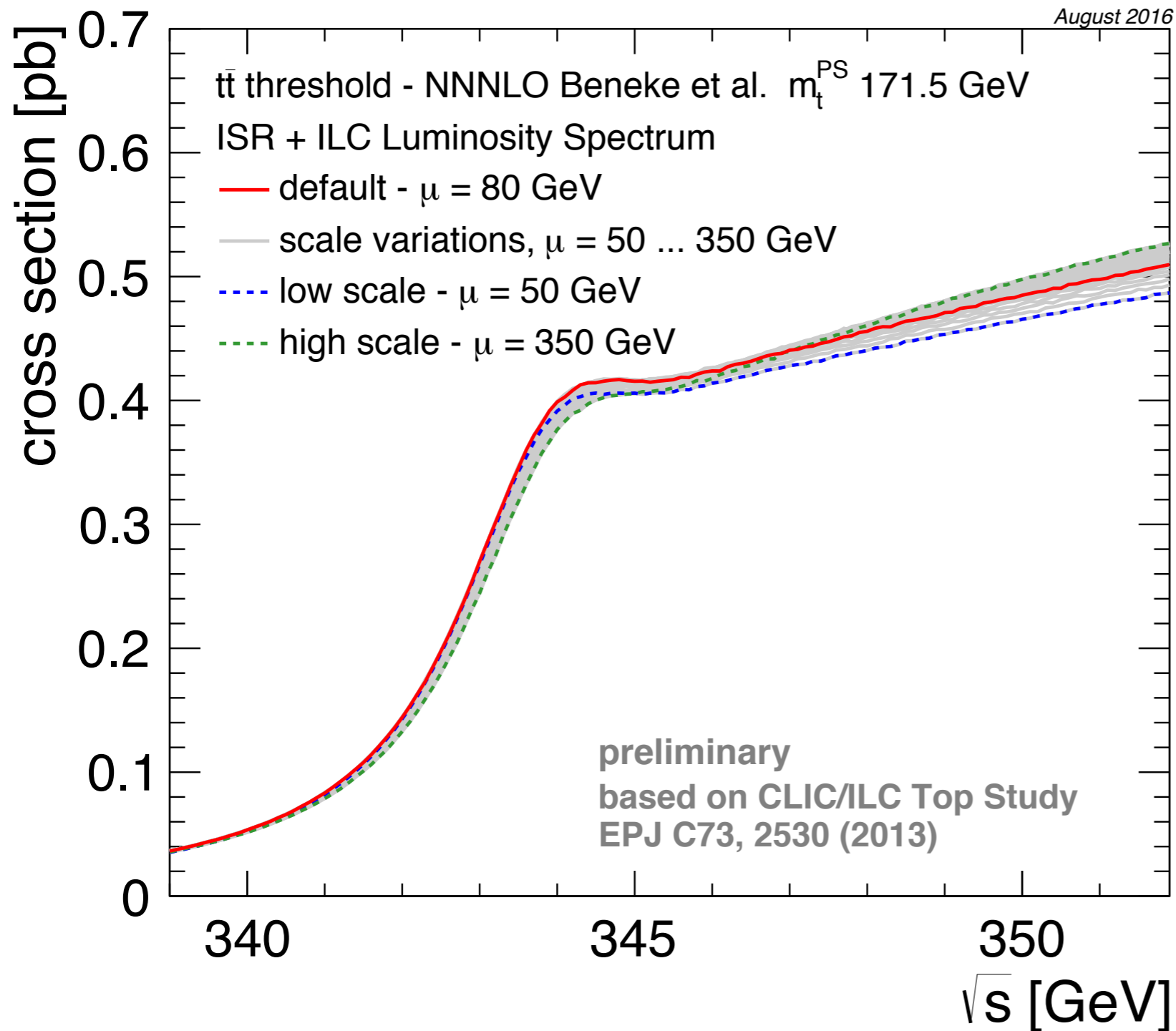


- 1D mass resolution (assuming def.  $y_t$ )  
**18 MeV (ILC)**  
**21 MeV (CLIC)**  
**16 MeV (FCCee)**
- 1D Yukawa resolution (assuming def.  $m_t$ )  
**0.067 (ILC)**  
**0.067 (CLIC)**  
**0.057 (FCCee)**
- Extension of  $1 \sigma$  contour:  
 **$m_t +49 -45 \text{ MeV}$  (ILC)**  
 **$y_t +0.168 -0.182$**   
 **$m_t +55 -50 \text{ MeV}$  (CLIC)**  
 **$y_t +0.168 -0.182$**   
 **$m_t +40 -36 \text{ MeV}$  (FCCee)**  
 **$y_t +0.130 -0.140$**

# The Influence of Scale Uncertainties

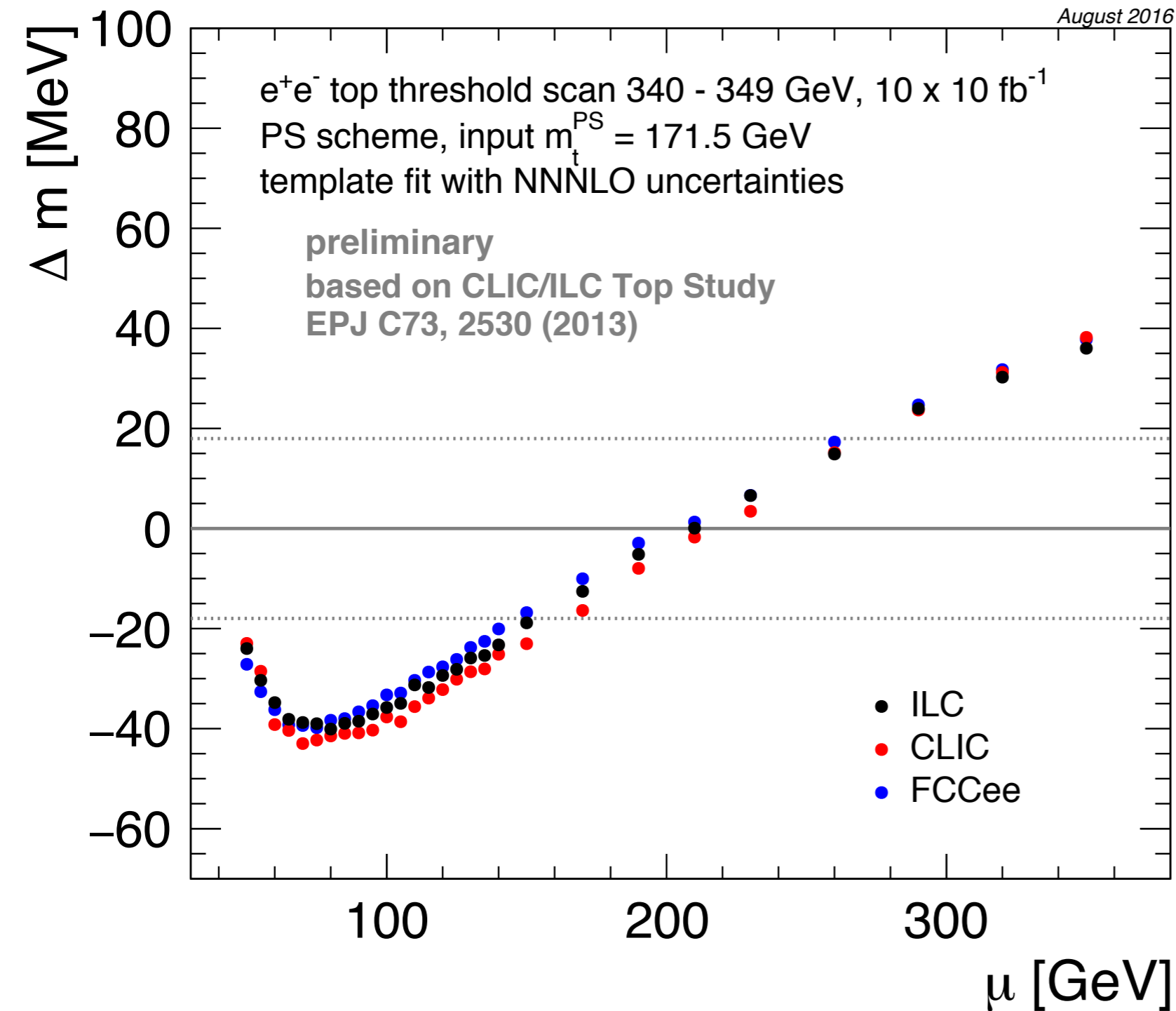
- Varying the scale  $\mu$  from 50 to 350 GeV

# The Effect of Scale Variations on the Cross Section



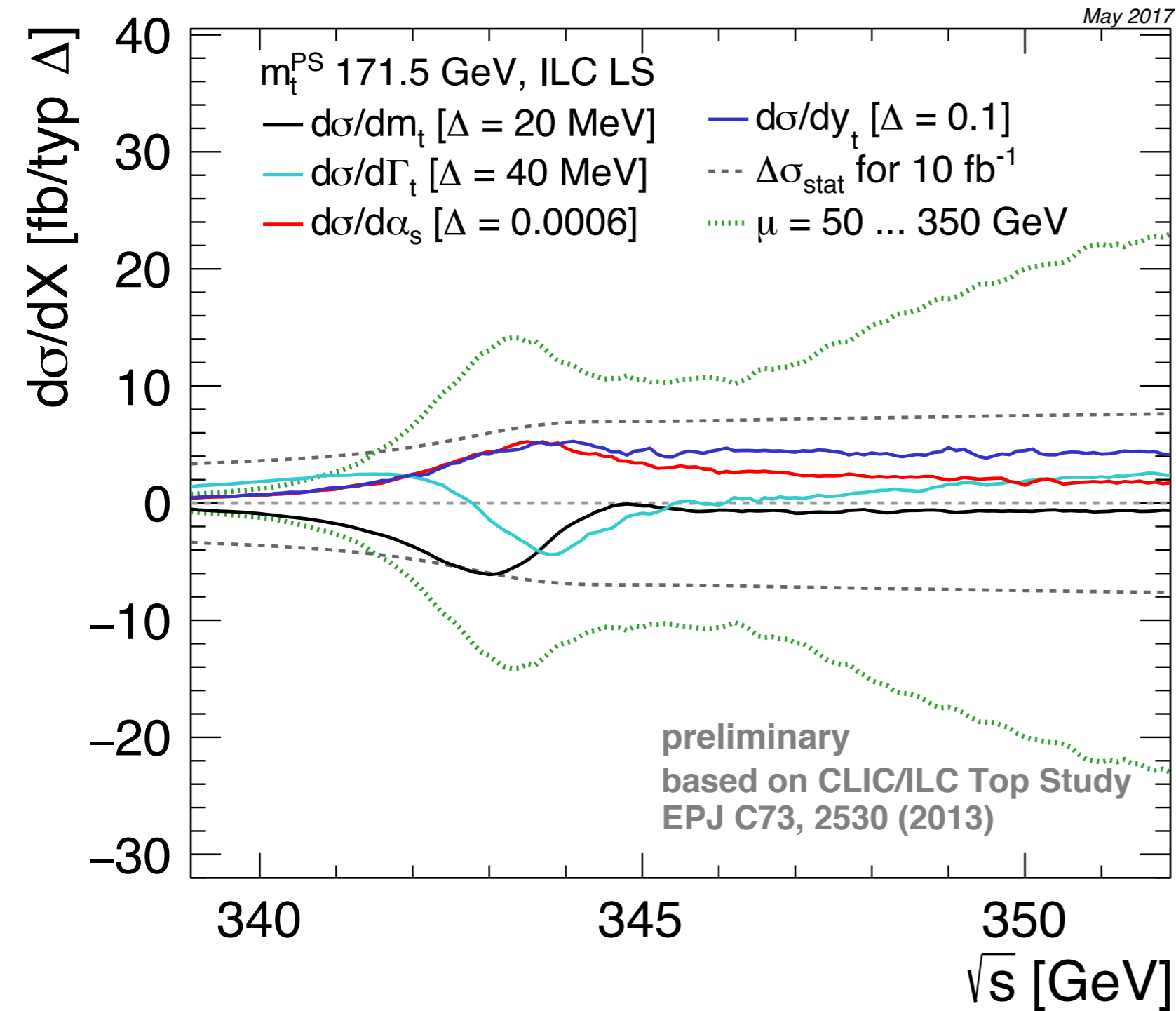
- The impact on the 1D mass measurement was discussed at the last workshop - results for different colliders documented in [arXiv:1611.03399](https://arxiv.org/abs/1611.03399)

# The Effect of Scale Variations on the Cross Section



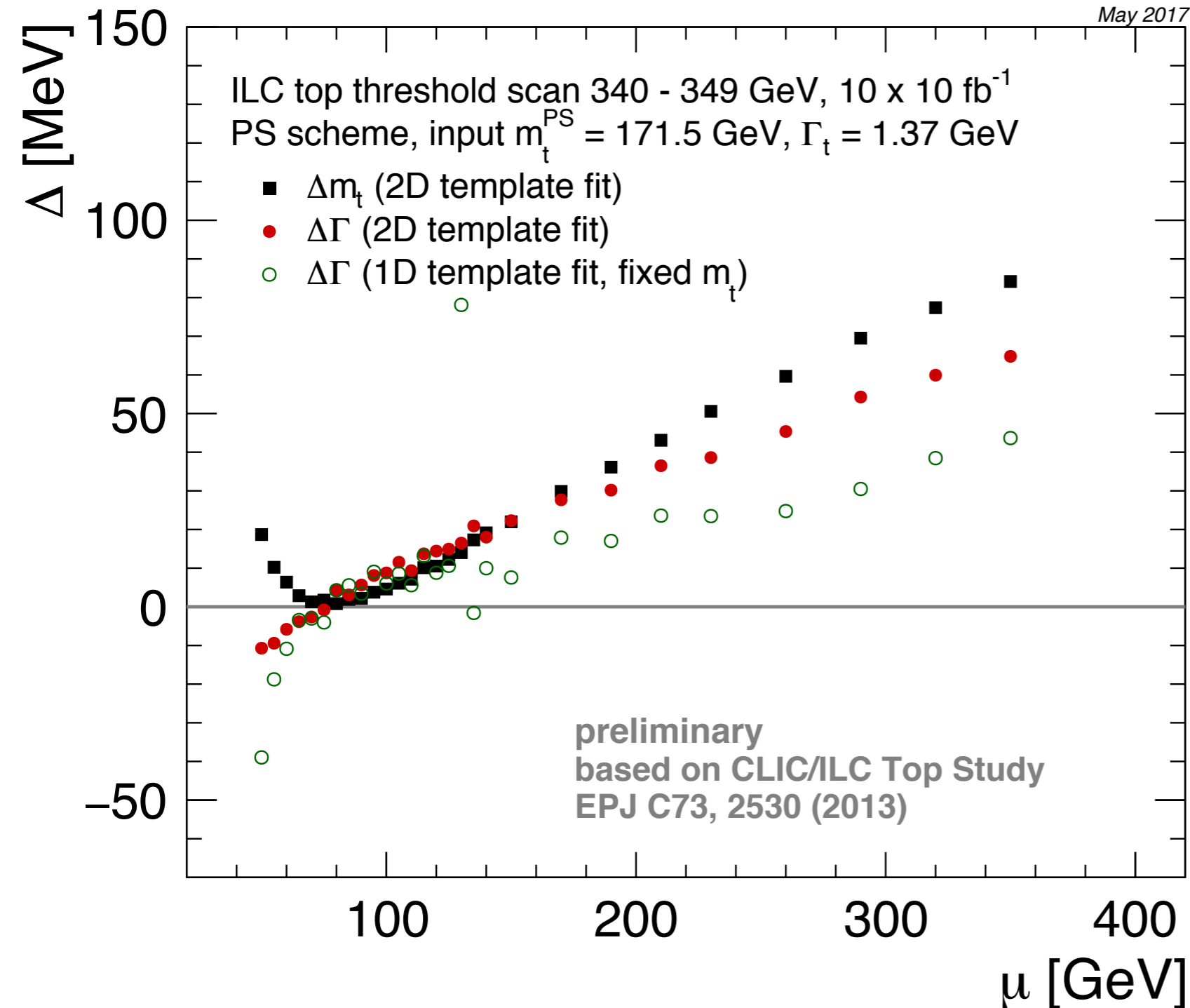
- The impact on the 1D mass measurement was discussed at the last workshop - results for different colliders documented in [arXiv:1611.03399](https://arxiv.org/abs/1611.03399)
- Bottom line:  $\pm 40 \text{ MeV}$  uncertainty (after correcting a “trivial” offset of  $\sim 40 \text{ MeV}$  originating from fitting procedure with uncertainty bands) irrespective of collider type

# Scale Variations: Illustrating the Impact



- Scale uncertainties (much) larger than variations according to expected statistical uncertainties
- Strong correlation of  $y_t$  and  $\alpha_s$
- Overlapping regions of sensitivity of  $m_t$  and  $\Gamma_t$  and of  $m_t$  and  $y_t$

# Scale Variations: Mass & Width



- 1D mass resolution (assuming def.  $\Gamma_t$ )  
**18 MeV**

- 1D width resolution (assuming def.  $m_t$ )  
**43 MeV**

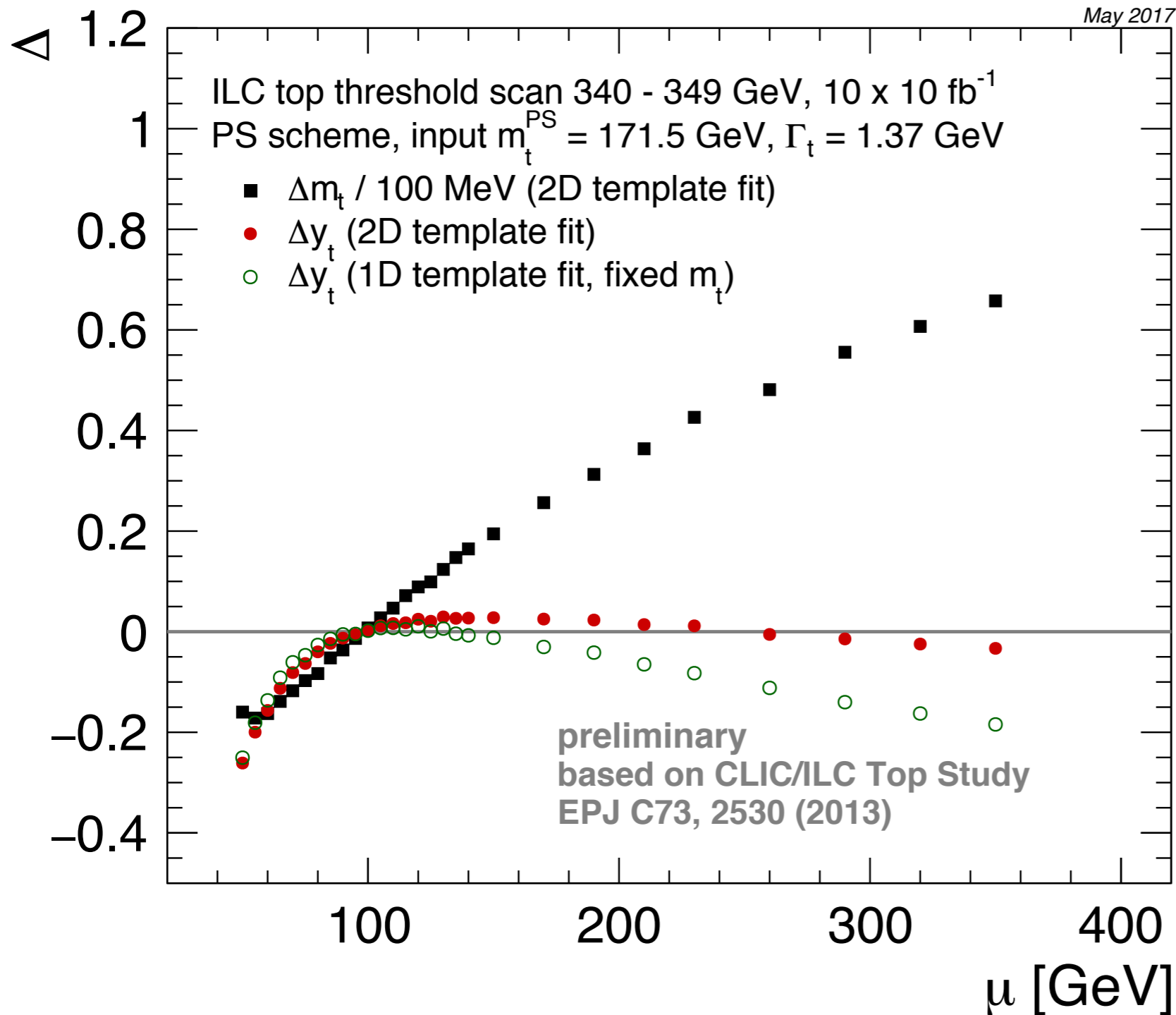
- Range of scale variations:  
 **$m_t +85 -0 \text{ MeV (2D)}$**   
 **$\Gamma_t +64 -10 \text{ MeV (2D)}$**   
 **$\Gamma_t +78 -39 \text{ MeV (1D)}$**

(remember: 1D mass:  
 $+80 \text{ MeV} - 0 \text{ MeV}$ )

1D results fluctuating somewhat - still unclear conclusions for different machines, in general rather comparable effects



# Scale Variations: Mass & Yukawa Coupling



- 1D mass resolution  
 (assuming def.  $\Gamma_t$ )  
**18 MeV**
  - 1D  $y_t$  resolution  
 (assuming def.  $m_t$ )  
**0.067**
  - Range of scale variations:  
 **$m_t +66 -17 \text{ MeV}$  (2D)**  
 **$y_t +0.03 -0.26$  (2D)**  
 **$y_t +0.01 -0.25$  (1D)**
- (remember: 1D mass:  
 +80 MeV - 0 MeV)

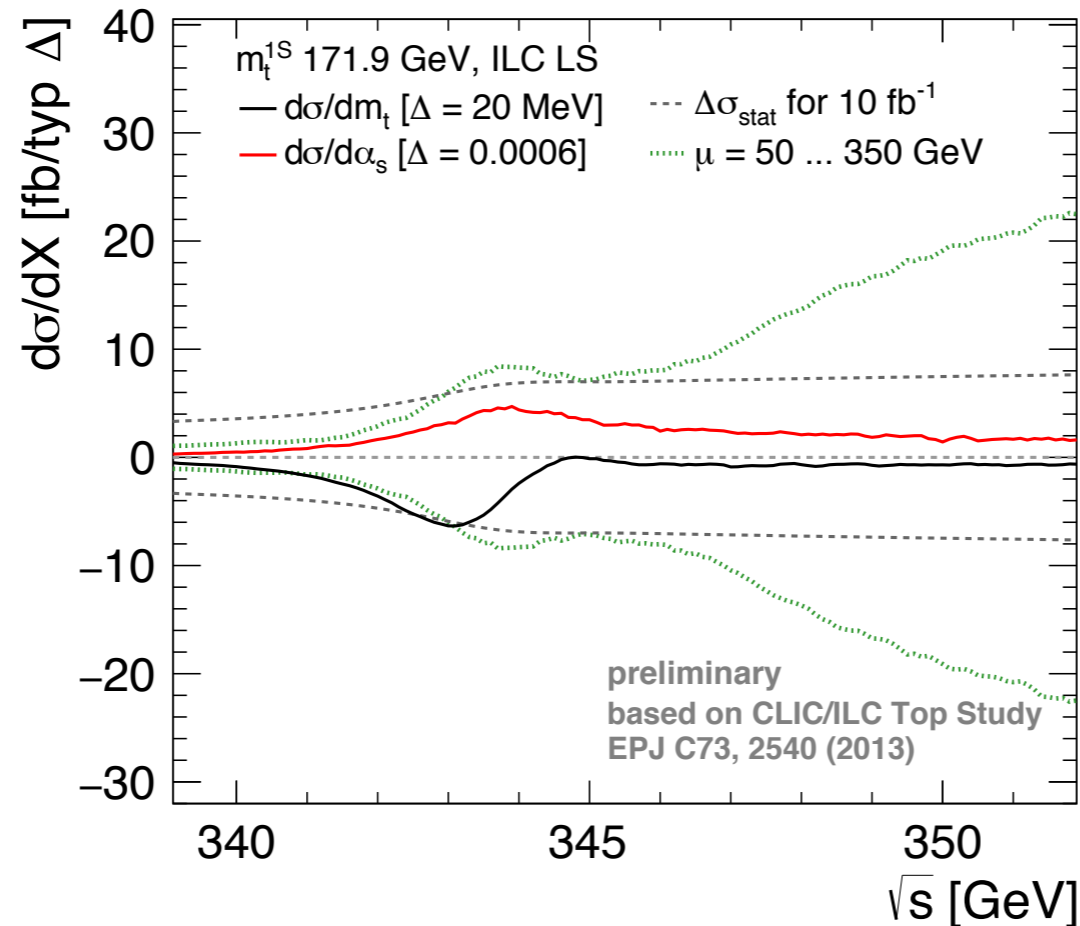
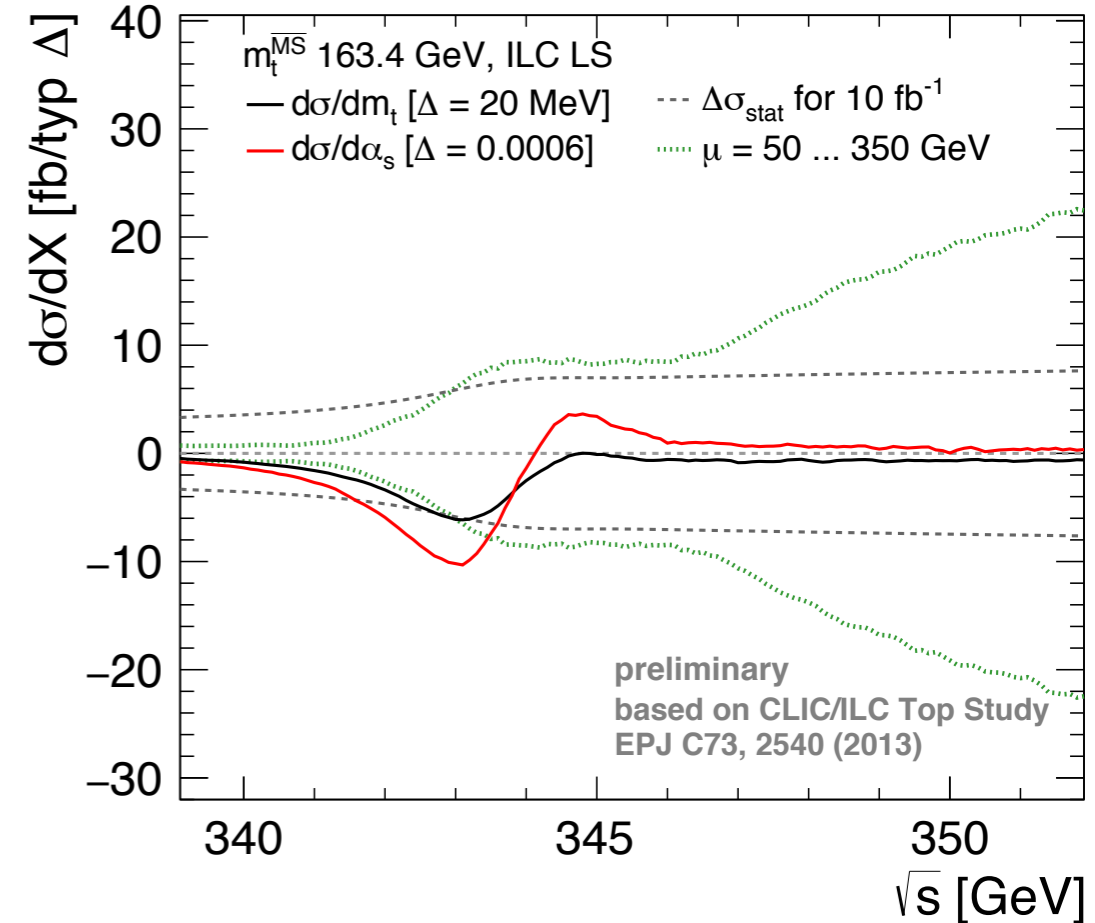
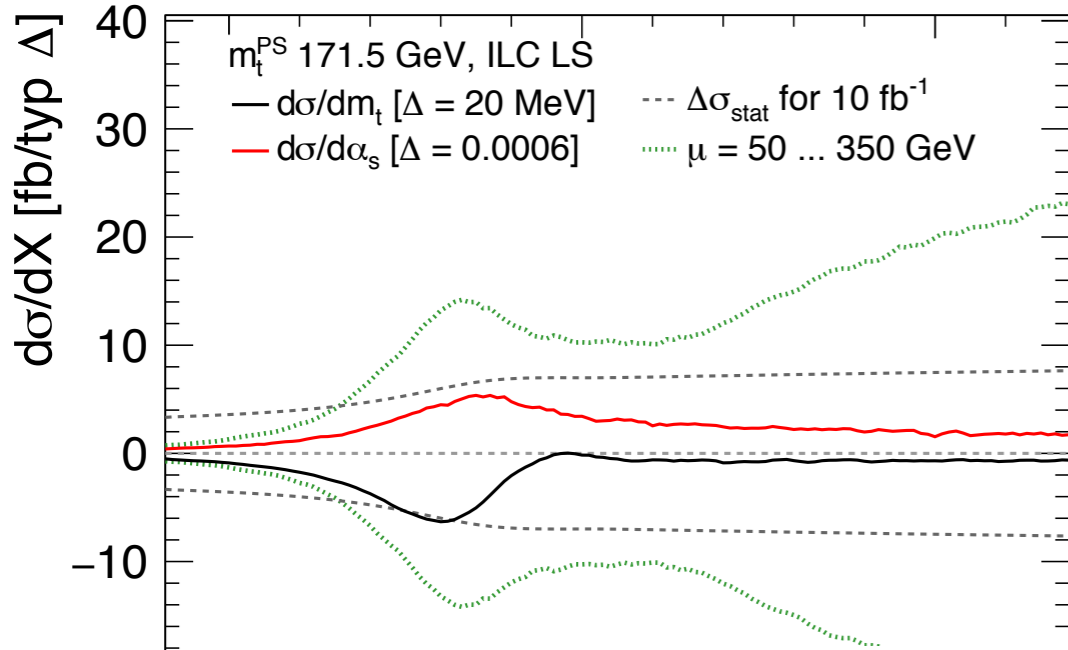
little dependence on luminosity spectrum -  $\sim 10\%$  larger uncertainties for FCCee



# The Effect of the Mass Scheme

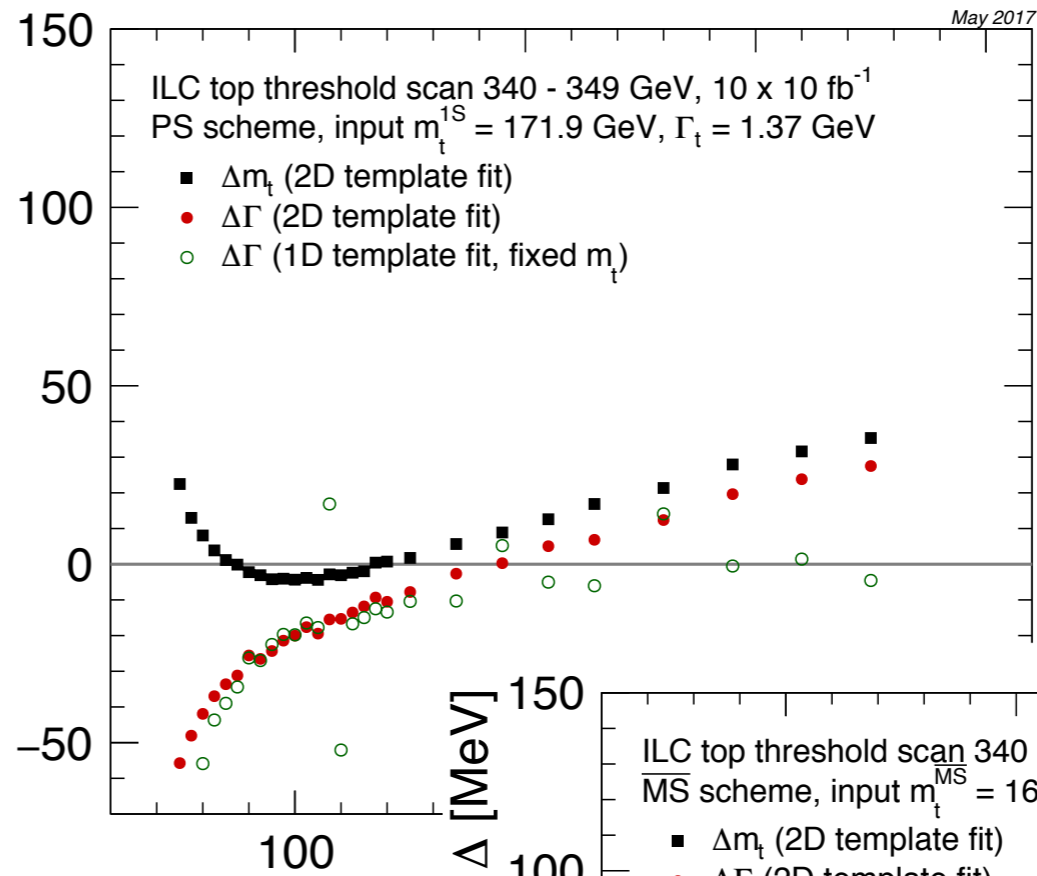
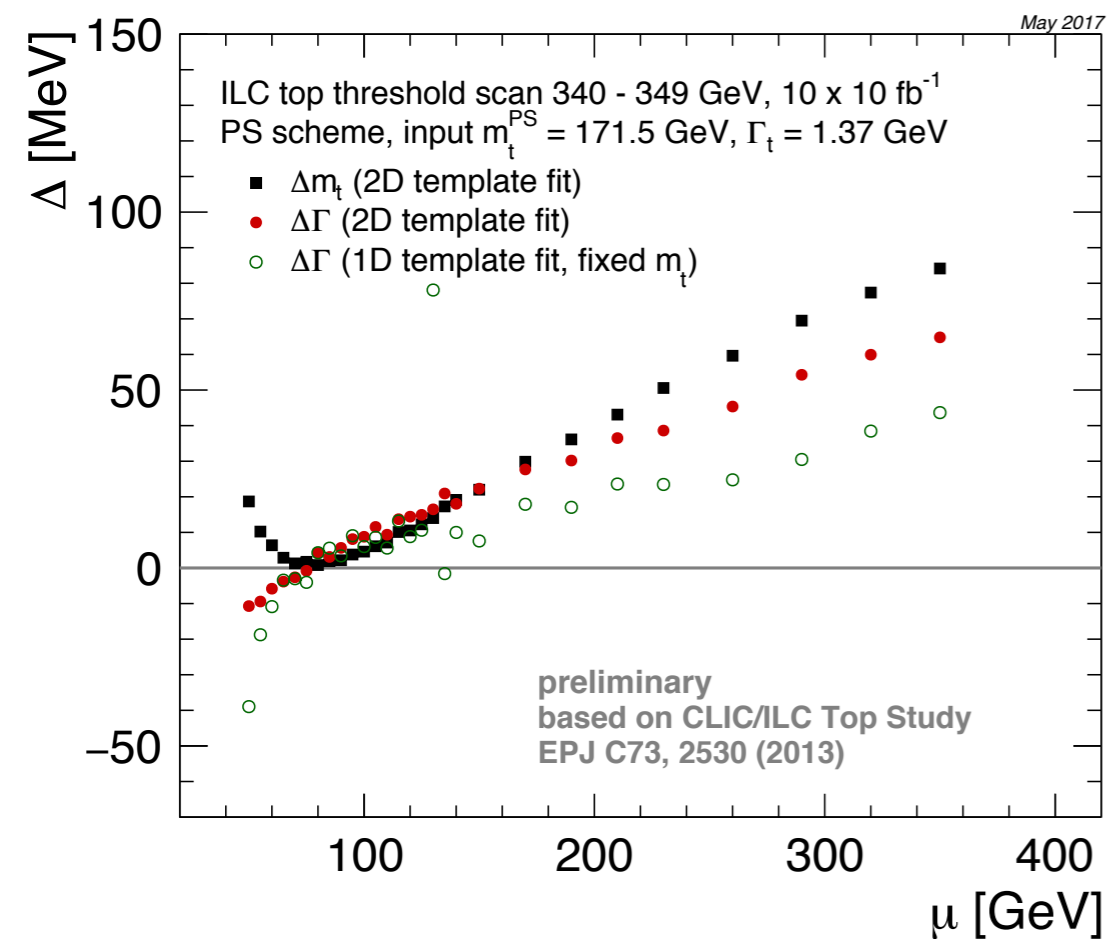
- Investigating the impact of using different mass scheme:
  - PS
  - 1S
  - $\overline{\text{MS}}$

# Reminder: Scale Uncertainties depend on Scheme

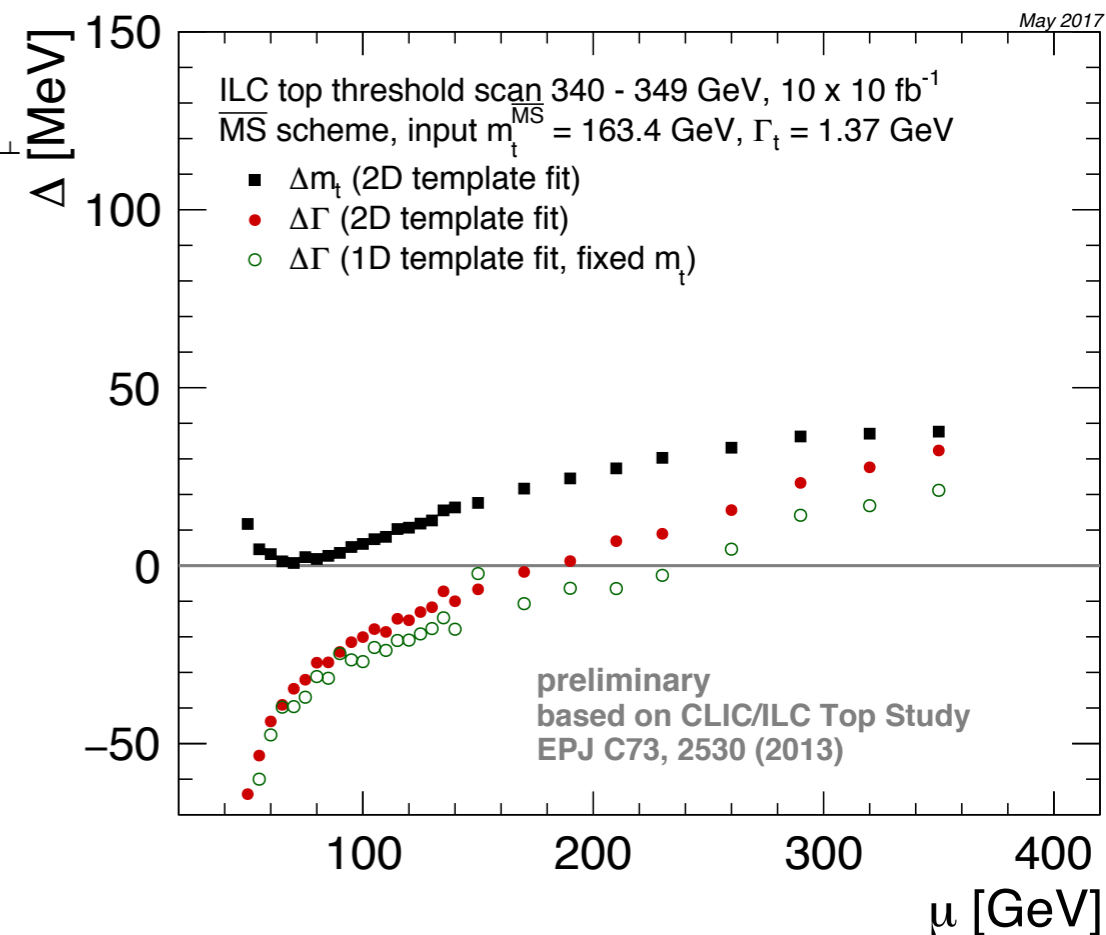


- Scale uncertainties in the region most sensitive to mass (and width) substantially larger in PS than in 1S and MSbar schemes
- Impact on mass uncertainty:  
 ~ 40 MeV (PS); ~ 20 MeV (1S, MSbar)

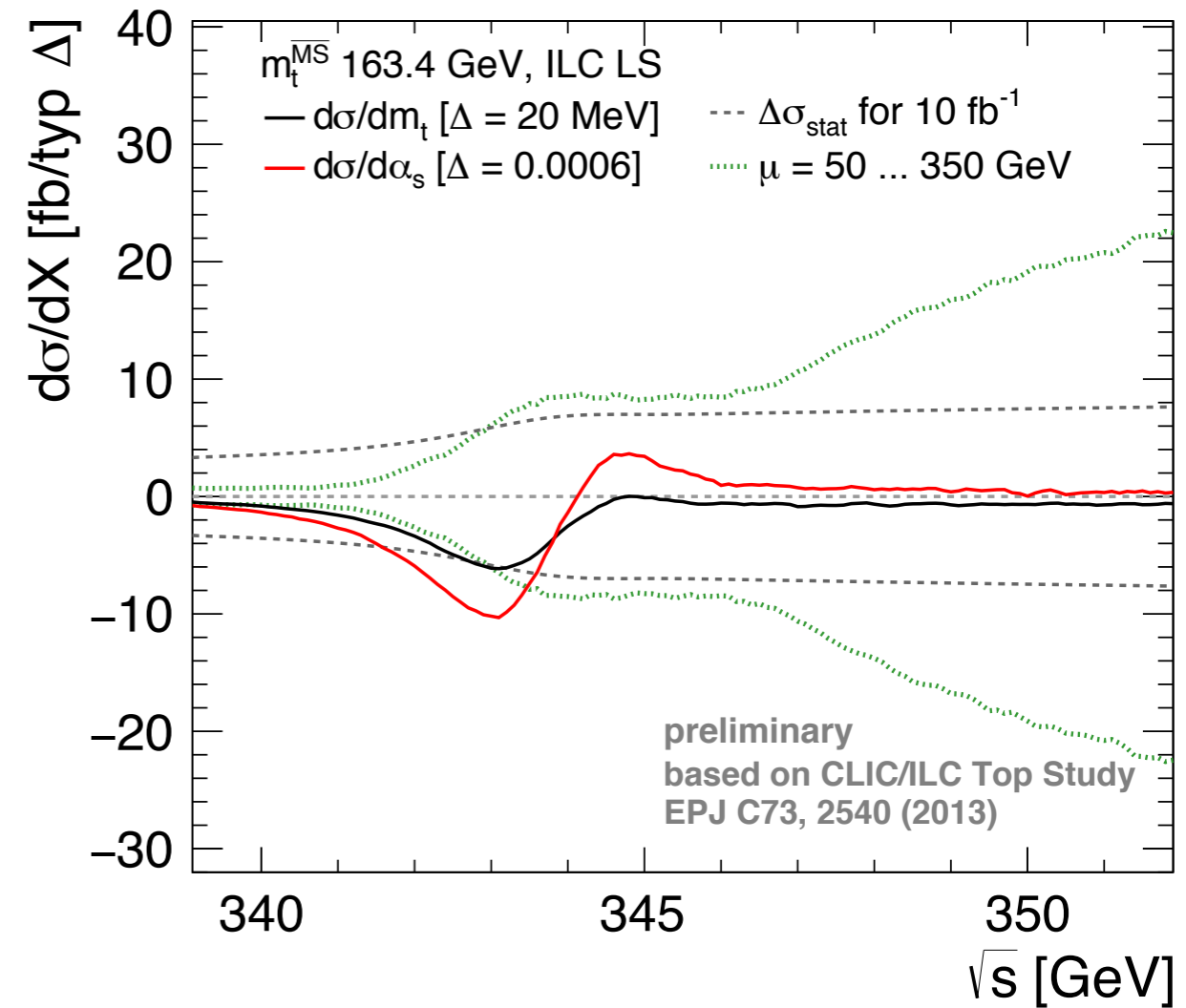
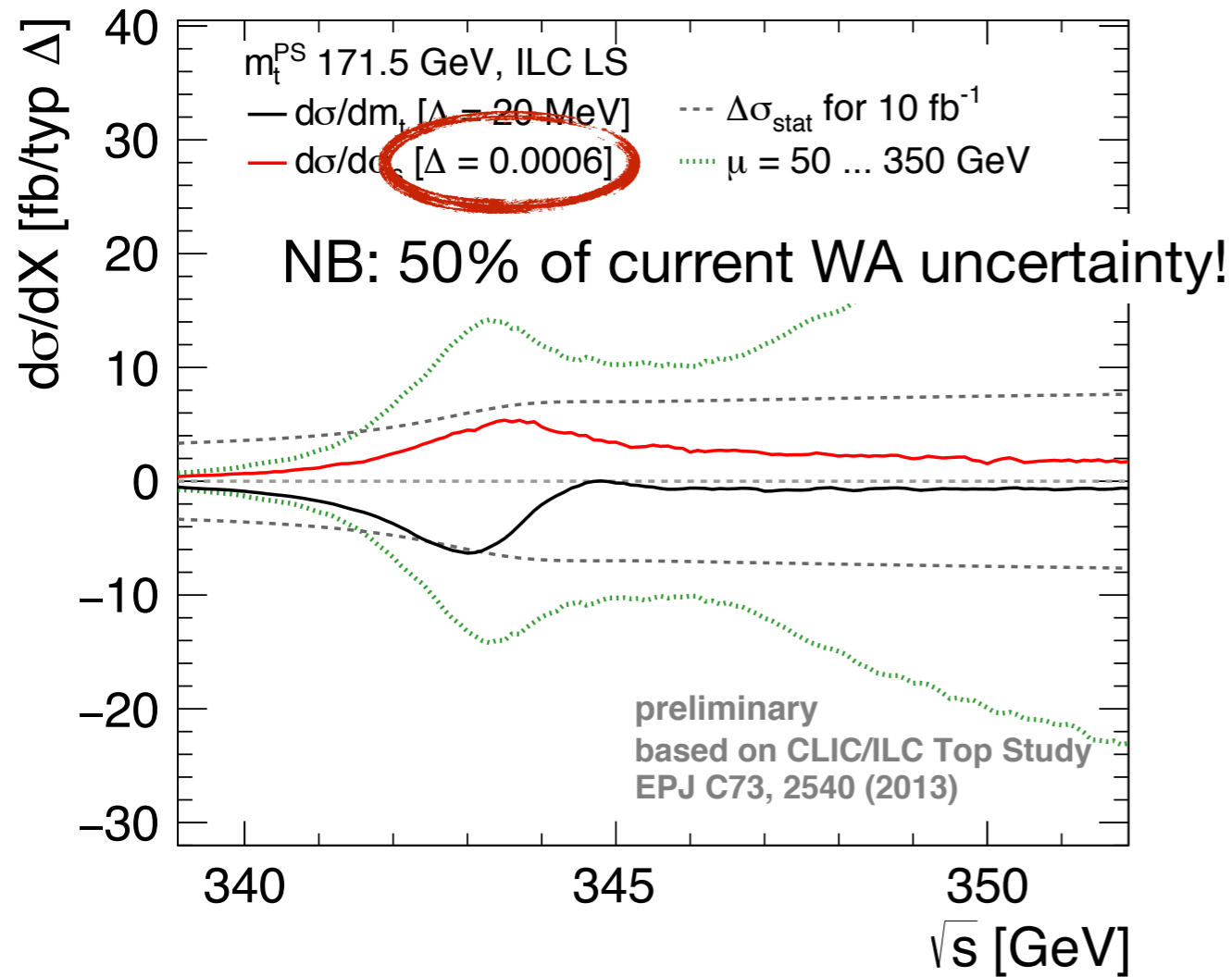
# Also seen in 2D Fits - Width



- Substantially reduced uncertainty on the mass
  - on the width all schemes result in similar uncertainties - with maybe a slight advantage for PS in 2D fit



# $\alpha_s$ - A follow-up to this morning's discussion



- MSbar scheme: Strong  $\alpha_s$  dependence, highly correlated with mass influence on cross-section -  $\alpha_s$  treated as external input / systematic in the fit:
  - PS - scheme:  $\Delta m_t = 2.6 \text{ MeV} / 10^{-4}$  uncertainty in  $\alpha_s$
  - MSbar - scheme:  $\Delta m_t = 5.3 \text{ MeV} / 10^{-4}$  uncertainty in  $\alpha_s$

# Comparing to the Current State of Theory

Experimental study:

- PS scheme:  $\Delta m_t = 2.6 \text{ MeV} / 10^{-4}$  uncertainty in  $\alpha_s$
- MSbar scheme:  $\Delta m_t = 5.3 \text{ MeV} / 10^{-4}$  uncertainty in  $\alpha_s$

Theory status on conversion (N<sup>4</sup>LO in QCD)

P. Marquard *et al.* PRL 114, 142002 (2015)

$$\frac{m_t(m_t)}{\text{GeV}} = 163.643 \pm 0.023 + 0.074\Delta_{\alpha_s} - 0.095\Delta_{m_t}^{\text{PS}}, \quad \Rightarrow 7.4 \text{ MeV} / 10^{-4} \text{ in } \alpha_s$$

$$\frac{m_t(m_t)}{\text{GeV}} = 163.643 \pm 0.007 + 0.069\Delta_{\alpha_s} - 0.096\Delta_{m_t}^{\text{1S}}, \quad \Rightarrow 6.9 \text{ MeV} / 10^{-4} \text{ in } \alpha_s$$

with  $\Delta\alpha_s$  in units of  $10^{-3}$  changes relative to default  $\alpha_s(m_Z)$

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with  $\Delta\alpha_s$  in units of  $10^{-3}$  changes relative to default  $\alpha_s(m_Z)$

$\Rightarrow$  Direct extraction of MSbar mass wins - also in terms of scale uncertainties ?

# Schemes: A Question to Theorists

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- Are there strong arguments one way or another to use a specific scheme?
- **1S** gives smaller scale uncertainties than **PS** - is this “physical”?
- Would a direct extraction of the  **$\overline{MS}$**  mass be preferred?

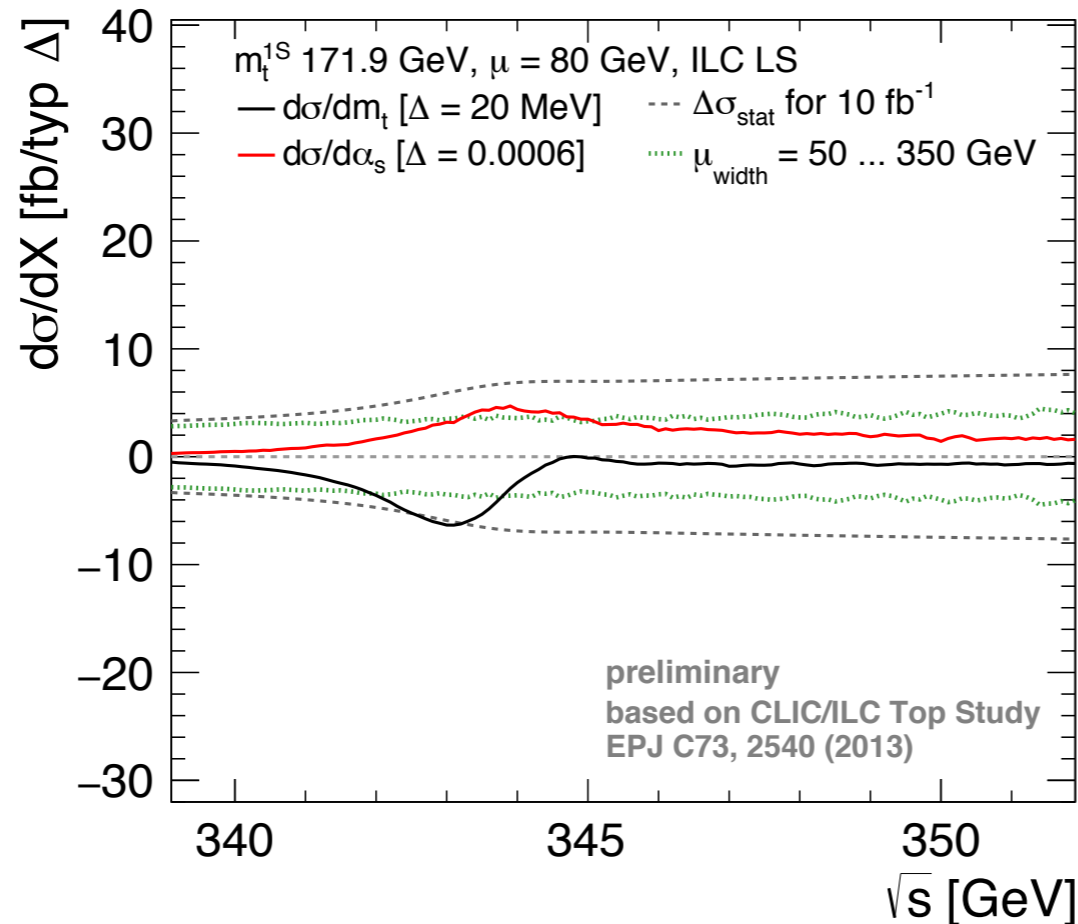
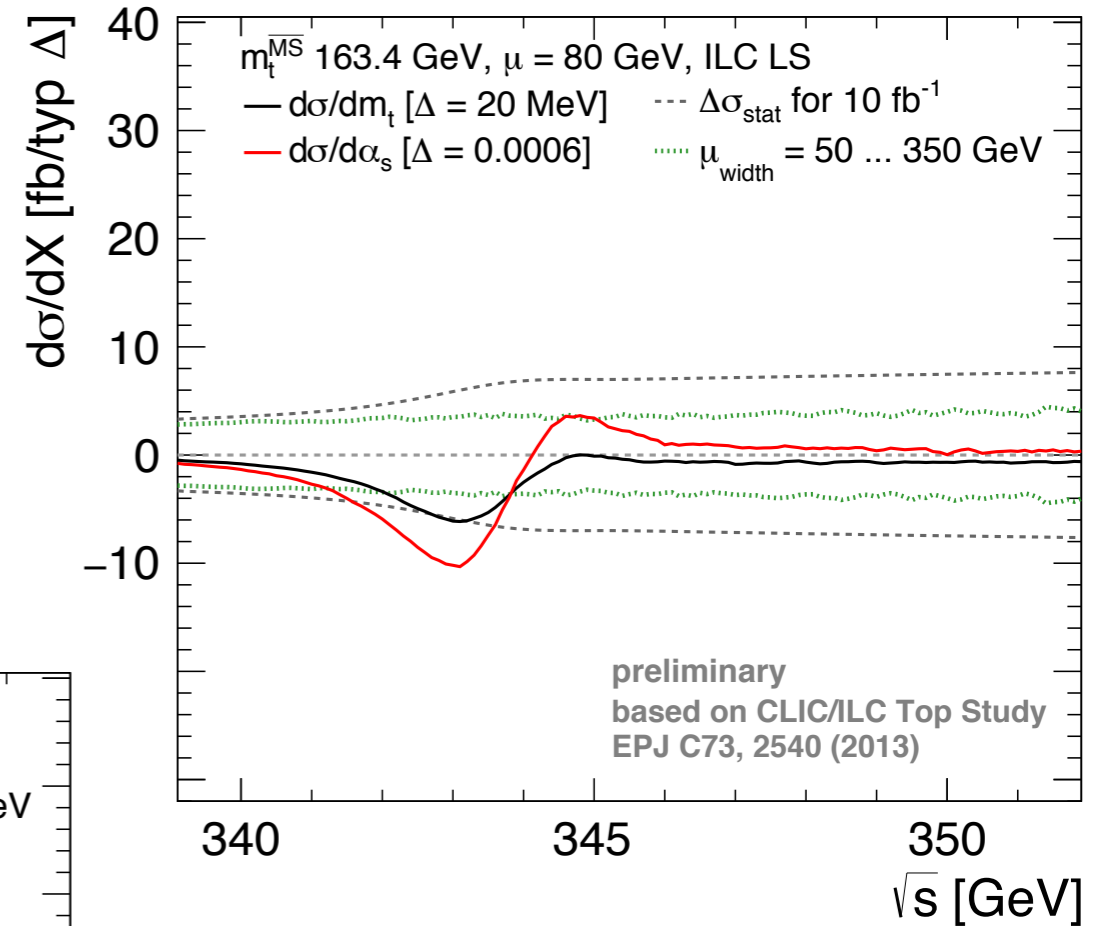
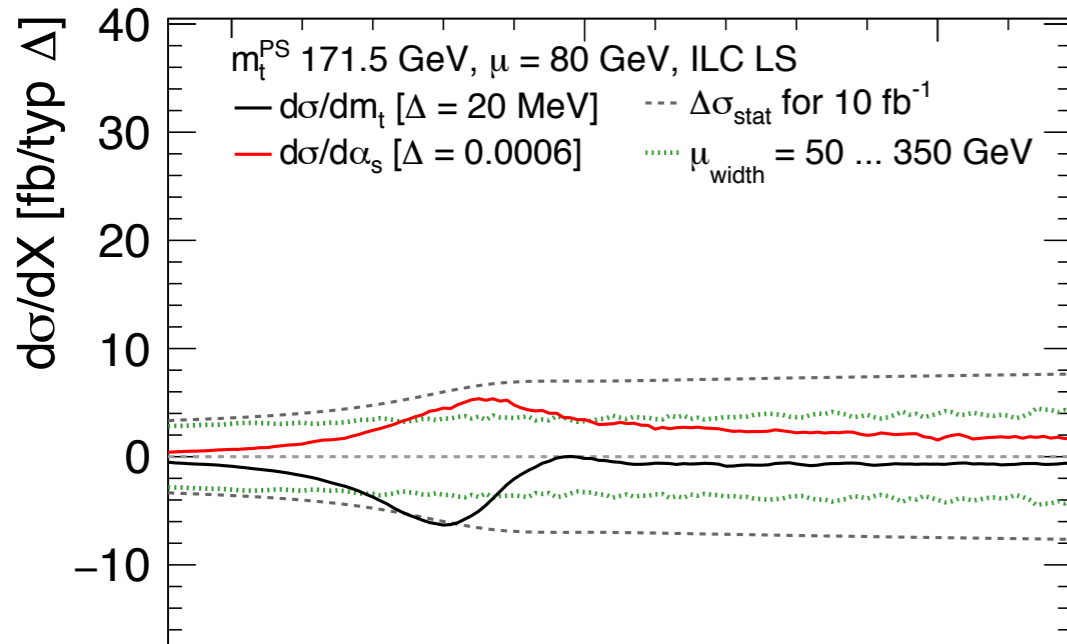


# Conclusions

- Extended threshold study to true 2D analysis of parameters
  - $m_t$  and  $\Gamma_t$  - 1D  $\Gamma_t$  resolution  $\sim 45$  MeV
  - $m_t$  and  $y_t$  - 1D  $y_t$  resolution  $\sim 0.067$
- In both cases: substantial increase of uncertainty due to correlations in 2D fit
  - potentially some optimization possible with specific selection of scan points
- FCCee results somewhat better, CLIC results somewhat worse, no excessive differences
- Further investigation of scale uncertainties
  - $\sim 40$  MeV (symmetrized) on  $m_t$
  - $\sim 60$  MeV (symmetrized) on  $\Gamma_t$
  - $\sim 0.13$  (symmetrized) on  $y_t$
- The size of the scale uncertainties (and with that the impact on the mass measurement) depends on the choice of the mass scheme -  $\Gamma_t$  less scheme dependent,  $y_t$  still to be studied

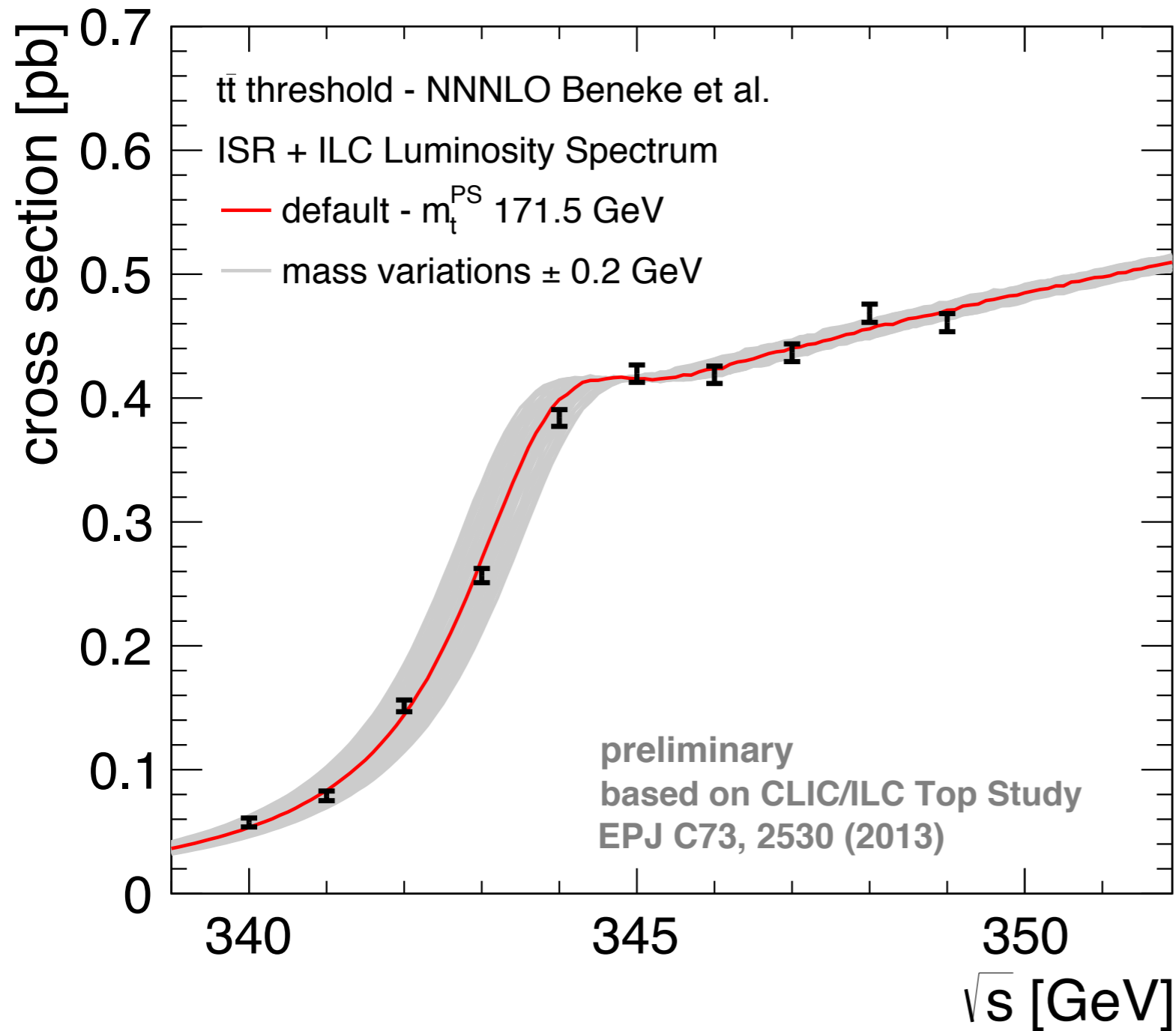
# Backup

# Another Scale Parameter: $\mu_{\text{width}}$



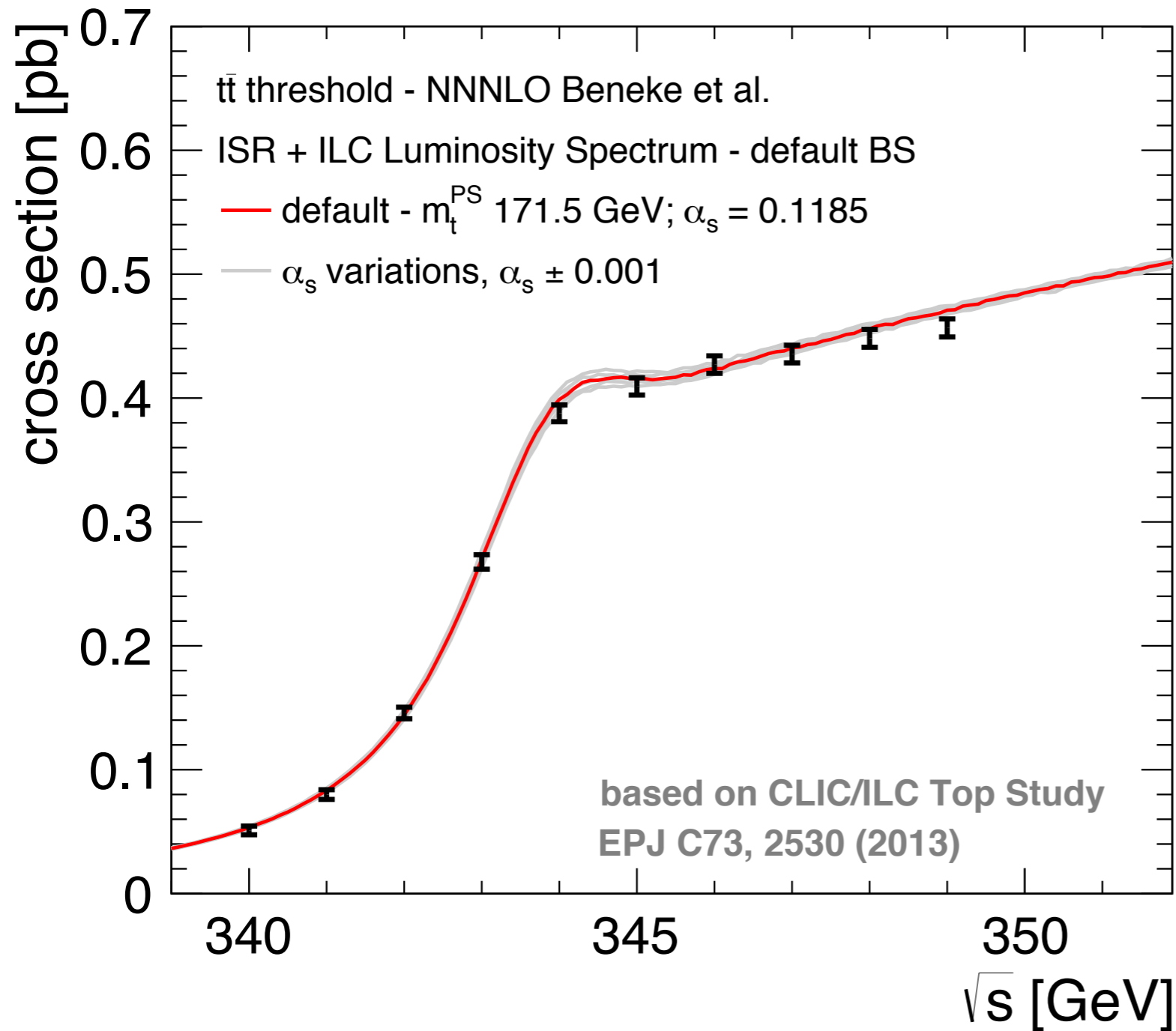
- Independent of mass scheme
- relatively small impact:  
 $< 20 \text{ MeV}$  uncertainty on mass

# Threshold Scan - Sensitivity to Mass Variations



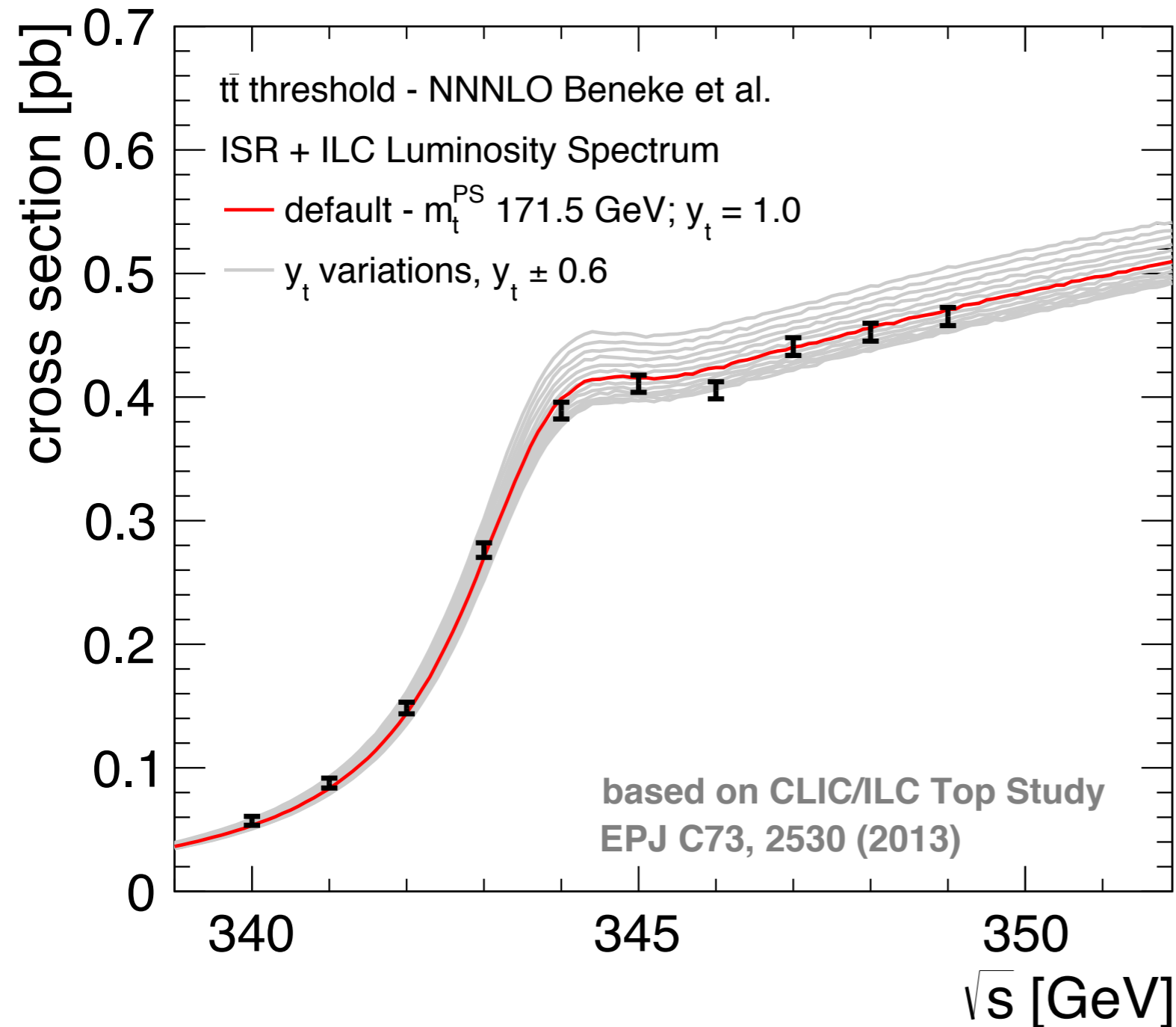
The assumption:  
 $10 \times 10 \text{ fb}^{-1}$ , points spaced by  
1 GeV from 340 to 349 GeV

# Threshold Scan - Sensitivity to $\alpha_s$ Variations



The assumption:  
 $10 \times 10 \text{ fb}^{-1}$ , points spaced by  
1 GeV from 340 to 349 GeV

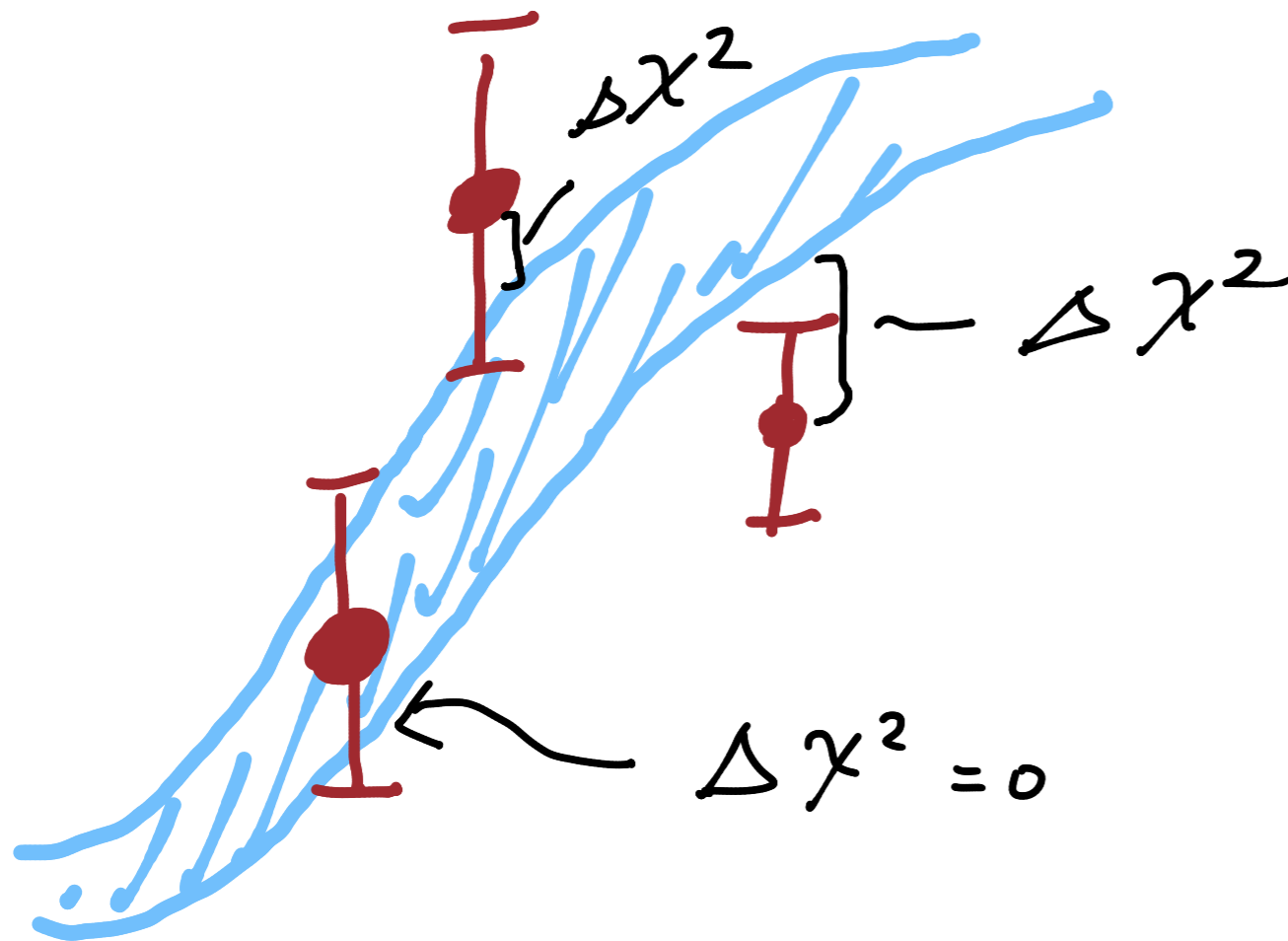
# Threshold Scan - Sensitivity to Yukawa Variations



The assumption:  
 $10 \times 10 \text{ fb}^{-1}$ , points spaced by  
1 GeV from 340 to 349 GeV

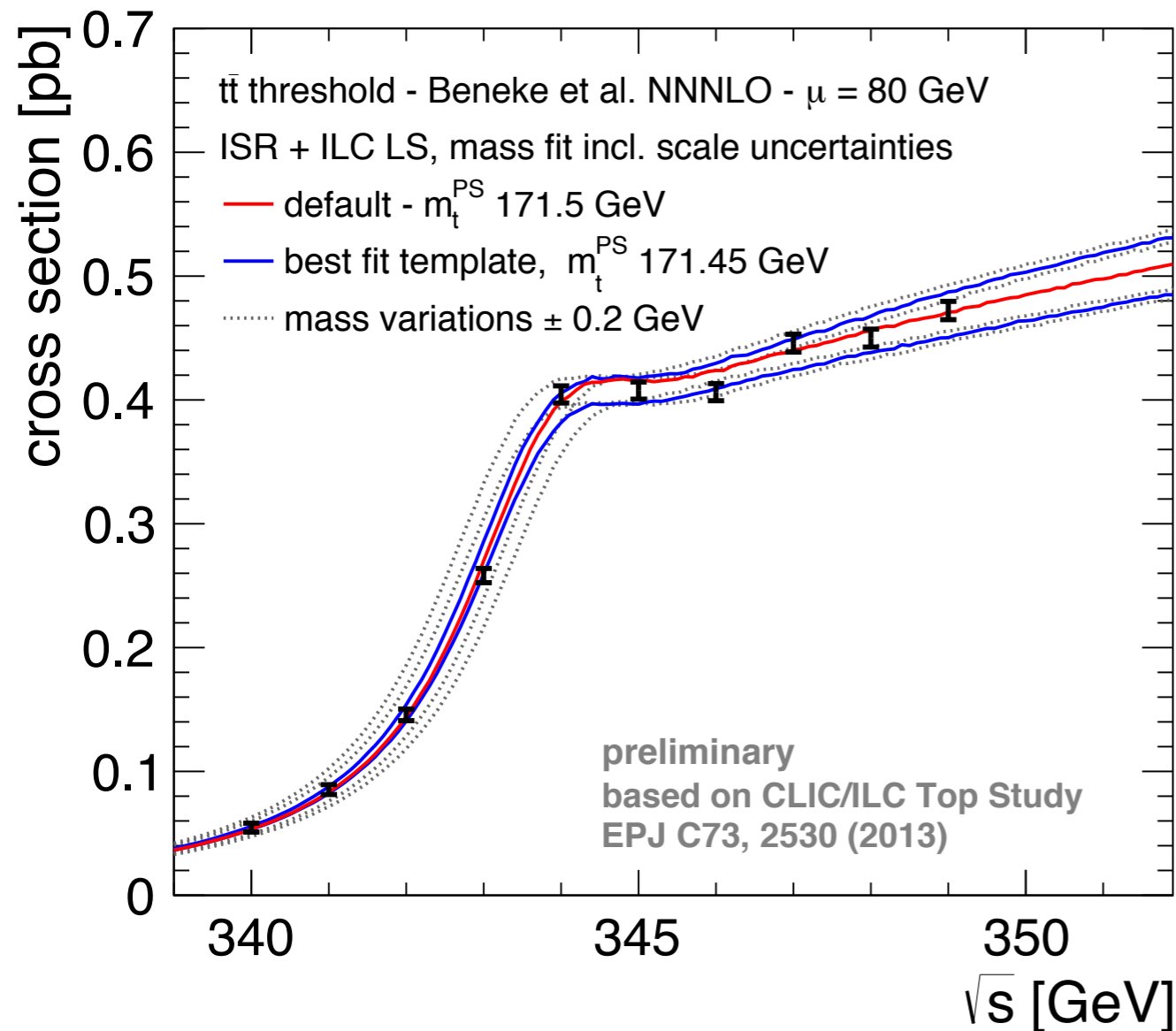
# Mass at Threshold: Fitting with Scale Uncertainties

- The underlying assumption made here: The range of cross section predictions given by the scale variations reflects the uncertainty of the calculation, within the “band” given by the range of variations all values are assumed to be equally probable



- The fit procedure: A template fit: Theoretical expectations for different top masses, for each the  $\chi^2$  of the data points is calculated, the mass is determined by a parabolic fit to the  $\chi^2$  distribution as a function of template mass
- ⇒ When including scale uncertainties in the templates, the  $\chi^2$  contribution of data points within the band is zero

# Mass at Threshold: Fitting with Scale Uncertainties



- Since the default  $\mu$  gives a cross section close to the maximum over the most relevant energy range, the fit with bands results in a (trivial) bias of 40 MeV
- The fit uncertainty is determined on an “event-by-event” basis from the  $\chi^2$  distribution taking the variations in mass required to get the minimum  $\chi^2 + 1$  (or  $\chi^2 = 1$  in cases where the parabolic fit of the  $\chi^2$  distribution gives a minimum  $< 0$ )