Workshop on top physics at the LC 2017

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CERN

NNLO non-resonant and other electroweak corrections at threshold

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Outline

I Top-pair production at linear colliders near threshold - QCD and EW corrections

II Non-resonant contributions @NLO

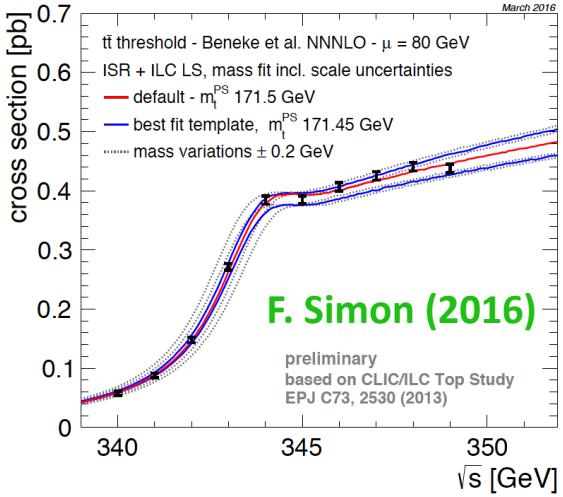
III Non-resonant contributions @NNLO

IV Results

V Summary

Top-pair production near threshold

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \simeq 350\ \text{GeV}$ will produce lots of $t\bar{t}$ pairs, allowing for a threshold scan of the top cross section



$\hookrightarrow$ Precise determination of the top mass m_t,
the width Γ_t and the Yukawa coupling λ_t

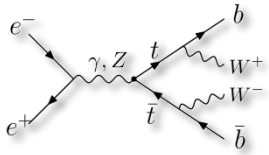
$$\rightarrow \quad \delta m_t \simeq \pm 45\ \text{MeV}$$

$\rightsquigarrow$ m_t is a crucial input for electroweak precision observables!

Requires also precise theoretical prediction $\Rightarrow$ $\delta\sigma/\sigma \sim 3\%$

QCD corrections are known up to NNLL/NNNLO, but electroweak contributions due to top decay not fully known at NNLO

Note! once EW effects are turned on, the physical final state is $W^+W^-b\bar{b}$



$\Rightarrow$ $\sigma(e^+e^- \to W^+W^-b\bar{b})$ in the $t\bar{t}$ threshold region

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5\ \text{GeV} \gg \Lambda_{\text{QCD}}$
$\Rightarrow t\bar{t}$ is perturbative at threshold

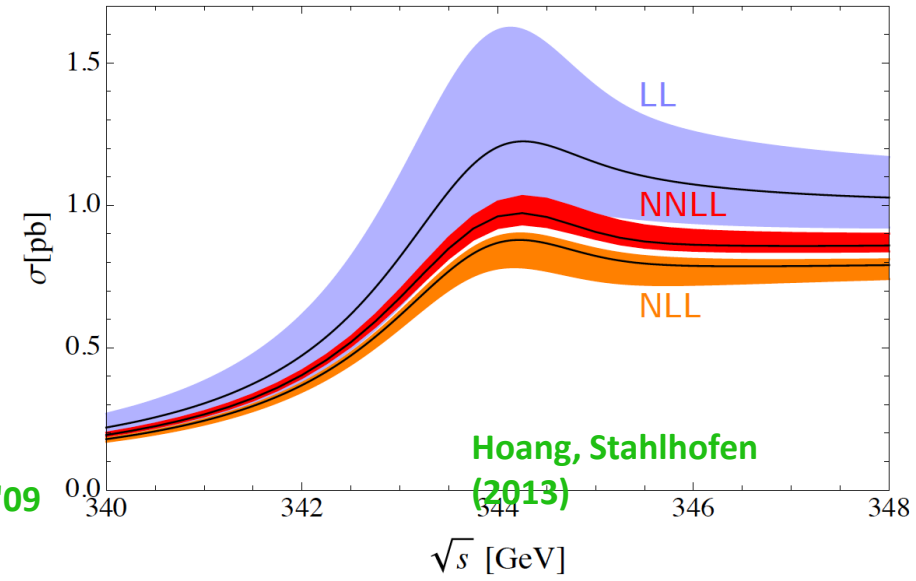
Top quarks move slowly near threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

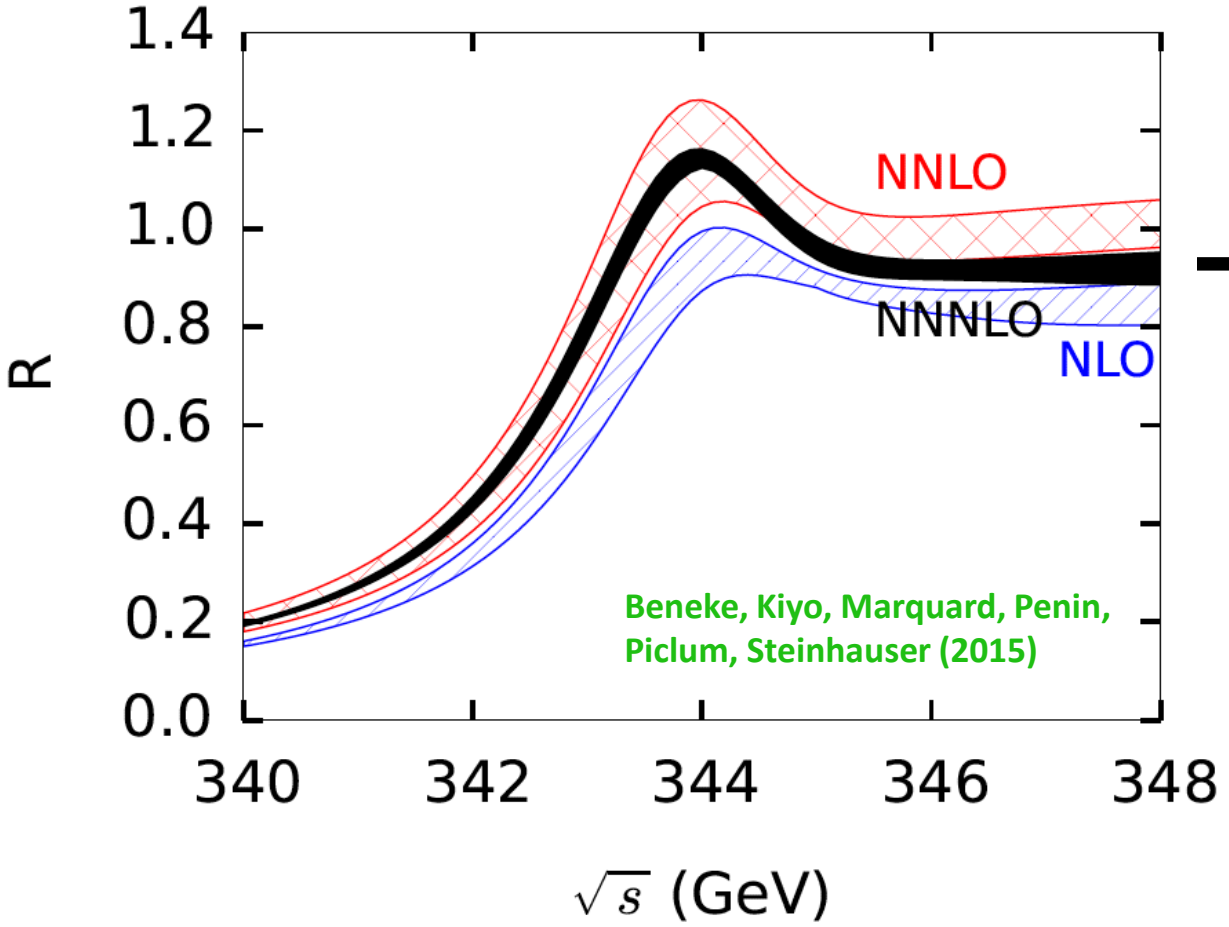
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders Fadin, Khoze '87-88 Strassler, Peskin '91 $\rightarrow$ **NRQCD**

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \ldots\right)$$

RG improvement by summing also $(\alpha_s \ln v)^m$: LL, NLL, ... $\rightarrow$ **vNRQCD/pNRQCD**

- NNLO
  Hoang, Teubner '98-99; Melnikov, Yelkhovsky '98
  Yakovlev ´98; Beneke, Signer, Smirnov '99
  Nagano, Ota, Sumino '99; Penin, Pivovarov '98-99
- NNLL
  Hoang, Manohar, Stewart, Teubner '00-01
  Hoang ´03; Pineda, Signer '06
  Hoang, Stahlhofen '06-13
- NNNLO
  Beneke, Kiyo, Schuller '05-08; Anzai, Kiyo, Sumino '09
  Smirnov, Smirnov, Steinhauser '08-10
  Marquard, Piclum, Seidel, Steinhauser '06-14

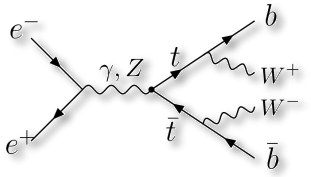




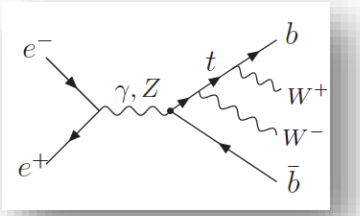
$\delta\sigma_{t\bar{t}}/\sigma_{t\bar{t}} \sim \pm 3\%$

- QQbar_threshold ⟶ *see A. Maier's talk*
- EW effects beyond LO and specially non-resonant effects give contributions at the level of the QCD uncertainty

- **Resonant contributions** to $e^+e^- \to W^+W^-b\bar{b}$ top and antitop close to the mass-shell

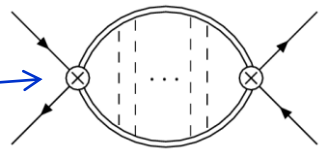


- Non-resonant (hard) corrections: account for the production of the bW pairs by highly virtual tops or diagrams with only one or no top

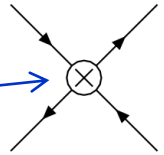


- power counting for finite width effects: $\frac{\Gamma_t}{m_t} \sim \alpha_{\rm EW} \sim \alpha_s^2 \sim v^2 \ll 1$

Effective field theory (EFT) for pair production of unstable particles near threshold, based on separation of resonant and nonresonant fluctuations

$t\bar{t}$ production operators $\mathcal{O}_p^{(k)}$ → 

resonant contributions

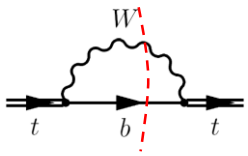
4-electron operators $\mathcal{O}_{4e}^{(k)}$ → 

non-resonant contributions

→ Extract cross section for $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude

Electroweak effects at LO

Fadin, Khoze (1987)



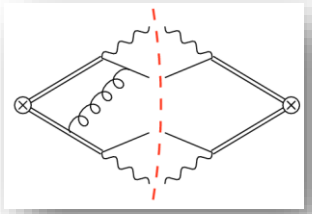
- Replacement rule: $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$

  $\Rightarrow$ unstable top propagator $\dfrac{i}{p^0 - \mathbf{p}^2/(2m) + i\Gamma_t/2}$ $\longleftarrow$ $\delta\mathcal{L} = \sum_{\mathbf{p}} \psi^\dagger_{\mathbf{p}}\, i\,\dfrac{\Gamma_t}{2}\,\psi_{\mathbf{p}}$

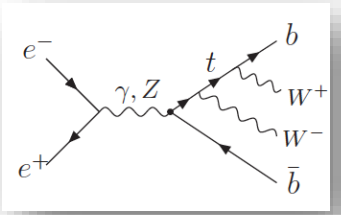
Electroweak effects at NLO

- Exchange of "Coulomb photon": trivially extension of QCD corrections

- Gluon exchange involving the bottom quarks in the final state $\Rightarrow$ these contributions vanish at NLO for the total cross section, Fadin, Khoze, Martin; Melnikov, Yakovlev (1994)
  also negligible if loose top invariant-mass cuts are applied; remains true at **NNLO** Hoang, Reisser (2005); Beneke, Jantzen, RF (2010)



- Non-resonant corrections to $e^+e^- \to W^+W^-b\bar{b}$ which account for the production of the Wb pairs by highly virtual tops or with only one or no top

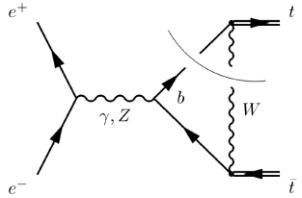


$\hookrightarrow$ $\Delta\sigma_{\text{non-res}} = \dfrac{1}{s}\sum_k \text{Im}\left[C^{(k)}_{4e}\right]\langle e^+e^-|\mathcal{O}^{(k)}_{4e}|e^+e^-\rangle$

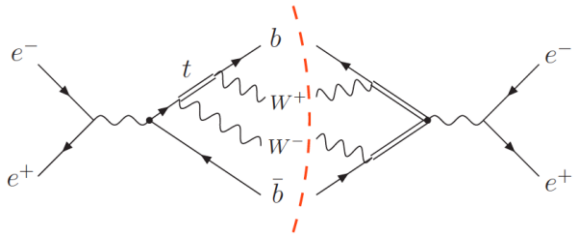
Beneke, Jantzen, RF (2010)

Electroweak effects at NNLO

- absorptive parts in the 1-loop matching coeffs. of the production currents (arising from bW cuts) Hoang, Reisser (2006)



$\Rightarrow$ reproduce interferences between double and single top (resonant) amplitudes



$$j_v = c_v(\mu)\,\phi^\dagger \sigma^i \chi$$

$$c_v = 1 + c_v^{(1)}\left(\frac{\alpha_s}{4\pi}\right) + c_v^{(2)}\left(\frac{\alpha_s}{4\pi}\right)^2 + iC_{bW,\text{abs}} + C_{\text{EW}} + \dots$$

- real part of hard one-loop EW corrections Guth, Kuhn (1992); Hoang, Reisser (2006)
- Higgs exchange, short-distance and potential contributions (N^{3}LO)
  Strassler, Peskin (1991); Beneke, Maier, Piclum, Rauh (2015)
- initial state QED radiation: usually taken into account in exp. studies. Produces big effects, analysis at NLL is needed ($e^\pm$ structure functions) *w.i.p*
- **NNLO non-resonant** contributions (**gluon corrections** to NLO ones) Partial results available Hoang, Reisser, PRF (2010); Jantzen, PRF (2013); PRF (2014) → *this talk*

II. Non-resonant NLO contributions

Non-resonant NLO contributions

Beneke, Jantzen , RF (2010)

$\Rightarrow$ cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:

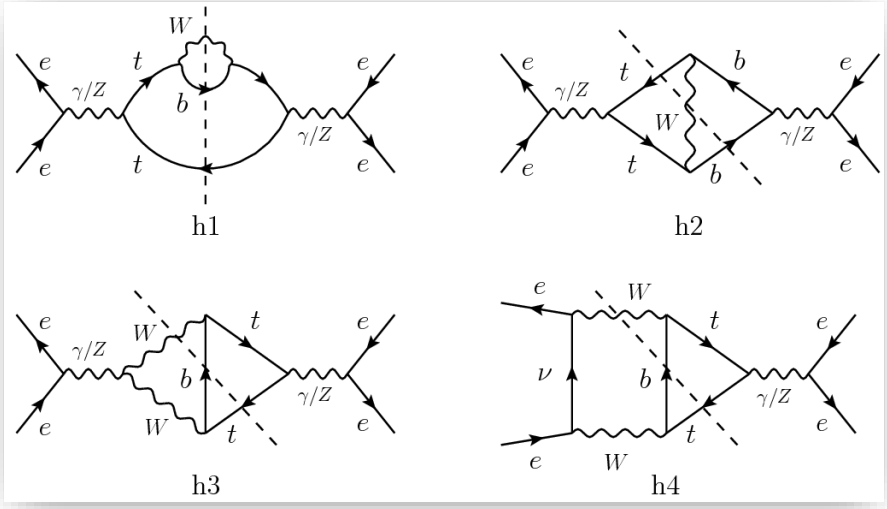
  $p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \gg \Sigma(p_t^2) \sim m_t^2 \alpha_{\text{EW}}$

  $\rightarrow \Gamma_t = 0$

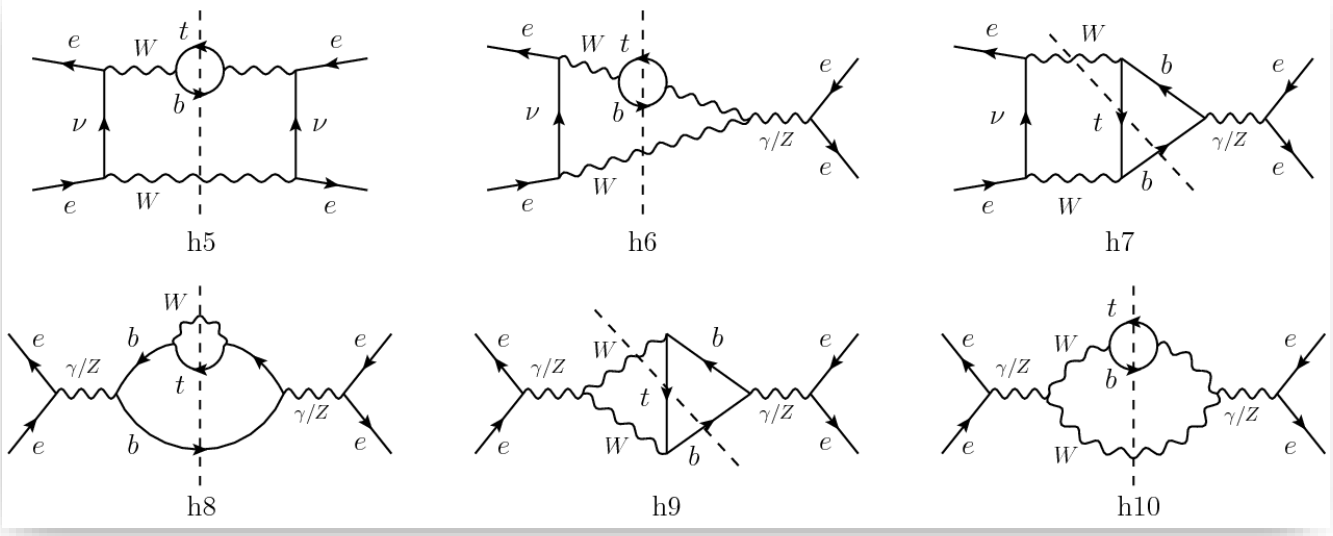
- suppressed w.r.t. LO $(\sim v)$ by

  $\alpha_{\text{EW}}/v \sim \alpha_s$

bW^+ from highly virtual top



bW^+ without intermediate top



In terms of the invariant mass of the bW^+ system, $p_t^2 = (p_b + p_{W^+})^2$, ($p_t \to$ also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section

Applying top invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

$\hookrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (corresponding to $\Delta M_t \gg \Gamma_t$)

$\rightsquigarrow$ cut has no effect in the resonant contributions

[In contrast: for tight cuts with $\Lambda^2 \sim m_t\Gamma_t$ ($\Delta M_t \sim \Gamma_t$), non-resonant contributions vanish and cuts only affect the resonant contributions]

III. Non-resonant NNLO contributions

Finite-width divergences in the resonant contributions

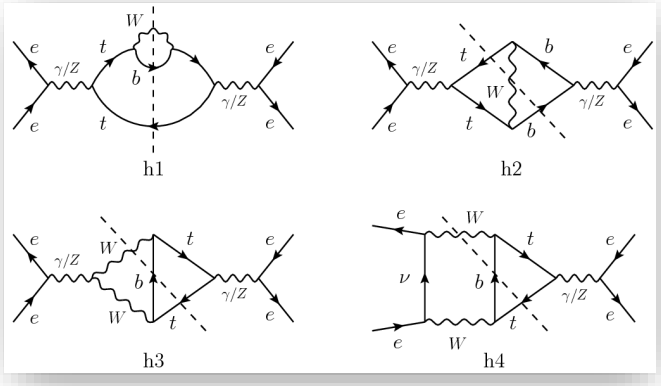
Resonant contributions obtained by assuming the top quarks are nearly on-shell (potential), but integrated over all momenta

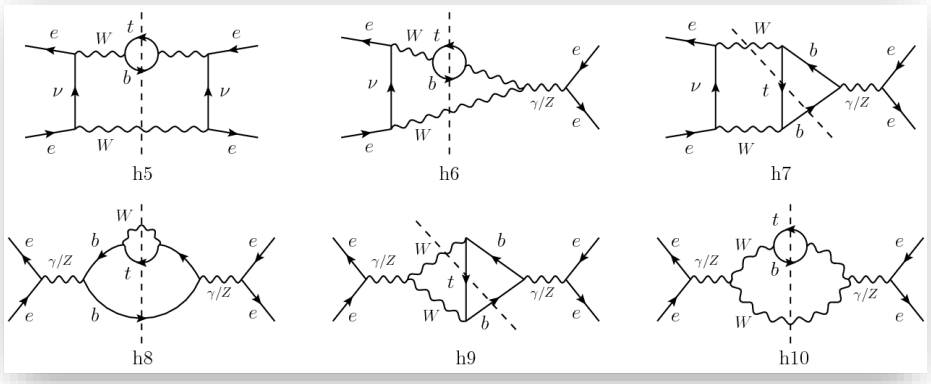
→ uncancelled **UV-singularity** from hard momenta: $\Delta\sigma^{\text{NNLO}} \sim m_t^2 \, \frac{\alpha_s \Gamma_t}{\epsilon}$

Related to finite top width in EFT cut propagator

- for stable top $\to \pi\,\delta\big(p^0 - \frac{\vec{p}^{\,2}}{2m_t}\big)$,
- for unstable top $\to \frac{\Gamma_t/2}{(p^0 - \vec{p}^{\,2}/2m_t)^2 + (\Gamma_t/2)^2}$ Breit–Wigner, UV-behaviour changed!

These divergences must cancel with non-resonant NNLO terms, which arise from **gluon corrections** to NLO non-resonant diagrams h1-h10

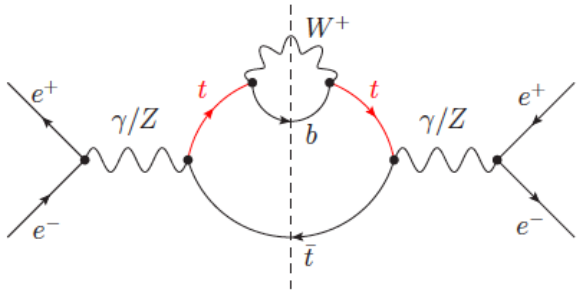




Endpoint divergences in the non-resonant contributions

Endpoint divergences of the phase-space integration at $p_t^2 \to m_t^2$ (because $\Gamma_t = 0$ here):

NLO:

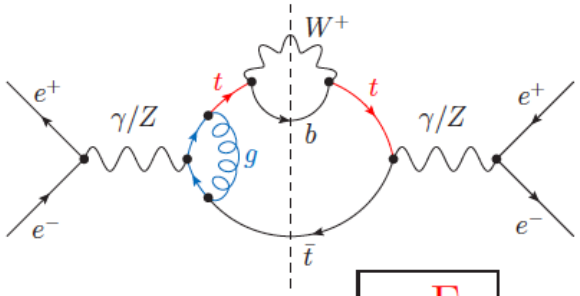


$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+\epsilon}} \quad \text{with } n = \tfrac{3}{2}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence <u>finite</u> in dim. reg.:

$$\int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{\frac{3}{2}+\epsilon}} = -\frac{2}{\Lambda} + \mathcal{O}(\epsilon)$$

NNLO:



$$\sim \int^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{n+a\epsilon}} \quad \text{with } n = 2, \tfrac{3}{2}, \mathbf{1}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ endpoint divergence $\propto \boxed{\alpha_s \frac{\Gamma_t}{\epsilon}}$ from $n = 1$:

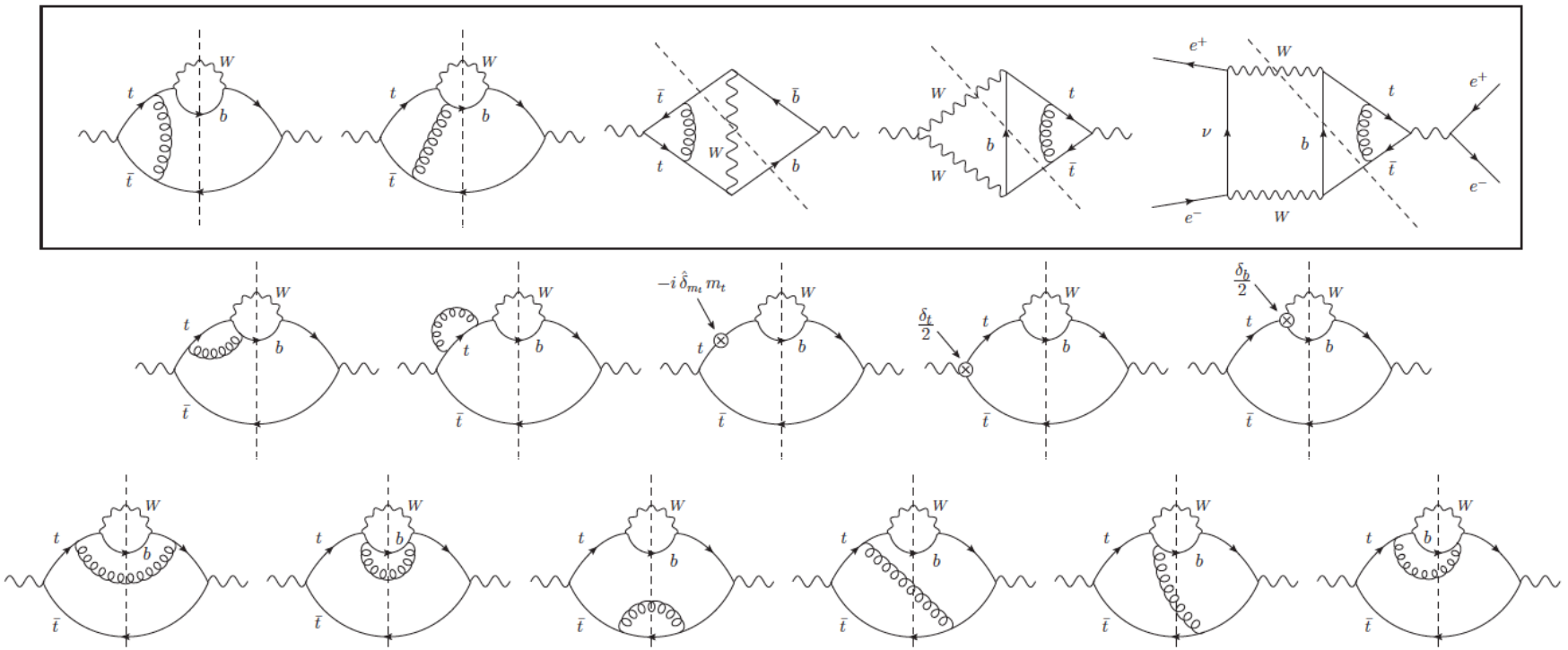
$$\mu^{4\epsilon} \int_{m_t^2-\Lambda^2}^{m_t^2} \frac{\mathrm{d}p_t^2}{(m_t^2 - p_t^2)^{1+2\epsilon}} = -\frac{1}{2\epsilon} + \ln\frac{\Lambda^2}{\mu^2} + \mathcal{O}(\epsilon)$$

Expand integrand in $(m_t^2 - p_t^2)/m_t^2$ $\iff$ asymptotic expansion of result in Λ/m_t

Leading terms in the expansion for $p_t^2 \to m_t^2$ of the integrands obtained using the expansion by regions approach

Jantzen, PRF (2013)

~100 diagrams, but only 15% of them contain endpoint divergences



boxed diagrams $\rightsquigarrow$ endpoint-singular $\dfrac{1}{\epsilon} - 2\ln\dfrac{\Lambda^2}{\mu^2}$ terms from potential gluons

$+$ "finite" endpoint-divergent $\dfrac{m_t}{\Lambda}$ & $\dfrac{m_t^2}{\Lambda^2}$ terms from hard & potential gluons

Computing non-resonant NNLO contributions

The integrands g_{ix} of the endpoint divergent contributions regulated dimensionally → loop integrals cannot be expanded in $d-4$

Expressions for scalar one-loop integrals in general *d*-dim (up to four external legs) are available, but their phase-space integration becomes complicated

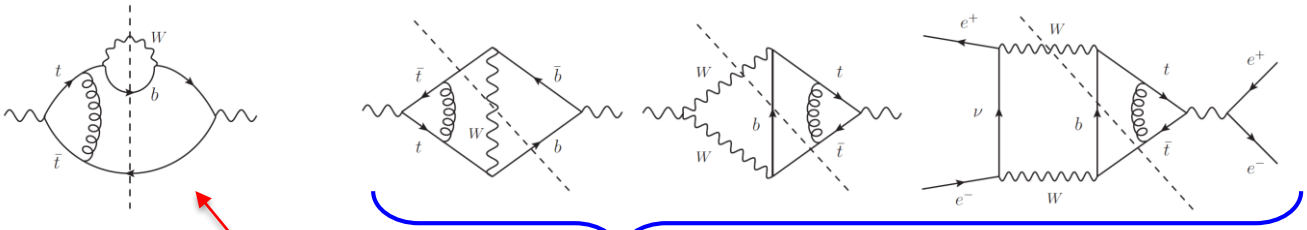
Different strategy used: use endpoint divergent terms obtained in **Jantzen, PRF '13** as subtractions to the integrands

$$t \equiv p_t^2/m_t^2 \quad , \quad y \equiv 1-\Lambda^2/m_t^2$$

$$\int_y^1 dt\, g_{ix}(t) = \int_y^1 dt \left[g_{ix}(t) - \sum_{a=1,\frac{3}{2},2} \sum_b \frac{\hat{g}_{ix}^{(a,b)}}{(1-t)^{a+b\epsilon}} \right] + \sum_{a=1,\frac{3}{2},2} \sum_b \frac{\hat{g}_{ix}^{(a,b)}(1-y)^{1-a-b\epsilon}}{1-a-b\epsilon}$$

$\mathcal{O}((1-t)^{-\frac{1}{2}}) \rightarrow t$-integration finite,
integrand can be expanded in ϵ

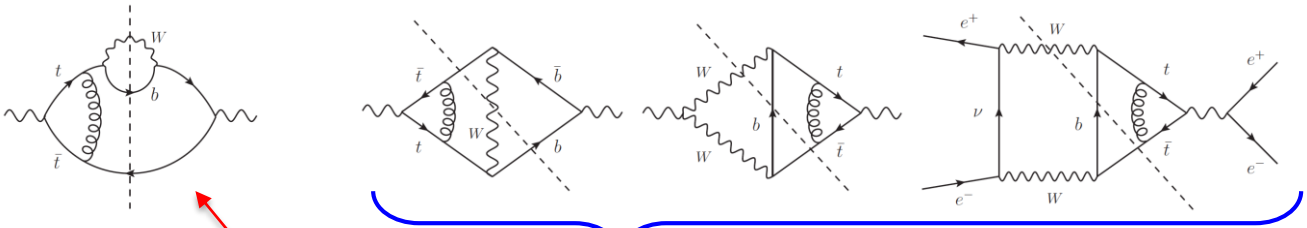
For the rest of contributions (endpoint finite diagrams) we use MadGraph



$$\sigma^{\mathrm{NNLO}}_{\mathrm{non-res}} = \sigma_{\mathrm{sq}} + \sigma_{\mathrm{int}} + \sigma_{\texttt{MadG}}$$

$$= \underbrace{\sigma_{\mathrm{sq}} + \sigma^{(\mathrm{EPdiv})}_{\mathrm{int}}}_{\text{UV and IR finite, but endpoint divergent}} + \underbrace{\left[\sigma^{(\mathrm{EPfin})}_{\mathrm{int}} + \sigma_{\texttt{MadG}}\right]}_{\text{finite}}$$

$\rightarrow$ cancels against $\frac{\alpha_s \Gamma_t}{\epsilon}$ in resonant part

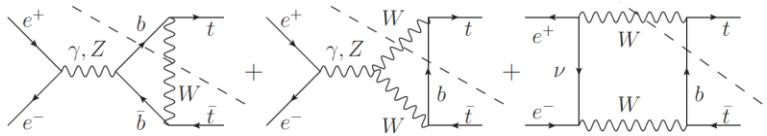


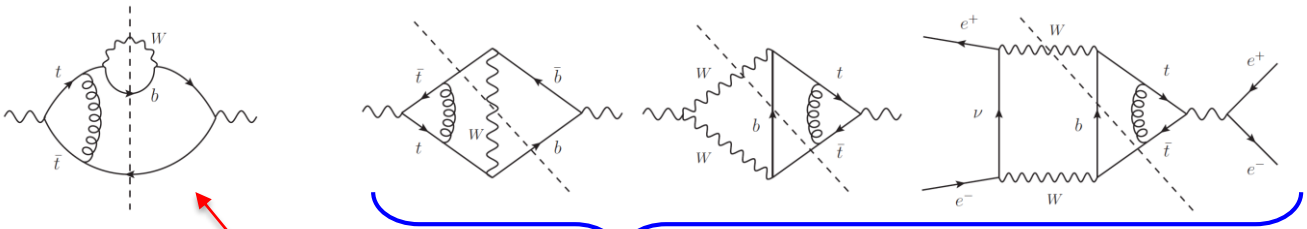
$$\sigma^{\text{NNLO}}_{\text{non-res}} = \sigma_{\text{sq}} + \sigma_{\text{int}} + \sigma_{\texttt{MadG}}$$

$$= \underbrace{\sigma_{\text{sq}} + \sigma^{(\text{EPdiv})}_{\text{int}}}_{\text{UV and IR finite, but endpoint divergent}} + \underbrace{\left[\sigma^{(\text{EPfin})}_{\text{int}} + \sigma_{\texttt{MadG}}\right]}_{\text{finite}}$$

$\rightarrow$ cancels against $\frac{\alpha_s \Gamma_t}{\epsilon}$ in resonant part

$$\sigma^{\text{NNLO}}_{\text{res}} = \sigma_{\text{res,rest}} + \sigma_{C^{(k)}_{\text{Abs,bare}}}$$





$$\sigma^{\rm NNLO}_{\rm non-res} = \sigma_{\rm sq} + \sigma_{\rm int} + \sigma_{\texttt{MadG}}$$

$$= \sigma_{\rm sq} + \sigma^{\rm (EPdiv)}_{\rm int} + \left[\sigma^{\rm (EPfin)}_{\rm int} + \sigma_{\texttt{MadG}}\right]$$

$$\sigma^{\rm NNLO}_{\rm res} = \sigma_{\rm res,rest} + \sigma_{C^{(k)}_{\rm Abs,bare}}$$

part (III)

part (I)

part (II)

$\Longrightarrow$ each of the finite parts can be evaluated in a different computational scheme:

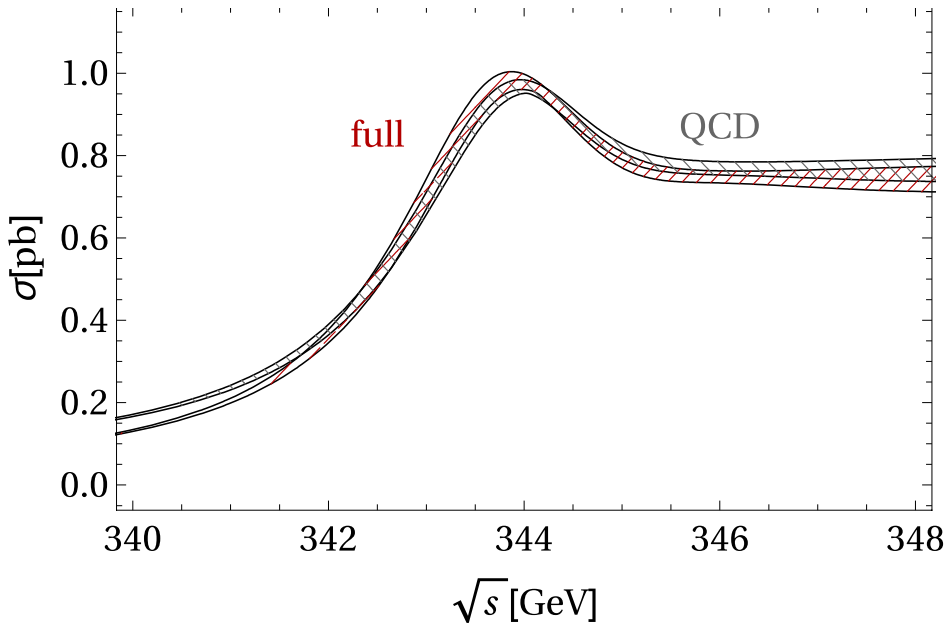
(I): in d-dim. and NDR scheme for γ_5

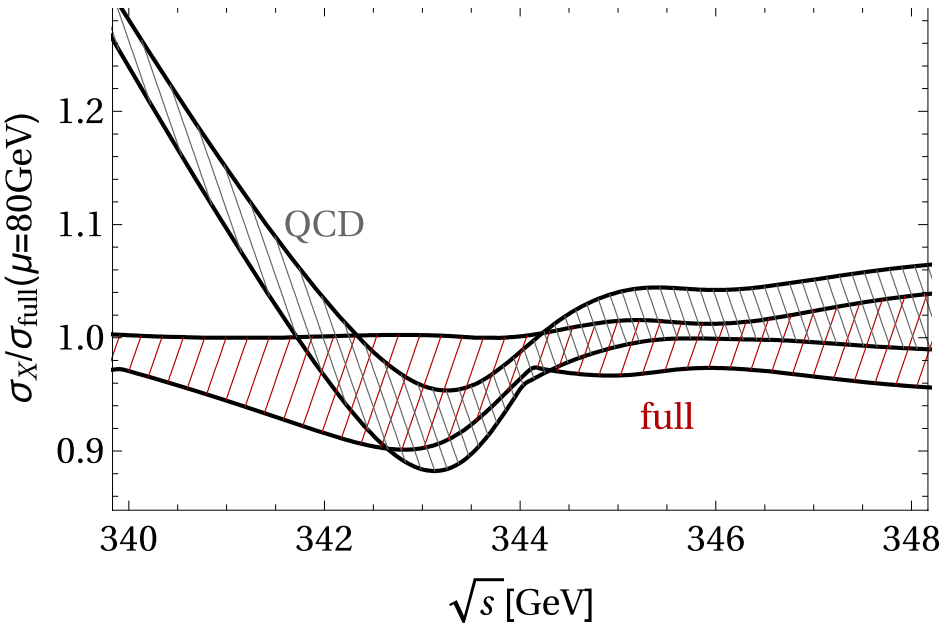
(II): perform Dirac algebra and inner loop in 4 dim., NDR scheme for γ_5

(III): scheme fixed by `MadGraph` conventions ($\sigma_{\texttt{MadG}}$ is UV divergent)

IV. Results

Inclusive top-pair production cross section





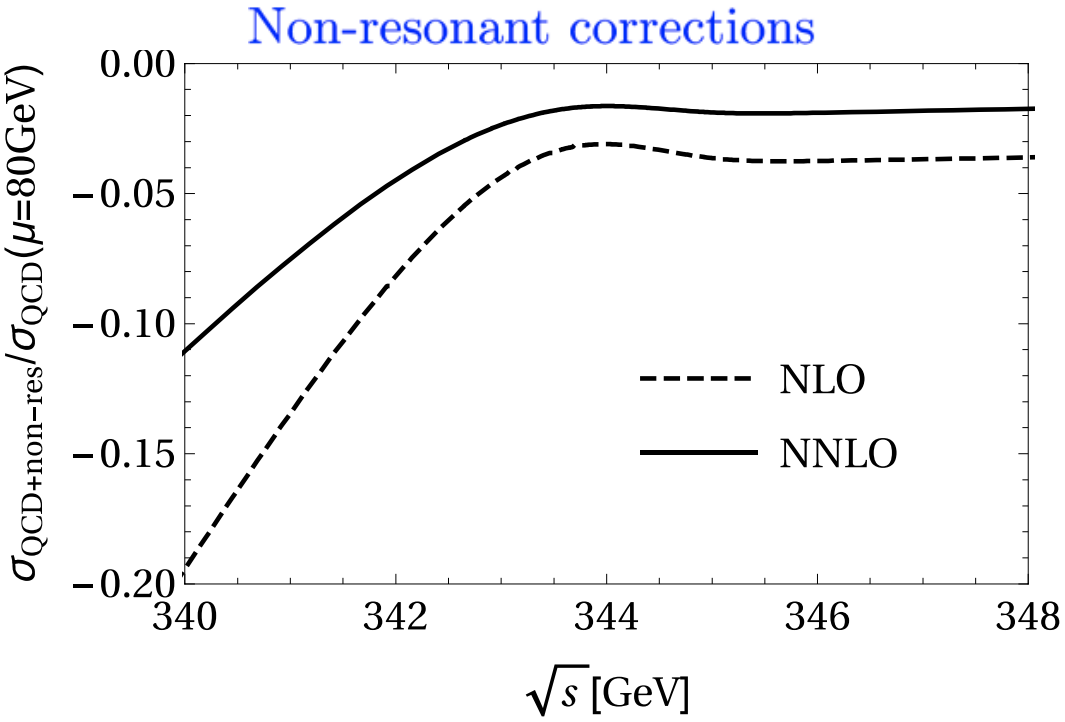
$m_t^{\rm PS} = 171.5$ GeV

$\mu_r = 50 - 350$ GeV (renormalization scale of α_s)

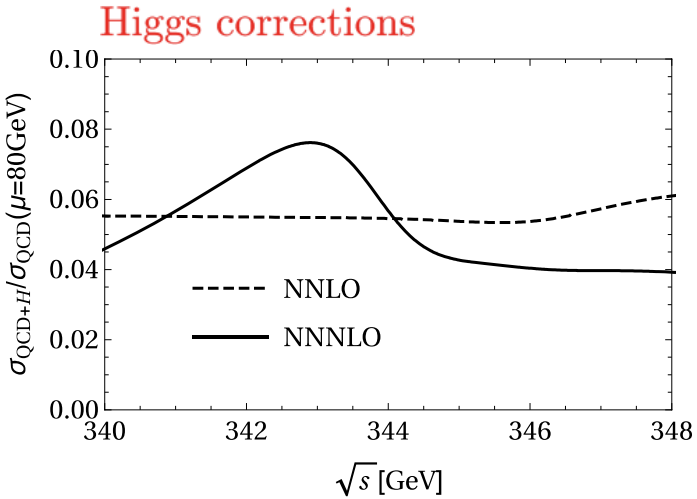
$\mu_w = 350$ GeV (scale associated to separation of resonant and nonresonant)

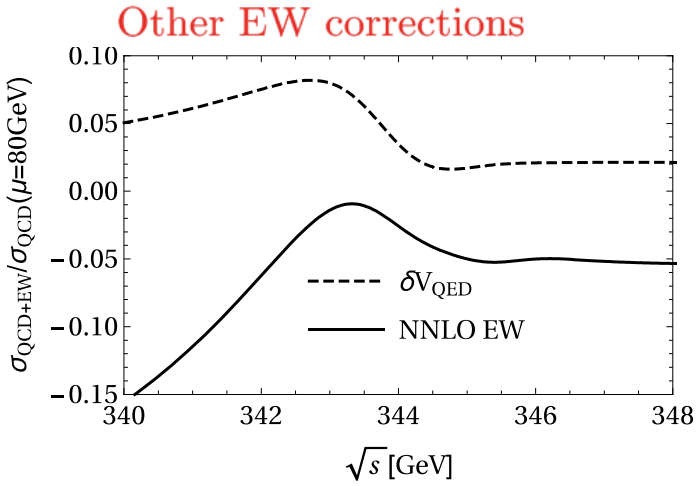
- Uncertainty band somewhat increased by EW effects: $\pm 5\%$ directly below the peak, $\pm 3\%$ in the rest. Due to Higgs potential insertion **Beneke, Maier, Piclum, Rauh (2015)**
- Largest effect of EW corrections below the peak, dominated by abs. parts of matching coeffs. and non-resonant

Size of non-resonant and other non-QCD corrections



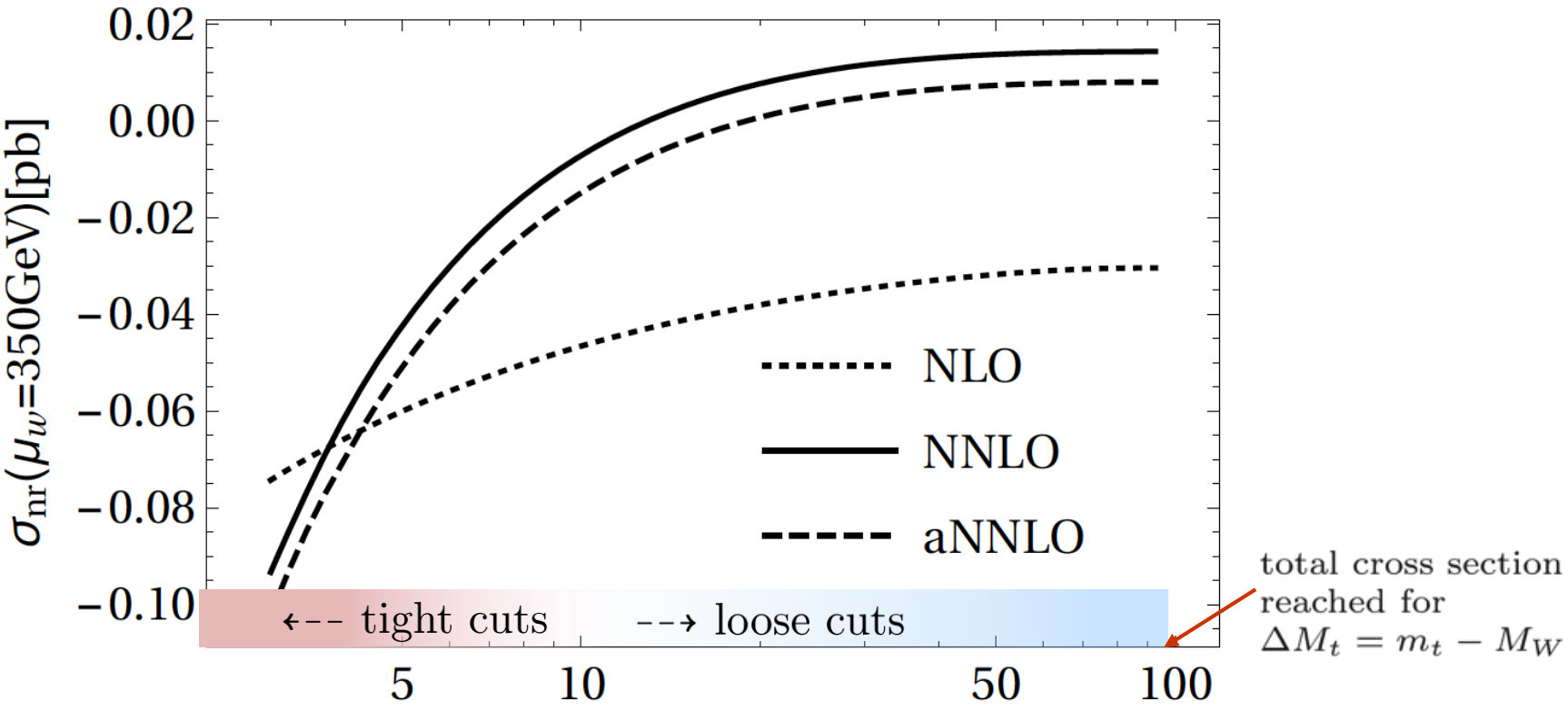
- Nearly energy-independent, shape $\propto \sigma_{\rm QCD}^{-1}$
- NLO: $-(3-4)\%$ near and above the peak, up to -20% below
- NNLO corrections compensate about half of the NLO result





Non-resonant corrections: dependence on the cut

Cut on bW invariant masses of the form $|\sqrt{p^2_{t,\bar{t}}} - m_t| \leq \Delta M_t$



- Tighter cuts increase importance of non-resonant corrections. Resonant contributions unaffected by the cut if $\Delta M_t \gg \Gamma_t$
- Dashed line: approximate result that includes only the endpoint divergent terms $\rightarrow m_t^2/\Lambda^2$, m_t/Λ , $\ln(\Lambda/m_t)$ ($\Lambda^2 \simeq 2m_t\Delta M_t$)

V. Summary and outlook

Non-resonant corrections

- accounts for *bW* pairs produced from virtual tops or with only one or no top near the ttbar threshold

✓ **computed at NNLO** for the total cross section and with top invariant-mass cuts

✓ required to make the QCD prediction consistent (cancellation of finite width divergencies)

- +2% effect above threshold, up to +10% below the peak; cancels about half of the NLO non-resonant correction

✓ **NNLO electroweak accuracy achieved!**

Outlook: combine NNNLO with NNLL QCD, incorporate initial state radiation in theory prediction, estimate NNNLO electroweak corrections...

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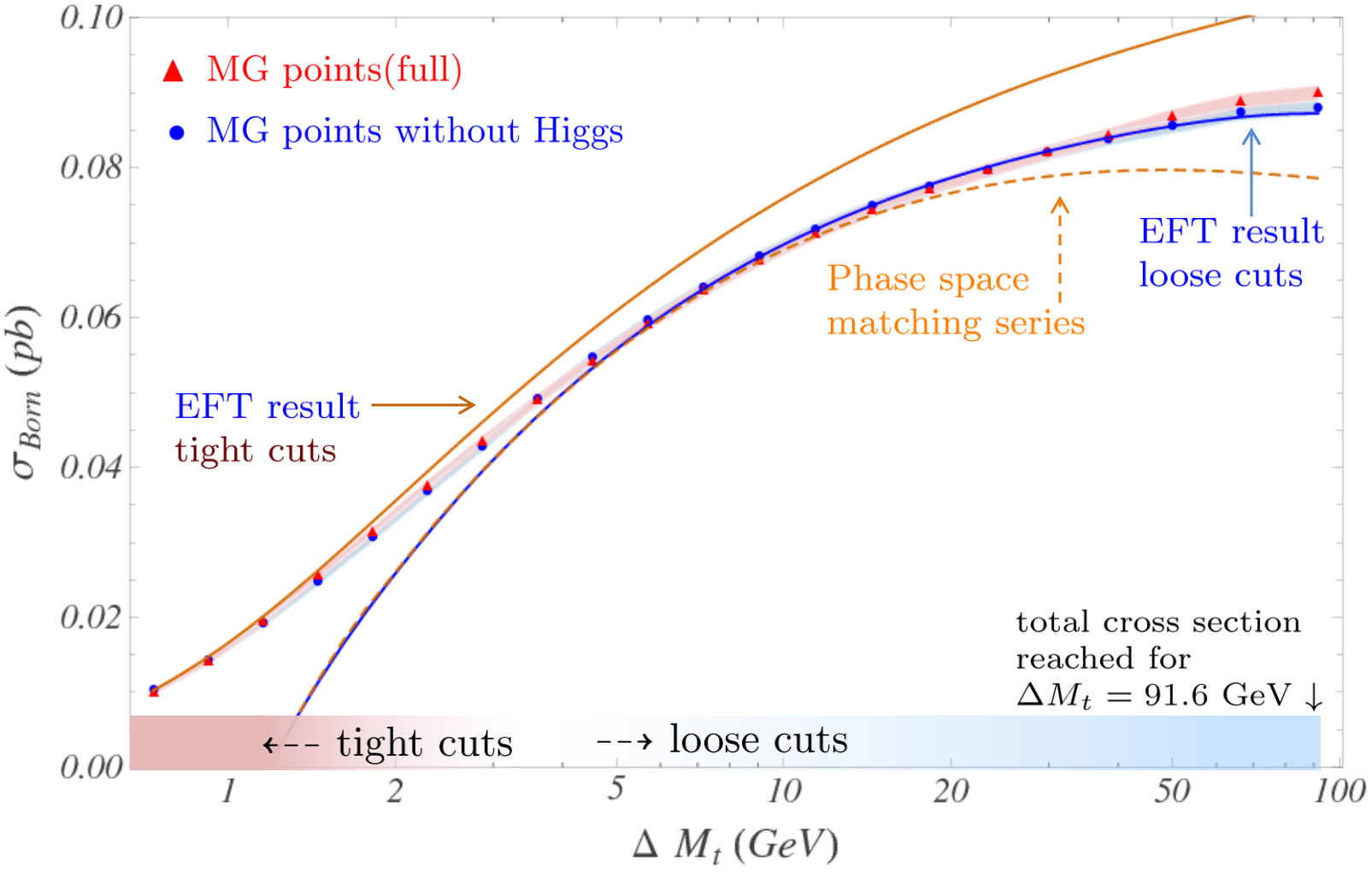
Thank you!

Backup slides

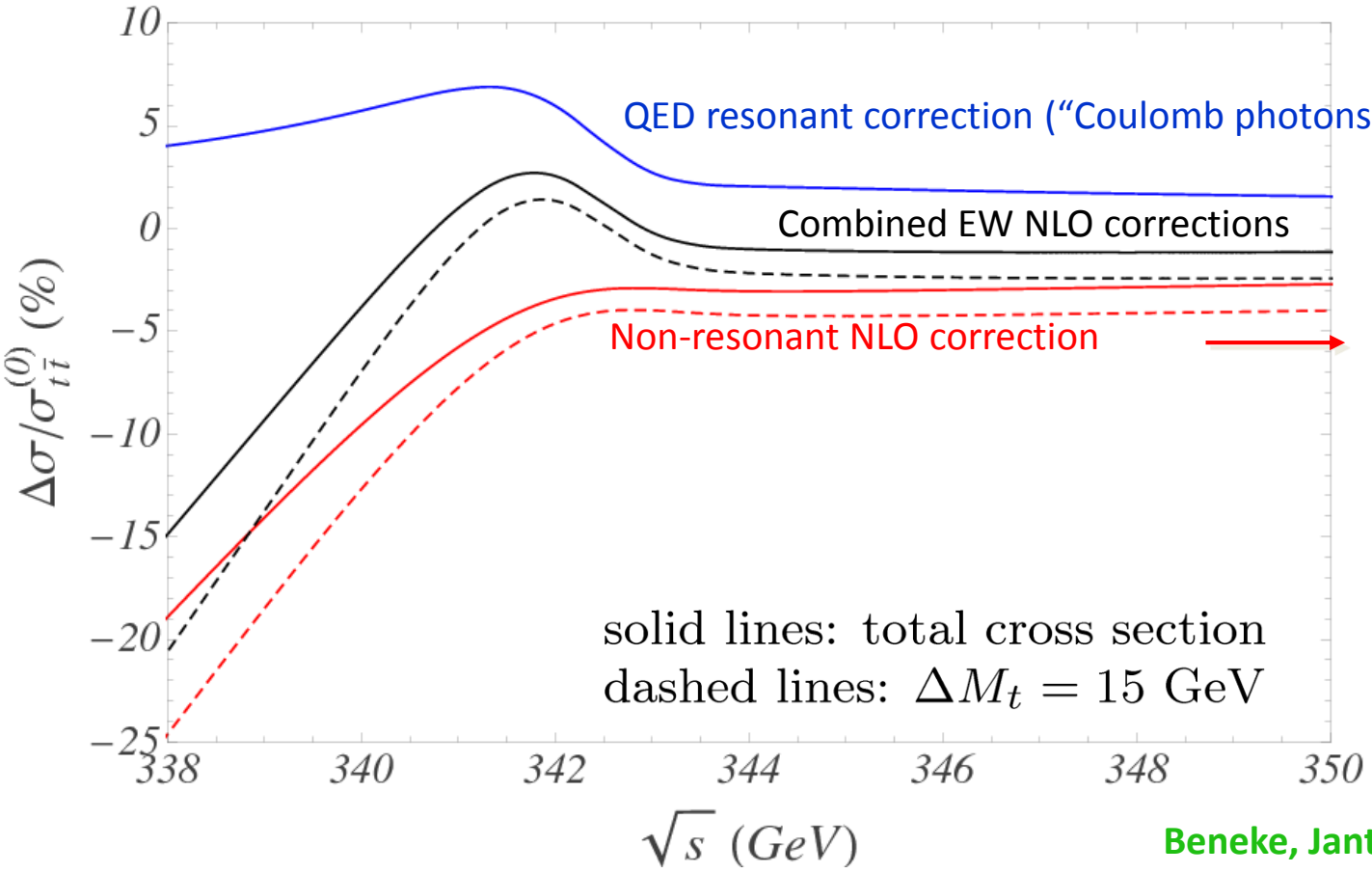
↪ generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$ with MadGraph (MG) for $s = 4m_t^2$, and analyzed dependence on the bW invariant-mass cut ΔM_t

EFT result: resonant LO+NNLO ($\alpha_s = 0$) + non-resonant NLO



Relative sizes of EW NLO corrections with respect LO

LO includes resummation of Coulomb gluons $\propto (\alpha_s/v)^n$ $[\alpha_s^{\overline{\rm MS}}(30\,{\rm GeV}) = 0.142]$



~ -30 fb
(-3% above and
up to -20%
below threshold)

Beneke, Jantzen , RF (2010)

Phase space matching

Alternative approach to compute non-resonant contributions

Hoang, Reisser, PRF (2010)

- Non-resonant contributions obtained for moderate invariant-mass cuts, $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$, as a series:

$$\frac{\Gamma_t}{\Lambda}\sum_{n,\ell,k}\left[\left(C(\alpha_s)\times\frac{m_t\Gamma_t}{\Lambda^2}\right)^n\times\left(\frac{\Lambda^2}{m_t^2}\right)^\ell\right]\times\left(\alpha_s\frac{m_t}{\Lambda}\right)^k \qquad n,\ell,k=0,1,\ldots$$

- NLO, NNLO and (partial) N^{3}LO contributions obtained (counting $\Lambda\sim m_t$) ✓

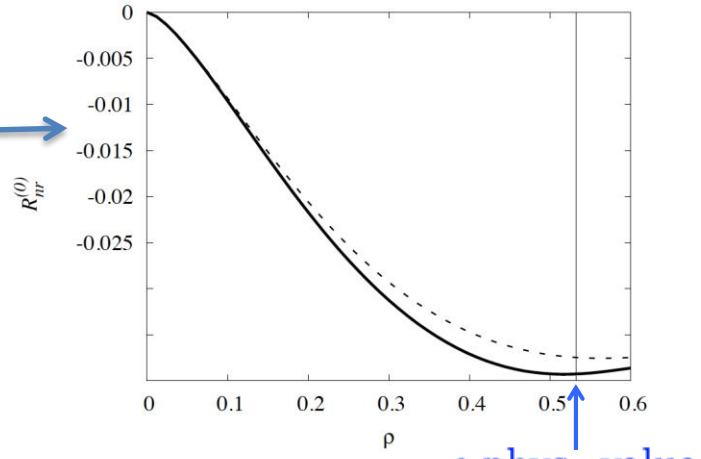
- Beyond NLO, phase space matching approach cannot be applied to larger cuts up to the total cross section ×

- Expansion of full NLO non-resonant contributions in $(\Lambda/m_t)^n$ agrees with first two terms in series above
  [higher powers receive contributions from diagrams h5-h10 with no top, not taken into account in the psm approach → remainder contributions, small at NLO ✓]

Non-resonant NNLO contribution for total cross section

Alternative framework [Penin, Piclum, 2012] computes non-resonant contributions to the total cross section by expanding in

$$\rho = 1 - M_W/m_t \approx 0.5$$



- at NLO, the first term in ρ deviates from the exact result by less than 5%
- at NNLO

$$R_{nr}^{(1)} = \frac{N_c C_F \alpha_s}{\pi^2 \rho} \frac{\Gamma_t}{m_t} \left\{ \left[Q_e^2 Q_t^2 + \frac{2 Q_e Q_t v_e v_t}{1-x_Z} + \frac{(a_e^2+v_e^2)v_t^2}{(1-x_Z)^2} \right] \times \left[\left(3L_E + \frac{3}{2} + 6\ln 2 \right) \pi^2 + (18 + 24\ln 2)\,\rho^{1/2} \right] + \frac{1}{s_w^4} \left[\frac{22}{3} + \frac{17\pi^2}{6} - \frac{17}{2}\ln 2 + \left(2 - 3\pi^2 + 9\ln 2\right) \frac{3\sqrt{2}}{4} \ln\left(1+\sqrt{2}\right) - \frac{27\sqrt{2}}{8} \left(\ln^2\left(1+\sqrt{2}\right) + \mathrm{Li}_2\left(2\sqrt{2}-2\right) \right) \right] \rho^{1/2} + \mathcal{O}(\rho) \right\}.$$

$L_E = \ln\left(\frac{\sqrt{E^2+\Gamma_t^2}}{\rho m_t}\right)$ → infrared regularization dependent term. But dim. reg. used for the resonant contributions, not clear how to combine both

Moreover: infrared structure does not agree with that our (exact in ρ) result

→ diagram missing in this computation

Endpoint-divergent non-resonant NNLO contribution

↪ dominant contribution for small Λ (or small ΔM_t)

$$\left[\begin{array}{c} x = M_W^2/m_t^2 \\ C_{\ldots}(s) = \gamma/Z\text{-prop. \& } e^{\pm}\text{-coupl.} \end{array}\right]$$

$$\begin{aligned}
\sigma^{(2),\mathrm{ep}}_{\mathrm{non-res}} &= \frac{64\pi^2\alpha^2}{s}\,\frac{\Gamma_t^{\mathrm{Born}}}{m_t} \\
&\times \left\{ \left[Q_t^2\, C_{\gamma\gamma}(s) - 2Q_t v_t\, C_{\gamma Z}(s) + v_t^2\, C_{ZZ}(s) \right] \left\{ 4N_c C_F\, \frac{\alpha_s}{\pi}\, \frac{m_t^2}{\Lambda^2} + \frac{6\sqrt{2}}{\pi^2} \left(\delta\Gamma_t^{(1)} - \frac{4C_F}{\pi}\alpha_s \right) \frac{m_t}{\Lambda} \right\} \right. \\
&\quad + N_c C_F\, \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon_{\mathrm{ep}}} + 2\ln\frac{\mu_s^2}{\Lambda^2} \right) \left\{ \left[Q_t^2\, C_{\gamma\gamma}(s) - 2Q_t v_t\, C_{\gamma Z}(s) + v_t^2\, C_{ZZ}(s) \right] \frac{7+7x+22x^2}{6(x-1)\,(1+2x)} \right. \\
&\qquad + \frac{1}{3} a_t^2\, C_{ZZ}(s) + \frac{1}{2} Q_t a_t\, C_{\gamma Z}(s) - \frac{1}{2} v_t a_t\, C_{ZZ}(s) \\
&\qquad + \left[Q_t Q_b\, C_{\gamma\gamma}(s) - \left(Q_t\,(v_b + a_b) + Q_b v_t \right) C_{\gamma Z}(s) + v_t\,(v_b + a_b)\, C_{ZZ}(s) \right] \frac{1-5x-2x^2}{6(1+x)\,(1+2x)} \\
&\qquad + \left[Q_t\, C_{\gamma\gamma}(s) - \left(v_t + Q_t\, \frac{c_w}{s_w} \right) C_{\gamma Z}(s) + v_t\, \frac{c_w}{s_w}\, C_{ZZ}(s) \right] \frac{2+5x-2x^2}{6x\,(1+2x)} \\
&\qquad \left. \left. - \left[Q_t\, C_\gamma(s) + v_t\, C_Z(s) \right] \left[\ln\left(\frac{2}{x} - 1 \right) + \frac{(1-x)\,(1-2x-23x^2)}{12x^2} \right] \frac{x}{4(1-x)^3\,(1+2x)} \right\} \right\} \\
&\quad + \text{finite } \Lambda\text{-independent terms} + \mathcal{O}(\Lambda/m_t)
\end{aligned}$$

Jantzen, PRF (2013)

- UV and IR singularities cancelled between diagrams ✓
- $1/\epsilon$ endpoint singularities & finite-width divergences cancel each other ✓
- comparison to HRR result: m_t^2/Λ^2 ✓, m_t/Λ ✓, $\Lambda^0\,\ln(\Lambda^2)$ ✓