# The `QQbar_threshold` code

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M. Beneke, Y. Kiyo, AM, J. Piclum arXiv:1605.03010

M. Beneke, AM, T. Rauh, P. Ruiz-Femenía arXiv:17??.?????

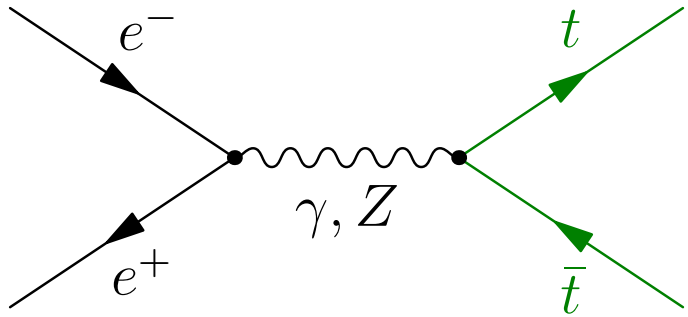
# What is `QQbar_threshold`?

- Library for heavy quark $(t\bar{t})$ production near threshold
- Not a Monte Carlo generator:
  limited number of observables, e.g. total cross section
- Not a program:
  C++ library + Mathematica package

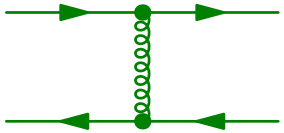
```cpp
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_xsection(
     344., {80., 350.}, {171.5, 1.33}, N3LO
  ) << '\n';
}
```

```
Needs["QQbarThreshold`"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
   TTbarXSection[
      sqrts, {80., 350.}, {171.5, 1.33},
      "N3LO"
   ],
   {sqrts, 340, 348}
]
```

# $e^+e^- \to t\bar{t}$ near threshold




- Kinematics: $v \ll 1, \quad E_{\text{kin}} \sim m_t v^2, \quad |\mathbf{p}| \sim m_t v$
- Dominant interaction:



$\Rightarrow$ Colour Coulomb potential $-\frac{C_F \alpha_s}{r}$

- $t\bar{t}$ "decays during bound state formation":

$$v \sim \alpha_s \Rightarrow E_{\text{kin}} \sim m_t \alpha_s^2 \sim -E_1$$
$$\alpha \sim \alpha_s^2 \Rightarrow \Gamma_t \sim m_t \alpha \sim -E_1$$

$\hookrightarrow$ effective theory description

# Effective theory description

Effective theory:

- Non-relativistic bound states described by Potential Non-Relativistic QCD (PNRQCD)
- Combine with top decays: Unstable Particle Effective Theory

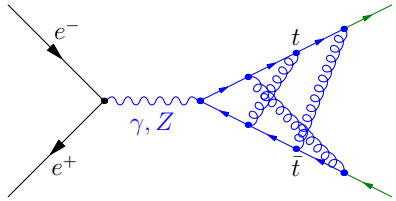
Power counting: $v \sim \alpha_s \sim y_t \sim \sqrt{\alpha} \ll 1$

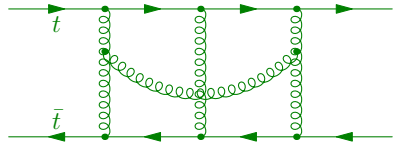
$$\sigma \sim \alpha^2 v \sum_k \left(\frac{\alpha_s}{v}\right)^k \times \begin{cases} 1 & \text{LO} \\ \alpha_s, v, \alpha/v & \text{NLO} \\ \alpha_s^2, \alpha_s v, v^2, y_t^2, \ldots & \text{N}^2\text{LO} \\ \alpha_s^3, \ldots & \text{N}^3\text{LO} \end{cases}$$

# Corrections in `QQbar_threshold`

- N$^3$LO QCD

[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]

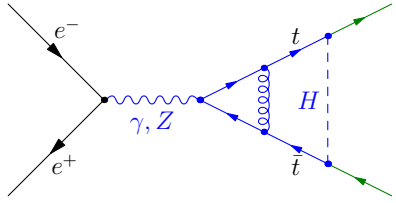


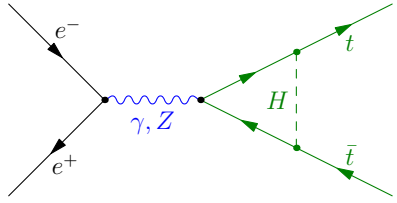




$\cdots$

- N$^3$LO Yukawa

[Eiras, Steinhauser 2006; Beneke, Maier, Piclum, Rauh 2016]



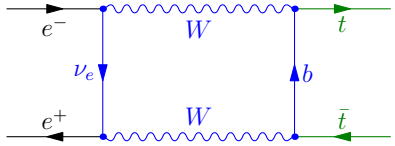

$\cdots$

- N$^2$LO electroweak

[Grzadkowski, Kühn, Krawczyk, Stuart 1986]

[Guth, Kühn 1991]

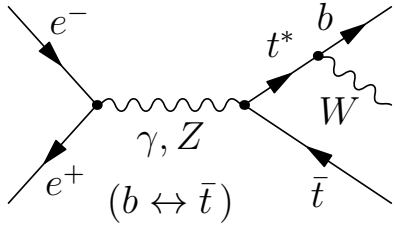
[Hoang, Reißer 2004 & 2006]

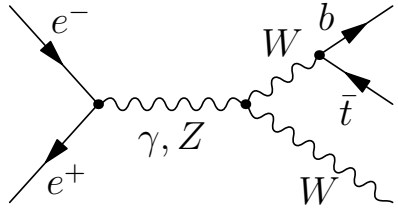


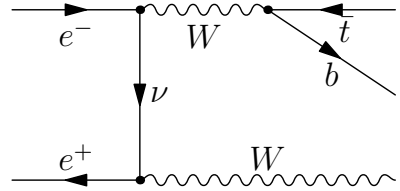

$\cdots$

- NLO non-resonant ($\sim \alpha/v$)

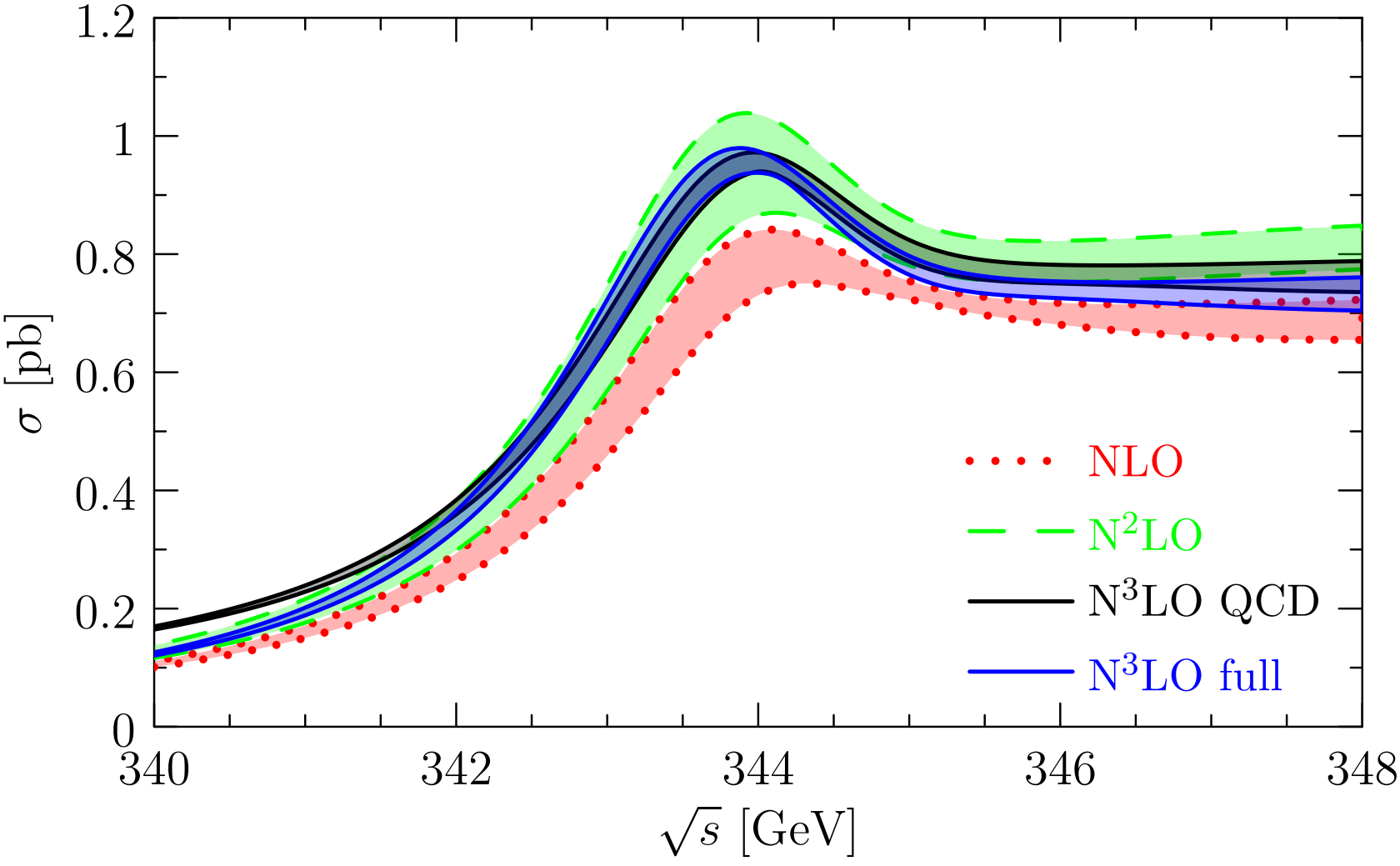
[Beneke, Jantzen, Ruiz-Femenía 2010]



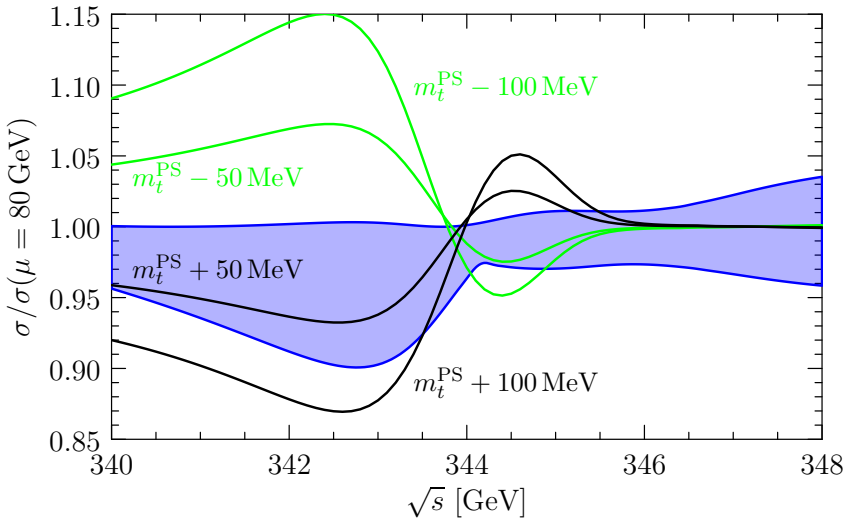
# Top-pair production cross section

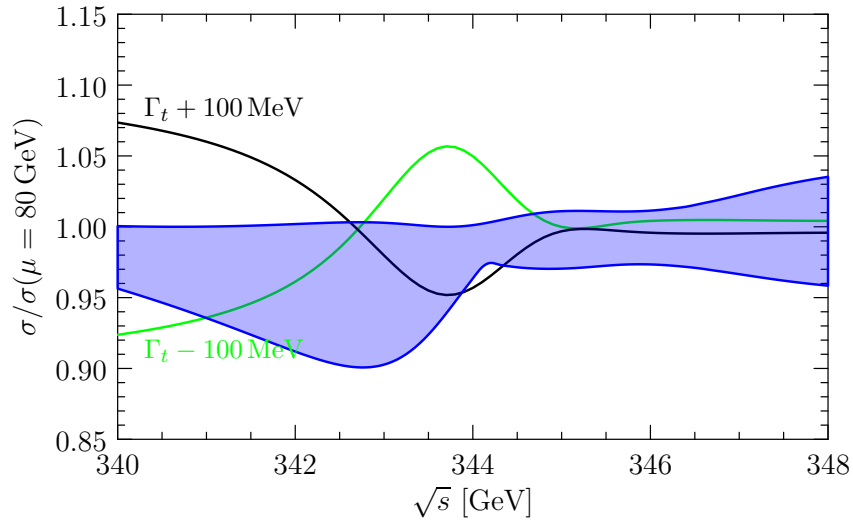


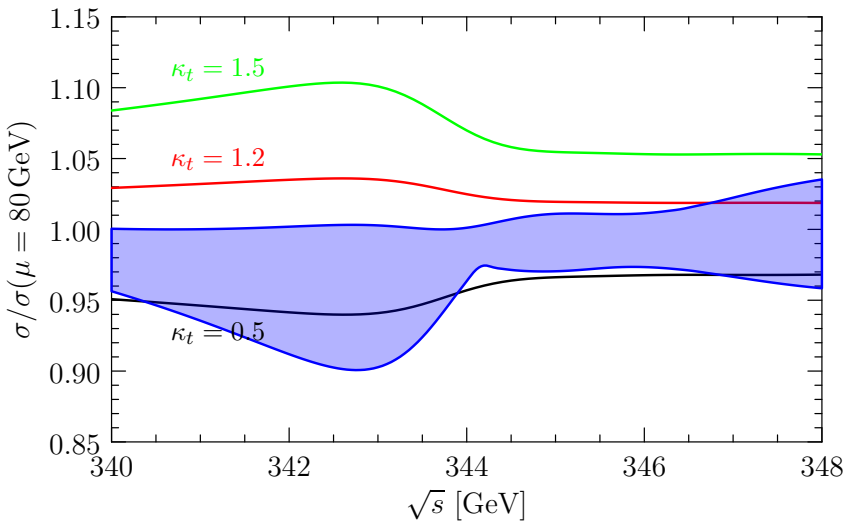

$$m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}\,, \qquad \Gamma_t = 1.33\,\mathrm{GeV}\,, \qquad \alpha_s(m_Z) = 0.1177\,,$$
$$\mu_w = 350\,\mathrm{GeV}\,, \qquad m_H = 125\,\mathrm{GeV}\,, \qquad \alpha(m_Z) = 1/128.944$$
$$50\,\mathrm{GeV} < \mu < 350\,\mathrm{GeV}\,, \qquad m_W, m_Z$$

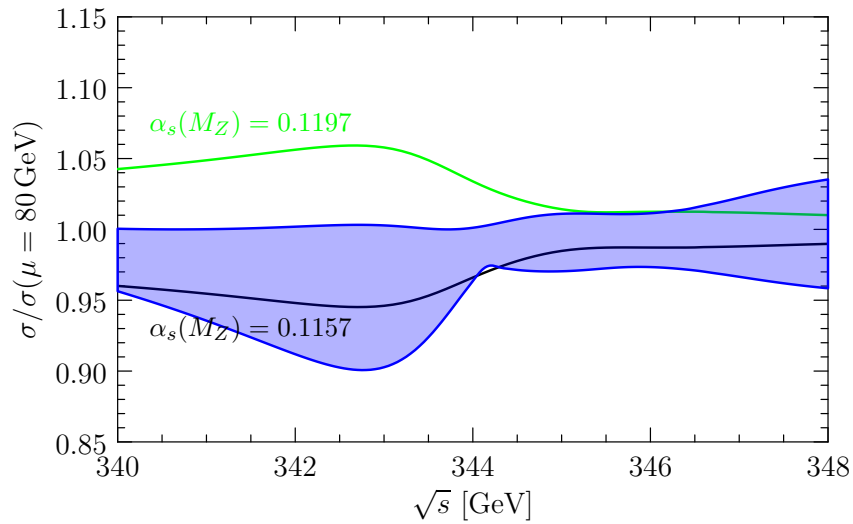
# Sensitivity to parameter variation

⟶ Frank Simon's talk



# Mass schemes

- Pole mass
- Potential-subtracted (PS) mass

$$m_t^{\mathrm{PS}}(\mu_f) = m_t - \frac{1}{2}\int_{|\mathbf{q}|<\mu_f} \frac{d^3\mathbf{q}}{(2\pi)^3}\, V_{\mathrm{Coulomb}}(\mathbf{q})$$

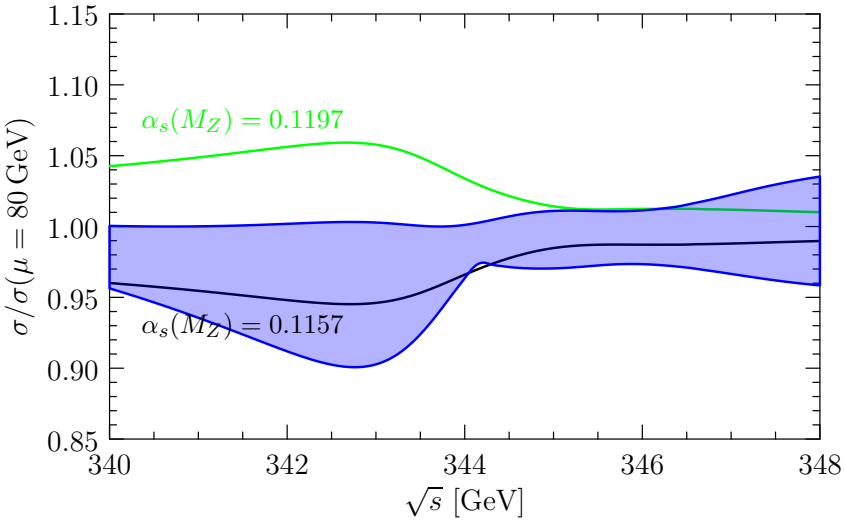
- 1S mass

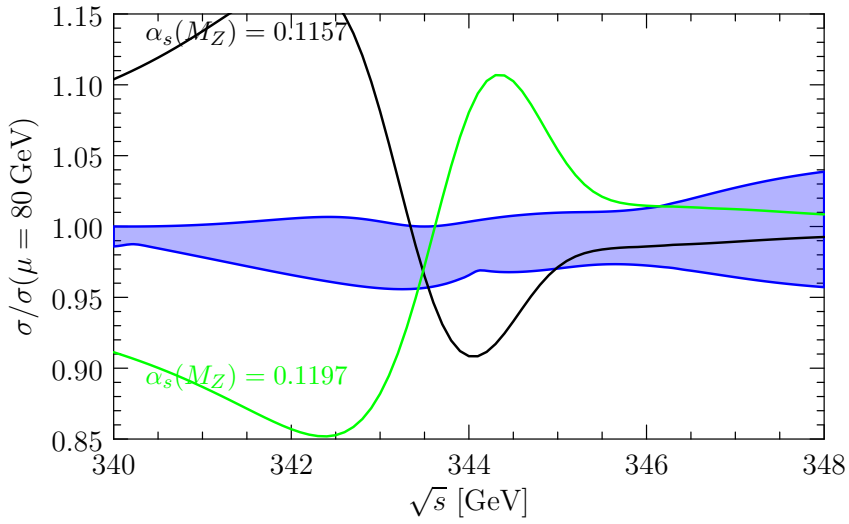
$$m_t^{\mathrm{1S}} = m_t - \frac{E_1}{2}$$

- $\overline{\mathrm{MS}}$ mass

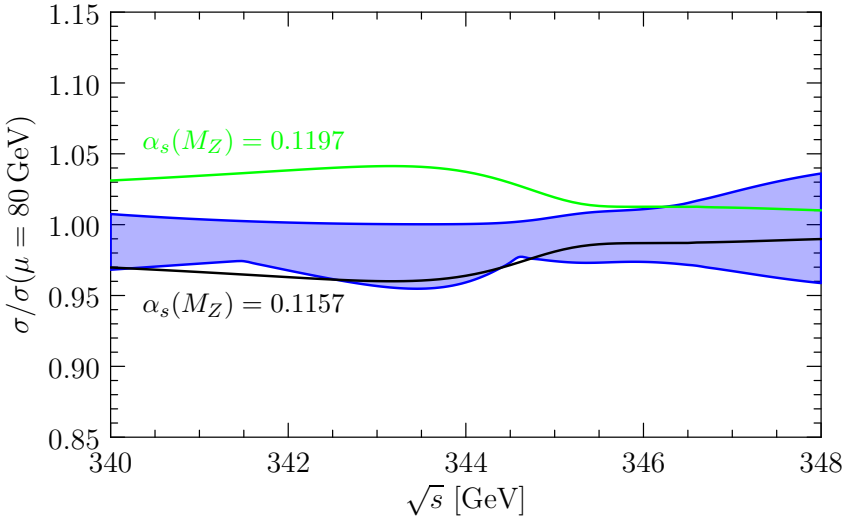
# Sensitivity to parameter variation

## Mass schemes




$m_t^{\mathrm{PS}}(20\,\mathrm{GeV}) = 171.5\,\mathrm{GeV}$ $\qquad$ $\overline{m}_t(\overline{m}_t) = 163.3\,\mathrm{GeV}$

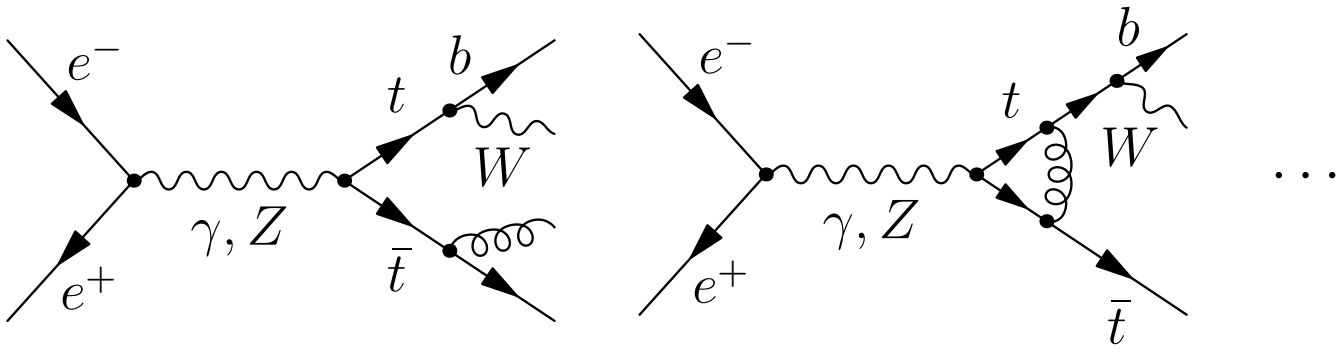



$m_t^{\mathrm{1S}} = 171.9\,\mathrm{GeV}$

# Work in Progress

# QQbar_threshold 2

## Non-resonant production at N$^2$LO




- Endpoint divergences as $m_{Wb},\ m_{Wbg} \to m_t$, cancel against resonant contribution
- Manual calculation of divergent parts, automatised (modified MadGraph 5) finite contributions
- Changed calculational scheme, small numeric differences in resonant contribution

$\hookrightarrow$ Pedro Ruiz-Femenía's talk

# QQbar_threshold 2

## Width corrections

Position of complex pole: $p_t^2 = M_*^2 \equiv m_t^2 - i m_t \Gamma_t$

Non-relativistic expansion:

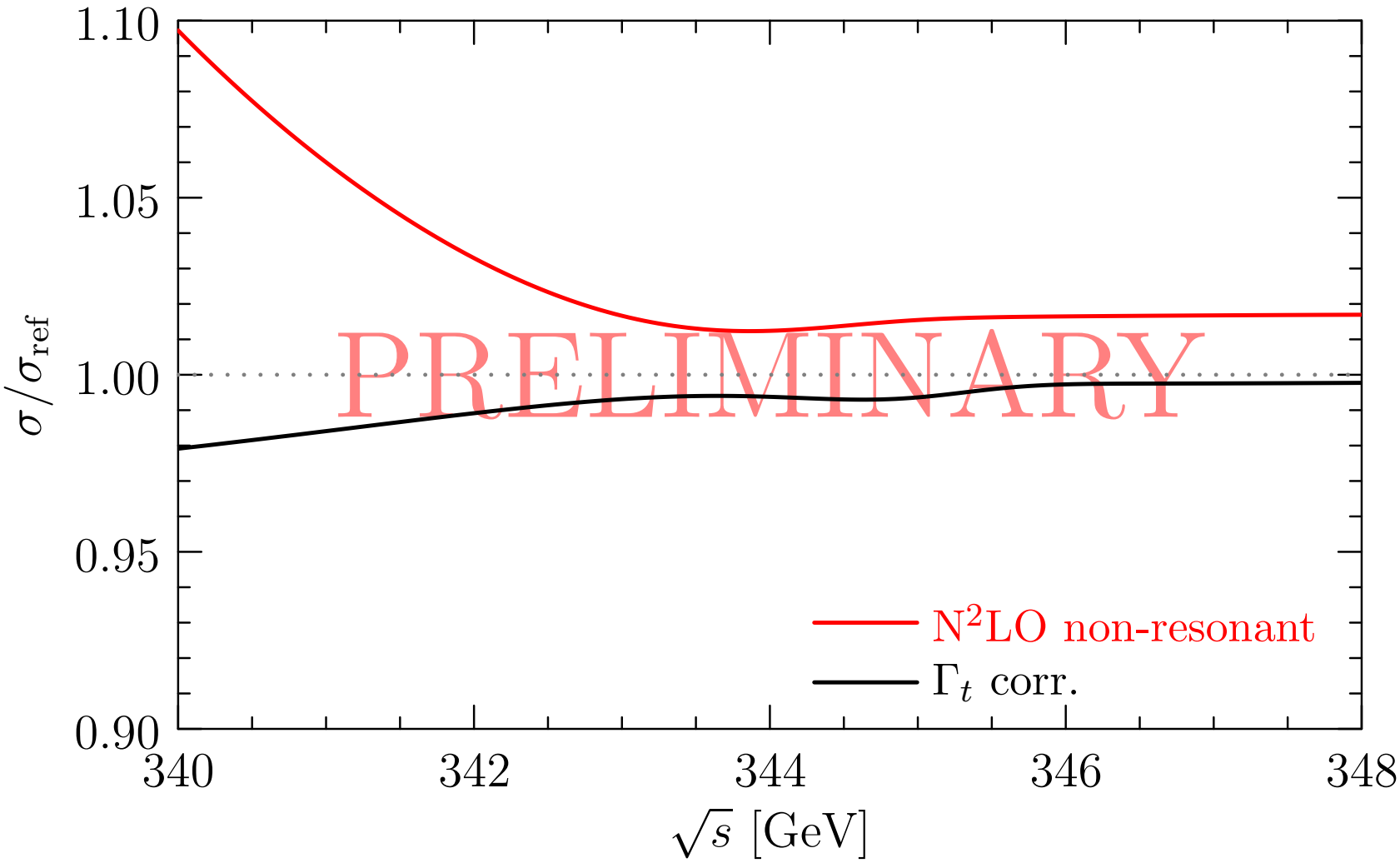
$$p_t^0 = m_t + \frac{\mathbf{p}_t^2 - i m_t \Gamma_t}{2 m_t} - \frac{\left(\mathbf{p}_t^2 - i m_t \Gamma_t\right)^2}{8 m_t^3} + \cdots$$

Time dilatation: $\Gamma_{t\bar{t}} < 2\Gamma_t$

Small effect on cross section

# QQbar_threshold 2

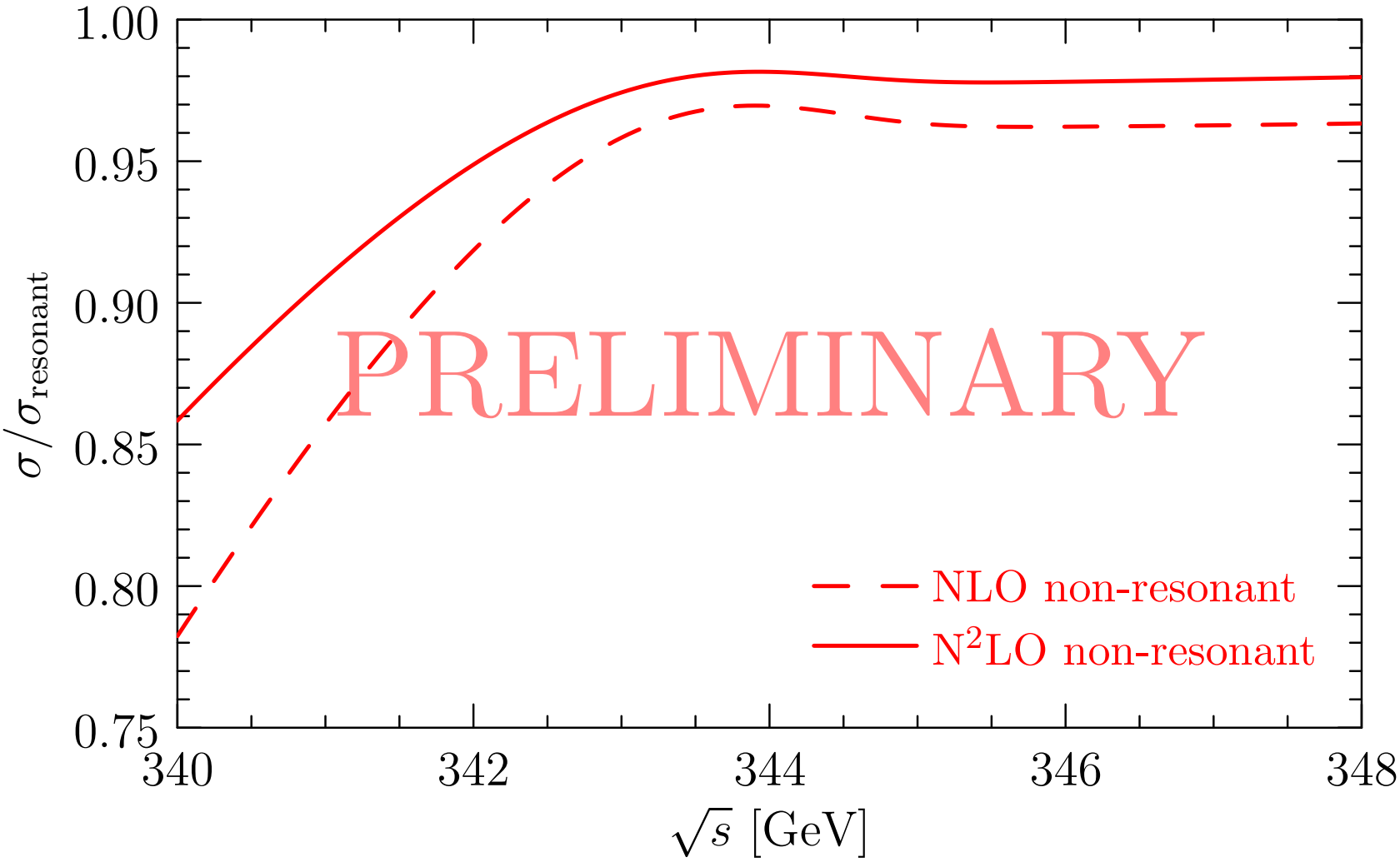
## Impact on cross section




$\sigma_{\rm ref}$: all corrections in current QQbar_threshold version

# QQbar_threshold 2

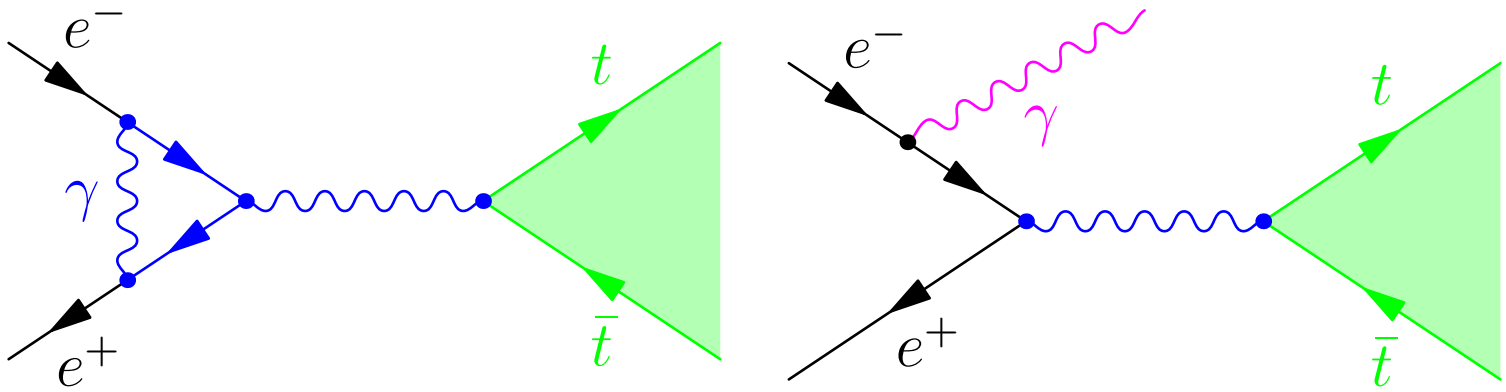
## Non-resonant production



# QQbar_threshold 2

## Initial state radiation

Photon corrections to initial state:

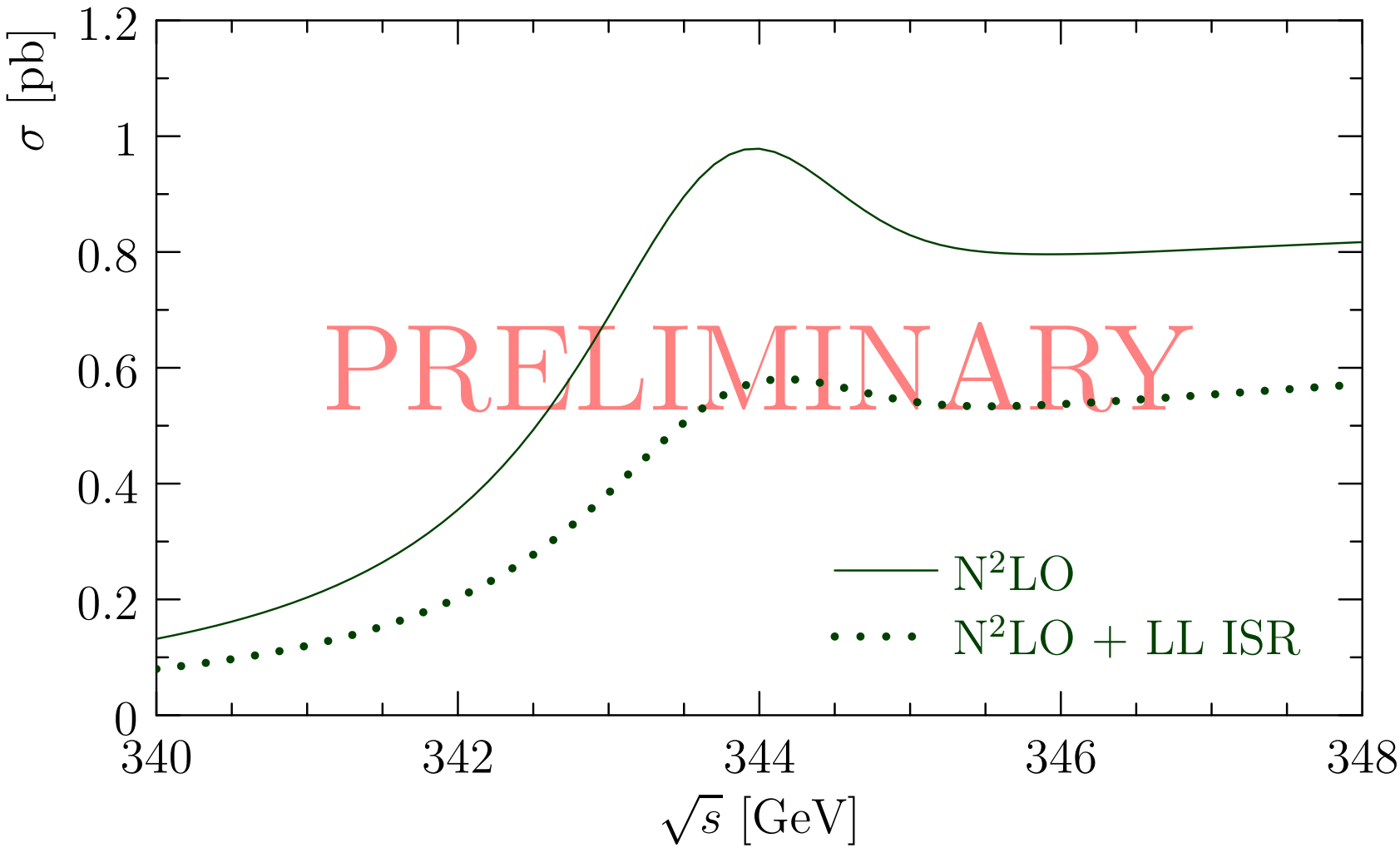



$\hookrightarrow$ large logarithms $\log^2 \frac{m_t}{m_e}$, resummed into structure functions

[Fadin, Kuraev 1985; Fadin, Khoze 1987]

$$\sigma(s) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\, \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\hat{\sigma}(x_1 x_2 s) + \{\mathrm{N^2LO}\}$$

# Initial state radiation



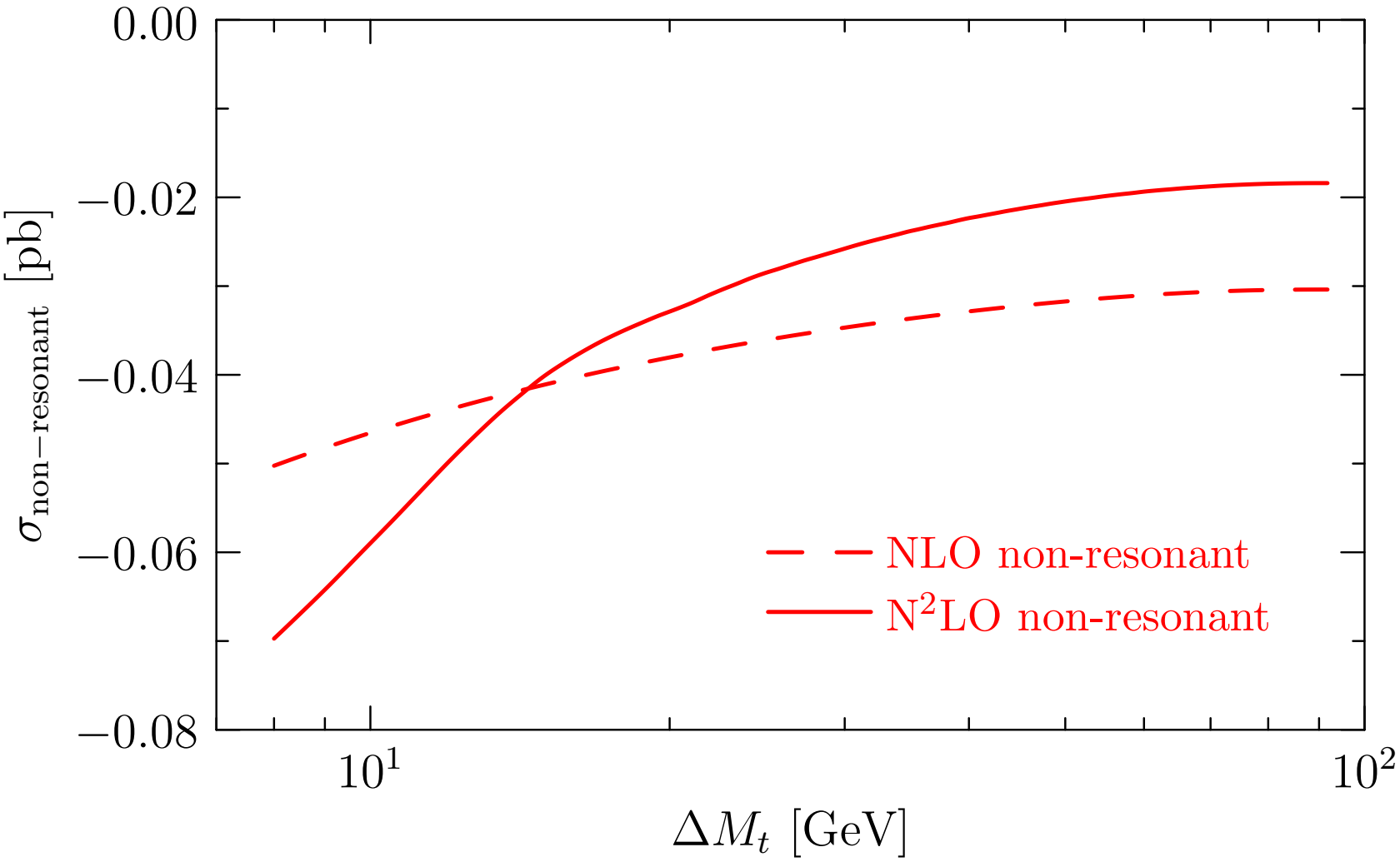
# Conclusion

- `QQbar_threshold` provides state-of-the-art prediction for $\sigma(e^+e^- \to t\bar{t})$:
  - N$^3$LO QCD + Yukawa corrections
  - N$^2$LO electroweak corrections
  - NLO non-resonant production
- Work in progress:
  - N$^2$LO non-resonant corrections
  - Initial state radiation

# Backup

# QQbar_threshold 2

## Non-resonant production



# Finite-width divergences

- Divergences $\sim \Gamma_t/\epsilon$ in QCD part,
  cancel at each order against electroweak & non-resonant corrections
- Incomplete N$^2$LO, N$^3$LO corrections $\Rightarrow$ residual divergences
- $\overline{\text{MS}}$ subtraction $\Rightarrow$ spurious dependence on $\log \frac{\mu_w^2}{m_t^2}$

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