Lectures on SM

Literature: "Ten Lectures on the EW interactions"
by Riccardo Barbieri (arXiv: 0706.0684v1)

"Flavor physics and CP violation"
by Yossef Nir (arXiv: 1010.2666)

Preface:

SM is a "reference theory" of particle physics

Experimentally well established:

⇒ all predicted elementary d.o.f.s ~~discov~~ confirmed (modulo dynamics behind ν-mass)

⇒ all key predictions confirmed (modulo ν masses)
↳ ~~found~~ single extension

⇒ accomodates all* physics phenomena measured in laboratories (experiments)

⇒ reference scenario against all competing theories are tested: "NP" = "BSM" or "SM extensions".

* some observed deviations, none significant (>5σ)

Content of Lectures:

1. From Fermi theory to SM ; key predictions (2-4)
2. Precision tests of SM : EW sector
3. Precision tests of SM : fermionic sector
4. SM Puzzles as motivation for NP

Disclaimer: No QCD ; Higgs tests

Unit conventions: $\hbar = c = 1$
$\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$

1. From Fermi theory to SM

1.1. Fermi theory of β-decay (Fermi 1934)

$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}} \overset{\sim 1 \text{ (later)}}{\cos\theta_c} \left[\bar{p}\gamma_\mu (1+\alpha\gamma_5) n\right]\left[\bar{e}\gamma_\mu (1-\gamma_5)\nu\right] + h.c.$$

$$[G_F] = (GeV)^{-2} \qquad \left([\psi] = (GeV)^{3/2}\right)$$

$$\Rightarrow \Gamma_\beta = \frac{G_F^2 \Delta^5}{60\pi^3} \cos^2\theta_c \left(1+3\alpha^2\right) \Phi$$

m_e effects ; $\Phi = 0.47$

$\Delta = (m_p - m_n) = 1.29\,MeV$

Angular dependence:

$$\frac{d\Gamma_\beta}{d\Omega_e} \propto \left(1 + \frac{1-\alpha^2}{1+3\alpha^2}\, \vec{v}_e \cdot \vec{v}_\nu\right)$$

Experimentally: $\tau_n = \frac{1}{\Gamma_\beta} \approx 885\,sec$, $\alpha \approx -1.27$

$\Rightarrow G_F^{-1/2} \approx 250\,GeV$ "Fermi scale"

Analogy with electrodynamics: exchange of (charged) vector boson $W^\pm$

$$\mathcal{L}_{int} = \frac{g}{\sqrt{2}} W^+_\mu J^-_\mu + h.c.$$

$$J^-_\mu = \cos\theta_c\, \bar{p}\gamma_\mu \frac{1+\alpha\gamma_5}{2} n + \bar{\nu}\gamma_\mu \frac{1-\gamma_5}{2} e$$

$$\Rightarrow \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

for $g \lesssim 1$: $M_W \lesssim 100\,GeV$

In more modern language: terms of quarks & leptons

$$J^-_\mu = \cos\theta_c\, \bar{u}\gamma_\mu P_L d + \bar{\nu}\gamma_\mu P_L e \quad ; \quad P_{R,L} = \frac{1\pm\gamma_5}{2}$$

Consistent framework within perturbative QFT:

Vector bosons ⟺ gauge interaction

$$A_\mu^a \iff T^a \quad \text{- generators of gauge group } G \quad ; \; a = 1 \ldots \underset{\substack{\uparrow \\ \text{size of group algebra}}}{N}$$

$$[T^a, T^b] = i f_{abc} T^c$$

Interacting scalar, fermionic fields : representations of G : ψ_i, ϕ_i ; $i = 1 \ldots \underset{\substack{\uparrow \\ \text{size of repr.}}}{M}$

Gauge transformations: $\omega(x)$

$$A_\mu \to \omega^{-1}(x)\left(A_\mu + \partial_\mu\right)\omega(x)$$

$$\phi \to T(\omega(x))\phi$$

Gauge kinetic term: $\mathcal{L}_{gauge} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, C^{abc} A^b_\mu A^c_\nu$$

Scalar & Fermionic covariant derivatives : $\mathcal{L}_\phi = \left(D_\mu \phi\right)^\dagger \left(D_\mu \phi\right) - V(\phi)$

$$D_\mu \phi_i = \partial_\mu \phi_i - i g T^a_{ij} A^a_\mu \phi_j$$

$$\mathcal{L}^{kin}_\psi = i \bar\psi \not{D} \psi = i \bar\psi \not{\partial} \psi + g A^a_\mu \underbrace{J^a_\mu}$$

$$J^a_\mu = \bar\psi_i \gamma^\mu T^a_{ij} \psi$$

→ Simplest (non-abelian) / Lie group : SU(2)

$$T^a = \frac{\sigma^a}{2}$$

→ Chiral structure: ψ = chiral (left-handed) Weyl fermions

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \qquad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$J^\pm_\mu = \bar Q \frac{\sigma^\pm}{2} \gamma_\mu Q + \bar L \frac{\sigma^\pm}{2} \gamma_\mu L$$

SU(2) has 3 generators $\sigma^\pm, \boxed{\sigma^3}$: $J^3_\mu = \bar Q \frac{\sigma_3}{2} \gamma_\mu Q + \bar L \frac{\sigma_3}{2} L$

⇒ existence of neutral currents & neutral gauge boson Z^0

⇒ EM interaction: u_L, d_L / $\nu_L\, e_L$ have different EM charges ; EM is vector-like $Q_{em}(u_L) = Q_{em}(u_R)$

$$Q_{em} = T_3 + \underset{\substack{\uparrow \\ U(1)_Y}}{Y}$$

⇒ QCD for quarks : $SU(3)_c$

Complete gauge-symmetric structure: $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\underset{\substack{\uparrow \\ \text{left-handed} \\ \text{Weyl fermions}}}{\psi} = \{ Q \sim (\underset{\substack{\uparrow \\ SU(3)_c}}{3}, \underset{\substack{\nwarrow \\ SU(2)_L}}{2})_{\underset{\substack{\nwarrow \\ U(1)_Y}}{1/6}} , L \sim (1,2)_{-1/2} , u^c \sim (\bar{3},1)_{-2/3} , d^c \sim (\bar{3},1)_{1/3} , e^c \sim (1,1)_1 \}$$

(ν - masses : $N \sim (1,1)_0$ - one possibility, not unique)

1.2 Construction of SM Lagrangian

Missing ingredient: W, Z masses

fermion masses (3 - generations, neutrino masses?)

W, Z masses

=> Spontaneous symmetry breaking - Higgs mechanism

$$\phi \sim (1,2)_{1/2} \quad \text{- Higgs field} \quad \langle \phi \rangle \neq 0 \ \text{at} \ \min_{\phi} V(\phi)$$

$$= \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

Choice allows interaction with fermionic fields => fermion masses as well
(Yukawa)

$$\mathcal{L}_{Yuk} = - \phi [Q_i Y^U_{ij} u^c_j + L_i Y^N_{ij} N_j] - \phi^+ [Q_i Y^D_{ij} d^c_j + L_i Y^E_{ij} e^c_j] + h.c.$$

(N - masses: (Majorana) $\mathcal{L}_M = - \frac{1}{2} N_i M_{ij} N_i \quad + h.c$)

$$\mathcal{L}_{\nu SM} = \mathcal{L}_{gauge} + \mathcal{L}^{kin}_{\psi} + \mathcal{L}_{\phi} + \mathcal{L}_{Yuk} + \mathcal{L}_M$$

$$= \mathcal{L}_{SM} + \underbrace{i \bar{N} \not{\partial} N - \left(\phi L_i Y^N_{ij} N_j + \frac{1}{2} N_i M_{ij} N_j \ + h.c \right)}_{\text{"right - handed" neutrino" Lagrangian}}$$

"The puzzle of SM charge quantization"

- in non-Abelian gauge groups interactions of arbitrary charged fields fixed in terms of single universal parameter (gauge coupling) + representation ⇒ gauge "charges" fixed

- in $U(1)$ individual charges arbitrary

- In SM: Why $Q(e) = -Q(p)$? $\quad (Q = \underset{\substack{\uparrow \\ SU(2)_L}}{T_3} + \underset{\substack{\uparrow \\ U(1)_Y}}{Y})$

  Tested experimentally to $\mathcal{O}(10^{-20})$!!

Anomaly cancellation in SM

- preservation of classical symmetries by quantum effects:

$$\underset{\substack{\uparrow \\ \text{totally symmetric triple product}}}{Tr[T^a T^b T^c]_S} = 0$$

for $\forall$ a, b, c, T - representations of <u>chiral fermions</u>

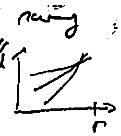
In SM: $\quad Tr[(\underset{\substack{\uparrow \\ SU(2)_L}}{T_3})^2 Q] = 0$ - charges of $SU(2)$ doublet need to add to zero

$$= \frac{1}{4} \times \underset{\substack{\uparrow \\ U_L}}{3} \times \left(\underset{\substack{\uparrow \\ u_L}}{\frac{2}{3}} - \underset{\substack{\uparrow \\ d_L}}{\frac{1}{3}}\right) + \frac{1}{4} \times \left(\underset{\substack{\uparrow \\ \nu_L}}{0} - \underset{\substack{\uparrow \\ e_L}}{1}\right) = 0$$

Connects charges of quarks & leptons.

Miracle?

⇒ Embedding of $U(1)_Y$ into non-abelian gauge groups ($GUTs$, $SU(5) \supset SU(3) \times SU(2) \times U(1)$) corroborated by RGE running of gauge couplings



1.3 Main predictions of SM

- gauge symmetry breaking & particle masses

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

for $\mu^2 > 0$: minimum of $V(\phi)$ at $\phi^\dagger\phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$

$\rightsquigarrow$ $\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2$ (4-sphere)

Can choose particular configuration

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \leftarrow \underset{= q_{em}}{T_3 + Y} \text{ unbroken}$$

$$\phi = \langle \phi \rangle + \ldots$$

↑ Goldstone & Higgs boson fluctuations

a) Gauge boson masses:

$$(D_\mu \phi)^\dagger D^\mu \phi = \frac{1}{2} \frac{v^2}{4} \left[g^2 \left((W_\mu^1)^2 + (W_\mu^2)^2 \right) + \left(-g W_\mu^3 + g' B_\mu \right)^2 \right] + \ldots$$

↑ $SU(2)_L$ gauge coupling, gauge fields, ↑ $U(1)_Y$ gauge field, $U(1)_Y$ gauge coupl.

define: * $W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (W_\mu^1 \pm W_\mu^2)$ ← charged weak massive bosons

$$m_W = \frac{g v}{2} \quad ; \quad G_F^{-1} = \sqrt{2} v^2$$

* $Z_\mu \equiv \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu$; $\theta_W = \arctan \frac{g'}{g}$

↑ neutral weak massive boson

$$\boxed{m_Z = \frac{m_W}{\cos\theta_W}}$$

(← if $g' \to 0$: $m_Z = m_W$ due to $SO(4)$ symmetry of $V(\phi)$, $\langle\phi\rangle$)

* $A_\mu \equiv \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu$ ← Photon (massless)

b) Fermion masses:

$$m_{ij}^\psi \equiv Y_{ij}^\psi \frac{v}{\sqrt{2}}$$

↑ can be diagonalised with bi-unitary transformation

$$m_{ii} = U_L^\psi m_{ij}^\psi U_R^{\psi\,-1}$$

$$\psi \to U_L^{\psi\,-1} \psi$$

$$\psi^c \to U_R^\psi \psi^c$$

c) Couplings of fermions to gauge bosons

- gauge covariant derivative:

$$D_\mu = \partial_\mu - ig W^a_\mu T^a - ig' Y B_\mu$$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^- T^-_\mu\right) - i\frac{g}{\cos\theta_W} Z_\mu \left(T^3 - \sin^2\theta_W Q\right)$$

$$- i\boxed{\frac{g}{\sin\theta_W}} A_\mu Q$$

$e = \frac{g}{\sin\theta_W} = \frac{g'}{\cos\theta_W}$ EM gauge coupling

($\alpha = \frac{e^2}{4\pi}$ fine structure constant)

d) Higgs boson

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix}\phi_1 + i\phi_2 \\ v + h + i\theta_3\end{pmatrix} = e^{i\frac{\tau_i \theta_i}{v}}\begin{pmatrix}0 \\ \frac{v+h}{\sqrt{2}}\end{pmatrix}$$

↑ can be removed via SU(2)$_L$ gauge transformation

$$V(h) = \frac{1}{2} m_h^2 h^2 + \sqrt{\frac{\lambda}{2}} m_h h^3 + \frac{\lambda}{4} h^4$$

$$m_h = \sqrt{2}\mu = \sqrt{2\lambda}\, v$$

- couplings to vector bosons:

$$(D_\mu\phi)^\dagger(D_\mu\phi) = \dots + \left[m_W^2 W^+_\mu W^-_\mu + \frac{1}{2} m_Z^2 Z_\mu Z_\mu\right]\left(1 + \frac{h}{v}\right)^2$$

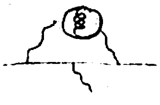
- couplings to fermions

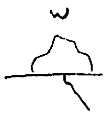
$$-\phi\bar{\psi} Y^\psi \psi^c = \dots + m_\psi \bar{\psi}\psi\left(1 + \frac{h}{v}\right)$$

2. Precision tests of SM : EW sector

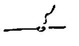
- overwhelming number of precision measurements performed over the past ~40 years

- example: $(g-2)_{e,\mu}$





det of α_{em} ↗ ↖ tests of QCD, EW, NP

 $= ie\left[\bar{e}\gamma^{\mu}e + \frac{\overset{\overset{(g-2)_e}{\downarrow}}{\Delta a_e}}{2m_e}\bar{e}\sigma_{\mu\nu}\partial^{\nu}e + \dots\right]$

(known to 10th order in QED)

most precise extraction of α_{em}

$$\begin{cases} \delta(a_e)_{exp} = 2.8\times10^{-13} \\ \delta(a_e)_{SM} = 7.6\times10^{-13} \end{cases}$$

$$\left.\begin{array}{l} \delta(a_\mu)_{SM} = 4.5\times10^{-10} \\ \delta(a_\mu)_{exp} = 6.3\times10^{-10} \end{array}\right\} \; a_\mu^{SM} - a_\mu^{exp} = (28.7 \pm 8.0)\times10^{-10}$$

(3.6σ)

2.1 Parity violation in atomic physics

- one of first manifestations of parity violation in neutral currents

- non-relativistic parity violating electron-nucleon contact interaction (nucleon no/spin effect)

$$H_{PV}^{eN} = \underset{\substack{\uparrow \\ \text{effective weak} \\ \text{(axial) charge}}}{Q_W}\frac{G_F}{4\sqrt{2}}\;\underset{\substack{\uparrow\\ \text{spin}}}{\vec{\sigma}_e}\cdot\underset{\substack{\uparrow\\ \text{velocity}}}{\vec{v}_e}\;\delta^3(\underset{\substack{\uparrow\\ \text{position}}}{\vec{r}_e}) + h.c.$$

- effective electron-quark interaction from Z-exchange

$$\mathcal{L}_{PV}^{eq} = \frac{G_F}{\sqrt{2}}\sum_{q=u,d}\left[C_{1q}(\bar{e}\gamma_\mu\gamma_5 e)(\bar{q}\gamma^\mu q) + C_{2q}(\bar{e}\gamma_\mu e)(\bar{q}\gamma^\mu\gamma_5 q)\right]$$

$$C_{1u} = -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \qquad C_{2u} = -C_{2d} = -\frac{1}{2} + 2\sin^2\theta_W$$

$$C_{1d} = \frac{1}{2} - \frac{2}{3}\sin^2\theta_W$$

↑ for non-rel. electron & point-like nucleus only $\mu=0$ component contributes

$$\Rightarrow \quad Q_W = -2\,(C_{1u}N_u + C_{1d}N_d)$$

↖ ↗ quark occupation numbers

$N_u = 2Z + N \qquad N_d = Z + 2N$

Exp: optical transitions between quasi-degenerate states of opposite parity (mixed by H_{PV})

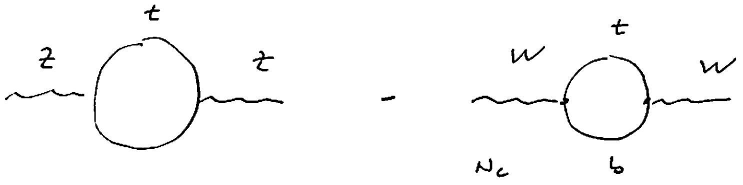
Cs $(Z = 55, N = 78)$: $Q_W(Cs) \simeq -73.16(29)_{exp}(20)_{theory}$

$\Rightarrow \sin^2\theta_W \simeq 0.239$ (at low scale $\sim 10\,MeV$)

2.2. Leading correction to the $\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W}$ parameter

- key test : measured with precision of $\sim 10^{-6}$!

- tests of EW quantum corrections : sensitive to top mass at 1% level !

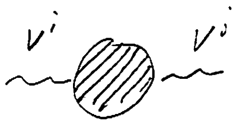


$$\rho = 1 + \frac{3 G_F m_t^2}{8 \sqrt{2} \pi^2} = 1 + \frac{\overset{N_c}{3} \lambda_t^2}{4\pi^2} \leftarrow \text{top yukawa}$$

$$\simeq 1.01$$

(Also important $Z f \bar{f}$ vertex, corrections, e.g. $\frac{ig}{\cos\theta_W}\left[\frac{1}{2}(1+\tau) - \frac{1}{3}\sin^2\theta_W\right] Z_\mu \bar{b}_L \gamma^\mu b_L$ $\quad \tau = -\frac{G_F m_t^2}{4\pi^2\sqrt{2}}$

- More general study of vacuum polarization amplitudes



$$\mathcal{L}_{vac.-pol.} = -\frac{1}{2} W_\mu^3 \Pi_{33} W_\mu^3 - \frac{1}{2} B_\mu \Pi_{00} B_\mu - W_\mu^3 \Pi_{30} B_\mu - W_\mu^+ \Pi_{WW} W_\mu^-$$

In momentum space (g.f.) define $\overset{EW}{/}$ "pseudo-observables"
$\Pi_i(q^2)$

$$\hat{S} \equiv \frac{g}{g'} \Pi_{30}'(0) \quad \leftarrow Z \text{ lineshape} \quad (W^3\text{-}B \text{ mixing})$$

$$\hat{T} \equiv \frac{\Pi_{33}(0) - \Pi_{WW}(0)}{m_W^2} = \rho - 1$$

$$\hat{U} = \Pi_{33}'(0) - \Pi_{WW}'(0) \quad , \hat{V} , \hat{W} , \hat{X} , \hat{Y}$$

...

Sensitivity to (heavy) physics affecting structure of EWSB

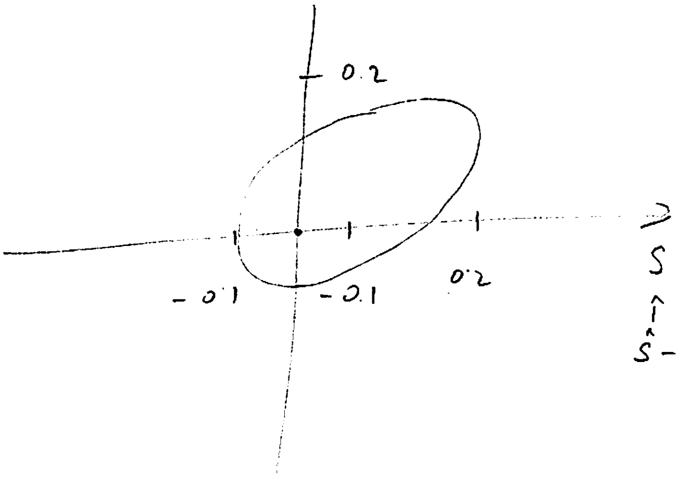
=> Sensitivity to Higgs mass in SM (only logarithmic)

$$\hat{S}_h = \frac{G_F m_W^2}{12\sqrt{2}\pi^2} \log\frac{m_h^2}{m_Z^2} = \frac{g^2}{192\pi^2} \log\frac{m_h^2}{m_Z^2} \quad ; \quad \hat{S}_t = -\frac{G_F m_W^2}{6\times\sqrt{2}\pi^2} \log\frac{m_t}{m_t}$$

$$\hat{T}_h = -\frac{3 G_F m_W^2}{4\sqrt{2}\pi^2} \tan^2\theta_W \log\frac{m_h^2}{m_Z^2} = \frac{3\alpha_{em}}{16\pi\cos^2\theta_W} \log\frac{m_h^2}{m_Z^2}$$

$\hat{T} - \hat{T}_{SM} \sim (-0.1 \pm 0.6) \times 10^{-3}$

$T \equiv \frac{1}{\alpha}\hat{T} \simeq 130\,\hat{T}$

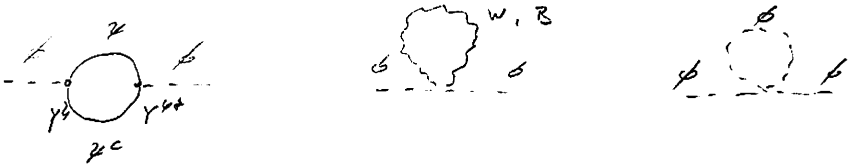


$S \equiv \frac{4\sin^2\theta_W}{\alpha}\hat{S} \simeq 120\,\hat{S}$

$\hat{S} - \hat{S}_{SM} \sim (-0.1 \pm 0.6) \times 10^{-3}$

(Slide)!

Vacuum polarization amplitudes for Higgs field



- Correction (renormalization) of the Higgs potential: $V(\phi)$

$$\mu^2 = \mu_0^2 + \delta\mu^2$$

If computed in Λ - hard UV cut-off regularization:

$$\delta\mu^2_{SM} = \frac{3\Lambda^2}{32\pi^2 v^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_h^2 + \dots\right)$$

No measurable consequence! (Regularization artifact)

But: If SM extended at high scales (GUTs, N-neutrinos, ...):

$$\delta\mu^2_{UP} \sim \frac{g^2_{UP}}{16\pi^2} M^2_{UP}$$

why $G_F^{-1/2} \ll M_{GUT}, M_{Planck}$? "EW hierarchy puzzle"

3. Precision tests of SM: Flavor physics & CPV

In SM matter fields (quarks & leptons) appear in 3 copies of same gauge representation.

"Flavor physics" refers to interactions that distinguish between flavors

=> Within SM: weak & Yukawa interactions

=> All flavor phenomena predicted in terms of:

- 9 masses of charged fermions
- 4 quark mixing parameters (3 angles + phase) - $W^+ \bar{u}^i d^j$ interactions

——— Adding Majorana mass terms for ν - masses ———

- 3 ν masses
- 6 mixing parameters (3 angles + 3 phases) - $W^+ \bar{\nu}^i e^j$ interactions

3.1 Discrete symmetries of SM

- Local Lorentz invariant QFT: CPT => CP violation = T violation

In SM C & P violated maximally

- C & P change chirality of fermion fields
- In SM left-handed & right-handed fields are different gauge representations

=> Independently of SM parameter values, $\not{C}$ & $\not{P}$ maximal in SM

In SM $\not{CP}$ depends on parameters (Yukawas)

- Hermiticity of $\mathcal{L}$: $Y_{ij} \bar{\psi}_{L_i} \phi \psi_{e_j} + Y_{ij}^* \bar{\psi}_{R_j} \phi^\dagger \psi_{L_i}$

$$\Downarrow \text{CP}$$

$$Y_{ij} \bar{\psi}_{e_j} \phi^\dagger \psi_{L_i} + Y_{ij}^* \bar{\psi}_{L_i} \phi \psi_{e_j}$$

CP symmetric only if $Y_{ij} = Y_{ij}^*$!

- More precisely: $\mathrm{Im}\left(\det\left[Y^d Y^{d\dagger}, Y^u Y^{u\dagger}\right]\right) \equiv J \neq 0$

Jarlskog invariant

Other sources of CPV in SM?

- Higgs potential is CP invariant (for single Higgs doublet transforms as $\phi(\vec{x},t) \overset{CP}{\to} \phi(-\vec{x},t)$ (CP-even scalar)
- Gauge kinetic terms:

$$F_{\mu\nu} \equiv F^a_{\mu\nu} t^a \xrightarrow{CP} -F^T_{\mu\nu}(-\vec{x},t)$$

Can write Lagrangian term: $\varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu}_a F^{\alpha\beta}_a = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} \mathrm{Tr}\left[F^{\mu\nu}F^{\alpha\beta}\right]$

$$= \partial_\mu J^\mu$$

$$J_\alpha \equiv 2\varepsilon_{\mu\nu\alpha\beta} \mathrm{Tr}\left[A_\beta \partial_\mu A_\nu + \frac{2i}{3} A_\mu A_\beta A_\nu\right]$$

Total derivative → contributes only through surface integrals of pure gauge configurations (boundary terms)

- non-perturbative effect in Non-Abelian YM theories

Possible terms in SM:

1. $SU(2)_L$: WW ~ only observable in B+L violating processes
2. $U(1)_Y$: Abelian (boundary terms vanish exactly)
3. $SU(3)_c$: $\Delta\mathcal{L}_{QCD} = \theta_{QCD}\,\varepsilon_{\mu\nu\alpha\beta}\, G^a_{\mu\nu} G^a_{\alpha\beta}$

- Leads to observable effect in neutron EDM: $\theta_{exp} < 10^{-10}$

"Strong CP puzzle"

3.2 SM weak interactions in the mass basis

Higgs vev: $\mathrm{Re}(\phi^0) \to \frac{v+h^0}{\sqrt{2}}$

Yukawa interactions → fermion mass matrices

$$m^q = \frac{v}{\sqrt{2}} Y^q$$

"mass basis" → m^q = diag :

$$Q \to U_L^{Q^{-1}} Q \qquad u^c \to U_R^u u^c \qquad d^c \to U_R^d d^c$$

In general: $m^u_{diag} = U_L^{Q(u)} m^u U_R^{u\,-1}$

$$m^d_{diag} = U_L^{Q(d)} m^d U_R^{d\,-1}$$

- $U_R^{u,d}$ can be absorbed into the definition of u^c, d^c (basis)

- Since $[m^u, m^d] \neq 0$: $U_L^{Q(u)} \neq U_L^{Q(d)}$

$$U_L^{Q(u)} U_L^{Q(d)\,-1} \equiv V_{CKM} \neq 0$$

SM flavor Lagrangian

Neutral current (γ, Z, G): No FCNC's at leading order in SM

$$\mathcal{L}_{flavor} = (\bar{q}^i \not{D} q^j \delta^{ij})_{NC} + \frac{g}{\sqrt{2}} \bar{u}_L^i \gamma^\mu V_{CKM}^{ij} d_L^j W_\mu^+ +$$

"flavor universal" "flavor changing", only LH fields

$$+ u_L^i \frac{m^u_{ii}}{v} u_R^i (v+h^0) + \bar{d}_L^i \frac{m^d_{ii}}{v} d_R^i (v+h^0)$$

"flavor diagonal", not universal!

3.3. Testing the CKM

Parametrization of V_{CKM}: (in mass ordered basis)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Strong hierarchical pattern of off-diagonal entries

$|V_{ud}| \sim |V_{cs}| \sim |V_{tb}| \sim 1$

$|V_{us}| \sim |V_{cd}| \sim 0.2$

$|V_{cb}| \sim |V_{ts}| \sim 0.04$

$|V_{ub}| \sim |V_{td}| \sim 5 \times 10^{-3}$

+ hierarchical quark masses

$m_u \sim 5\,\text{MeV}$ $m_t \sim 173\,\text{GeV}$

$m_d \sim 10\,\text{MeV}$ $m_b \sim 4.2\,\text{GeV}$

"SM flavor puzzle"

- Explicit in Wolfenstein expansion ; $\lambda \equiv |V_{us}| \simeq 0.22$

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Free parameters : $\lambda, \underbrace{A, \rho, \eta}_{\mathcal{O}(1)} \leftarrow$ phase

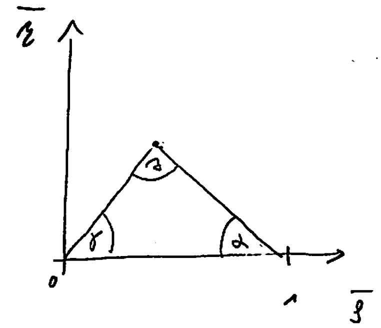
Unitarity of CKM : $\sum_k V_{ik}^* V_{jk} = \delta_{ij}$ $\sum_k V_{ki}^* V_{kj} = \delta_{ij}$

- particularly interesting for $i=1$ $j=3$ in terms of Wolfenstein parameters

all terms of same order in λ:

$$A\lambda^3\left\{-\left[\bar\rho + i\bar\eta\right] - \left[1-\bar\rho - i\bar\eta\right] + 1\right\} = 0 \quad ; \quad \begin{aligned}\bar\rho &\equiv \rho\left(1-\tfrac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4) \\ \bar\eta &\equiv \eta\left(1-\tfrac{\lambda^2}{2}\right) + \mathcal{O}(\lambda^4)\end{aligned}$$

- Defines triangle in a plane



Angles & sides - Observable quantities
(invariant under phase transformations of quark fields)

Area = measure of CPV

Precision tests of CKM (Slide)

- $|V_{us}|$ (λ) from $K \to \pi e \nu$ (3‰) $\lambda = 0.2253(9)$
- $|V_{cb}|$ (A) from $B \to X_c e \nu$ (2%) $A = 0.822(12)$

- $|V_{ub}|^2 \propto \bar{\rho}^2 + \bar{\eta}^2$ from $B \to \pi e \nu$
- time-dependent CP asymmetry in $B \to \psi K_S$ $\left(S_{\psi K_S} = \sin 2\beta = \frac{2\bar{\eta}(1-\bar{\rho})}{(1-\bar{\rho})^2 + \bar{\eta}^2} \right)$
- rates of $B \to DK$ depend on $e^{i\gamma} = \frac{\rho + i\eta}{\rho^2 + \eta^2}$
- rates of $B \to \pi\pi, \rho\pi, \rho\rho$ depend on $\alpha = \pi - \beta - \gamma$
- mass-differences in B_d & B_s neutral meson system: $\frac{\Delta m_d}{\Delta m_s} \propto \frac{|V_{td}|^2}{|V_{ts}|^2} = \lambda^2 \left[(1-\bar{\rho})^2 + \bar{\eta}^2 \right]$
- CPV in $K \to \pi\pi$ (ε_K) (complicated expression)

Lead to impressive agreement with best-fit ranges

$$\rho = 0.130 \pm 0.024$$

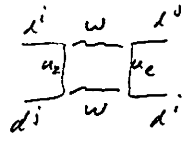
$$\eta = 0.362 \pm 0.014 \leftarrow \text{CKM phase is } \mathcal{O}(1)$$

$\Rightarrow$ CPV in flavor-changing processes is dominated by CKM.

In SM FCNCs at higher orders in pert. theory (quantum effects)

- calculable: several suppression factors
- example: $\Delta F = 2$ 4-quark amplitudes contributing to neutral meson oscillations: $|\psi(t)\rangle = a(t)\,|M^0\rangle + b(t)\,|\overline{M^0}\rangle + \ldots$

$$i \frac{d}{dt} \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = H \begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$



$$M_{12}^{SM} = \operatorname{Re}\left[H_{12}^{SM} \right] = \underbrace{\frac{G_F^2 m_t^2}{16\pi^2}}_{\substack{\text{loop} \\ + \text{GIM}}} \underbrace{\left(V_{ti} V_{tj}^* \right)^2}_{\substack{\text{CKM} \\ \text{suppression}}} \underbrace{\frac{\langle \overline{M} | (\bar{d}^i \gamma_\mu P_L d_j)^2 | M \rangle}{2 m_M}}_{\text{hadronic matrix el.}} \underbrace{F\left(\frac{m_t^2}{m_W^2} \right)}_{\mathcal{O}(1)} + \ldots$$

Resulting suppression:

$$\frac{\lambda_c^2}{64\pi^2}|V_{cd}^* V_{cs}|^2 \sim 5\times10^{-9} \quad \frac{\Delta m_K}{m_K} \sim 7\times10^{-15}$$

Hadronic effects of known to $\mathcal{O}(1)$

$$\cancel{\lambda}^2 \quad \frac{\Delta m_D}{m_D} \sim 9\times10^{-15}$$

Hadronic effects known to $\mathcal{O}(10)$

$$\frac{\lambda_t^2}{64\pi^2}|V_{td} V_{ts}^*|^2 \sim 9\times10^{-8} \quad \frac{\Delta m_B}{m_B} \sim 6\times10^{-14}$$

$$\frac{\lambda_t^2}{64\pi^2}|V_{ts} V_{tb}^*|^2 \sim 3\times10^{-6} \quad \frac{\Delta m_{B_s}}{m_{B_s}} \sim 2\times10^{-12}$$

Hadronic factors computable on Lattice
current relative precision $\lesssim 10\%$!!

$\Delta m_d = 0.55(3)\ ps^{-1}$ $\quad \Delta m_d^{exp} = 0.5065(19)\ ps^{-1}$

$\Delta m_s = 16.8(4)\ ps^{-1}$ $\quad \Delta m_s^{exp} = 17.757(21)\ ps^{-1}$

3.4 Fermion Number conservation in SM

In absence of $\mathcal{L}_{Yuk}$ SM contains large global matter symmetry:

$$G^{fam}_{SM} = U(3)^5 = \underbrace{SU(3)^3_q}_{} \times \underbrace{SU(3)^2_\ell}_{} \times \underbrace{U(1)^5}_{}$$

$$SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c}$$

$$SU(3)_L \times SU(3)_{e_c}$$

$$U(1)_B \times \underbrace{U(1)_L \times U(1)_{e^c}}_{} \times U(1)_Y \times U(1)_{PQ}$$

$U(1)_B$: $\uparrow$ $\frac{u^c + d^c + Q}{3}$

$U(1)_L \times U(1)_{e^c}$: $\downarrow$ broken by Y^e to $U(1)_{\underbrace{L+e^c}_{\tilde{L}}}$

$U(1)_Y$: $\uparrow$ gauged, broken by $\langle\phi\rangle$

$U(1)_{PQ}$: $\uparrow$ residual $U(1)$ broken by $[Y^u, Y^d] \neq 0$

$[Y^u, Y^e] \neq 0$

Furthermore:

In absence of ν-masses Y^e can be defined diagonal ($U^e_{L,R}$ not-physical)

$$Y^e \sim \mathrm{diag}\left(\frac{m_e}{v}, \frac{m_\mu}{v}, \frac{m_\tau}{v}\right) \leftarrow \text{invariant under } U(1)_e \times U(1)_\mu \times U(1)_\tau$$

SM (without ν-masses) has $U(1)_B \times \underbrace{U(1)_e \times U(1)_\mu \times U(1)_\tau}_{U(1)_{\tilde{L}}}$

global / symmetry

classical

Broken by quantum effects: $\Rightarrow$ $U(1)_{B-L}$

$$\mathrm{Tr}[\underbrace{T^a T^b}_{SU(2)_L} B] = 3\,\mathrm{Tr}[T^a T^b L_i] \neq 0$$

$\Rightarrow$ At perturbative level SM without ν-masses has <u>no charged LFV</u>

Confirmed exp.:

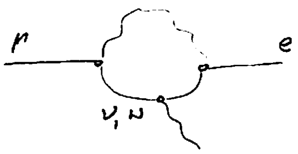
$$Br(\mu \to e\gamma) < 4 \times 10^{-13}$$

$$Br(\mu \to eee) < 10^{-12}$$

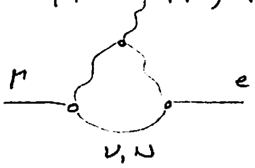
$$\frac{\Gamma(\mu \to e \text{ in Au})}{\Gamma(\mu\text{-capture in Au})} < 7 \times 10^{-13}$$

Lepton numbers (flavor) conservation in presence of m_ν

ν-mass through N^0: $\mathcal{L}^{\nu\text{-mass}} = -\frac{1}{2}\left(\nu^T\ N^T\right)\begin{pmatrix} 0 & Y^N v \\ Y^{NT} v & M \end{pmatrix}\begin{pmatrix}\nu \\ N\end{pmatrix}$; for $M \gg v$:



+



$\mathcal{L}^{\nu\text{-mass}} \approx \frac{v^2}{2}\nu^T Y^N \frac{1}{M} Y^{NT}\nu + \ldots$

"Majorana", "See-saw"

for $M \ll v Y_N$:

$\mathcal{L}^{\nu\text{-mass}} = -\frac{v}{\sqrt{2}} Y^N \nu^T N + h.c.$

"(pseudo-)Dirac"

$$\mathcal{L}_{\mu\to e\gamma} \simeq e\,\frac{\alpha}{\pi}\,\frac{v^3}{m_W^4}\left(Y^N \min\left(1, \frac{m_W^2}{M^2}\right) Y^{N\dagger}\, Y^e\right)_{\mu e}\left(\bar{\mu}\,\sigma_{\mu\nu}\, e\right) F_{\mu\nu}$$

related to ν-masses

related to e^+-masses

$$\left(Y^N\right)^2 \simeq \frac{m_\nu^2}{v^2}\max\left(1, \frac{M}{m_\nu}\right)$$ up to accidental cancelations

$$\simeq A^{\mu e}\left(\bar{\mu}\,\sigma_{\mu\nu}\, e\right) F_{\mu\nu}\quad ;\ A^{\mu e} \lesssim e\,\frac{\alpha}{\pi}\,\frac{m_\mu}{m_W^2}\,\frac{m_\nu}{m_W}$$

completely negligible