The Higgs sector in the SM

Literature: 1406.1786

Electroweak symmetry breaking

$SU(2)_L \times U(1)_Y \to U(1)_{QED}$

1. $W^\pm, Z$ – massive γ – massless

* gauge invariance forbids gauge boson mass term!

$\cancel{\mathcal{L} \supset \frac{1}{2} m^2 A_\mu A^\mu}$ $(A_\mu \to A_\mu + \frac{1}{g}\partial_\mu \alpha)$

2. Chiral fermions, three copies (generations)

	left-handed		right-handed		
	$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	u_R	d_R	e_R
$SU(2)_L \times U(1)_Y$:	$(2, \frac{1}{6})$	$(2, -\frac{1}{2})$	$(1, \frac{2}{3})$	$(1, -\frac{1}{3})$	$(1, -1)$

* $Q = T_3 + Y$

* Lorentz group $SO(1,3) \simeq SU(2) \times SU(2)$ representations

Weyl spinors $\psi_L \in (\frac{1}{2}, 0)$ $\psi_R \in (0, \frac{1}{2})$

Under Lorentz transformation:

Exercise: show L.I.

$$\psi_{L,R} \to e^{(-i\vec{\theta} \mp \vec{\eta}) \frac{\vec{\sigma}}{2}}\, \psi_{L,R}$$

(rotations: $\vec{\theta}$, boosts: $\vec{\eta}$)

Dirac mass term $\cancel{\mathcal{L} \supset -m\, \psi_L^\dagger \psi_R + h.c.}$

Majorana mass term $\cancel{\mathcal{L} \supset -m\, \psi_L^T i\sigma_2 \psi_L}$

* Both violate gauge invariance!

So how do elementary particles get mass?

* All fermions are massive (except maybe one neutrino)

→ via interactions

The SM Higgs mechanism

- Introduce a single $SU(2)_L$-doublet scalar field, which causes spontaneous breaking

representation $(2,\frac{1}{2})$

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi_0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\,\phi_2 \\ \phi_3 + i\,\phi_4 \end{pmatrix}$$

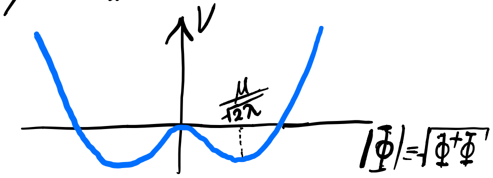
$$\mathcal{L}_\phi = \underbrace{(D_\mu \Phi)^\dagger (D^\mu \Phi)}_{1} - \underbrace{V(\Phi)}_{2} + \underbrace{\mathcal{L}_{Yukawa}}_{3}$$

gauge invariant Lagrangian (up to dim 4)

2 Scalar potential $V(\phi) = -\mu^2\, \Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$

$\lambda > 0$ – vacuum stability

If $\mu^2 > 0 \rightarrow$ EWSB



Minimisation:

$$\Phi^\dagger\Phi = \frac{1}{2}\left(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2\right) \equiv \frac{\mu^2}{2\lambda} \quad \text{– sphere in 4D}$$

The global accidental symmetry (custodial symmetry)

$$\begin{array}{ccc} SO(4) & \simeq & SU(2)_L \times SU(2)_R \\ \downarrow & & \downarrow \\ SO(3) & \simeq & SU(2)_V \end{array}$$

$$\langle \phi_3 \rangle = v = \frac{\mu}{\sqrt{\lambda}}$$

$$\langle \phi_1 \rangle = \langle \phi_2 \rangle = \langle \phi_4 \rangle = 0$$

$(2,2) \rightarrow 3 \oplus 1$

3 → Goldstone triplet; 1 → Higgs boson

define:

$$\Sigma \equiv \begin{pmatrix} \phi_0^* & \phi^+ \\ -\phi^- & \phi_0 \end{pmatrix} \; ; \quad \Sigma \rightarrow U_L\, \Sigma\, U_R^\dagger$$

$$\underbrace{\det(\Sigma)}_{\text{invariant under } SU(2)_L\times SU(2)_R} = |\phi_0|^2 + \phi^+\phi^- = \Phi^\dagger\Phi = \underbrace{\frac{1}{2}\sum_i \phi_i^2}_{\text{invariant under } SO(4)}$$

VEV ($\simeq 246$ GeV)

Choose a vacuum: $\langle\phi_3\rangle \equiv v = \sqrt{\frac{\mu^2}{\lambda}}$ $\quad \langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0$

Define: $\phi_3(x) = h(x) + v$ (expand around vacuum)

Calculate quadratic terms in the potential:

$$V = -\frac{\mu^2}{2}\left(\phi_1^2+\phi_2^2+(h+v)^2+\phi_4^2\right) + \frac{\lambda}{4}\left(\phi_1^2+\phi_2^2+(h+v)^2+\phi_4^2\right)^2$$

$$= \underline{0\cdot\phi_1^2 + 0\cdot\phi_2^2 + 0\cdot\phi_4^2} + \underline{\lambda v^2 h^2} + \dots$$

massless modes (Goldstone boson)

* three broken generators

massive mode (real scalar field spin - 0)

Higgs boson

$m_h = \sqrt{2\lambda v^2} \simeq 125$ GeV

- Field redefinitions

Exercise: Expand exp up to linear order

$$\phi(x) = \frac{1}{\sqrt{2}}\, e^{i\frac{\xi^a(x)\sigma^a}{v}} \begin{pmatrix} 0 \\ v+h(x) \end{pmatrix} \quad ; \quad \phi^\dagger\phi = \frac{1}{2}(v+h)^2$$

- Use gauge "rotations" to remove Goldstones from theory — unitary gauge

$$SU(2)_L: \quad \phi(x) \to e^{i\lambda_L^a(x)\frac{\sigma^a}{2}}\,\phi(x)$$

$$\lambda_L^a(x) \equiv -2\frac{\xi^a(x)}{v}$$

Higgs self-couplings

$$\mathcal{L}_V \supset -V(\phi) = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

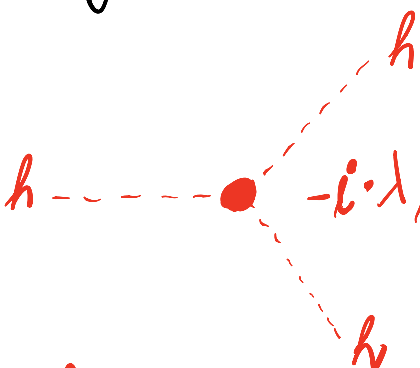
$$\supset -\lambda v^2 h^2 - \lambda v h^3 - \frac{\lambda}{4} h^4$$

$\phi^\dagger \phi = \frac{1}{2}(v+h)^2$

$\mu^2 = \lambda v^2$

$-\lambda v^2 = -\frac{m_h^2}{2}$

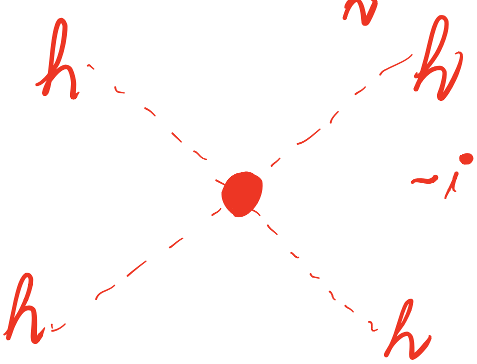
Feynman rules:



$$-i\cdot\lambda v\cdot 3! \equiv -3i\frac{m_h^2}{v}$$

*Not observed at the LHC!

h^3: HL-LHC ?



$$-i\frac{\lambda}{4}\cdot 4! = -3i\frac{m_h^2}{v^2}$$

Gauge boson - Higgs sector

$$\mathcal{L} \supset (D_\mu \Phi)^\dagger (D^\mu \Phi) \qquad \Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

Covariant derivative $D_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - i\frac{g}{2}W_\mu^a \tau^a$

In the unitary gauge:

$$D_\mu \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{i}{2} g \left(W_\mu^1 - i W_\mu^2\right)(v+h) \\ \partial_\mu h + \frac{i}{2}\left(g W_\mu^3 - g' B_\mu\right)(v+h) \end{pmatrix}$$

Exercise: Plug in

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) + \frac{1}{8} g^2 (v+h)^2 \left(W_\mu^1 - i W_\mu^2\right)\left(W^{1\mu} + i W^{2\mu}\right)$$

$$+ \frac{1}{8}(v+h)^2 \left(-g' B_\mu + g W_\mu^3\right)^2$$

Complex vector field: $W_\mu^+ = \dfrac{W_\mu^1 - i W_\mu^2}{\sqrt{2}}$ $\quad W_\mu^- = (W_\mu^+)^*$

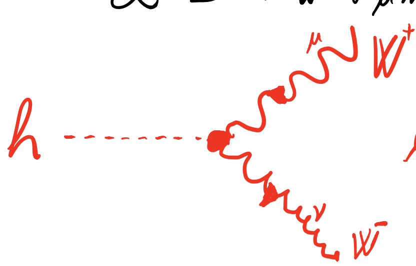
-Sign from $\underset{-\frac{2}{3}}{\bar{u}_L} \gamma^\mu \underset{-\frac{1}{3}}{d_L} \underset{+1}{W_\mu^+}$ charged weak interaction

hWW: $\mathcal{L} \supset m_W^2 \left(1+\frac{h}{v}\right)^2 W_\mu^+ W^{-\mu}$

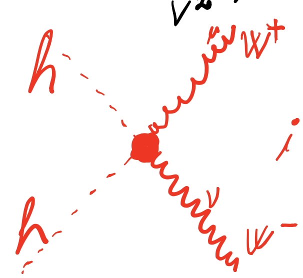
$$\boxed{m_W = \frac{gv}{2}}$$

W mass proportional to VEV and electroweak coupling strength

$$\mathcal{L} \supset m_W^2 W_\mu^+ W^{-\mu} + 2\frac{m_W^2}{v} h W_\mu^+ W^{-\mu} + \frac{m_W^2}{v^2} h^2 W_\mu^+ W^{-\mu}$$



$i\, 2 \frac{m_W^2}{v} g^{\mu\nu}$



$i\, 2 \frac{m_W^2}{v^2} g^{\mu\nu}$

How about Z ? (Photon is massless)

$$(g W_\mu^3 - g' B_\mu) = \sqrt{g^2+g'^2}\underbrace{\left(\frac{g}{\sqrt{g^2+g'^2}} W_\mu^3 - \frac{g'}{\sqrt{g^2+g'^2}} B_\mu\right)}_{Z_\mu}$$

$$\cos\theta_W \equiv c_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

- Weinberg mixing angle

$$\mathcal{L} \supset \frac{1}{8}(v+h)^2 \left(-g' B_\mu + g W_\mu^3\right)^2$$

$$\supset \frac{1}{2} m_Z^2 \left(1+\frac{h}{v}\right)^2 Z_\mu Z^\mu$$

$$\boxed{m_Z = \frac{\sqrt{g^2+g'^2}}{2} v}$$

Similar diagrams as for hWW

Higgs boson is electrically neutral!

* Couplings hWW and hZZ already measured at $\mathcal{O}(10\%)$

Fermion-Higgs interactions (Yukawa sector)

Left-handed field ~ doublets Right-handed ~ singlets

- Since Higgs is a doublet, one can write dim-4 operators (renormalizable)

• Leptonic sector:

$$\mathcal{L} \supset - y_e\, \bar{e}_R\, \phi^\dagger L_L - y_e^*\, \bar{L}_L\, \phi\, e_R$$

hyperchange: $+1$ $\quad -\frac{1}{2}$ $\quad -\frac{1}{2}$

$(\quad)^*(\;)$ ~ $SU(2)_L$ invariant

Expanding:

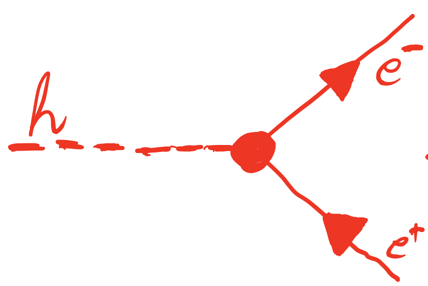
$$\mathcal{L} \supset -\frac{y_e}{\sqrt{2}}(v+h)\left(\bar{e}_L e_R + \bar{e}_R e_L\right)$$

$$\supset -\frac{m_e}{v}\left(1+\frac{h}{v}\right)\bar{e}e$$

$$\phi^\dagger Q_L = \left(0 \;\; \frac{v+h}{\sqrt{2}}\right)\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\boxed{m_f = \frac{y_f\, v}{\sqrt{2}}}$$

* Fermion masses proportional to vev and Yukawa coupling



$$-i\frac{m_e}{v}$$

$y_e \sim 2\cdot 10^{-6}$

$y_\tau \sim 7\cdot 10^{-3}$

$y_t \sim 1$

↳ SM flavour puzzle

Quark sector

Exercise: Show that $\tilde{\Phi} = i\sigma_2 \Phi^*$ transforms as Φ under $SU(2)_L$

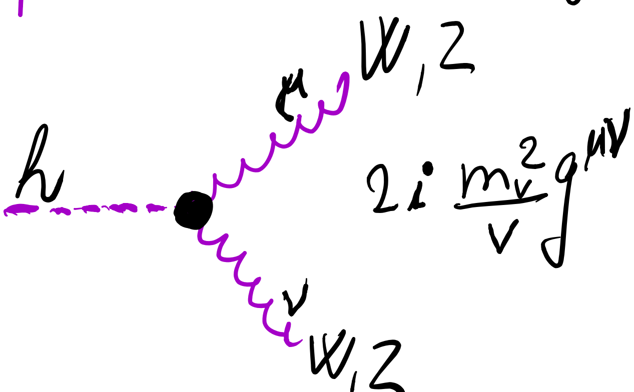
$$\mathcal{L} \supset -y_d^{ij}\, \bar{Q}_L^i\, \Phi\, d_R^j - y_u^{ij}\, \bar{Q}_L^i\, \tilde{\Phi}\, u_R^j + \text{h.c.}$$

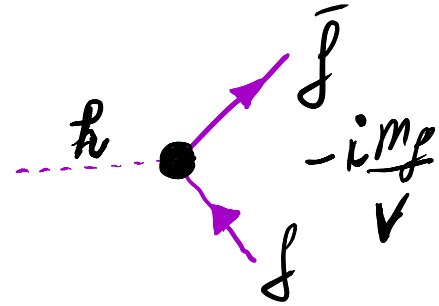
- diagonalizing 3×3 matrices $\mathcal{M}_{u,d} = -y_{u,d}\frac{v}{\sqrt{2}}$
- Higgs couplings: Flavour diagonal but non-universal

$h\bar{f}f$: $-i\frac{m_f}{v}$

Higgs boson characterization

Step 0: Check single Higgs couplings

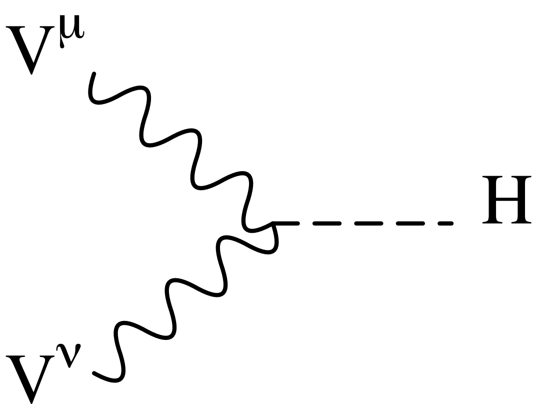




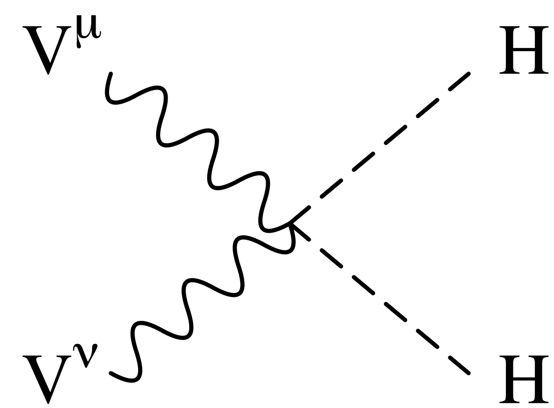
In pp collisions at the LHC!

Higgs couplings

- Higgs-Gauge boson couplings *(V=W,Z)*

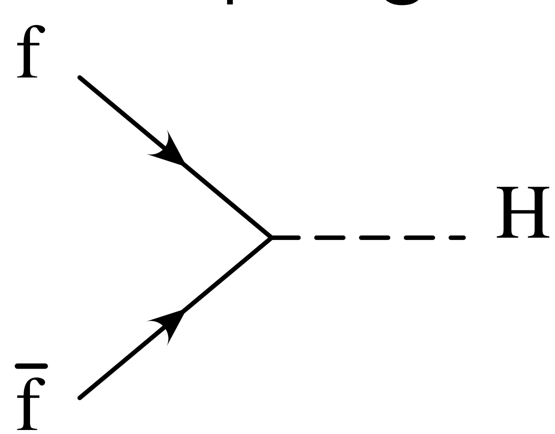


$$= 2i\frac{M_V^2}{v}g^{\mu\nu}$$



$$= 2i\frac{M_V^2}{v^2}g^{\mu\nu}$$

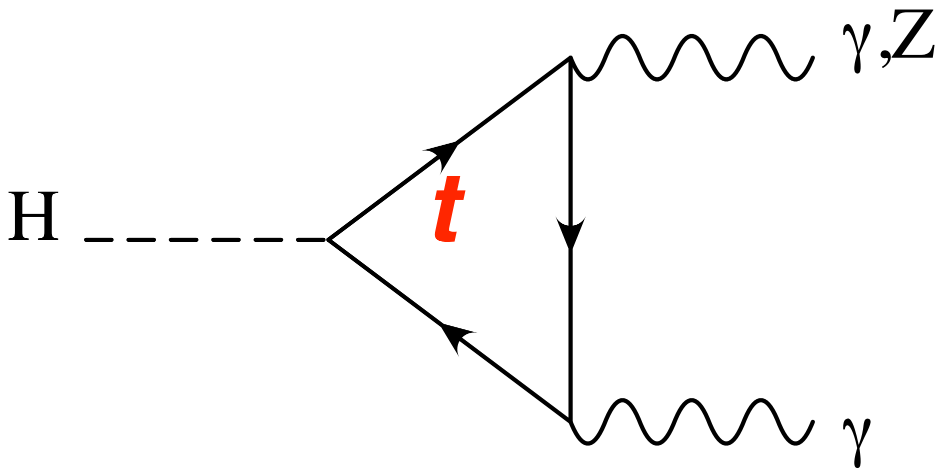
- Higgs-Fermion couplings

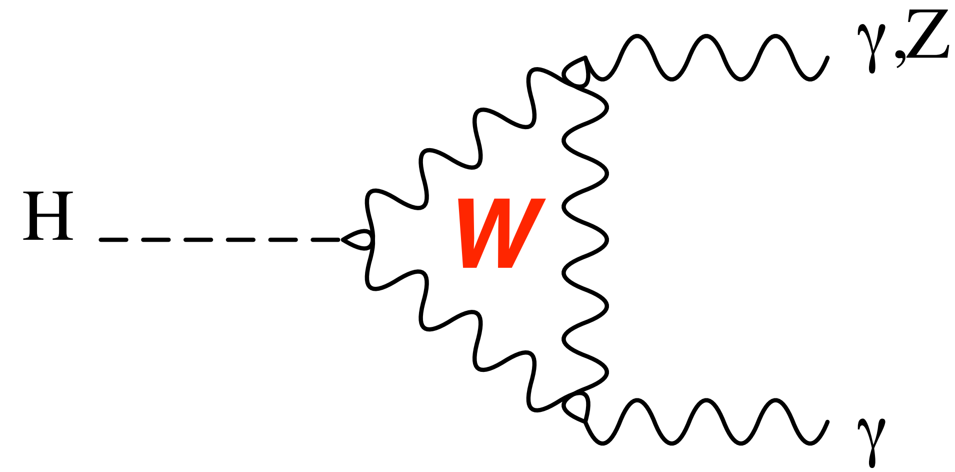


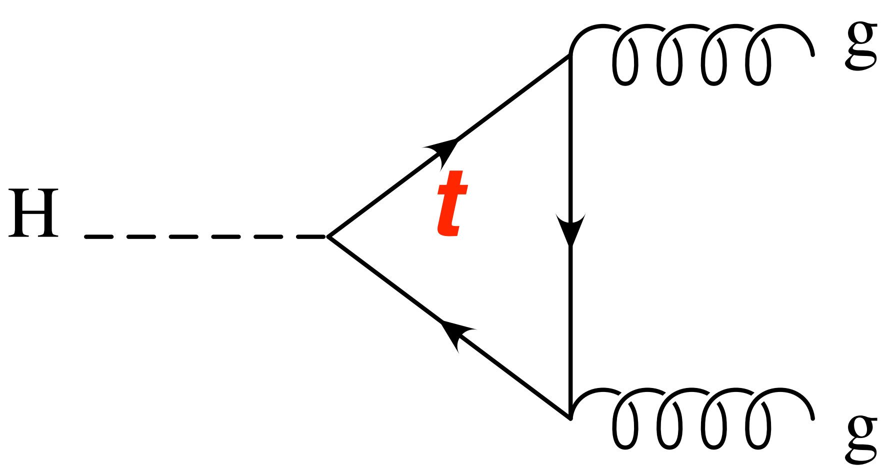
$$= -i\frac{m_f}{v}$$

Higgs couplings

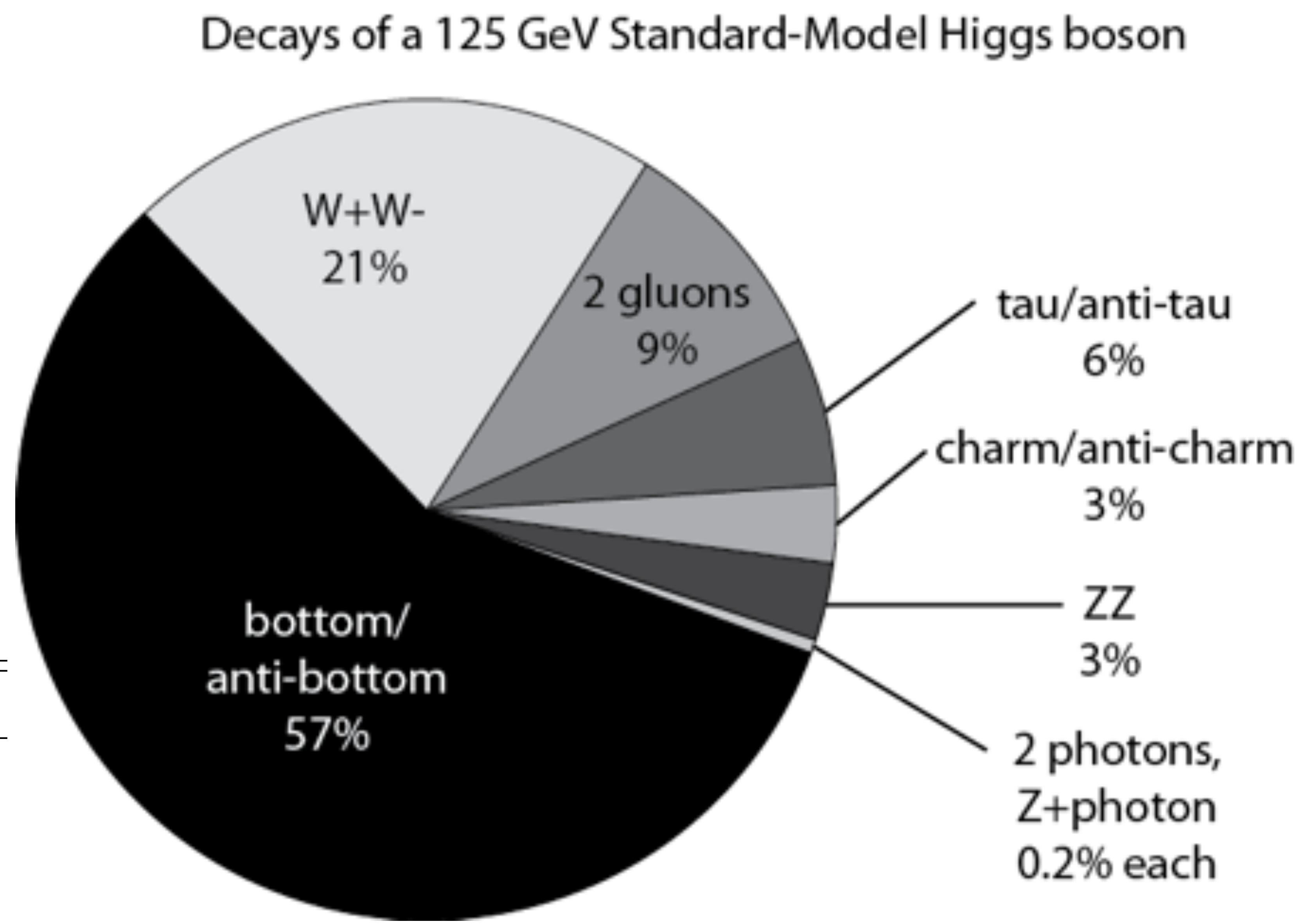
- Loop-induced Higgs couplings





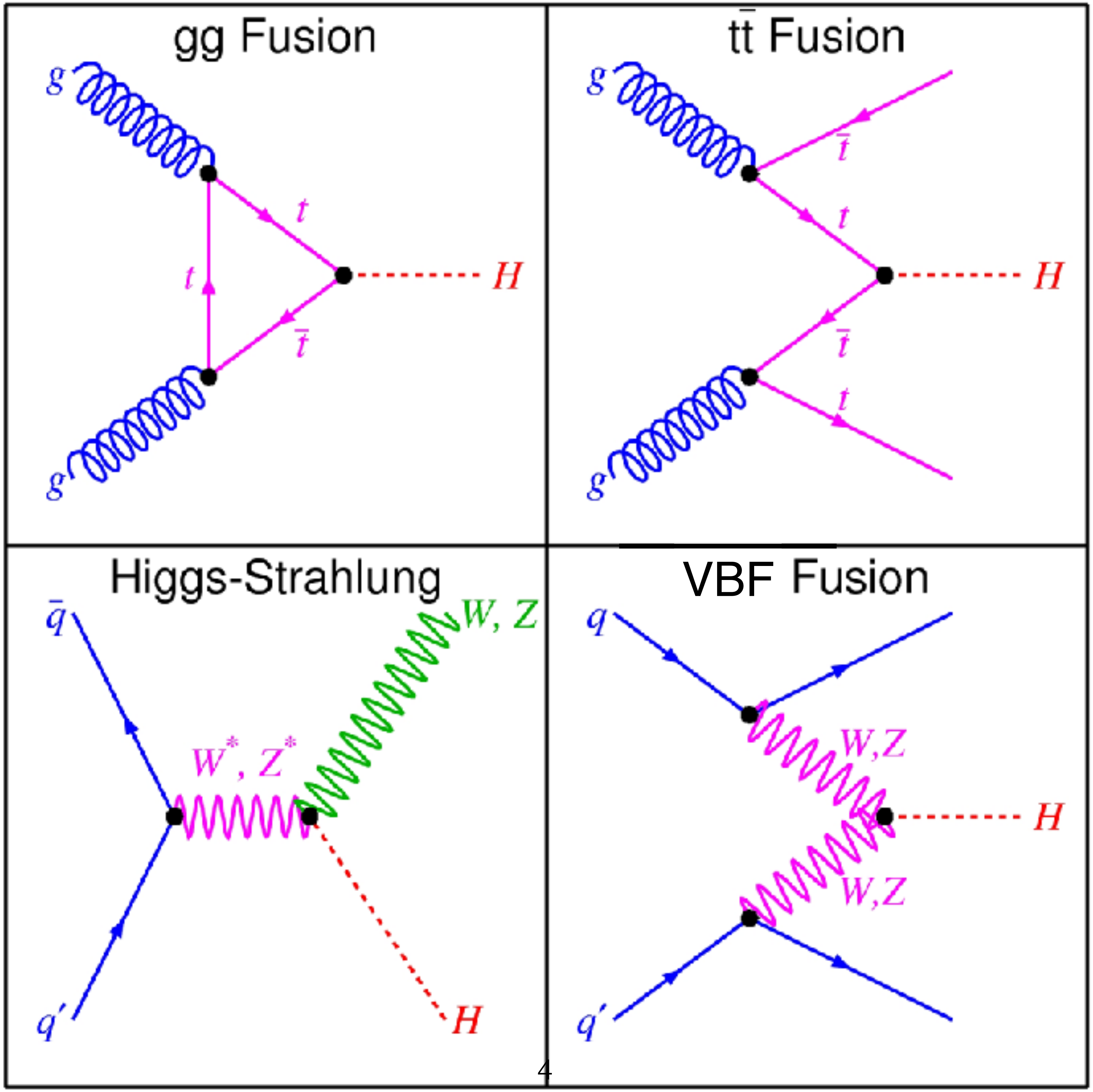


Higgs decay channels in the SM

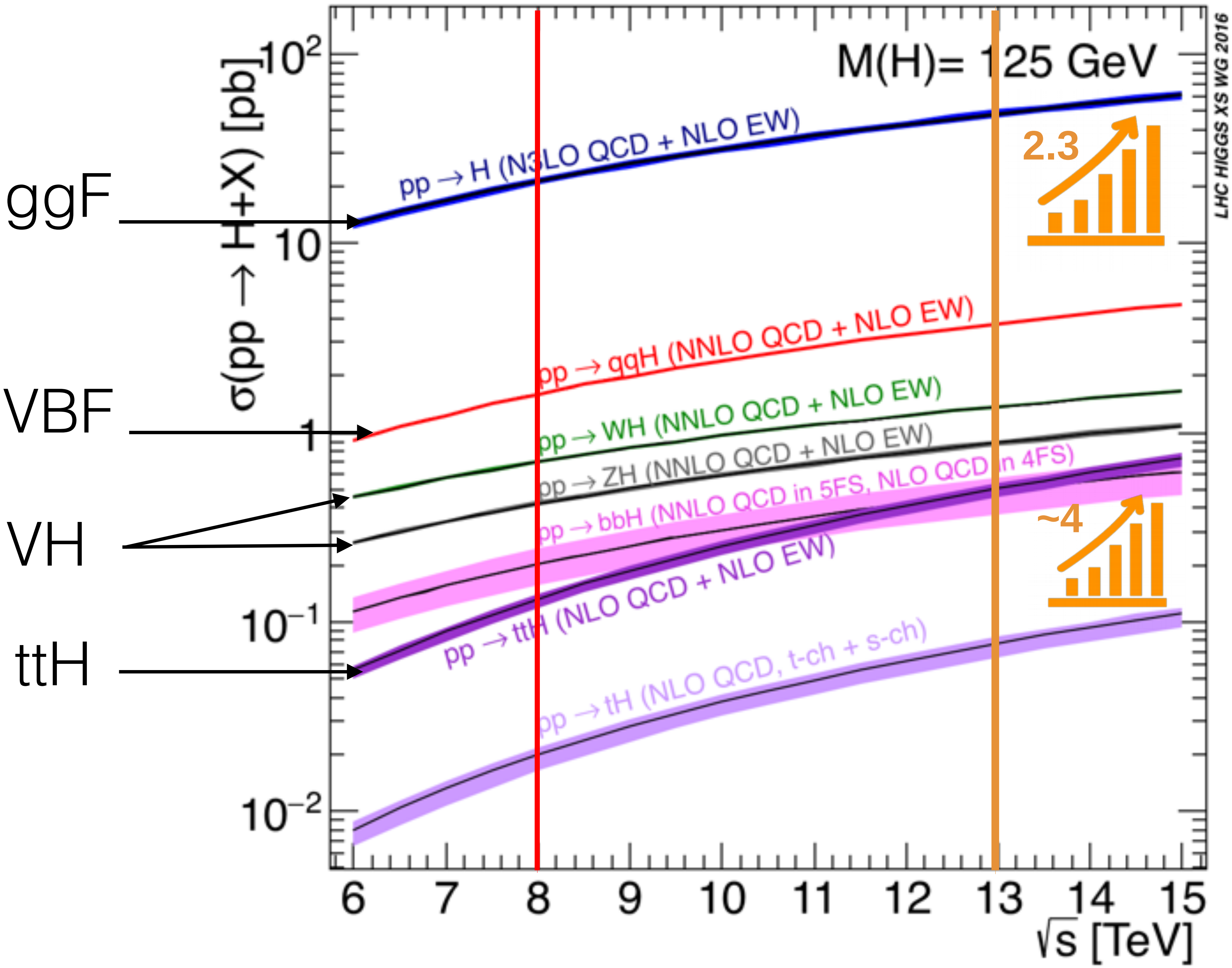


Decay mode	Branching fraction [%]
$H \to bb$	57.5 ± 1.9
$H \to WW$	21.6 ± 0.9
$H \to gg$	8.56 ± 0.86
$H \to \tau\tau$	6.30 ± 0.36
$H \to cc$	2.90 ± 0.35
$H \to ZZ$	2.67 ± 0.11
$H \to \gamma\gamma$	0.228 ± 0.011
$H \to Z\gamma$	0.155 ± 0.014
$H \to \mu\mu$	0.022 ± 0.001

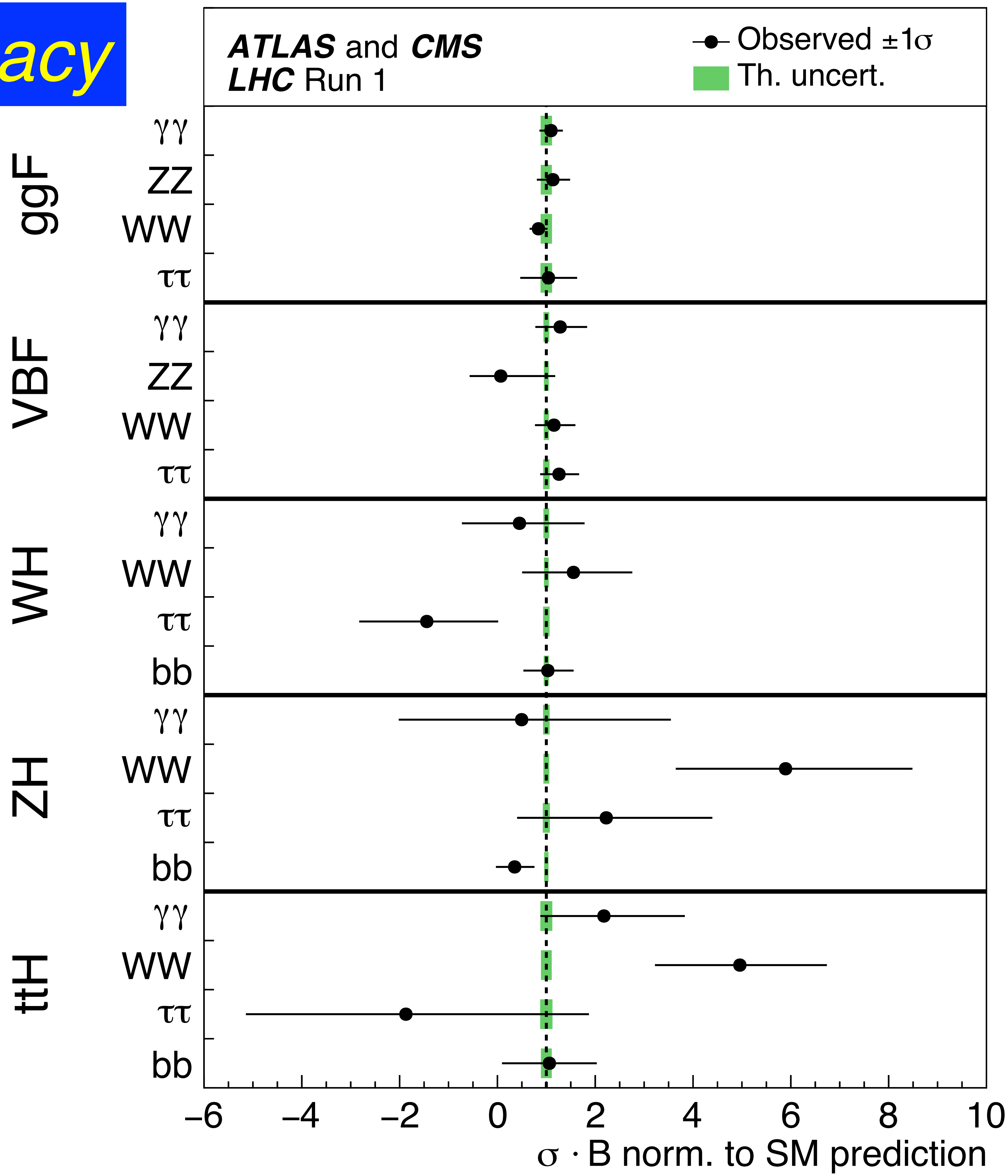
Higgs production at the LHC



Higgs production at the LHC

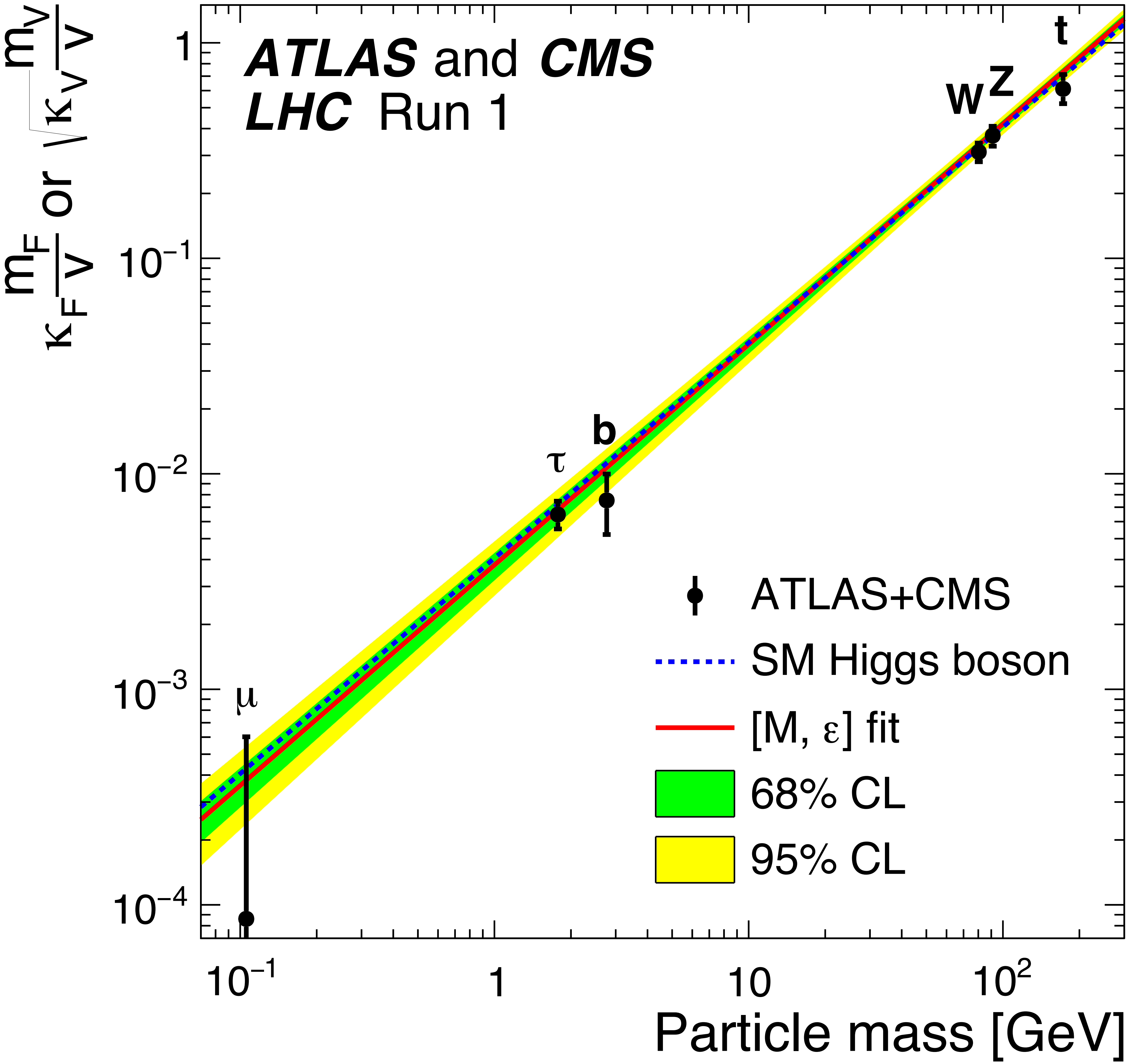


Run I legacy



ATLAS and CMS
LHC Run 1
Observed ±1σ
Th. uncert.
ggF
VBF
WH
ZH
ttH
γγ
ZZ
WW
ττ
bb
−6
−4
−2
0
2
4
6
8
10
σ · B norm. to SM prediction

Run I legacy

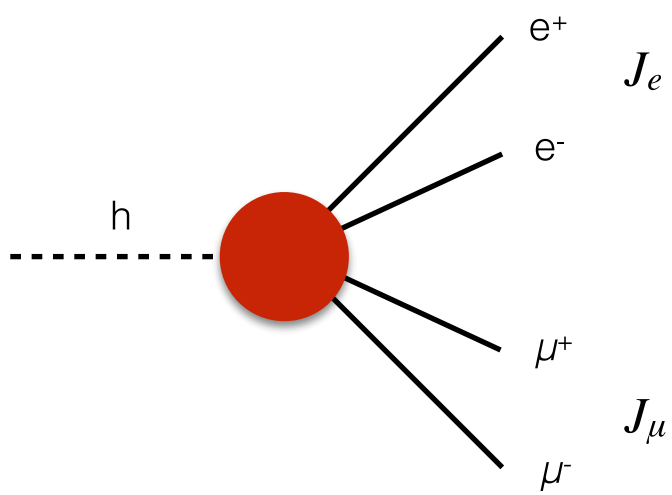

ATLAS and CMS
LHC Run 1
ATLAS+CMS
SM Higgs boson
[M, ε] fit
68% CL
95% CL
μ
τ
b
W
Z
t
Particle mass [GeV]
$\kappa_F \frac{m_F}{v}$ or $\sqrt{\kappa_V} \frac{m_V}{v}$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10
10^{2}

Going beyond coupling strength modifiers…

Higgs pseudo-observables

Example: h→2e2μ

Decomposition of the ***(helicity-conserving)*** *amplitude:*



$$\mathcal{A} = i\frac{2m_Z^2}{v_F}\sum_{e=e_L,e_R}\sum_{\mu=\mu_L,\mu_R}(\bar{e}\gamma_\alpha e)(\bar{\mu}\gamma_\beta\mu)\times$$
$$\left[F_1^{e\mu}(q_1^2,q_2^2)g^{\alpha\beta} + F_3^{e\mu}(q_1^2,q_2^2)\frac{q_1\cdot q_2\ g^{\alpha\beta} - {q_2}^{\alpha}{q_1}^{\beta}}{m_Z^2} + F_4^{e\mu}(q_1^2,q_2^2)\frac{\varepsilon^{\alpha\beta\rho\sigma}q_{2\rho}q_{1\sigma}}{m_Z^2}\right]$$

M. Gonzalez-Alonso, A. Greljo, G. Isidori and D. Marzocca, Eur. Phys. J. C **75**, no. 3, 128 (2015) [arXiv:1412.6038].
M. Gonzalez-Alonso, A. Greljo, G. Isidori and D. Marzocca, Eur. Phys. J. C **75**, no. 7, 341 (2015) [arXiv:1504.04018].

Higgs pseudo-observables

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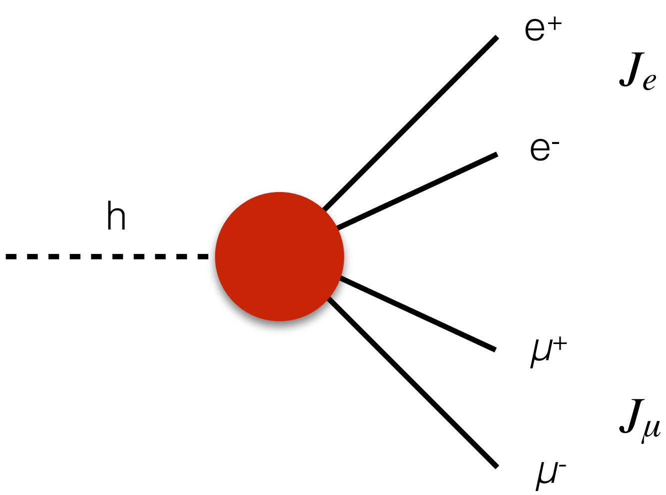
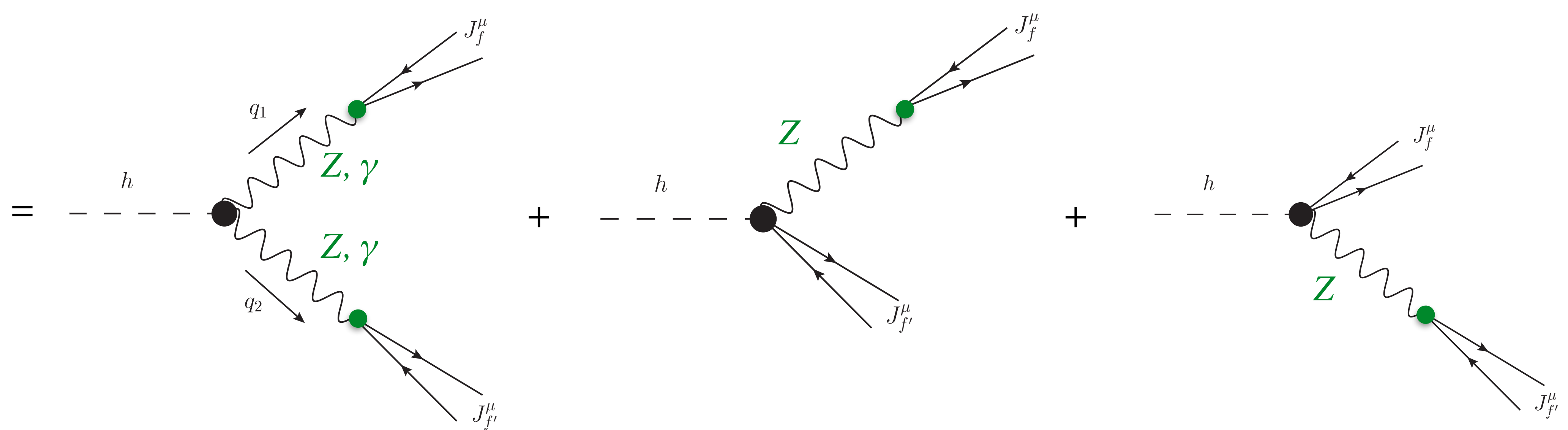
*Decomposition of the **(helicity-conserving)** amplitude:*



$$\mathcal{A} = i\frac{2m_Z^2}{v_F} \sum_{e=e_L,e_R} \sum_{\mu=\mu_L,\mu_R} (\bar{e}\gamma_\alpha e)(\bar{\mu}\gamma_\beta \mu) \times$$
$$\left[F_1^{e\mu}(q_1^2, q_2^2) g^{\alpha\beta} + F_3^{e\mu}(q_1^2, q_2^2) \frac{q_1 \cdot q_2 \; g^{\alpha\beta} - q_2{}^\alpha q_1{}^\beta}{m_Z^2} + F_4^{e\mu}(q_1^2, q_2^2) \frac{\varepsilon^{\alpha\beta\rho\sigma} q_{2\rho} q_{1\sigma}}{m_Z^2} \right]$$

Momentum expansion of the form factors around the physical poles:

- Smooth kinematical distortions from the SM (heavy NP)



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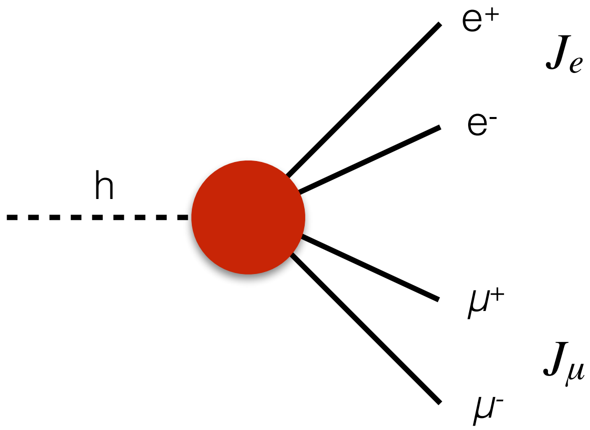
Higgs pseudo-observables

Example: h→2e2μ



$$\mathcal{A} = i\frac{2m_Z^2}{v_F} \sum_{e=e_L,e_R} \sum_{\mu=\mu_L,\mu_R} (\bar{e}\gamma_\alpha e)(\bar{\mu}\gamma_\beta \mu) \times \left[F_1^{e\mu}(q_1^2,q_2^2) g^{\alpha\beta} + F_3^{e\mu}(q_1^2,q_2^2) \frac{q_1\cdot q_2 \; g^{\alpha\beta} - q_2{}^\alpha q_1{}^\beta}{m_Z^2} + F_4^{e\mu}(q_1^2,q_2^2) \frac{\varepsilon^{\alpha\beta\rho\sigma} q_{2\rho} q_{1\sigma}}{m_Z^2} \right]$$

Momentum expansion

$$\begin{aligned}
\mathcal{A} =& i\frac{2m_Z^2}{v_F} \sum_{e=e_L,e_R} \sum_{\mu=\mu_L,\mu_R} (\bar{e}\gamma_\alpha e)(\bar{\mu}\gamma_\beta \mu) \times \\
& \left[\left(\kappa_{ZZ} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \frac{\epsilon_{Ze}}{m_Z^2} \frac{g_Z^\mu}{P_Z(q_2^2)} + \frac{\epsilon_{Z\mu}}{m_Z^2} \frac{g_Z^e}{P_Z(q_1^2)} \right) g^{\alpha\beta} + \right. \\
&+ \left(\epsilon_{ZZ} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \kappa_{Z\gamma} \epsilon_{Z\gamma}^{\text{SM-1L}} \left(\frac{eQ_\mu g_Z^e}{q_2^2 P_Z(q_1^2)} + \frac{eQ_e g_Z^\mu}{q_1^2 P_Z(q_2^2)} \right) + \kappa_{\gamma\gamma} \epsilon_{\gamma\gamma}^{\text{SM-1L}} \frac{e^2 Q_e Q_\mu}{q_1^2 q_2^2} \right) \frac{q_1\cdot q_2 \; g^{\alpha\beta} - q_2{}^\alpha q_1{}^\beta}{m_Z^2} + \\
&+ \left. \left(\epsilon_{ZZ}^{\text{CP}} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \epsilon_{Z\gamma}^{\text{CP}} \left(\frac{eQ_\mu g_Z^e}{q_2^2 P_Z(q_1^2)} + \frac{eQ_e g_Z^\mu}{q_1^2 P_Z(q_2^2)} \right) + \epsilon_{\gamma\gamma}^{\text{CP}} \frac{e^2 Q_e Q_\mu}{q_1^2 q_2^2} \right) \frac{\varepsilon^{\alpha\beta\rho\sigma} q_{2\rho} q_{1\sigma}}{m_Z^2} \right]
\end{aligned}$$

In the SM: $\kappa_X \to 1, \;\; \epsilon_X \to 0$

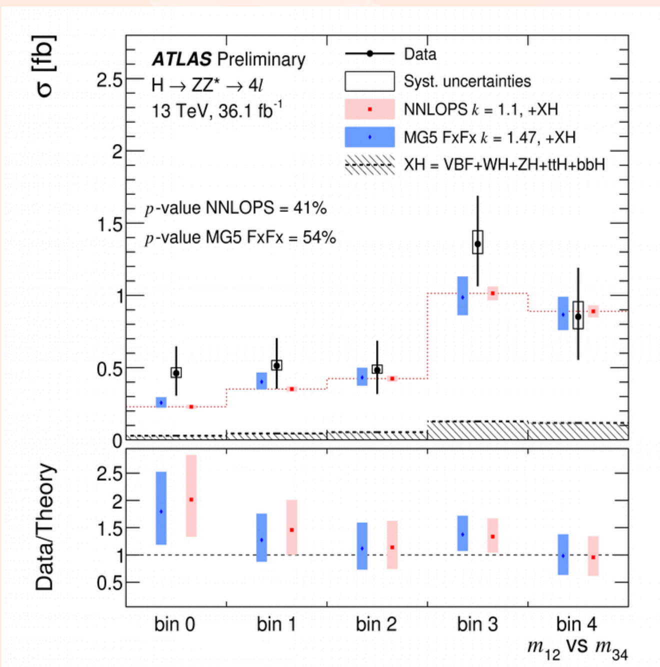
$$P_Z(q^2) = q^2 - m_Z^2 + i m_Z \Gamma_Z$$

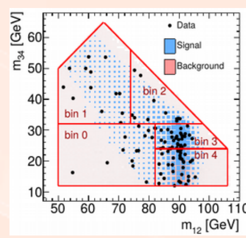
ATLAS collaboration, LHCP 2017 Shanghai, 15-20 May

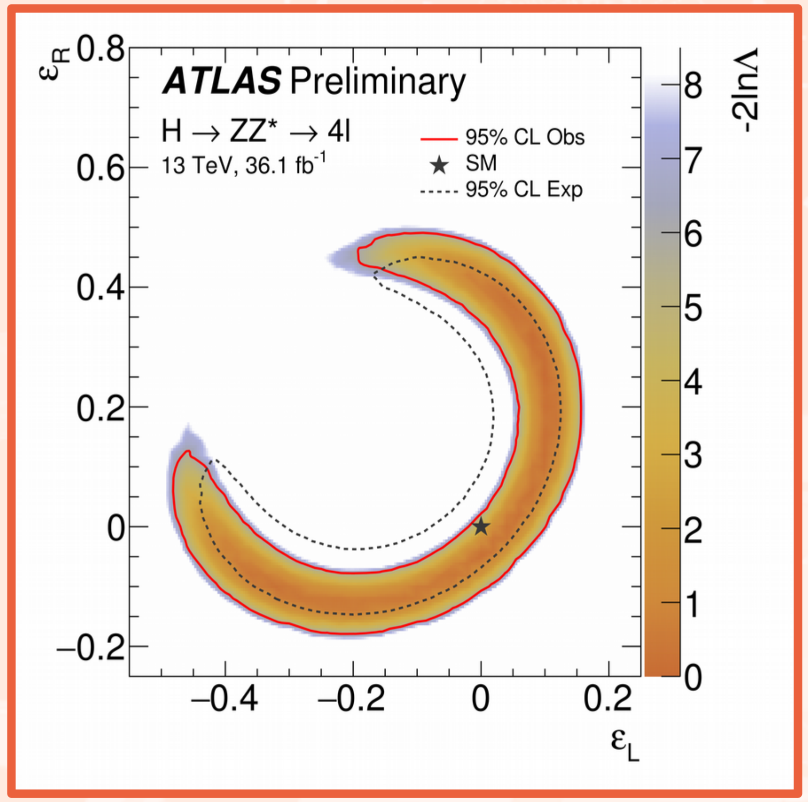
ATLAS-CONF-2017-032

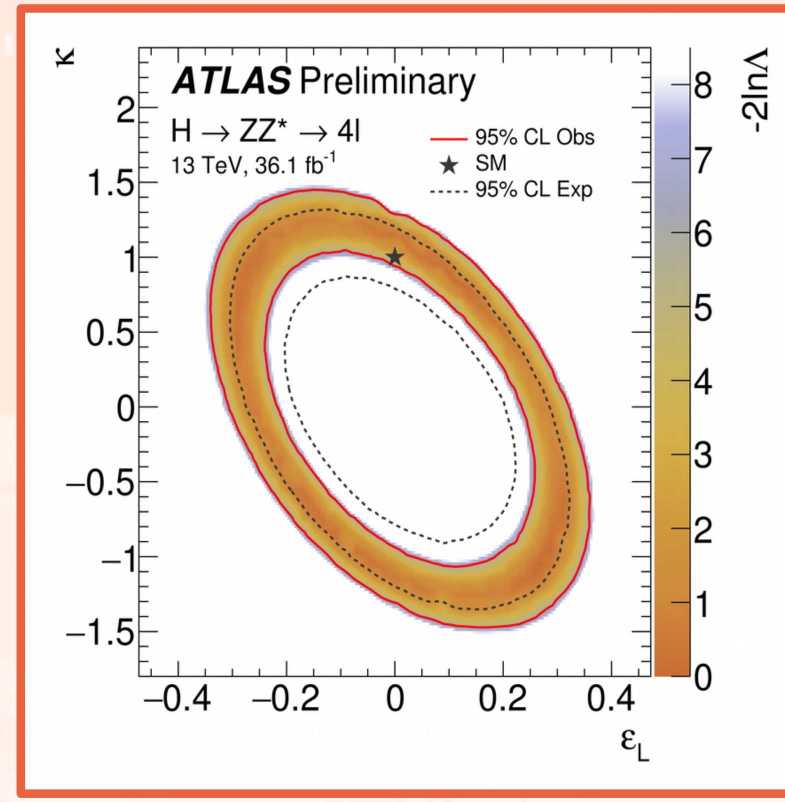
$36.1\ fb^{-1}$

Double differential m12 vs m34 [$H \to ZZ^* \to 4l$]









Interpreted into limits on modified interactions within the framework of POs

[Eur. Phys. J. C (2015) 75: 128 arxiv: 1504.04018]

limits on modified contact terms between H and L-/R-handed leptons [lepton universality imposed for contact terms]

limits on ε_L and modified coupling to Z boson [$\varepsilon_R = 0.48^* \varepsilon_L$]

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