Waves in non-neutral plasma

- Diocotron wave -simple infinite length model
 -finite length, finite amplitude corrections
- Plasma wave -Trivelpiece-Gould (TG) wave
 - -Thermally excited TG wave
- Cyclotron wave

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Diocotron mode

Where does the name comes from?

Diocotron comes from Greek "pursue, chase"

Hollow electron beam => **Diocotron instability**



This talk is about stable diocotron wave NOT diocotron instability of hollow plasma

Diocotron wave

Let's consider the simplest case: monotonically decreasing density profile



"Infinitely" long plasma column m_{θ} =1



Replace the wall by an equal and opposite **image charge** such that the potential at $r=R_w$ is constant



Potential of ∞ line charge

$$\phi(r,\theta) = -\frac{N_L q}{2\pi\varepsilon_0} \left[\ln \sqrt{r^2 + D^2 - 2rD\cos\theta} - \ln \sqrt{r^2 + S^2 - 2rS\cos\theta} \right]$$

image
$$= \frac{N_L q}{2\pi\varepsilon_0} \ln \left[\frac{r}{S} \frac{\sqrt{1 + \frac{D^2}{r^2} - \frac{2D}{r}\cos\theta}}{\sqrt{1 + \frac{r^2}{S^2} - \frac{2r}{S}\cos\theta}} \right]$$

choose $\frac{S}{R_w} = \frac{R_w}{D}$ then at $r = R_w$ $\phi(R_w, \theta) = -\frac{N_L q}{2\pi\varepsilon_0} \ln \left(\frac{D}{R_w}\right)$
 $S = \frac{R_w^2}{D}$ Independent of Θ

Electric field from a line charge

Using Gauss' law
$$E = \frac{\sum Q}{2\pi\varepsilon_0 rL} = \frac{N_L q}{2\pi\varepsilon_0 r}$$

The image charge electric field at r=0 is:

$$E_{i} = \frac{-N_{L}q}{2\pi\varepsilon_{0} S} = \frac{-N_{L}q D}{2\pi\varepsilon_{0} R_{W}^{2}}$$

The ExB drift velocity of the (real) charge in the electric field of the image charge is:

Assuming D/R_w <<1
$$V_d = \frac{E_i}{B} = \frac{-N_L q D}{2\pi \varepsilon_0 B_z R_w^2}$$

The infinite length small amplitude diocotron frequency is:

$$f_{dio} = \frac{V_d}{2\pi D} = \frac{N_L q}{4\pi^2 \varepsilon_0 B_z R_W^2}$$

The diocotron frequency is frequently expressed in term of the rotation frequency

$$f_{dio} = \frac{N_L q}{4\pi^2 \varepsilon_0 B_z R_W^2}$$

$$N_L = \int_0^{R_p} n(r) 2\pi r \, dr = n\pi R_p^2$$

$$= \frac{n\pi q R_p^2}{4\pi^2 \varepsilon_0 B_z R_W^2} = \frac{nq}{4\pi \varepsilon_0 B_z} \left(\frac{R_p}{R_W}\right)^2 = f_E \left(\frac{R_p}{R_W}\right)^2$$

The diocotron frequency is the rotation frequency of a plasma extending to the wall

Higher order diocotron mode

$$f_{m_{\theta}}^{dio} = f_E \left[m_{\theta} - 1 + \left(\frac{R_p}{R_W} \right)^{2m_{\theta}} \right] \qquad \text{Where} \qquad f_E = \frac{en}{4\pi\varepsilon_0 B}$$
"square profile"
plasma rotation frequency

$$f_{m_{\theta}=2}^{dio} \cong f_{ExB}$$
 The m _{θ} =2 mode is close to the rotation frequency

The diocotron plasma mode is a negative energy mode!

The image charge have opposite sign of the "real" charge



 $S = \frac{R_W^2}{D}$

Increasing D reduces S

The plasma is attracted towards its image charge. The electrostatic energy decreases as the mode amplitude increases. Kinetic energy is negligible.

How much electrostatic energy to displace the plasma by D in the image electric field?

$$W_{ES} = \int_{0}^{D} F \, dx = \int_{0}^{D} Q \cdot E_i \, dx = \int_{0}^{D} N_L q L_P \frac{\left(-N_L q \cdot x\right)}{2\pi\varepsilon_0 R_W^2} dx = \frac{-\left(N_L q\right)^2}{4\pi\varepsilon_0} \frac{D^2}{R_W^2} L_p$$

$$E_i = \frac{-N_L q D}{2\pi\varepsilon_0 R_W^2}$$
Negative energy

The diocotron mode can be destabilized by dissipation!



Use feedback circuit to damp the mode



Figure 3.4: Feedback growth rate versus phase shift.

Diocotron mode is a tool to move the plasma off axis

- phaser picture (before CCD image of plasma)
- load off axis multi-trap (Surko's multicell)

The diocotron frequency can be used to measure the line density

We have to be careful here

$$f_{\infty}^{dio} = \frac{N_L q}{4\pi^2 \varepsilon_0 B_z R_W^2}$$
 Valid for infinite length, small amplitude

For a measured
$$N_L$$
 and a measured diocotron frequency,
the infinite length equation gives a frequency too small by a factor of 2 or 3
for a plasma $L_p/R_p \sim 2$

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Large amplitude diocotron

For large displacement, the column distorts into an elliptical cross-section

Non-linear correction

$$f_{NL} = f_{\infty} + f_{\infty} \left(\frac{1 - 2(R_{p}/R_{W})^{2}}{\left[1 - (R_{p}/R_{W})^{2}\right]^{2}} \right) \left(\frac{D}{R_{W}}\right)^{2}$$





C.F. Driscoll and K.S Fine Phys. Fluids B, 2, 1359 (1990) K.S. Fine, C.F. Driscoll and J.H. Malmberg, PRL, 63, 2232, (1989)

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Confining potential push the plasma in the **z-direction**.

This result in a **radial force on a off axis plasma**.

Confining potential add to the force due to the image charge

$$F_{tot} = F_i + F_c$$

mage confinement voltage

$$\frac{f_{dio}}{f_{\infty}} = \frac{F_{tot}}{F_{i,\infty}} = 1 + \left[\frac{j_{01}}{2}\left(\frac{1}{4} + \ln\left(\frac{R_{W}}{R_{p}}\right) + \frac{T}{N_{L}e^{2}}\right) - 0.671\right]\left(\frac{R_{W}}{L_{p}}\right)$$
Plasma electrostatic pressure "
"Plasma kinetic pressure "
"finite length on image charge "

Experimental test of finite length effects



K.S Fine and C.F. Driscoll Phys .Plas. 5, 601,(1998)

Magnetron mode

For small short plasma the confining potential dominates

$$f = -\frac{E_r}{2\pi D B_z} = \frac{1}{2\pi D B_z} \left[\frac{\partial \phi_c}{\partial r} + \frac{\partial \phi_i}{\partial r} \right]_{r=D}$$
$$= \frac{1}{2\pi B_z} \left[1.15 \frac{V_c}{R_w^2} \frac{L}{R_w} - 1.0027 \frac{Q}{R_w^3} \right]$$
For example:
$$V_c = 10V$$
$$R_w = 1 \text{ cm}$$
$$L/R_w = 0.2$$
$$230 \text{ kHz}$$
$$1.4 \text{ kHz}$$
$$R_w = 1 \text{ cm}$$
$$Potential'''$$

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Plasma wave

Infinite plasma



Long plasma in a "Penning-Malmberg" trap Trivelpiece Gould mode



 $E_z=0$ at the wall E is radial at the wall

 E_z is reduce by the conducting the wall $f_{TG} < f_{Langmuir}$

Continuity Density perturbation: $\delta n(r) \exp\{i(m_{\theta}\theta + k_{z}z - \omega t)\}$

$$+\frac{\partial}{\partial z}n\cdot v_{z}=0 \qquad -i\omega \,\,\delta n+n \,\,ik_{z}\delta v_{z}=0 \qquad \delta n=\frac{n \,\,k_{z}\delta v_{z}}{\omega} -$$

Newton

 ∂t

$$m\frac{\partial v_z}{\partial t} = qE_z = -q\frac{\partial}{\partial z}\varphi \qquad -m \ i\omega \ \delta v_z = -q \ ik_z\delta\varphi \qquad \delta v_z = \frac{q \ k_z\delta\varphi}{m \ \omega}$$

Poisson

 $\nabla^2 \varphi = -4\pi q \, \delta n$ Keep c

$$-k^2\delta\varphi = -4\pi q \ \delta n$$

$$\delta n = \frac{n q k_z^2 \delta \varphi}{m \omega^2}$$

p all k in Poisson eq.
$$-k^{2}\delta\varphi = \frac{-4\pi q^{2}nk_{z}^{2}\delta\varphi}{m\omega^{2}}$$

$$\omega^2 = \frac{k_z^2}{k^2} \frac{4\pi q^2 n}{m} \qquad \qquad \omega^2 = \frac{k_z^2}{k^2} \omega_p^2$$

" cold Trivelpiece Gould mode"

With thermal pressure

$$\omega^2 = \frac{k_z^2}{k^2} \omega_p^2 + 3\overline{v}^2 k_z^2 \quad \text{Trivelpiece Gould mode"}$$

If we kept all k not only
$$k_z$$
, we would get

$$\omega^2 = \omega_p^2 \left(\mathbf{1} + \frac{3}{2} \mathbf{k}^2 \lambda_D^2 \right)$$

Standard plasma wave in unmagnetized, infinite plasma: "Langmuir wave"





In the radial direction the potential vanish at the wall.

$$k_{\perp} \text{ must satisfy:} \qquad k_{z}R_{p} \frac{I_{0}'(k_{z}R_{p})K_{0}(k_{z}R_{w}) - K_{0}'(k_{z}R_{p})I_{0}(k_{z}R_{w})}{I_{0}(k_{z}R_{p})K_{0}(k_{z}R_{w}) - K_{0}(k_{z}R_{p})I_{0}(k_{z}R_{w})} - k_{\perp}R_{p} \frac{J_{0}'(k_{\perp}R_{p})}{J_{0}(k_{\perp}R_{p})} = 0$$

S.A. Prasad and T.M. " Phys. Fluids 26, 665 (1983)

$$k_{\perp} = \frac{1}{R_{\rho}} \left(\frac{2}{\ln(R_{W}/R_{\rho})} \right)^{\frac{1}{2}}$$
 Valid for $R_{p} << R_{W}$

$$\omega^{2} = \frac{k_{z}^{2}}{k^{2}}\omega_{p}^{2} + 3\overline{v}^{2}k_{z}^{2} = \frac{k_{z}^{2}}{k_{z}^{2} + k_{\perp}^{2}}\omega_{p}^{2} + 3\overline{v}^{2}k_{z}^{2}$$

$$\omega = \sqrt{\frac{k_z^2}{k_\perp^2}} \omega_p^2 + 3\overline{v}^2 k_z^2$$

$$\sqrt{a+b} \approx \sqrt{a} + \frac{1}{2}\frac{b}{\sqrt{a}}$$
 for a \gg b

$$\omega \approx \frac{k_z}{k_\perp} \omega_p \left[1 + \frac{3}{2} \frac{\overline{v}^2 k_z^2}{\omega^2} \right] = \frac{k_z}{k_\perp} \omega_p \left[1 + \frac{3}{2} \frac{\overline{v}^2}{v_{ph}^2} \right]$$

$$\omega = \omega_p \frac{m_z \pi}{L_p} R_p \left(\frac{1}{2} \ln \left(\frac{R_w}{R_p} \right) \right)^{\frac{1}{2}} \left[1 + \frac{3}{2} \left(\frac{\overline{v}}{v_{ph}} \right)^2 \right]$$

Finite length TG modes

For a long column "acoustic" dispersion relation

$$\omega = \omega_p k_z R_p \left(\frac{1}{2} \ln \left(\frac{R_W}{R_p} \right) \right)^{\frac{1}{2}} \left[1 + \frac{3}{2} \left(\frac{\overline{v}}{v_{ph}} \right)^2 \right]$$
$$\omega - m_\theta \omega_E = \omega_p k_z R_p \frac{1}{j_{m_\theta m_r}} \left[1 + \frac{3}{2} \left(\frac{\overline{v}}{v_{ph}} \right)^2 \right]$$

Valid for
$$m_{\theta}=0$$

 $m_{\theta} \neq 0$ mode are Doppler shifted by the rotation frequency This can be useful for some rotating wall application

 $k_{eff} = k_z + \alpha_1 R_p + \alpha_2 R_W$

k_{eff} correspond to a longer wavelength than

$$k_z = \frac{m_z \pi}{L_p}$$

All these m



Higher M_z results in higher frequency

Higher M_{θ} results in lower ω - $M_{\theta}\omega_{E}$

Higher M_r results in lower frequency

Higher order TG modes



 $(m_{\theta}, m_{z}, m_{r})$

TG modes travel forward or backward on the rotating plasma column

Phys. Plasmas 7, 2776 (2000)

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TG modes are easy to excite



TG modes are excited at very low level by thermal fluctuations

Provide effective diagnostic tool.

Phys. Rev. Lett. 90, 115001 (2003)

Thermally excited TG plasma modes



As the plasma temperature T_p increases:

- Landau damping increases
- Mode frequency increases
- "Area under the mode increases"

Non-Neutral plasma can relax to a state of thermal equilibrium in the rotating frame.





$$\frac{V^2}{df} = 4k_B T \operatorname{Re}(Z)$$





Plasma temperature from thermally excited mode



The plasma temperature is non-destructively determined by "listening" to plasma fluctuations.

Phys. Plas. 10, 1556 (2003)

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Cyclotron modes are a useful tool for identification of impurities ion

Thermal cyclotron spectroscopy.

- Heat resonantly impurity ions at their cyclotron frequency
- Hot impurity ions heat Mg₂₄ through collisions
- Fluorescence of the Mg₂₄ cooling laser beam increases



Cyclotron wave

For single particle the cyclotron frequency is Ω_{c}

$$_{c} = 2\pi F_{c} = \frac{q B}{m}$$

Radial force balance ($m_{\theta} = 1$)



But E_r is generally non-zero due to space charge, image charge, and trap potentials

Center of Mass Mode Frequency ($m_{\theta}=1$)



Center of mass mode is downshifted from the "bare" cyclotron frequency

Surface Cyclotron Modes



Uniform Density Surface Wave Theory

Solve Vlasov-Poisson equation for an $cos(m\theta)$ perturbation in the f_E frame



$$\Delta f_1 = -f_E \left(\frac{R_p}{R_w}\right)^2 = -f_D$$

R. W. Gould, Phys. Plasmas 2, 1404, (1995) Dubin , Phys. Plasmas 20, 042120 (2013)

Cyclotron Mode Frequencies vs. $f_{\rm E}$



M. Affolter et al. Physics of Plasma (2015)

Summary

- Diocotron wave measure N_L
 - control position of the plasma in trap
- Plasma wave useful for RW application
 - Thermally excited: Temperature diagnostic
- Cyclotron wave measure the plasma composition

- measure the magnetic field

Publications can be found at nnp.UCSD.edu