Correlations in Trapped Plasma

Part 1

Effect of correlations on equilibrium properties

Correlations change the structure of confined ion plasma

Part 2

Effect of correlations on a dynamical property

Correlations increase the perpendicular to parallel collision rate Correlations increase collision rate in non-magnetized plasma

François Anderegg



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Strongly correlated plasmas

Thermodynamic state of an One Component Plasma determined by:

- Size
- Shape
- Coupling parameter

$$\Gamma = \frac{q^2}{a_{WS} k_B T}$$

with
$$\frac{4}{3}\pi a_{WS}^3 n = 1$$

$$a_{WS} = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}}$$

 $\Gamma \approx \frac{\text{potential energy between neighboring ions}}{\text{ion thermal energy}}$

 $\Gamma > 1 \Rightarrow$ strongly coupled OCP

$$\Gamma = \frac{q^2}{a_{WS}T}$$

$$a_{WS} = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}}$$

Non-neutral plasma	Giant planet interiors	White dwarf stars	Dusty plasma
T ~ 10 ⁻⁵ eV n ~ 2x10 ⁷ cm ⁻³ Γ ~ 10	T ~ 1 eV n ~ 10 ²⁴ cm ⁻³ Γ ~ 10	T ~ 100 eV n ~ 10 ³⁰ cm ⁻³ Γ ~ 10	T ~ 1 eV n ~ 1 cm ⁻³ q ~ 10 ⁴ e Γ ~ 10





 $\Gamma << 1$: plasma state $\Gamma > 1$: liquid state $\Gamma > 172$: bcc crystal



John Bollinger, NIST

Frontiers in High Energy Density Physics, NRC, (2003)

Plasmas vs strongly coupled plasmas



Even when Γ is large, the mean field $e\phi >>$

 $e\phi >> \frac{e^2}{a}$ (for N >>1)

The shape of plasma remain ~ unchanged by correlations

In the absence of correlations

Boltzman distribution "one particle distribution"

$$f(r, v) = C \exp\left(-\frac{1}{k_b T} \left[H + \omega P_{\theta}\right]\right) \qquad H = \frac{mv^2}{2} + e(\phi_T + \phi_P) \qquad \nabla^2 \phi_P = -4\pi e n_P$$

$$P_{\theta} = mv_{\theta}r + \frac{eB}{2c}r^2$$

In the presence of correlations

Gibbs distribution " N particles distribution"

$$\begin{split} f\left(r_{1}, \mathbf{v}_{1}, r_{2}, \mathbf{v}_{2}, \dots, r_{N}, \mathbf{v}_{N}\right) &= C \exp\left(-\frac{1}{k_{b}T} \left[H^{(N)} + \omega P_{\theta}^{(N)}\right]\right) \\ H^{(N)} &= \sum_{i=1}^{N} \frac{m \mathbf{v}_{i}^{2}}{2} + \mathbf{e}\left(\phi_{T}(r_{i}) + \sum_{j>i} \phi_{ij}\right) \\ \phi_{ij} &= \frac{e^{2}}{\left|r_{i} - r_{j}\right|} + \text{"image charge"} \\ P_{\theta}^{(N)} &= \sum_{i=1}^{N} m \mathbf{v}_{\theta i} r_{i} + \frac{eB}{2c} r_{i}^{2} \end{split}$$

$$= C \exp\left(-\sum_{i=1}^{N} \frac{m}{2k_b T} \left[v_i + \omega r_i \ \hat{\theta}_i\right]^2\right) \tilde{C} \exp\left(-\sum_{j=1}^{N} \frac{1}{k_b T} \left\{\frac{1}{2} \sum_{i \neq j} \frac{e^2}{|r_i - r_j|} + e\phi_T\right\}\right)$$

Reduced distributions

Spatial distribution $\rho^{(M)}(r_1, r_2, ..., r_M) = \int d^3 r_{M+1} ... d^3 r_N f(r_1, ..., r_N)$

Density

$$n(r) = N\rho^{(1)}(r)$$
 First reduced distribution

$$\rho^{(2)}(r_1, r_2) = \rho^{(1)}(r_1) \ \rho^{(2)}(r_2) \left[1 + g(r_1, r_2) \right]$$



 $g(r_1,r_2)$: correlation function measures the extra probability beyond what would be expected of a completely random distribution of finding particles at r_1 and r_2

Two body spatial correlation

Figure 2.3: Correlation function for one component plasma.

Coulomb interaction is a binary interaction, all thermodynamics quantities can be evaluated from $n(r) = g(r_1,r_2) = T$

Correlations with small plasmas

Dubin and O'Neil, Computer Simulation of Ion Clouds in a Penning Trap, PRL 60, 511 (1988)



FIG. 1. Density as a function of spherical radius for N = 100, $\Gamma = 140$.

Boundary effects dominate => Shell structure is observed

Observations of shell structure

• 1988 – shell structures in Penning traps NIST group

PRL 60, 2022 (1988)





• 1992 – 1-D periodic crystals in linear Paul traps MPI Garching

Nature 357, 310 (92)



 1998 – 1-D periodic crystals with plasma diameter > 30 a_{WS} Aarhus group

PRL <u>81</u>, 2878 (98)

See Drewsen presentation next week



Large plasma

Influence of surface is limited

Interior comparable to infinite size crystal

body centered cubic



face centered cubic



hexagonal close packed



Coulomb energies/ion of bulk bcc, fcc, and hcp lattices differ by $< 10^{-4}$

How large must a plasma be to exhibit a bcc lattice?

1989 - Dubin, planar model PRA <u>40</u>, 1140 (89) result: plasma dimensions ≥ 60 interparticle spacings required for bulk behavior $N > 10^5$ in a spherical plasma \Rightarrow bcc lattice

2001 – Totsji, simulations, spherical plasmas, N≤120 k PRL <u>88</u>, 125002 (2002) result: N>15 k in a spherical plasma ⇒ bcc lattice



NIST Penning trap – designed to look for "large" bcc crystals

John Bollinger NIST



Bragg scattering



Bragg scattering from spherical plasmas with N~ 270 k ions





Rotating wall control of the plasma rotation frequency

John Bollinger NIST



Phase-locked control of the plasma rotation frequency

John Bollinger NIST

Huang, et al., Phys. Rev. Lett. 80, 73 (98)

time averaged Bragg scattering



camera strobed by the rotating wall



- determine if crystal pattern due to 1 or multiple crystals
- enables real space imaging of ion crystals



Summary of correlation observations in approx. spherical plasmas

 $N \ge 2 \times 10^5$ observe bcc crystal structure



$1 \ge 10^5 > N > 2 \ge 10^4$

observe other crystal structures (fcc, hcp?, ...) in addition to bcc structure

N < 2 x 10⁴ Shell structure dominates





with planar plasmas all the ions can reside within the depth of focus

Planar structural phases can be 'tuned' by changing ω_r

Real space images

John Bollinger NIST

66.50 kHz



top-views

side-views

1 lattice plane, hexagonal order

2 planes, cubic order



Top- (a,b) and side-view (c) images of crystallized ⁹Be⁺ ions contained in a Penning trap. The energetically favored phase structure can be selected by changing the density or shape of the ion plasma. Examples of the (a) staggered rhombic and (b) hexagonal close packed phases are shown.

c

Mitchell, *et al.*, Science **282**, 1290 (98) Theoretical curve from Dan Dubin, UCSD



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Use Non Neutral Plasma to study properties of "stars"

- -- Non-neutral plasmas are low density, low temperature Nuclear reactions are NOT happening.
- -- But something analog to nuclear reaction IS happening.

Dubin proved nuclear fusion reaction in correlated plasma is isomorphic to enhanced **perp-to-parallel collisions** in a pure ion plasma at low temperatures.

Phys. Plasmas 15, 055705 (2008)

Phys. Rev. Lett. 94, 025002 (2005)

-- The correlation enhancement is directly analogous to correlation-enhanced fusion in the Sun with $\Gamma \sim 0.05$, and in white dwarfs with $\Gamma >>1$.

"Salpeter enhancement"

Perpendicular to Parallel Collisions:

$$\frac{d}{dt}T_{\perp} = \mathbf{v}_{\perp \prime \prime} (T_{\prime \prime} - T_{\perp})$$

Distance of closest approach $b = e^2 / T$

 $\mathbf{v}_{\perp \prime \prime} = \mathbf{n} \, \overline{\mathbf{v}} \, \mathbf{b}^2 \, 4\sqrt{2} \, \mathbf{I}(\overline{\mathbf{\kappa}}) \, g(\Gamma)$

Magnetization suppression

Correlation enhancement

 $\overline{\mathbf{K}} = \sqrt{2} \mathbf{b/r}_{c}$

Adiabaticity parameter

Correlation parameter



-- I($\bar{\kappa}$) Suppress $v_{\perp//}$ collisions in the "highly magnetized" regime of $r_c < b$. In this regime, only rare, energetic collisions mix E_{\perp} and $E_{//}$.

Dubin's lecture (yesterday)

-- $g(\Gamma)$ Enhances these rare collisions, due to particle correlations, in the cryogenic liquid regime of $\Gamma = 1 - 10$.

Overview



When $r_c < b$: $v_{\perp//}$ comes from rare energetic collisions

cross-section for
$$E_{I/} = E_{\perp}$$
 sharing

$$\sigma(E_{I/}) \propto e^{-\pi \left(\frac{b}{r_{c}}\right)} \propto e^{-\pi \left(\frac{e^{2} \Omega_{c}}{T v_{I}}\right)} \approx e^{-\pi \left(\frac{C}{E_{II}}\right)^{\frac{3}{2}}}$$

$$v_{\perp//} = \int dE_{II} \frac{1}{T} e^{-E_{II}/T} \sigma(E_{II})$$

$$\int e^{E_{II}/T} \sigma(E_{II}) \sim e^{-C/E_{II}} \frac{3/2}{Like rare fusion collisions aitch \sigma_{fusion}(E_{I/})}$$

$$v_{\perp//} = n\overline{v}b^{2} 4\sqrt{2} I(\overline{k}) g(\Gamma)$$

$$E_{Gamow} \approx 1.23 \overline{k}^{\frac{2}{5}} T$$

$$I(\overline{k}) \approx C \exp\left(-2.044 \overline{k}^{\frac{2}{5}}\right) \int_{E_{Gamow} for \overline{k} = 20 \text{ corresponds}}$$

$$\kappa = \sqrt{2} b/r_{c} \frac{b}{r_{c}} + \frac{1}{2} b/r_{c} \frac{c}{r_{c}} + \frac{1}{2}$$

Collision Enhancement from Shielding (correlation)

Debye shielding (correlation) reduces the energy barrier for close impact distances $\,\rho\,$



Measured Collision Rates at B = 3.Tesla



Low density:

Magnetization suppress $\nu_{\perp\!/\!/}$

No correlation, no $v_{\perp//}$ enhancement

High density: correlated at T<10⁻⁴, strong enhancement

Phys. Rev. Lett. **102**, 185001 (2009) Phys. Plasmas 17, 055702 (2010)

Enhancement versus Correlation Γ



Enhancement depends on correlation parameter Γ

But is independent of κ

Phys. Rev. Lett. **102**, 185001 (2009) Phys. Plasmas 17, 055702 (2010)

Summary of perpendicular to parallel colision

- Perp-to-parallel collisions are strongly *suppressed* in the "strong magnetization" regime of *r_c* < *b*.
 Only rare, energetic collisions cause E_⊥to E_{//} energy exchange.
- These rare, energetic collisions are strongly enhanced in the correlated liquid and crystal regimes.
- Enhancements up to 10⁹ over uncorrelated theory are observed.
- Same enhancement applies to rare energetic fusion collisions in hot, dense, correlated plasmas such as stars.

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Collisional Relaxation of Tagged Ions in Strongly Coupled Ultracold Neutral Plasmas

Thomas Killian RICE



•Optical pumping on $\Delta m = \pm 1$ transitions perturbs ion velocity distribution of each spin state. • Perturbed velocity distributions relax towards a Maxwellian through collisions. • We image only the m = +1 spin state

Evolution of Average Velocity Gives Collision Rate and Diffusion Constant Beyond Landau-Spitzer



Observe dramatic increase of relaxation rate over the case neglecting correlations

Bannasch et al.,, Phys. Rev. Lett. 109, 185008 (2012)

Summary

- Correlations change the internal structure of plasma:
 - small plasma : Shell structure
 - large plasma N> 10⁵ Crystals
 - planar plasma crystal (promising for quantum computing)
- Correlations increase perpendicular to parallel collision rate

$$v_{\perp //} = v_{\perp //}^{no \text{ cor}} \exp(\Gamma)$$

• Correlations increase the collision rate in non-magnetized plasma

Publications can be found at nnp.UCSD.edu