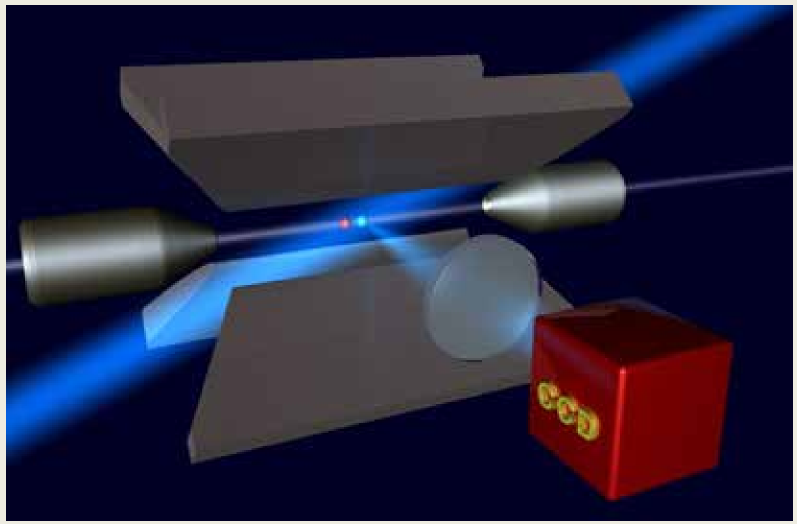
Optical Clocks I



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& Leibniz Universität Hannover

Les Houches Winter School, 19.-30.01.2015

Physikalisch-Technische Bundesanstalt


Hermann v. Helmholtz

Location Braunschweig: 1 km^2, approx. 1500 employees

- National Metrology Institute (subordinate of BMWi)
- **Tasks:** determination of fundamental constants, dissemination of SI units, development of measurement techniques,…
- ca. **1800 employees**, of which are **110 PhD candidates**
- 60% research: ~600 publications per year

Physikalisch-Technische Bundesanstalt

- “Clock Hall” @ PTB: Where the German second is made…

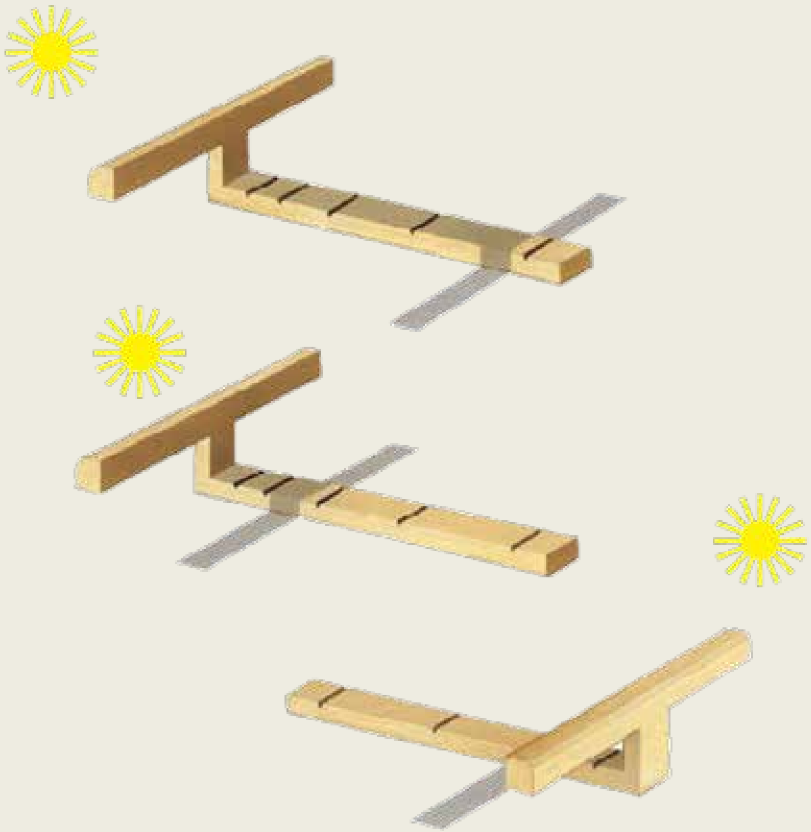
Overview Optical Clocks

- Introduction
  - brief history of time (keeping)
  - principle & characterization of clocks
- The atomic reference
  - ions & neutrals
  - frequency shifts
- The clock laser
  - Requirements or “Mission impossible”
  - Important design aspects

Overview Optical Clocks II

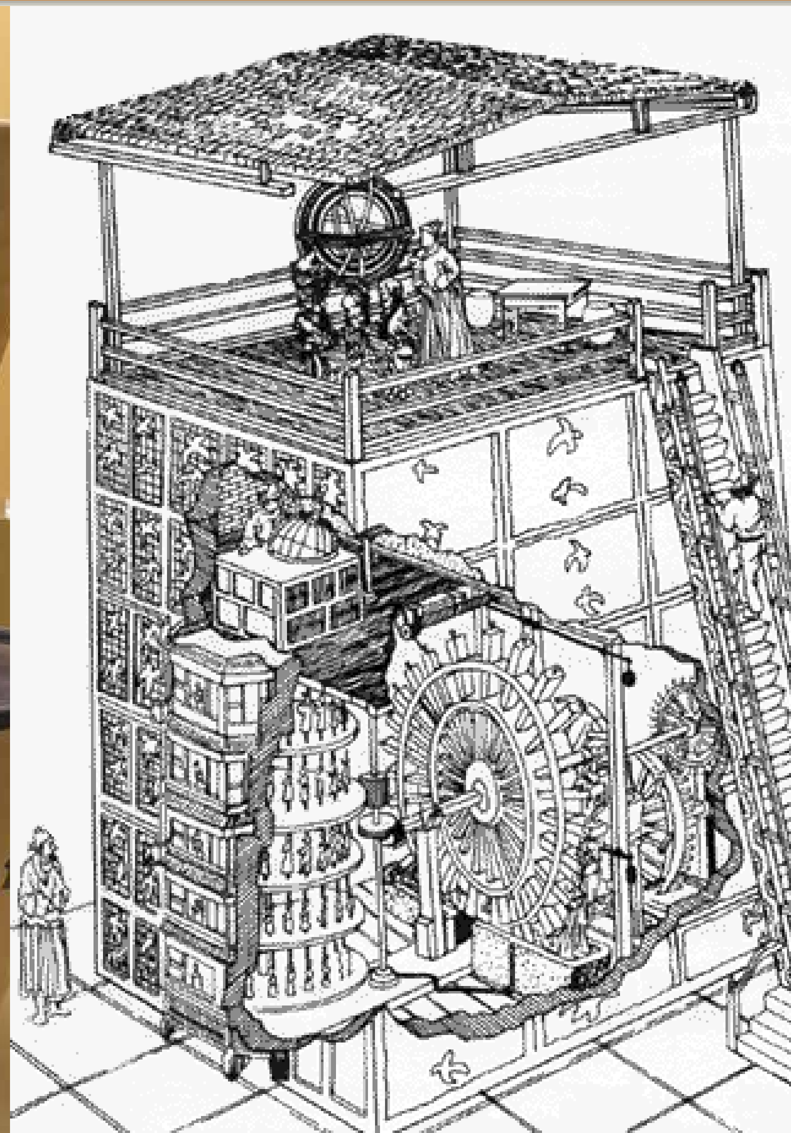
- Frequency comb and dissemination
  - counting optical cycles
  - getting the light somewhere useful
- Results from selected ion clocks
- Applications
  - relativistic geodesy
  - fundamental physics
- Future trends
  - improving the instability of ion clocks

Sun Dials



ca. 1450 BC

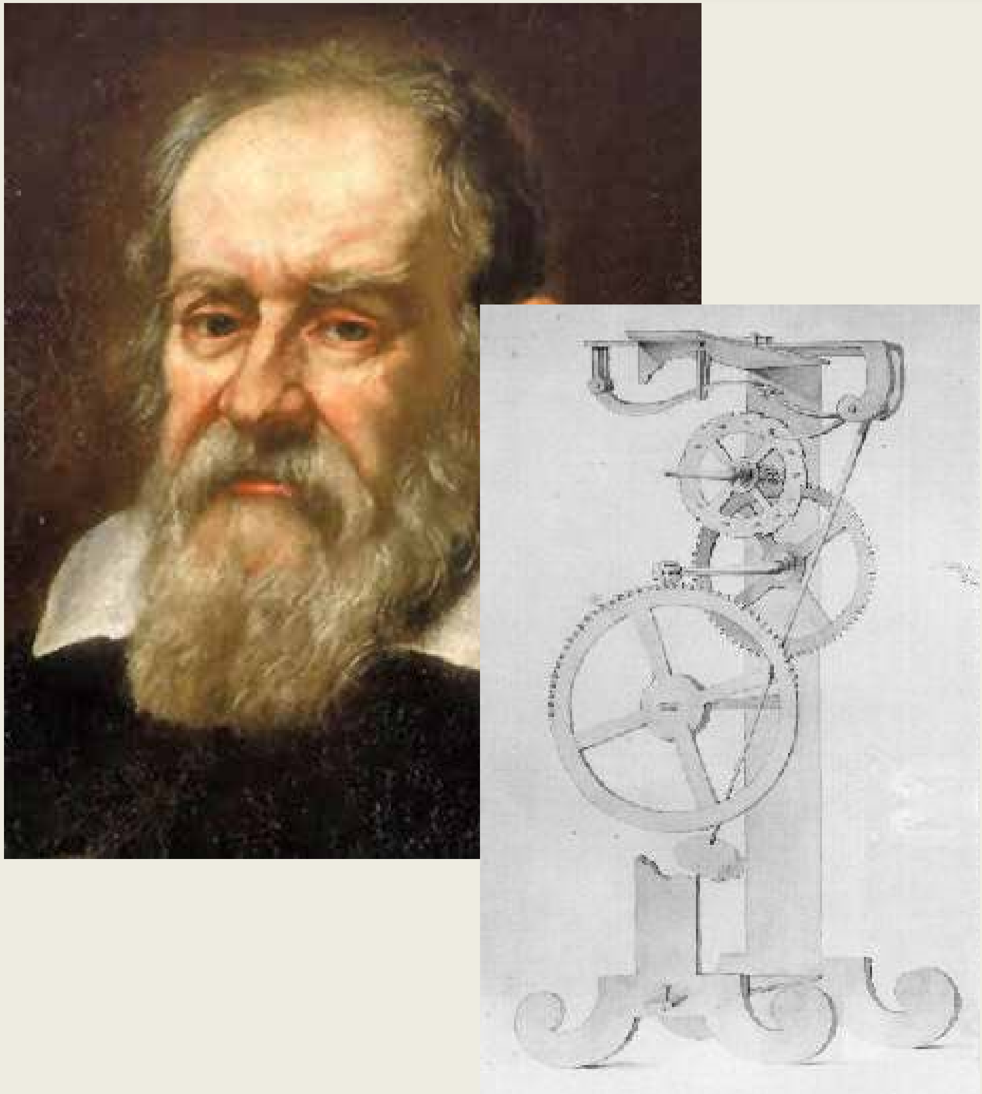
Water Clocks



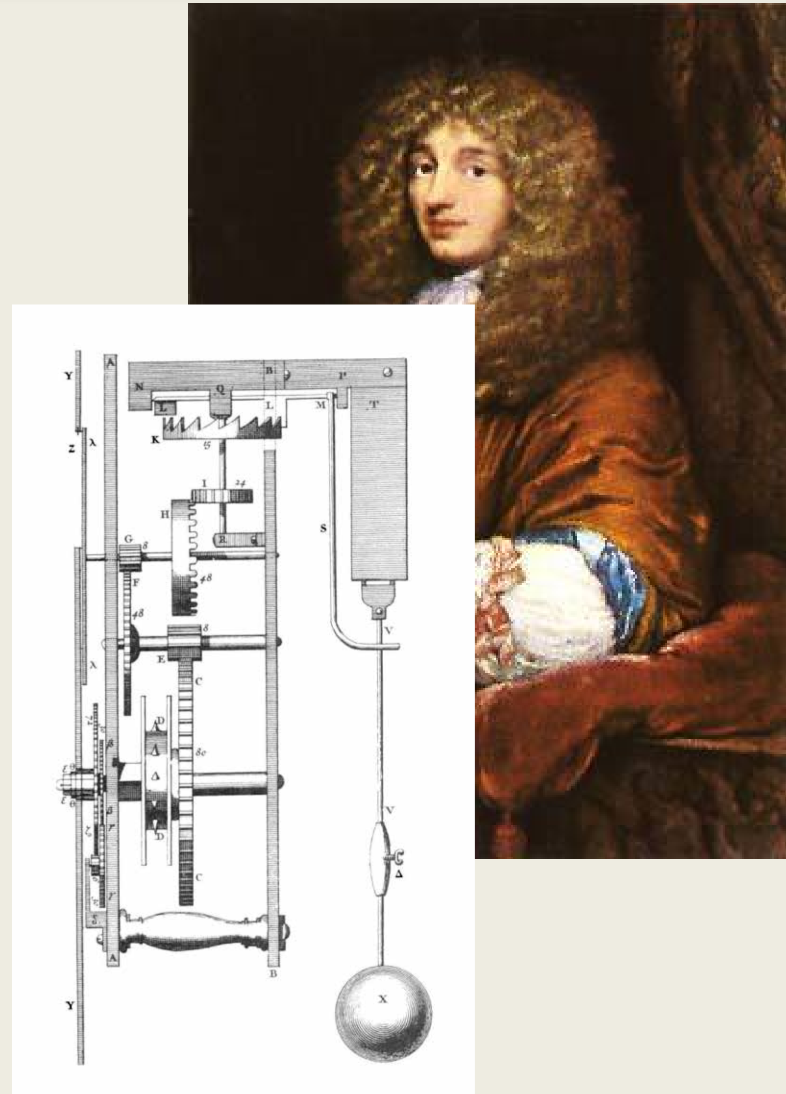
Su Sung water clock tower

Candle- and Sand Clocks

Pendulum Clocks



Galileo Galilei (1580)

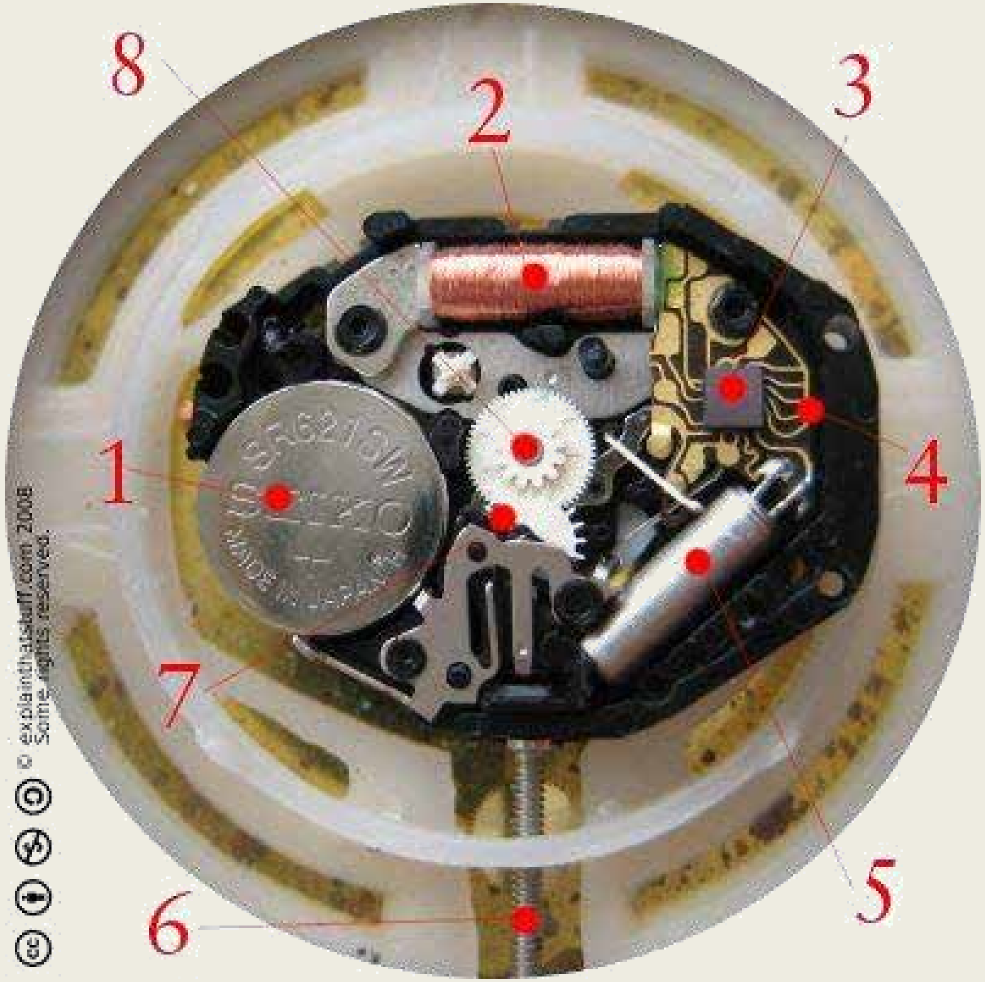


Christiaan Huygens (1656)

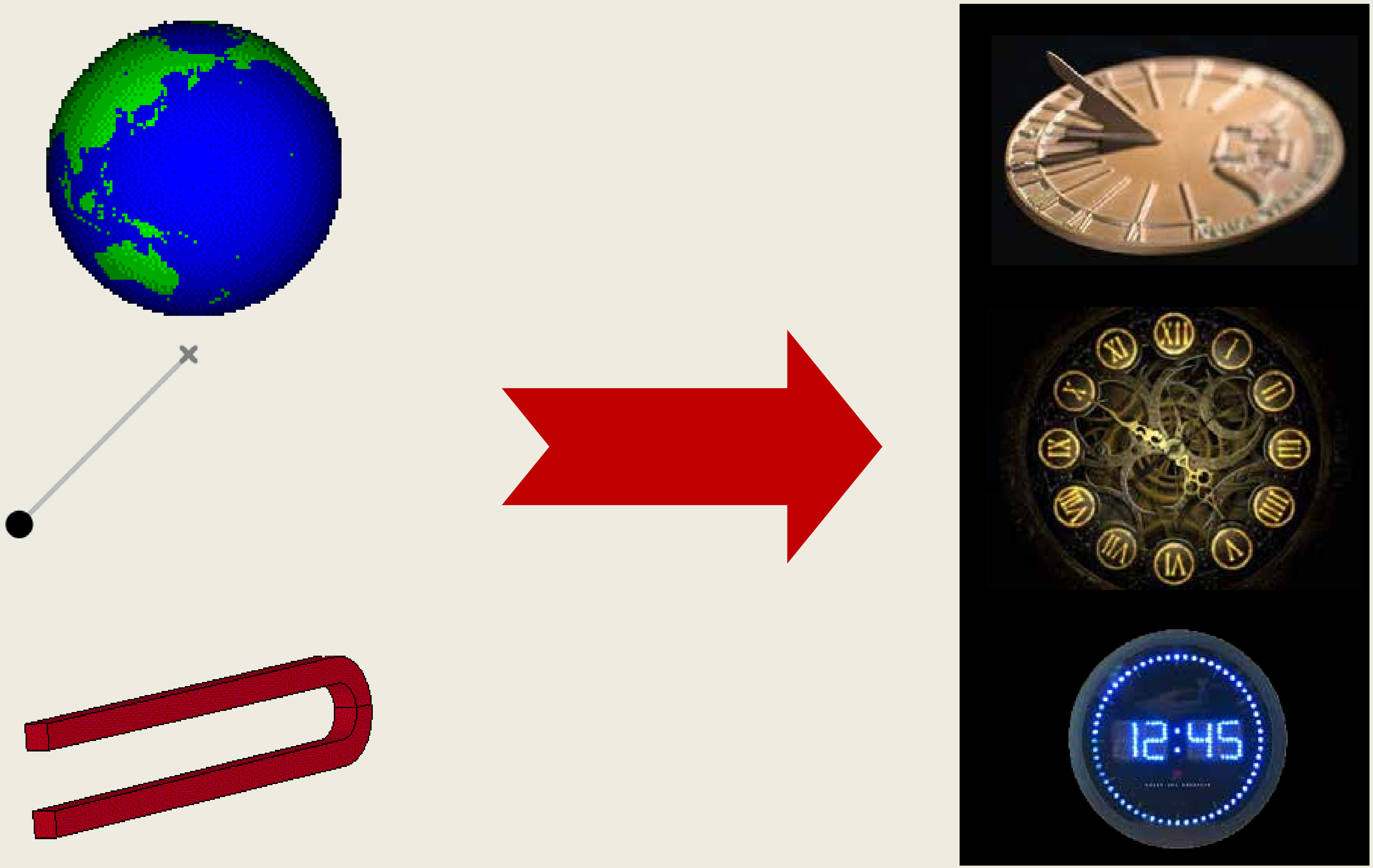
Clocks for Navigation

- Longitude Prize of the British Government: 3.5 M€
- John Harrison‘s H5 (1772)
- Instability: few seconds in a week

Modern Quartz Clocks



Principle of Clocks

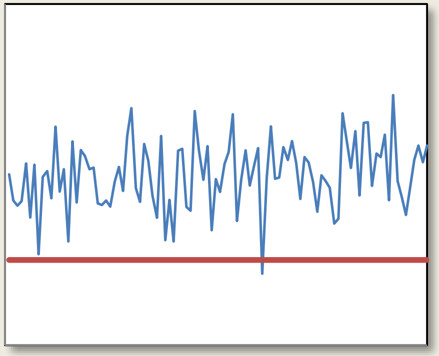


Oscillator

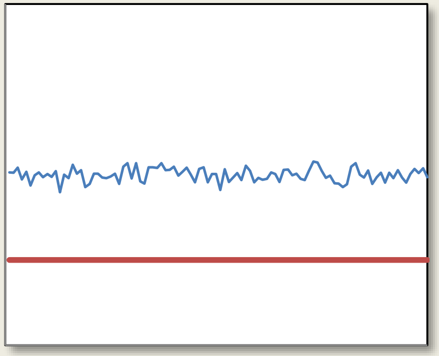
Counter

What's a Good Clock?

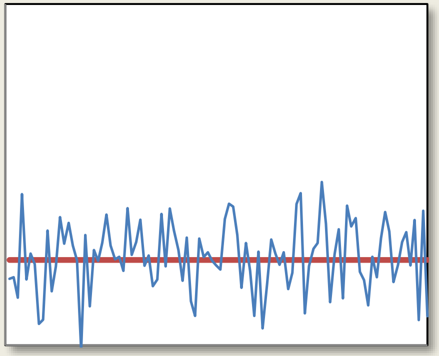
Two notions: Stability & Accuracy



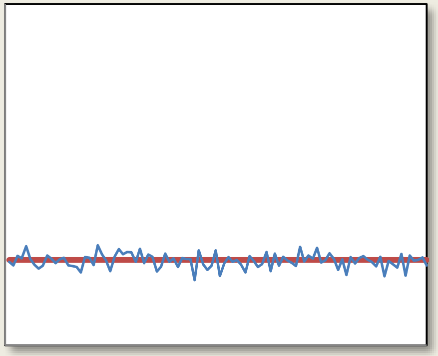
unstable,
inaccurate



stable,
inaccurate



unstable,
accurate

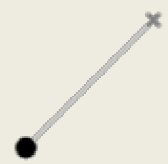


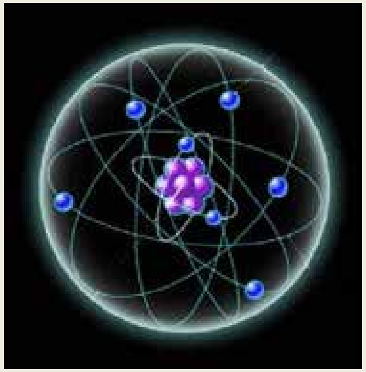
stable,
accurate

- Stability: How large is the scatter of data points?
  è Uncertainty Type A (u_A, only statistical)
- Accuracy: How well is the unperturbed reference reproduced?
  è Uncertainty Type B (u_B, outside information)

Problem: Value of unperturbed reference *a priori* not known!

What's a Good Clock?





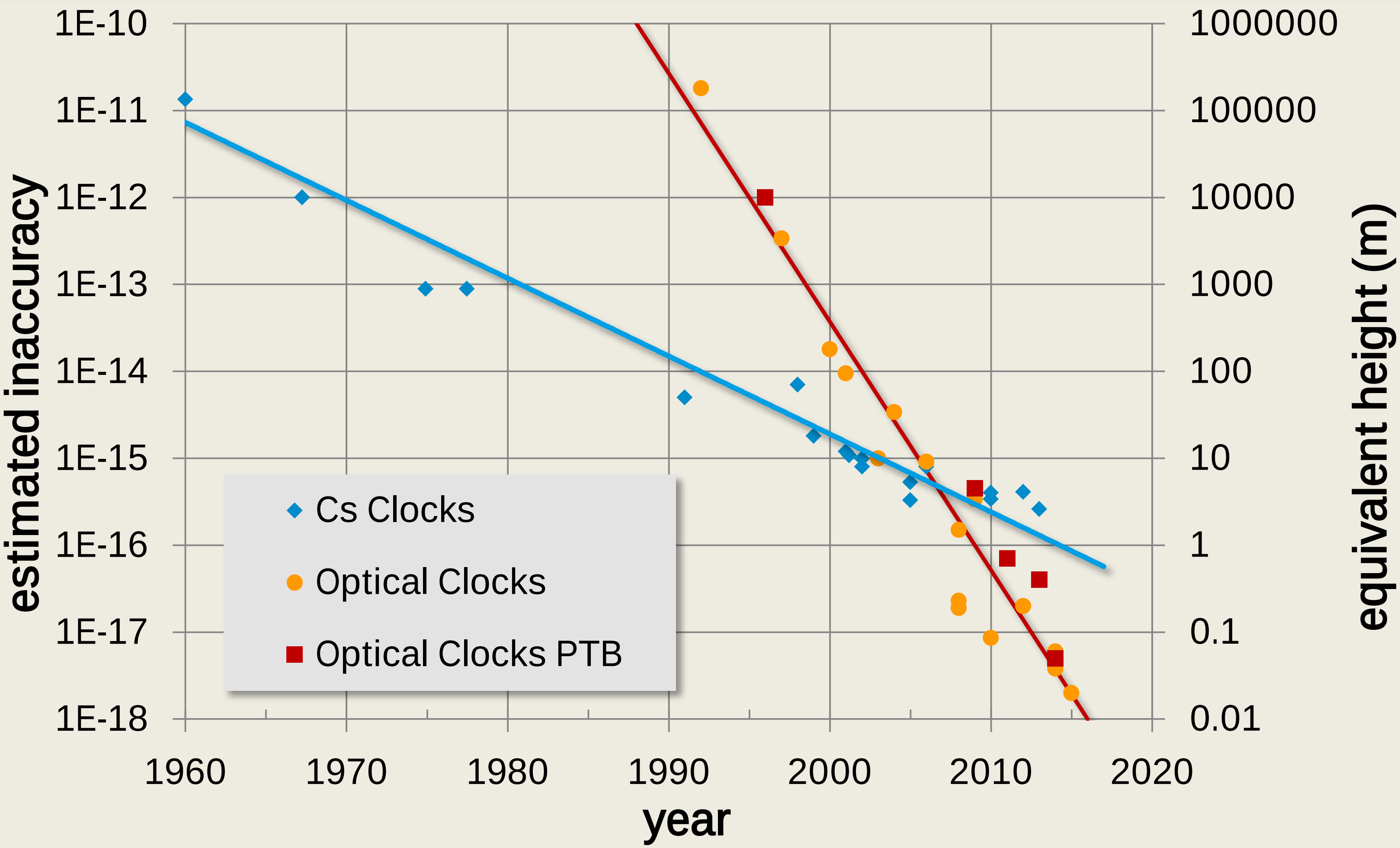
Stable, unperturbed reference required:

Atoms are ideal !

James Clerk Maxwell, 1870:

If, then, we wish to obtain standards of length, time, and mass which shall be absolutely permanent, we must seek them not in the dimensions, or the motion, or the mass of our planet, but in the wave-length, the period of vibration, and the absolute mass of these imperishable and unalterable and perfectly similar molecules.

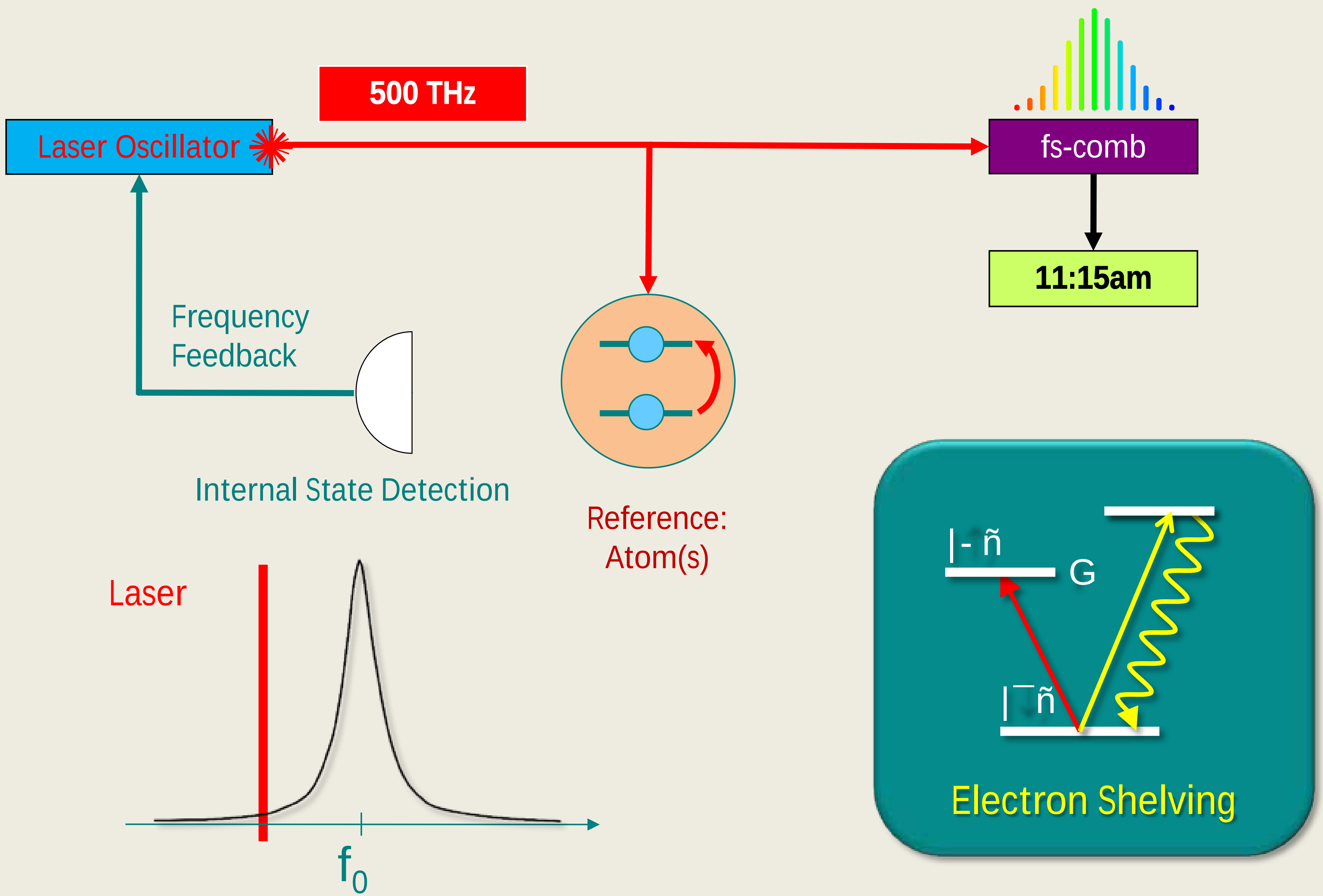
State of the art: accuracy



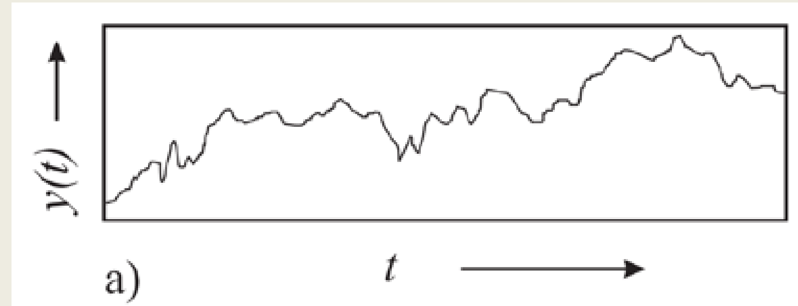
- Definition of the Second (since 1967):

One Second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Principle of Optical Clocks



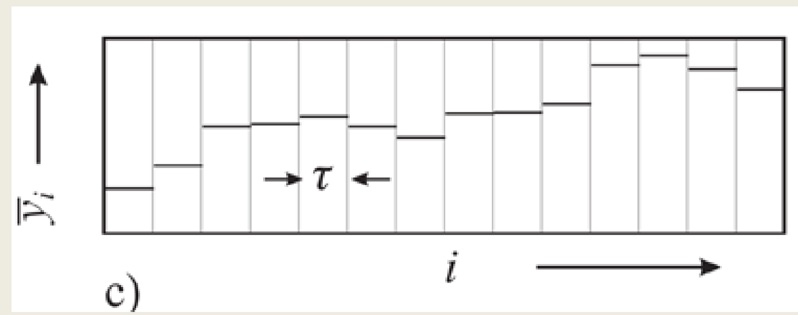
Characterization of Frequency Standards



- fractional frequency difference:

$$y(t) = \frac{f_m(t)-f_c}{f_0} = \frac{\Delta f}{f_0}$$

- i$^{\text{th}}$ measurement averaged over τ:

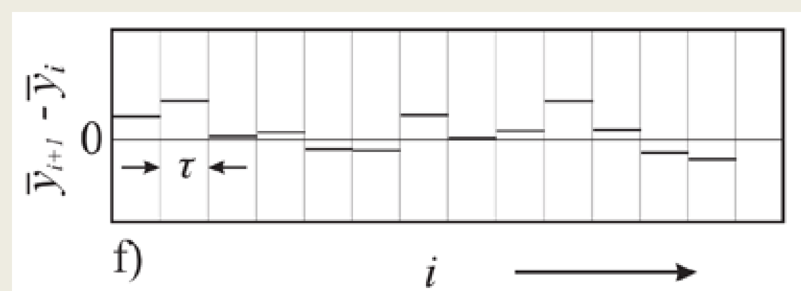


$$\bar{y}_i(\tau) = \int_{t_i}^{t_i+\tau} y(t)$$

- Allan variance:

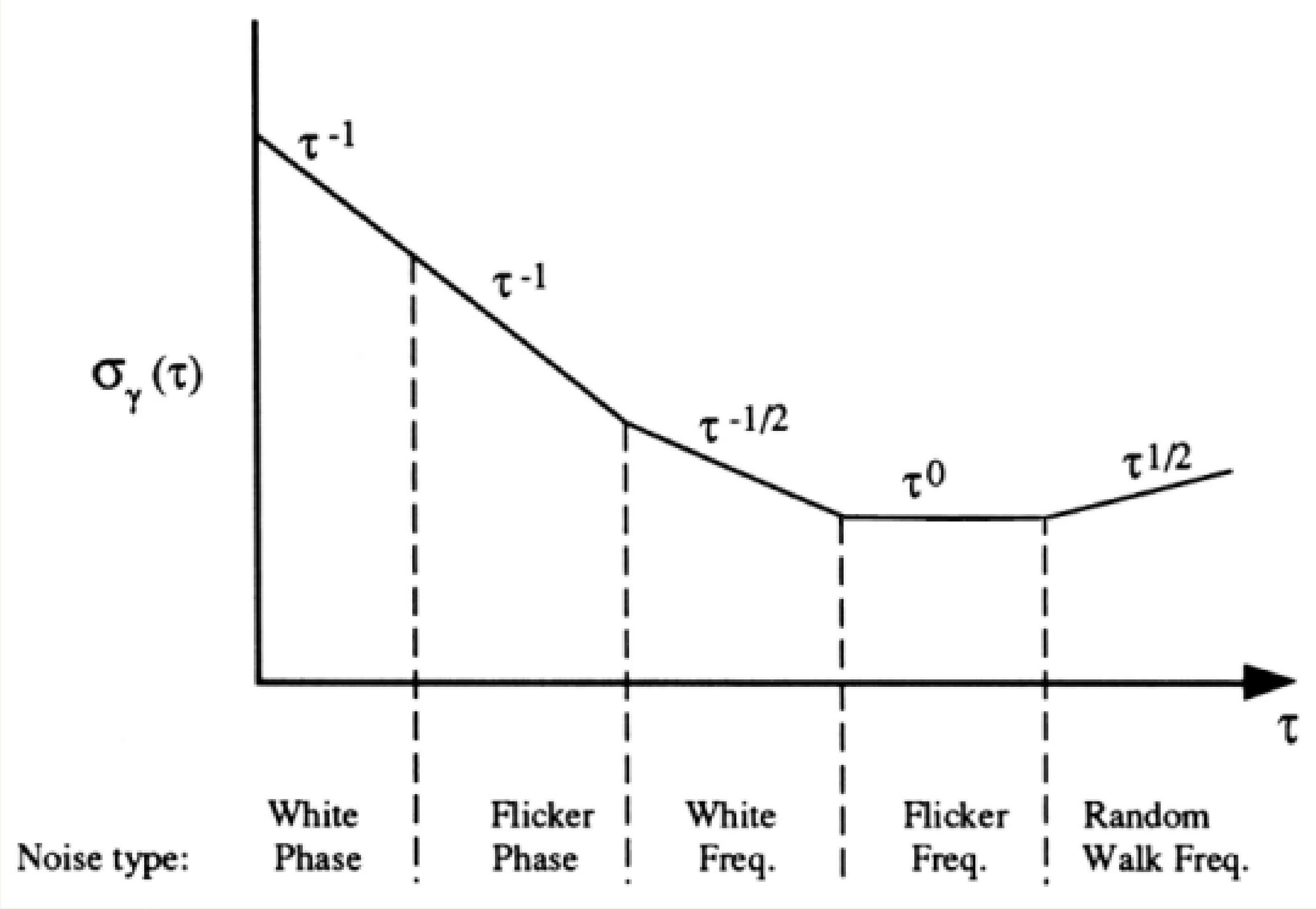
$$\sigma_y^2(\tau) = \frac{1}{2(M-1)} \sum_{i=1}^{M-1} [\bar{y}_{i+1}(\tau) - \bar{y}_i(\tau)]^2 = \frac{1}{2} \langle (\bar{y}_{i+1} - \bar{y}_i)^2 \rangle$$

- Allan deviation: $\sigma_y(\tau)$



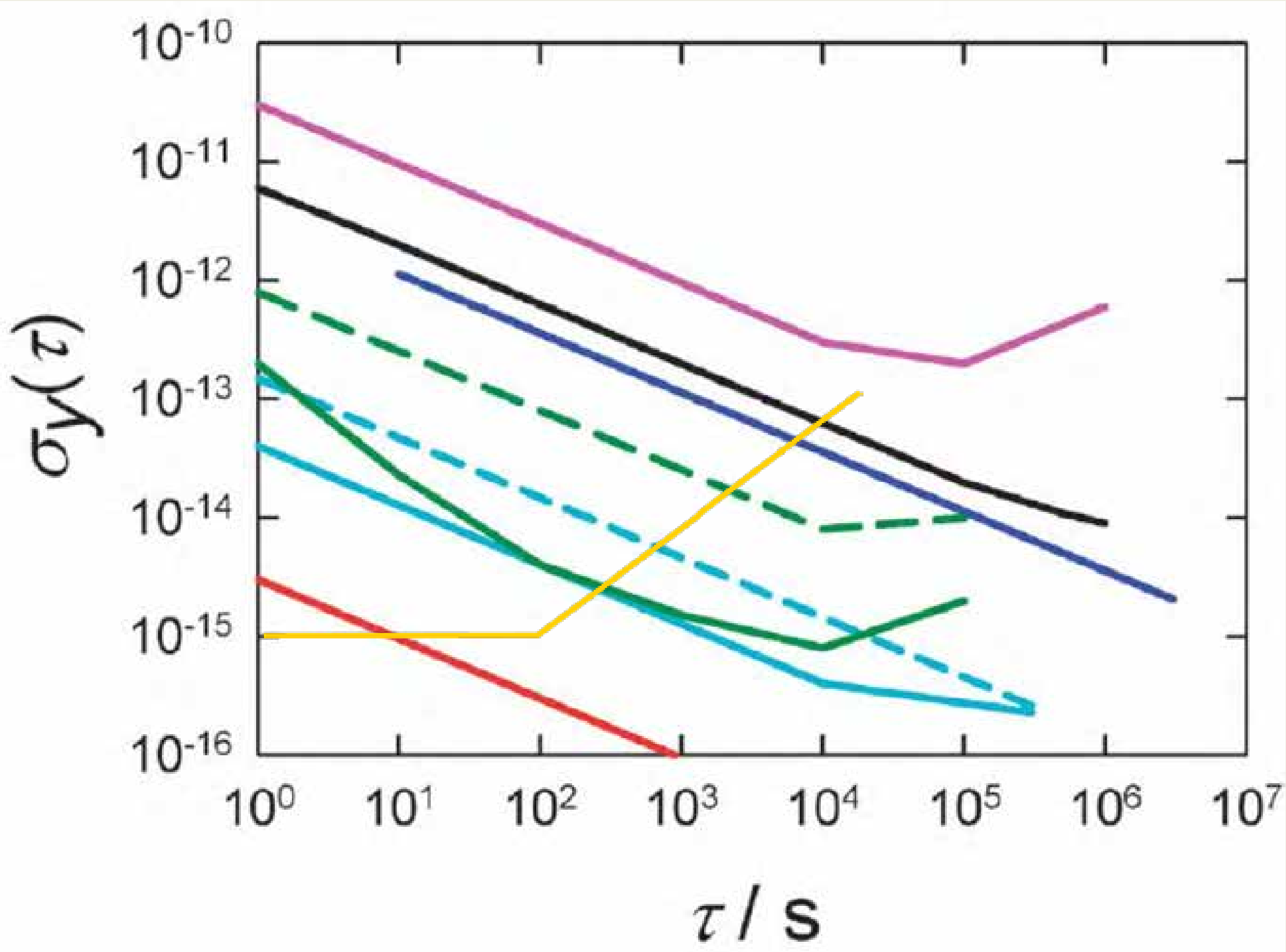
[Riehle, *Frequency Standards*]

Characterization of Frequency Standards



[NIST]

Typical Allan Deviations



commercial Rb

commercial Cs (5071A)

CS1 vs. CS2 (8 years)

passive H-maser

active H-maser

Cs fountains (CSF1, FO-2)

Sr lattice clock

laser

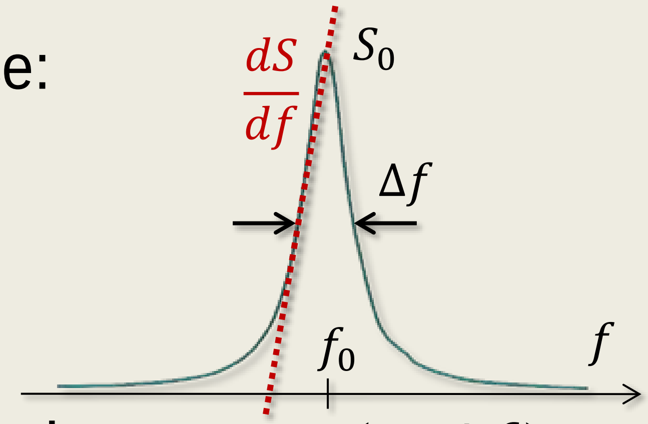
[courtesy of E. Peik]

Measurement noise

- compare laser frequency with reference and apply correction
- signal S_0; e.g. N independent spin-$\frac{1}{2}$ atoms: $S_0 = N$
- fluctuations δS in signal; e.g. QPN: $\delta S = \sqrt{Np(1-p)}$
- noise in determining frequency difference:

$$\delta y_1 = \left(\frac{\delta f}{f_0}\right)_1 = \frac{\delta S}{f_0(dS/df)} = \frac{\delta S}{S_0 Q \kappa_S}$$

with $Q = \frac{f_0}{\Delta f}, \kappa_S = \frac{dS}{df}\frac{\Delta f}{S_0} \sim 1$



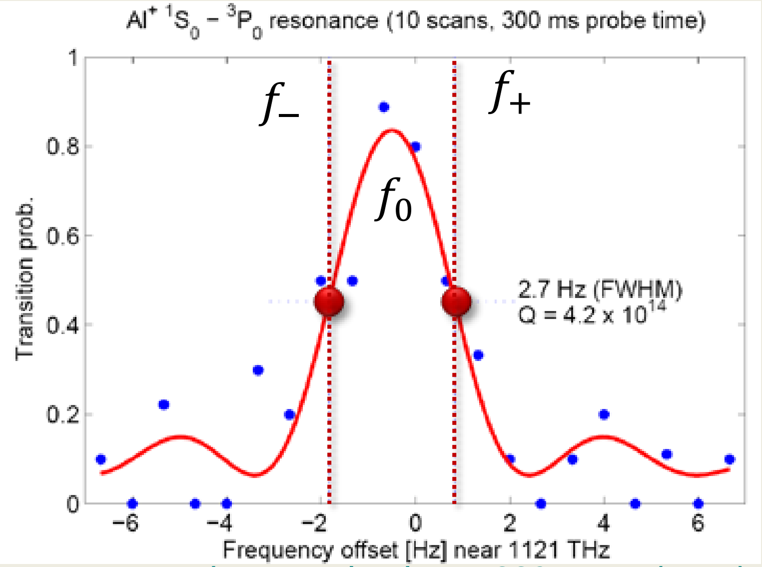
- $M = \tau / T_c$ Ramsey measurements of duration $T_c \sim 1/(2\pi\Delta f)$:

$$\sigma_y(\tau) = \left(\frac{\delta f}{f_0}\right)_1 \sqrt{\frac{1}{M}} = \frac{\delta S}{S_0 Q \kappa_S}\sqrt{\frac{T_c}{\tau}} = \frac{1}{2\pi f_0 \sqrt{N T_c \tau}}$$ (for white FN)

[Ludlow, Boyd, Ye, Peik, Schmidt, arXiv:1407.3493]

Feedback loop

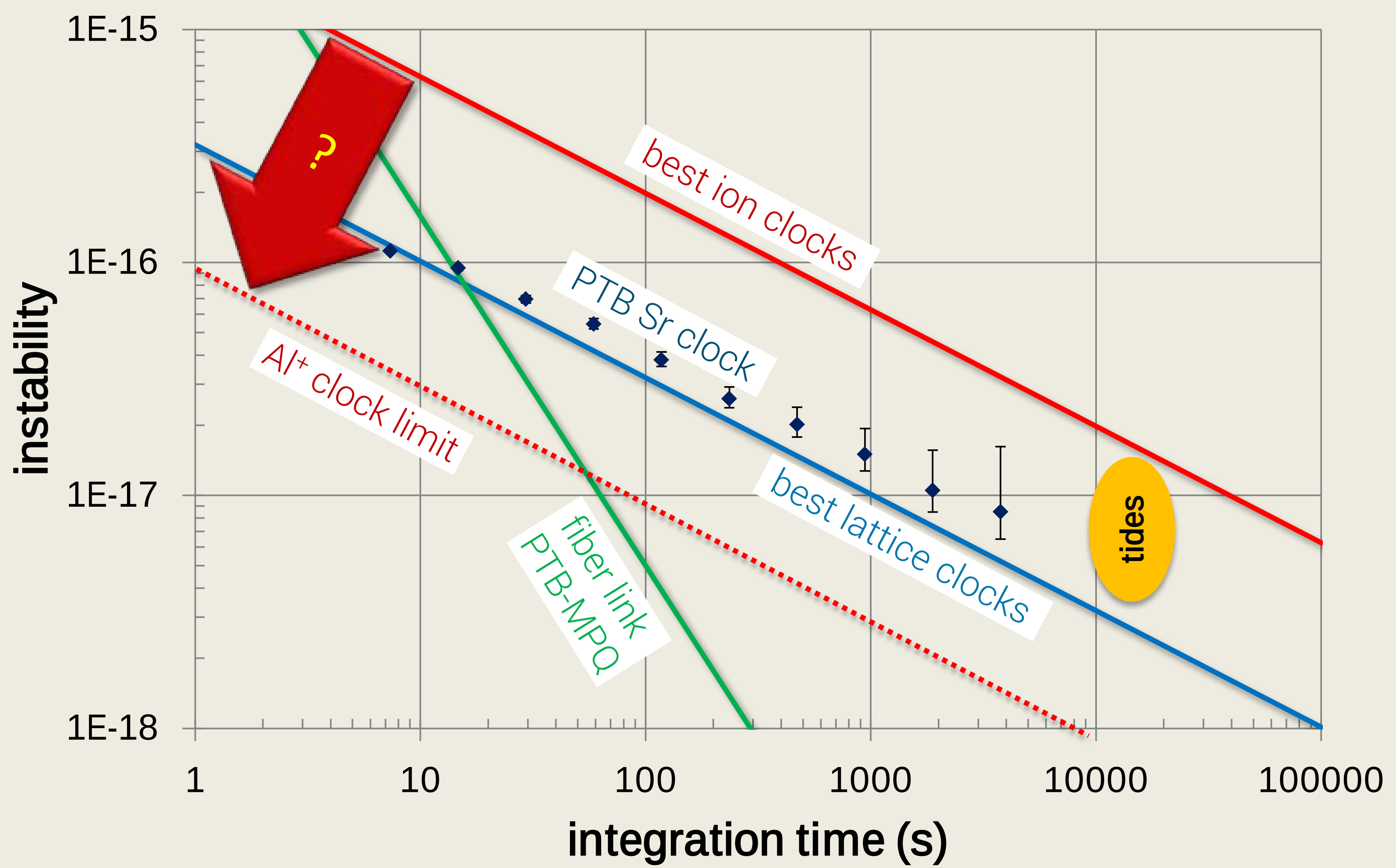
- Interrogate line at $f_\pm = f_0 \pm \delta_m$
- δ_m is chosen to achieve $\mathrm{p}_\pm \approx 0.5$
- error signal after $2n$ cycles:
  $e = \delta_m \frac{p_+ - p_-}{n}$
- frequency correction:
  $f' \to f + g \cdot e$
  with gain $g \leq 1$
  è implements a first order integrator
- additional correction for linear drift of laser (typ. $1 \ldots 100$ mHz/s)

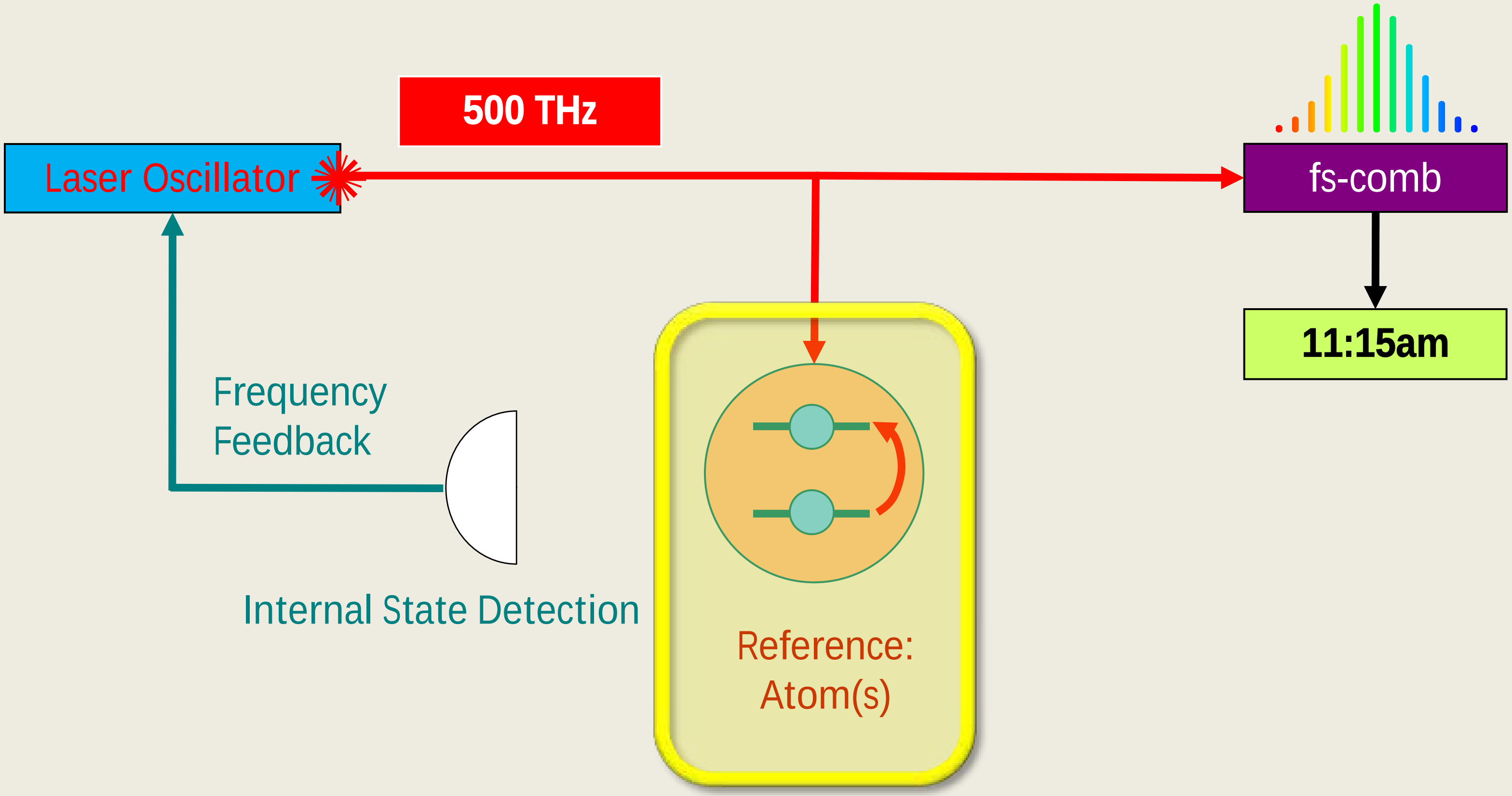


C.W. Chou, *et al.* Science **329**, 1630 (2010)

[Ludlow, Boyd, Ye, Peik, Schmidt, arXiv:1407.3493]

State of the art: stability





THE REFERENCE

[Ludlow, Boyd, Ye, Peik, Schmidt, arXiv:1407.3493]

Wishlist for Ideal Clock Atom

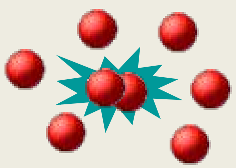
- **High stability**

  quantum projection noise limit: $\sigma_y(\tau) = \dfrac{1}{2\pi f_0\sqrt{NT_c\tau}}$

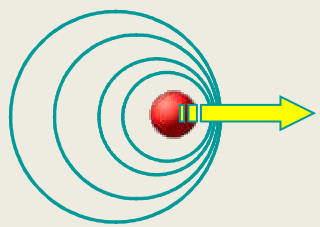
  è narrow optical transition, many atoms

- **High accuracy**

  è low sensitivity to resonance Shifts

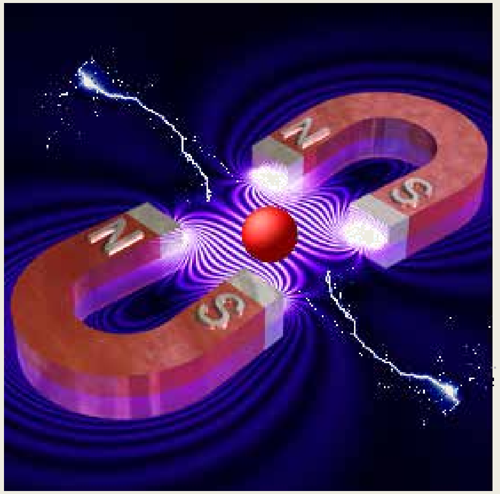


collisions



Doppler shifts

electric and magnetic fields



- **Many species to chose from:**

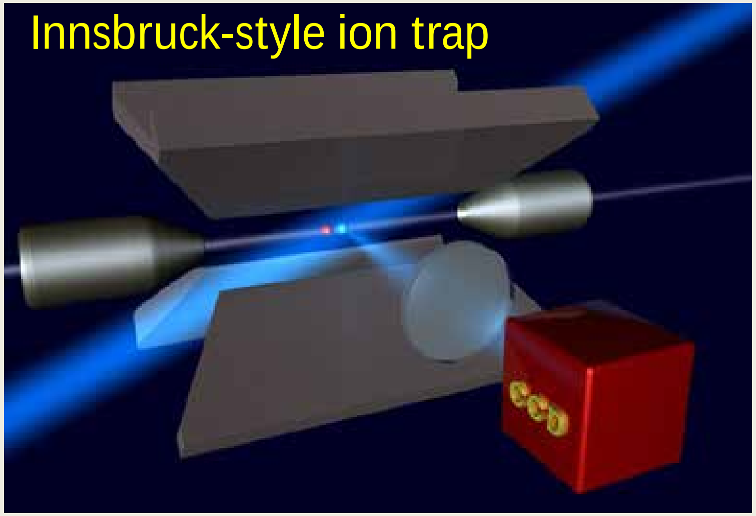
  Neutral atoms: Ca, Mg, Sr, Yb, Hg, Ag

  Ions: Hg^+, Sr^+, Yb^+, In^+, Ca^+, Al^+

Laser Spectroscopy of Trapped Ions

Dehmelt 70s & 80s:

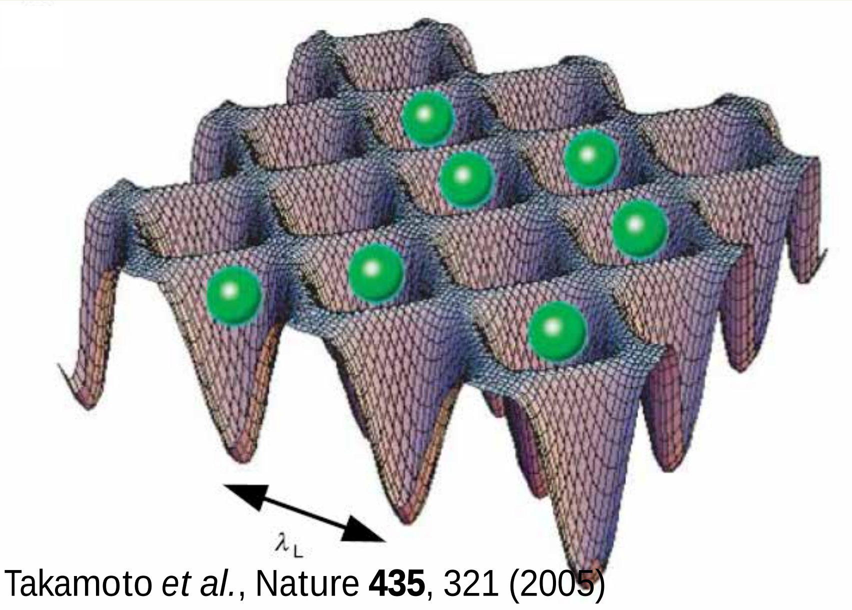
- large trap frequencies
  è recoil-free absorption
- long interrogation times
- trap ion in zero field
  è no trap induced shifts
- isolated from environment
  + laser cooling
  + no interactions
  è Highest accuracy



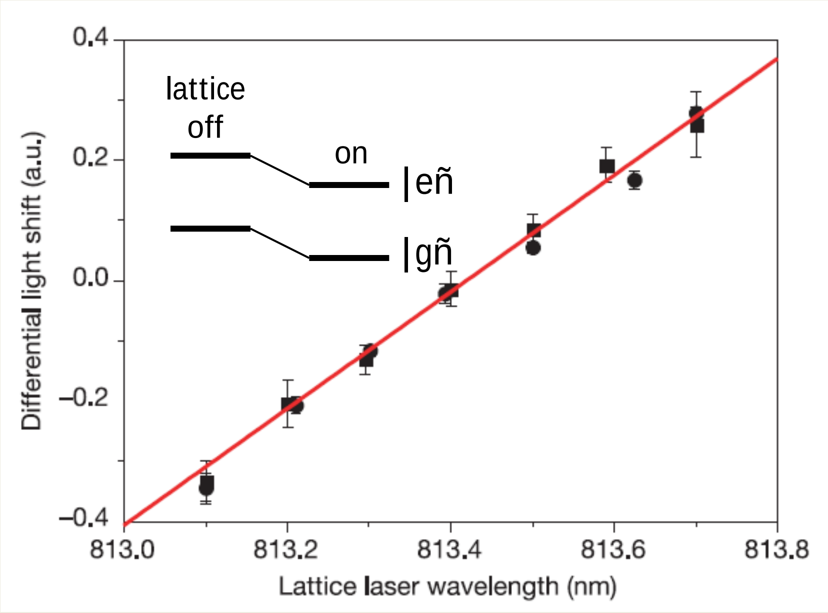
d ~ 0.8 mm; w_z ~ 2 MHz; w_r ~ 4 MHz

Neutral Atom Lattice Clocks

- Individual traps for atoms using magic wavelength lattice
- Lamb-Dicke regime è recoil-free spectroscopy
- Large number of atoms: $N \sim 10^3 \ldots 10^4$
  è potential for high stability
- Promising candidates: Hg, Yb, Sr



Takamoto *et al.*, Nature **435**, 321 (2005)



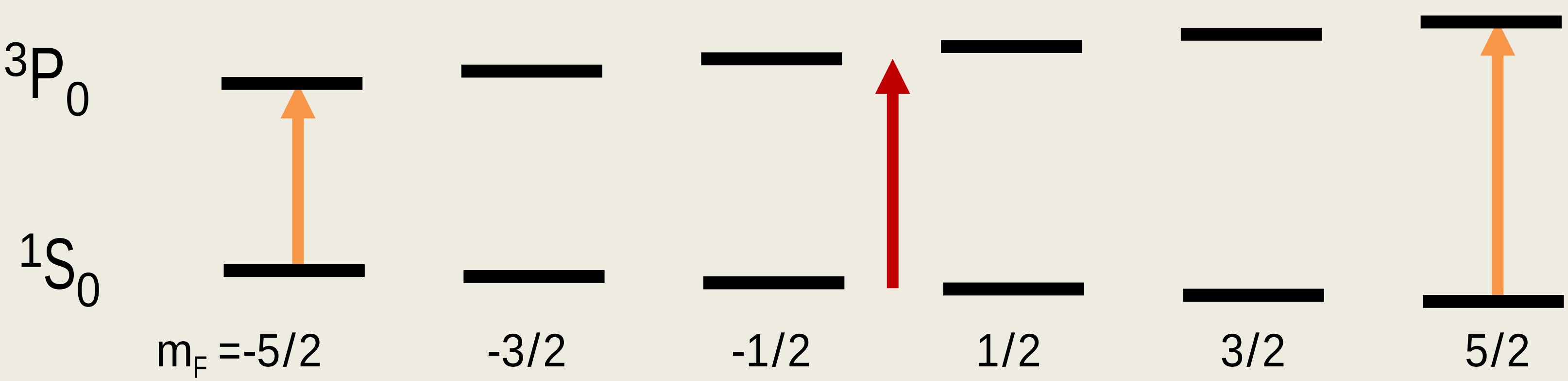
Frequency shifts: magnetic fields

- expansion for small $|B|$:

$$f - f_0 = \Delta f_M = C_{M1} B + C_{M2} B^2 + \cdots$$

linear Zeeman effect: $\Delta f = g_F m_F \mu_B B / h$

- for integer spin: $m_F = 0 \rightarrow m_F' = 0 \Rightarrow C_{M1} = 0$
- $J = 0$: $C_{M1} \sim \frac{\mu_N}{h} \sim 10\ \mathrm{MHz/T}$ (only nuclear spin)
  $J > 0$: $C_{M1} \sim \frac{\mu_B}{h} \sim 10^4\ \mathrm{MHz/T}$ (e- ang. momentum)
  è average two components with $\Delta f_{M-} = -\Delta f_{M+}$
  è *in situ* magnetic field measurement



Frequency shifts: magnetic fields

- expansion for small $|B|$:

$$f - f_0 = \Delta f_M = C_{M1}B + C_{M2}B^2 + \cdots$$

quadratic Zeeman effect:

- origin: decoupling of total angular momentum
- energy shifts found by diagonalizing the Hamiltonian:

$$H_S = hA_S \vec{I} \cdot \vec{J} + g_J(S)\mu_B \vec{J} \cdot \vec{B} + g_I' \mu_B \vec{I} \cdot \vec{B} + \cdots$$

  – A_S: hyperfine constant
  – $g_J(S)$: electronic g-factor
  – g_I': nuclear g-factor
  – $\mathrm{h}\Delta f \sim [g_J(S)\mu_B B]^2 / A_S$

è operate @ low and known B

e.g. Be$^+$ ground state $S_{1/2}, F = 1/2, F = 3/2$