Imperial College London

Penning Traps
Lecture 1

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Aims of the lectures

I hope that at the end of these 3 lectures you will:

- know what a Penning trap is
- understand the mathematical description of the trapping mechanism
  - but also have an intuitive understanding of how the Penning trap works
- know what features of the Penning trap make it attractive for certain types of applications
- have an overview of a range of experiments that have been carried out with Penning traps

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Outline of lecture 1

Lecture 1: Penning Trap Basics

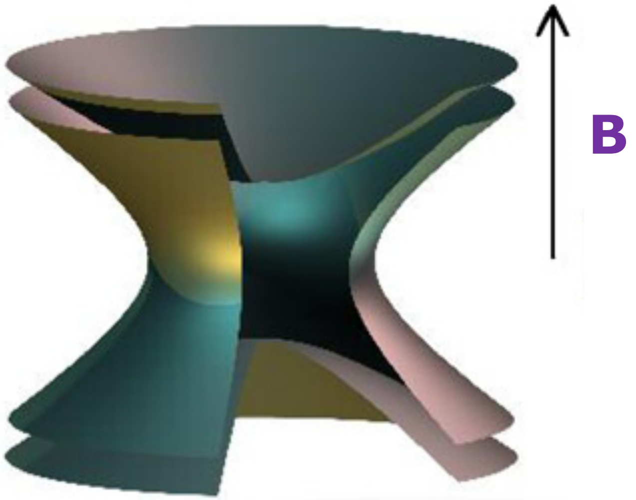
1. History of Penning Traps
2. Principles of Operation
3. Advantages and Disadvantages
4. Practical Aspects

Lecture 2: Review of Experiments

Lecture 3: Rotating Frame and Axialisation

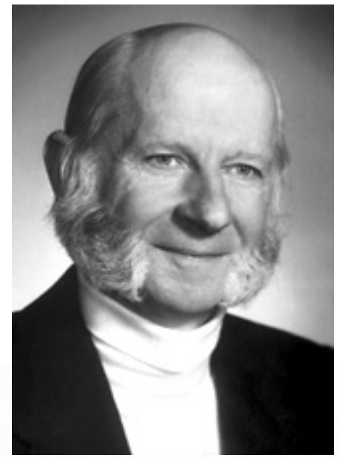
1. History of the Penning trap

- Penning's original experiments
- Development of the conventional Penning trap geometry
- Early experiments with Penning traps
- First demonstration of laser cooling



History of Penning Traps

- The name “Penning trap” was coined by Dehmelt
  - Penning (1936) didn’t actually trap charged particles
  - Electrons confined in discharges with B-field present



2.2. PENNING TRAP

Even though this scheme does not strictly belong to those admissible under our title, we will begin with a discussion of the Penning arrangement which we are using for the containment of electrons as well as cyclotron resonance and thermalization studies of these particles (Dehmelt, 1961). The confinement mechanism is that of the Penning discharge widely used in cold cathode ionization gages and ion-getter pumps. In our embodiment of this principle (Penning, 1936; Pierce, 1949) hyperbolic electrodes (see Fig. 1) create a dc electric potential of the form

$$\phi(x, y, z) = A(x^2 + y^2 - 2z^2), \qquad A = A_0 = \text{const}, \tag{2.3}$$

Dehmelt, H. G. (1961). Unpublished.

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History of Penning Traps

- Pierce (1949) analysed the 3-electrode trapping geometry with a magnetic field
- Showed that electrons are trapped by the fields

Thus, the condition under which electrons cannot get indefinitely far away from the axis is that

$$\omega_c > \sqrt{2}\omega_0 \qquad (4\cdot33)$$

We see that it is possible to obtain a pure sinusoidal motion of electrons trapped in this combination of electric and magnetic fields.

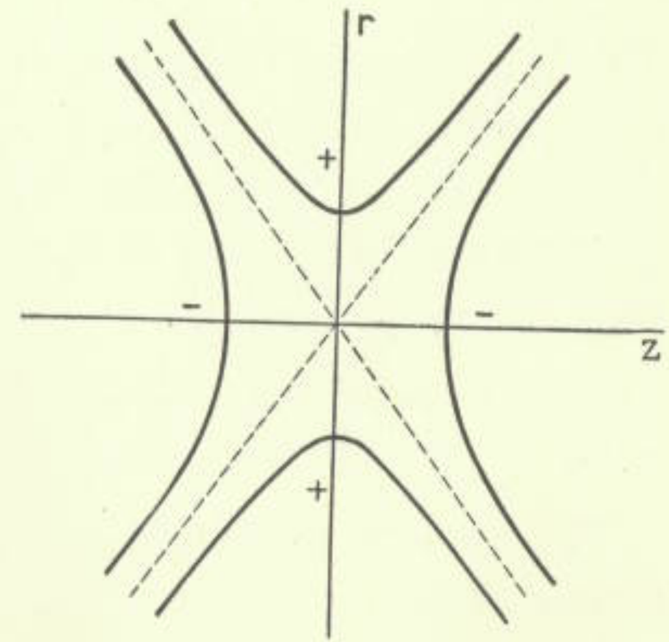

FIG. 4.7—Electron motion between hyperbolic electrodes may be limited to a certain region by use of an axial magnetic field.

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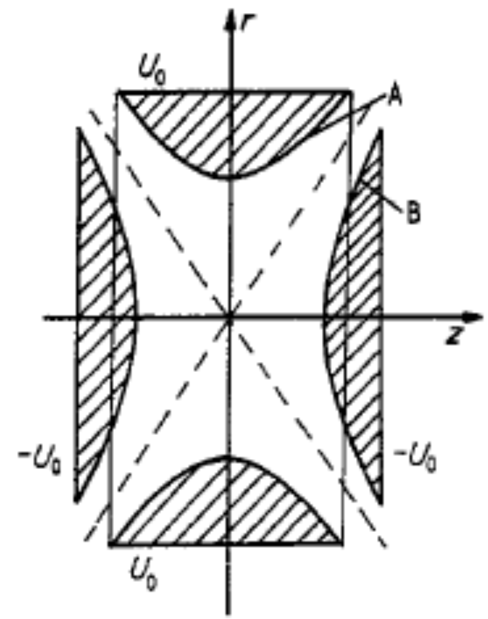
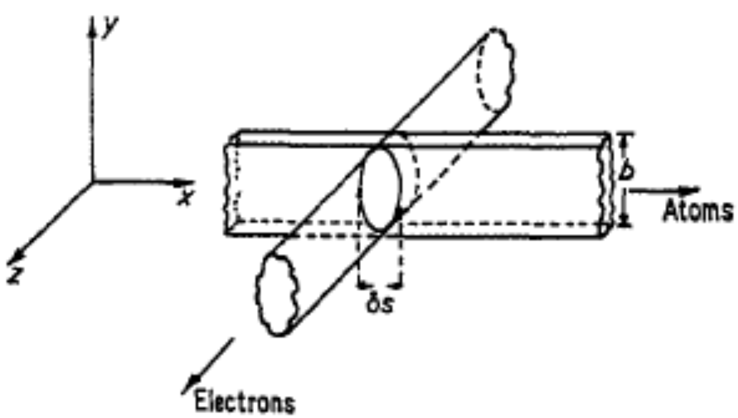
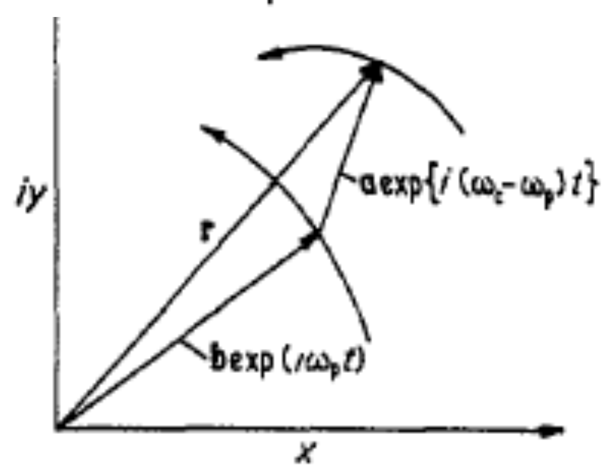
A trap for polarising electrons

- Byrne and Farago (1965) studied the 3-electrode arrangement with a **B**-field
- They polarised trapped electrons using a polarised atomic beam
- The electrons were released to make a pulsed polarised beam of electrons







Early experiments with Penning traps

- Gräff *et al.* (1968) measured cyclotron and spin resonances of trapped electrons
- Gräff *et al.* obtained electron *g*-factor value to 3 10^{-7} (0.3 ppm) in 1969

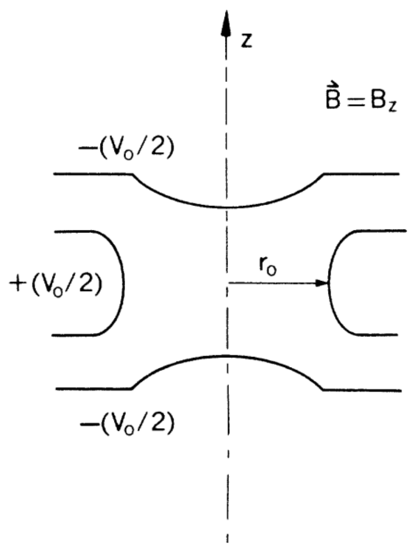


FIG. 1. The quadrupole electron trap used to contain free electrons in ultrahigh vacuum in order to observe their cyclotron- and spin-resonance frequencies.

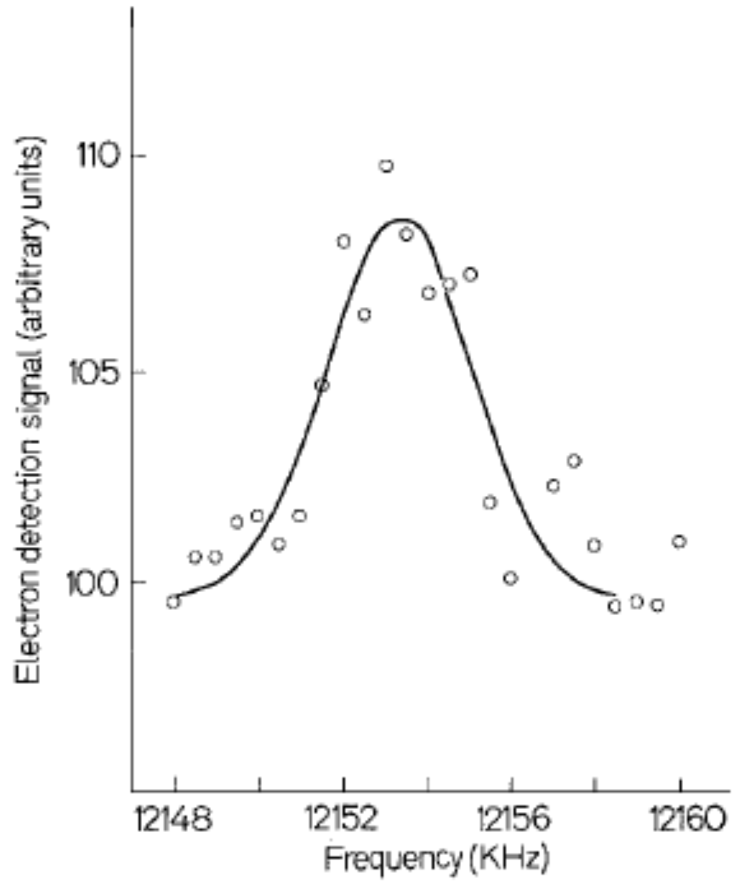
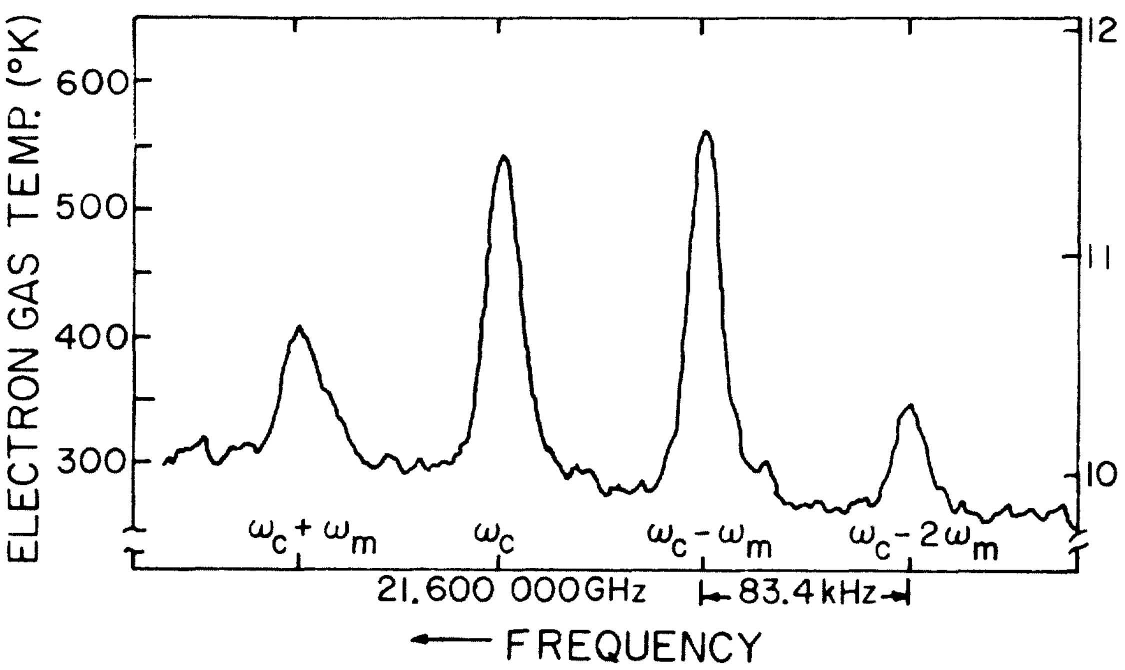


Fig. 2. The $(\omega'_L - \omega'_B)$ line

RF spectroscopy in the Penning trap

- Dehmelt and Walls (1968) used currents induced in an external circuit to monitor the temperature of electrons in a Penning trap – the “Bolometric Technique”



Electron temperature as the drive is swept through the cyclotron resonance at 21.6 GHz

Sidebands from magnetron motion at 83 kHz

First laser cooling demonstration

- First laser cooling demonstration (Wineland *et al* 1978) also used the Bolometric technique

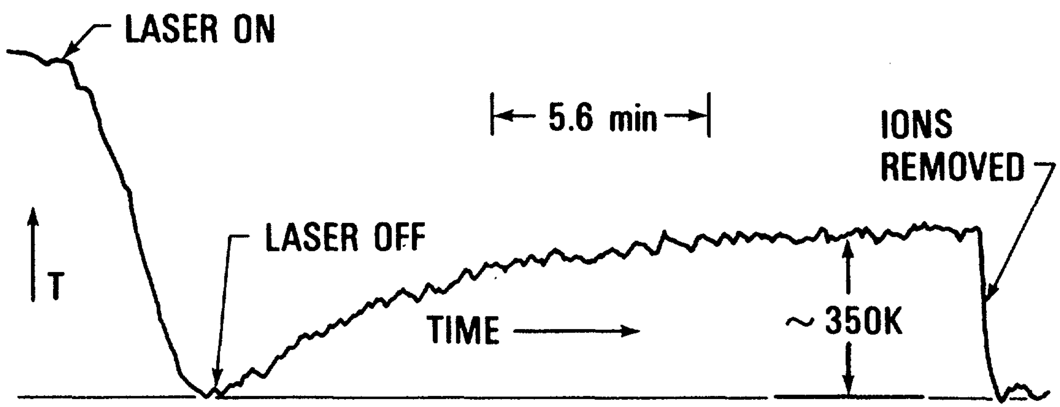


FIG. 2. Ion temperature vs time when laser cooling is applied for fixed $\nu_L - \nu_0$. The ions were initially heated above equilibrium temperature with the laser. Laser cooling was then applied on the $-\frac{1}{2} \leftrightarrow -\frac{3}{2}$ transition for a fixed time until a temperature approaching 0 K (< 40 K) was achieved. After the laser is turned off, the ions rethermalize to the ambient temperature.

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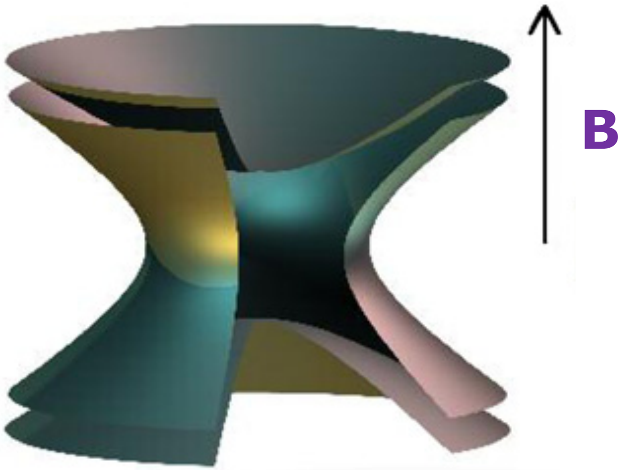
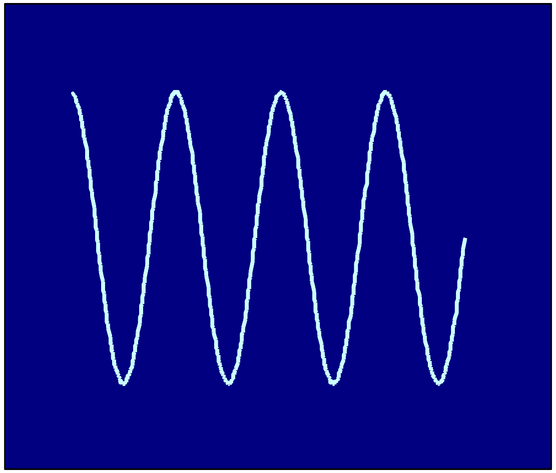
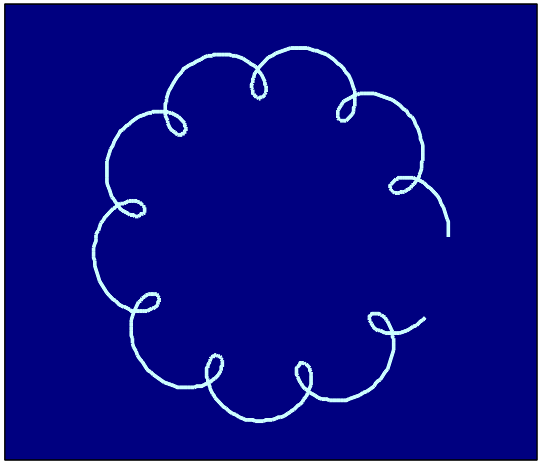
2. Principles of operation of Penning traps

- Electrode configuration
- Equations of motion
- Modes of oscillation
- Potential and kinetic energy
- Space charge effects





Axial motion



Radial motion

Earnshaw's theorem

- In order to trap a charged particle we would need a quadrupole potential with a three-dimensional minimum:

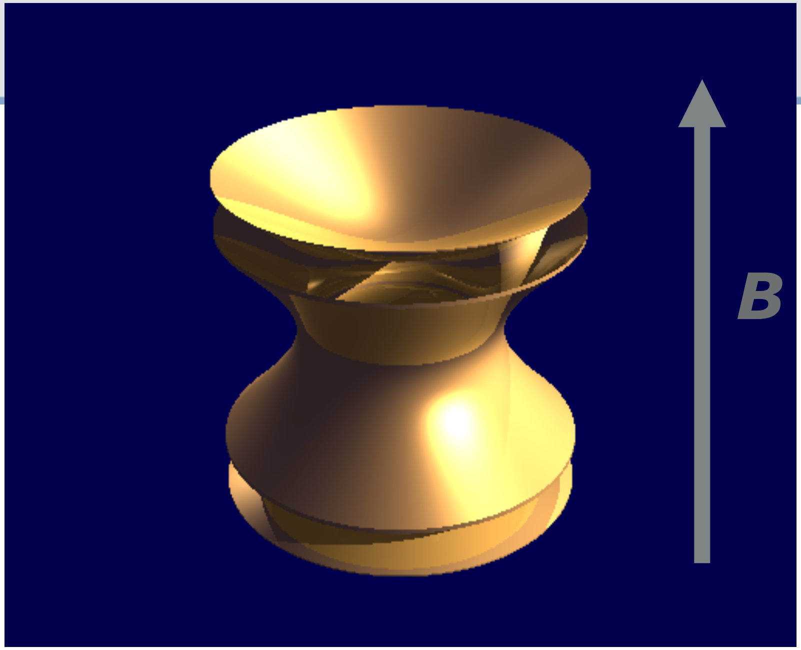
$$\phi(x,y,z) = Ax^2 + By^2 + Cz^2$$

  with *A*, *B* and *C* all positive
- However, Laplace's equation $\nabla^2\phi = 0$ requires that

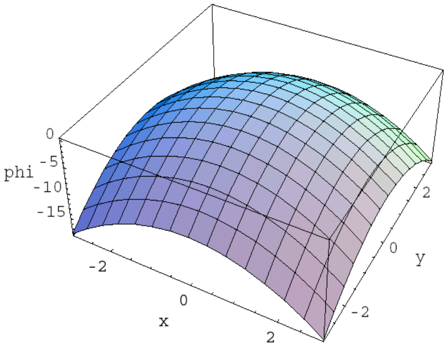
$$A + B + C = 0$$

- So three-dimensional trapping is not possible with static fields
- Two ways around this:
  - Add a magnetic field – the *Penning trap*
    - » these lectures
  - Use oscillating fields – the *Radiofrequency (Paul) trap*
    - » Martina Knoop's lectures

Principles of operation



- Three electrodes
  - Hyperboloids of revolution
  - Generate pure quadrupole potential
- DC potential applied between endcaps and ring
  - Traps ions in the axial direction
  - Radial motion unstable
- Strong **B** field applied along the axis
  - provides radial confinement (ions forced into cyclotron loops)
- Requires ultra-high vacuum
  - Avoids collisions leading to ion loss due to instability of radial motion



Radial potential

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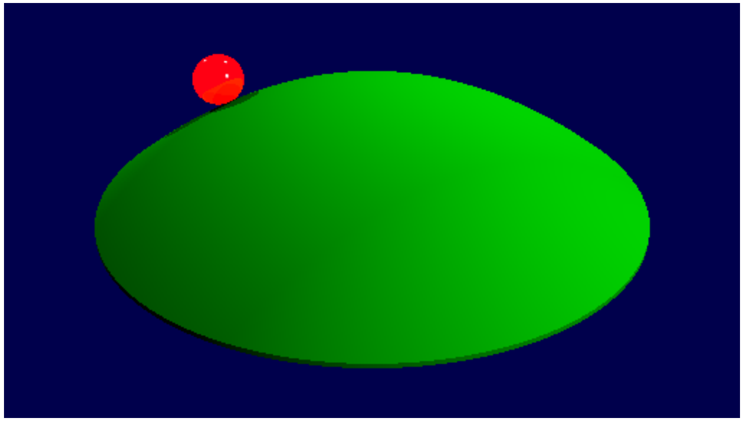
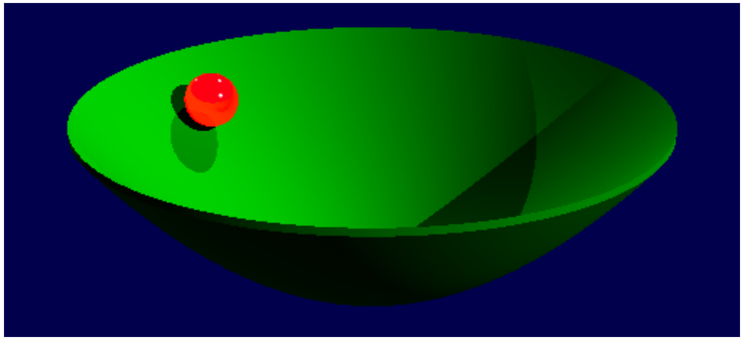
Radial motion in the Penning trap

- The axial motion is stable, as it is just simple harmonic motion in a quadratic potential
- In the plane of the ring the electric field pushes the ion out towards the ring
- The magnetic field stops it falling out by making it orbit round the centre of the trap in a new *effective* potential





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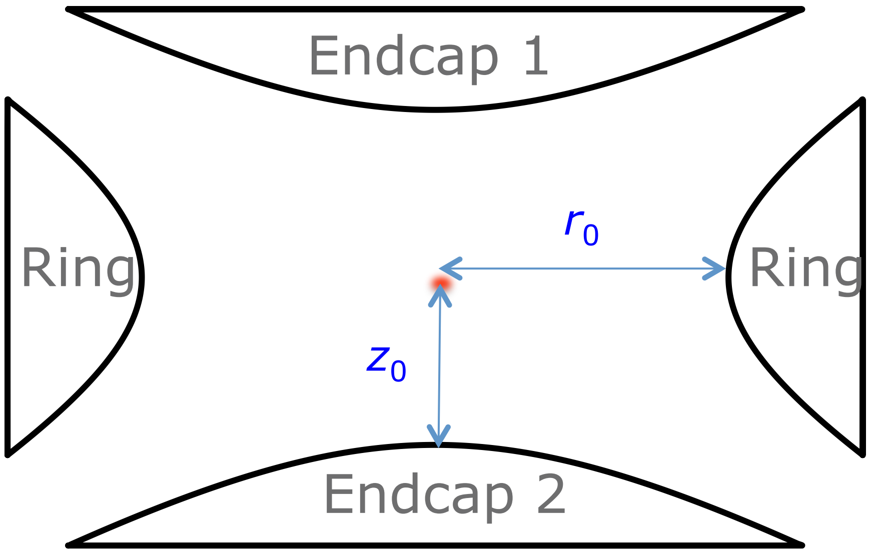
Ideal Penning trap electrodes



- The desired potential is $\phi(r,z) = A(2z^2 - r^2)$
- This can be generated with electrodes the shape of the equipotential surfaces of this potential, i.e. hyperboloids of revolution
- With r_0 and z_0 defined in the diagram we obtain

$$\phi(r,z) = V(2z^2 - r^2)/(r_0^2 + 2z_0^2)$$

Equations of motion

The Lorentz force on a charged particle in **E** and **B** fields is

$$m\ddot{\mathbf{r}} = e(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

Here $E = -\nabla\phi$ where $\phi(r,z) = V(2z^2 - r^2)/(r_0^2 + 2z_0^2)$.

The equations of motion for a single particle are therefore

$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2/2)x = 0$$

$$\ddot{y} - \omega_c \dot{x} - (\omega_z^2/2)y = 0$$

$$\ddot{z} + \omega_z^2 z = 0$$

here $\omega_c = eB/m$ and $\omega_z^2 = 4eV/mR^2$ where $R^2 = r_0^2 + 2z_0^2$.

For the radial motion it is convenient to write this in complex form:

$$\ddot{u} - i\omega_c \dot{u} - (\omega_z^2/2)u = 0$$

where $u = x + iy$

Motion in the Penning trap

We end up with three types of motion:

- Axial motion
  - » oscillation between the endcaps (~200 kHz)

$$\omega_z^2 = 4eV / mR^2$$

- Modified cyclotron motion
  - » orbit around a magnetic field line (~500 kHz)

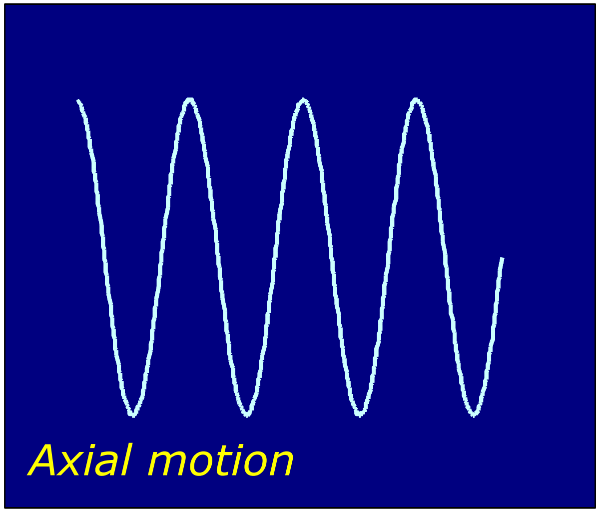
$$\omega'_c = \omega_c / 2 + \sqrt{(\omega_c / 2)^2 - \omega_z^2 / 2}$$

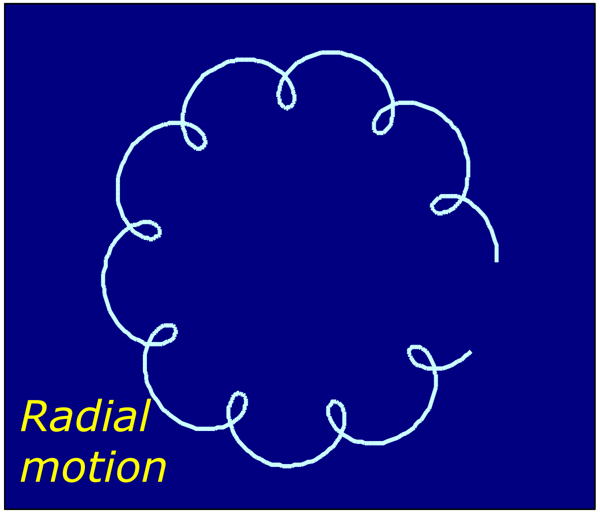
- Magnetron motion (**E×B** drift)
  - » slow orbit around trap centre (~50 kHz)

$$\omega_m = \omega_c / 2 - \sqrt{(\omega_c / 2)^2 - \omega_z^2 / 2}$$

  - » This motion is unstable (negative total energy)

- These values are for an atomic ion in a field of ~1 T
- Classical derivation but all motions can be treated quantum mechanically





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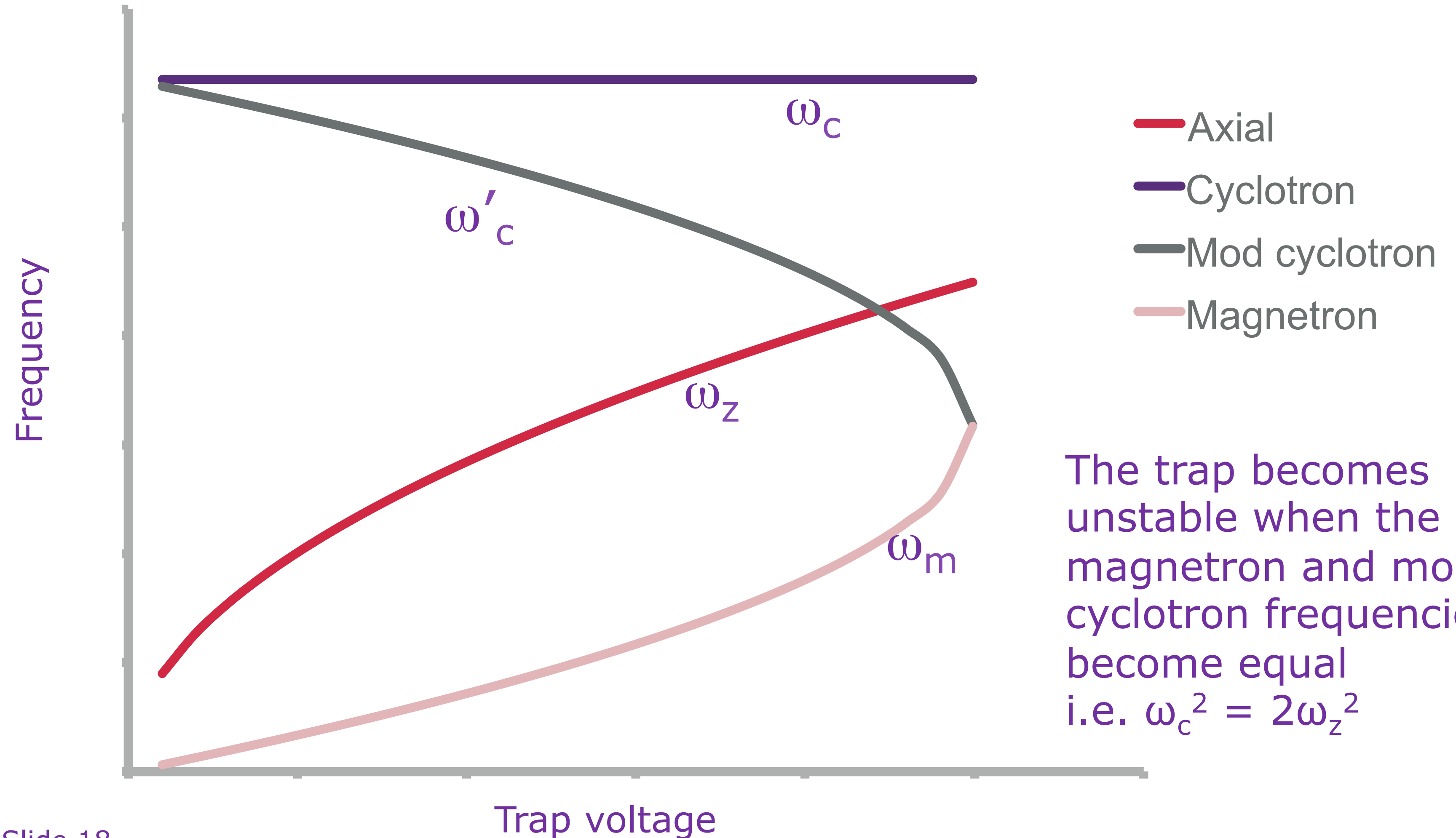
Oscillation frequencies



The trap becomes unstable when the magnetron and modified cyclotron frequencies become equal
i.e. $\omega_c^2 = 2\omega_z^2$

Invariance theorem

- From the expressions for the three oscillation frequencies, we find:

  $$\omega_c = \omega'_c + \omega_m$$

  and

  $$\omega_c^2 = \omega'^2_c + \omega_z^2 + \omega_m^2$$

- The last equation (discovered by Gabrielse) is still true for any small angle misalignment of the trap or quadratic imperfections of the field
  - It is therefore very important for experiments where accurate measurements of the true cyclotron frequency are necessary
- This is what allows very high precision mass comparisons and *g*-factor measurements to be made in a Penning trap

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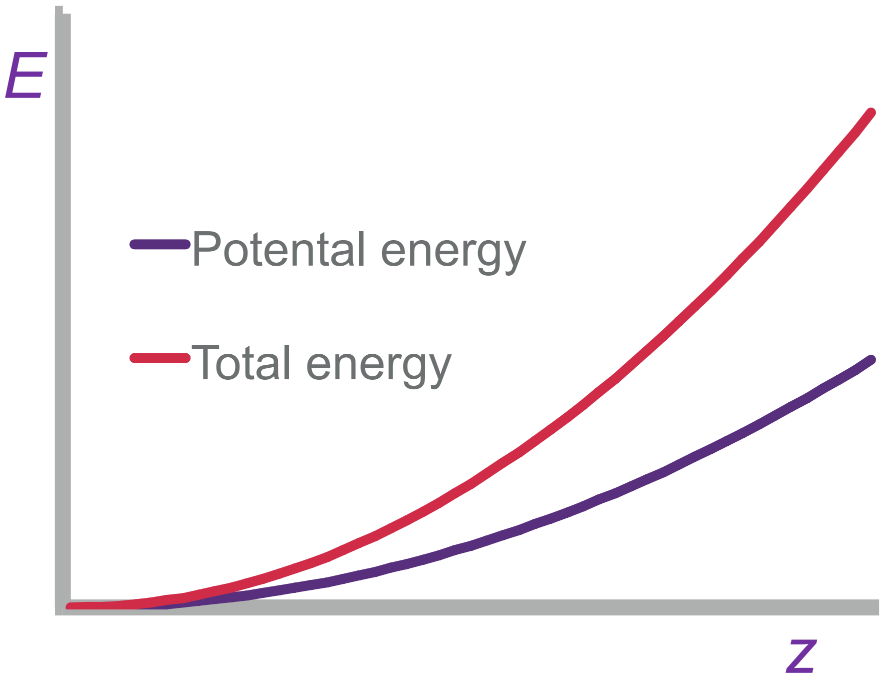
Potential and kinetic energy – axial



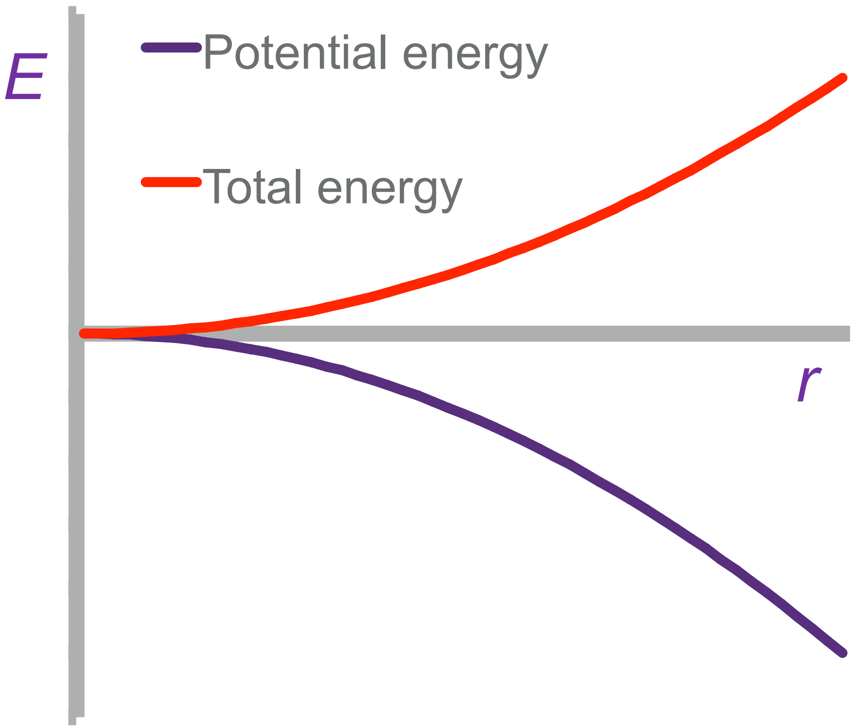
- For the **axial motion** the average potential energy and the average kinetic energy are equal

- $E = ½ [mz^2\omega_z^2 + mv_z^2]$
  $= ½ mz_0^2\omega_z^2$
  (z_0 is amplitude of motion)

- The usual principle of equipartition of energy applies ($<E_{pot}> = <E_{kin}> = ½kT_z$)

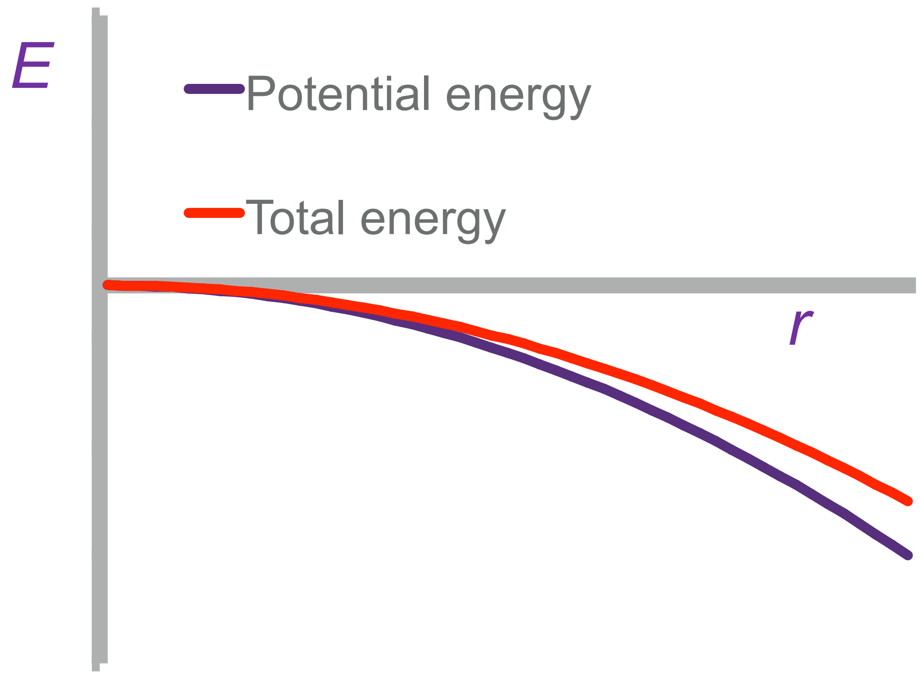
- In Quantum Mechanical terms: $E_z = n_z\hbar\omega_z$

Potential and kinetic energy – cyclotron



- For the **cyclotron motion** the average potential energy and the average kinetic energy are ***not*** equal
- $E = -\frac{1}{4} m r_c^2 \omega_z^2 + \frac{1}{2} m v_c^2$
  $= \frac{1}{2} m r_c^2 \omega'_c(\omega'_c - \omega_m)$
- Equipartition of energy does **not** apply here
- We define a "kinetic temperature" through $\langle E_{kin} \rangle = \frac{1}{2} k T_c$
- In Quantum Mechanical terms: $E_c = n_c \hbar \omega'_c$

Potential and kinetic energy – magnetron



The negative magnetron energy has several consequences for the applications of the Penning trap

- For the **magnetron motion** the potential energy and the kinetic energy are not equal
- $E = -\frac{1}{4}\, m r_m^2 \omega_z^2 + \frac{1}{2}\, m v_m^2$
  $= -\frac{1}{2}\, m r_m^2 \omega_m (\omega'_c - \omega_m)$
- Equipartition of energy does **not** apply here and the total energy is negative
- We define a "kinetic temperature" through $\langle E_{kin} \rangle = \frac{1}{2}\, k T_m$
- In Quantum Mechanical terms: $E_m = -n_m \hbar \omega_m$

Interactions between ions

If two ions are in the trap, the equations of motion for particle 1 are now:

$$\ddot{x}_1 + \omega_c \dot{y}_1 - (\omega_z^2/2)x_1 + (x_1 - x_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$\ddot{y}_1 - \omega_c \dot{x}_1 - (\omega_z^2/2)y_1 + (y_1 - y_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$\ddot{z}_1 + \omega_z^2 z_1 + (z_1 - z_2)e^2/4\pi\varepsilon_0 m r^3 = 0$$

$$r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

with a similar set of equations for particle 2.

These coupled equations can be separated:

$$\ddot{X} + \omega_c \dot{Y} - (\omega_z^2/2)X = 0$$

Centre of mass motion

Unchanged oscillation freqs.

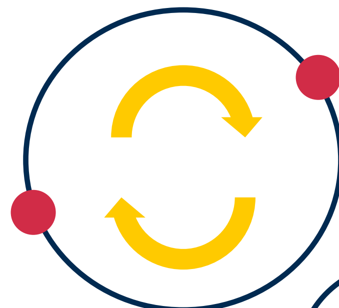
$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2/2)x + 2e^2 x/4\pi\varepsilon_0 m r^3 = 0$$

Relative motion

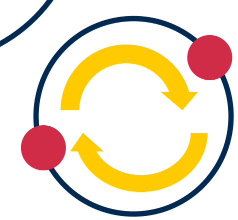
Modified oscillation freqs.

Motion of 2 ions

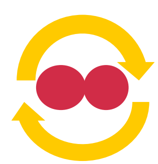
- The relative motion in the axial direction has an increased frequency.
  - If the ions are both on the z-axis their minimum energy is when $z_1^3 = e^2/16\pi\varepsilon_0 m\omega_z^2$. The relative oscillation frequency is then $\sqrt{3}\,\omega_z$
- If the ions are separated radially, they rotate about the z-axis at ω_R:
  - Large separation: $\omega_R \sim \omega_m$



  - Medium separation: $\omega_R > \omega_m$

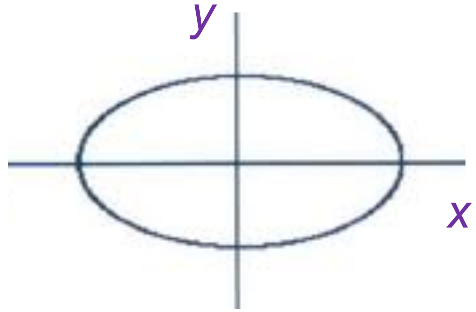


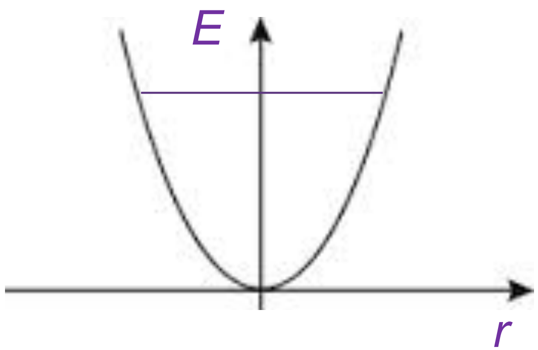
  - Minimum separation: $\omega_R = \omega_c/2$
    $r^3 = e^2/16\pi\varepsilon_0 m(\omega_c^2/4 - \omega_z^2/2)$



Space charge

- When large numbers of ions are present, the cloud of ions behaves like a plasma
  - Referred to as a *non-neutral plasma* or *single-component plasma*
- The main properties of this plasma are:
  - It has a uniform density *n*
  - It has sharp edges (at low temperature)
  - It has a ellipsoidal shape
  - It rotates at a uniform rate about the *z* axis
- We can think of the plasma as "filling up" the quadratic potential well
- The whole plasma moves at the centre of mass oscillation frequencies
- It has many internal modes of oscillation





Brillouin flow

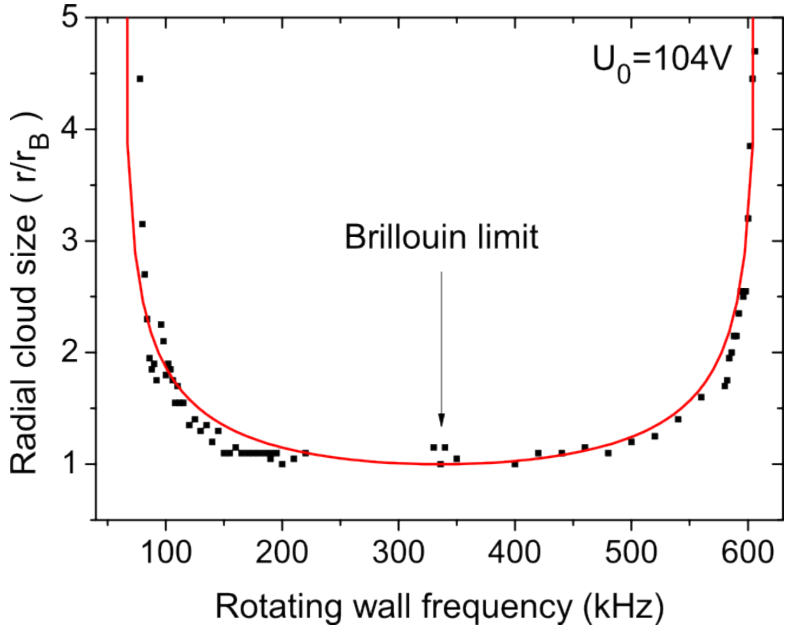
- When cold, the plasma behaves like a solid body rotating at a frequency ω_R and this is linked to the number density of the plasma *n*:

$$n = 2\varepsilon_0 m\omega_R(\omega_c - \omega_R)/e^2$$

- This means that the maximum density is achieved when $\omega_R = \omega_c/2$ and then

$$n = \varepsilon_0 m\omega_c^2/2e^2 = \varepsilon_0 B^2/2m$$

- This is called *Brillouin flow.*



At Brillouin flow, the total relativistic energy density of the particles (nmc^2) is equal to the energy density of the magnetic field ($B^2/2\mu_0$)

WHY IS THIS?

3. Advantages and disadvantages

Penning traps offer a very unusual environment for experiments.

They allow some experiments to be performed that would be difficult or impossible in other systems.

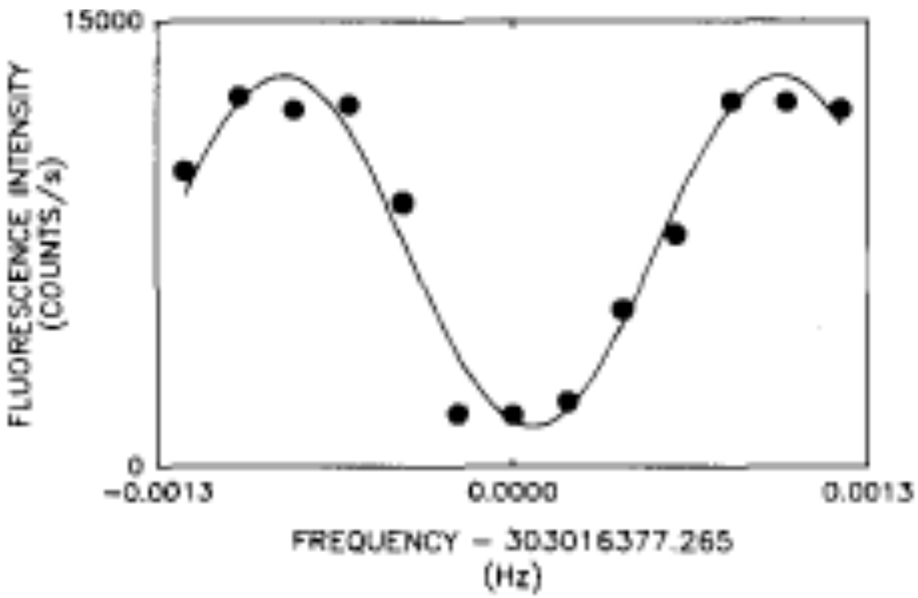
What are the particular advantages and disadvantages that they offer?

Advantages of Penning traps (1)

- **An isolated and stable environment**
  - The particles do not interact with the container walls or residual gas
    - » This means that exotic particles can be trapped
      - e.g. positrons, antiprotons, highly charged ions…
  - Only static electromagnetic fields are used
  - Free of unwanted perturbations

This RF Ramsey spectrum was taken with a 550 s interaction time giving a central fringe width of 1mHz (NIST)

It demonstrates that coherence between two HFS Zeeman states is preserved for very long times

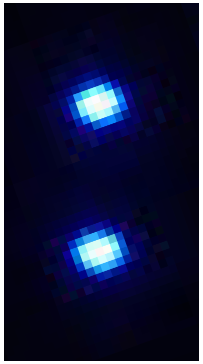


Advantages of Penning traps (2)

- **Tight localisation of the particle(s)**
  - Particles remain at the centre of the trap and cannot escape
  - Long interaction times possible
    - » Especially after laser cooling
  - Typical size of the thermal ion motion:
    - » At room temperature: 80 μm
    - » At the Doppler limit (1 mK): 0.15 μm
    - » In the ground state of the potential well: 0.02 μm

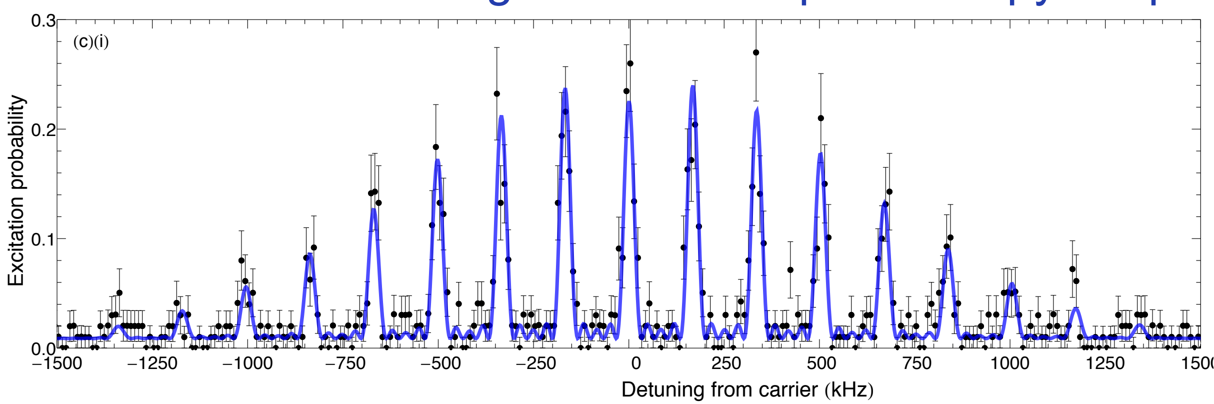
Image of two ions in a Penning trap after Doppler laser cooling.

The axial separation of the ions is about 30 μm



Advantages of Penning traps (3)

- **Magnetic field**
  - Provided usually by a superconducting magnet
    - » Very uniform and stable field
  - Useful for mass and g-factor measurements etc
- **Cooling**
  - There are lots of different cooling techniques available
  - Increases lifetime of trapped particles
  - Reduces or eliminates Doppler effect:
    - » ultra-high resolution spectroscopy in optical domain



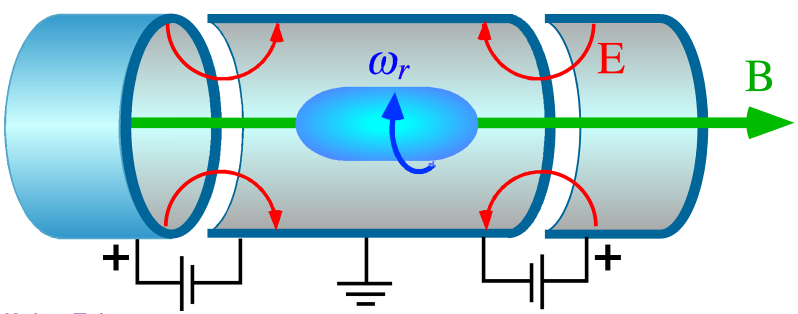
Doppler width after Doppler cooling to 1 mK is approximately 1 MHz

At room temperature it is typically 1 GHz

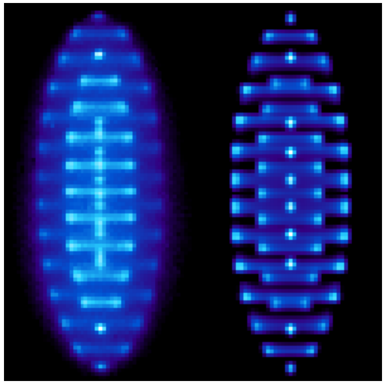
Advantages of Penning traps (4)

- **Plasma physics studies**
  - A cloud of ions in a Penning trap is known as a "single component plasma" or "non-neutral plasma"
  - The plasma has a uniform density in a quadrupole trap
  - It differs from normal plasmas in that the trap potential plays the role of the oppositely charged particles
  - This makes it a relatively simple system to study
    - » The properties (e.g. modes of oscillation) can be calculated easily
    - » In the limit of low temperatures the plasma has a sharp boundary
  - At very low temperatures, ion Coulomb crystals are formed

See lectures by Dan Dubin and others



Penning-Malmberg trap



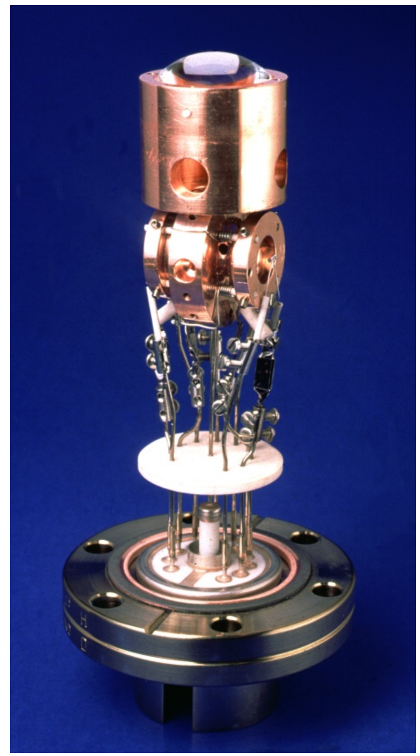
Ion Coulomb crystal

Disadvantages

- Magnetic field is expensive to provide
- Access to the ions is difficult
  - Superconducting magnet limits access severely
  - Trap electrodes make optical access difficult
  - All holes in electrodes distort the quadratic potential
- Vacuum system needs to be good for UHV ($<10^{-9}$ mbar)
- Magnetron motion is unstable
  - The energy associated with this motion is negative
  - Collisions and perturbations all tend to make the radius increase
  - Hard to cool efficiently
  - Leads to a rotation of the whole plasma

4. Practical Aspects

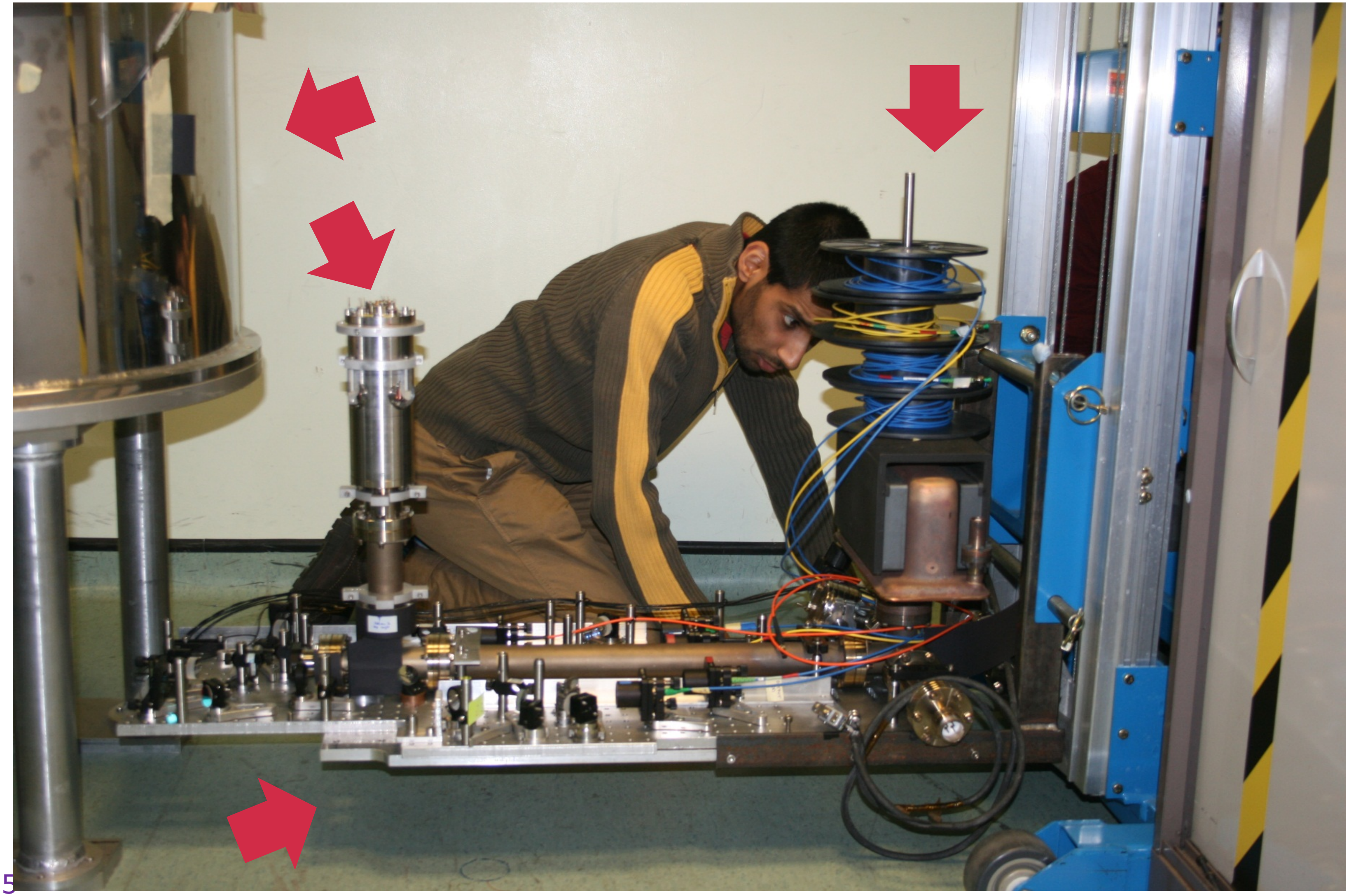
- Trap environment
- Loading ions in the trap
- Detection of ions
- Cooling of ions



Trap environment

- Magnetic field
  - We need a field of 1-10 T – generally requires a superconducting magnet
- Cryogenic environment
  - Some experiments require the whole trap to be at a few K
- Ultra-High Vacuum [see Martina Knoop's lecture]
  - Collisions will lead to heating up or loss of ions, so the trap has to be held under UHV conditions ($<10^{-9}$ mbar)
  - This also means that the trap components must be very clean to prevent degassing and contamination of the vacuum
  - Either create ions inside the trap or transport them in from outside

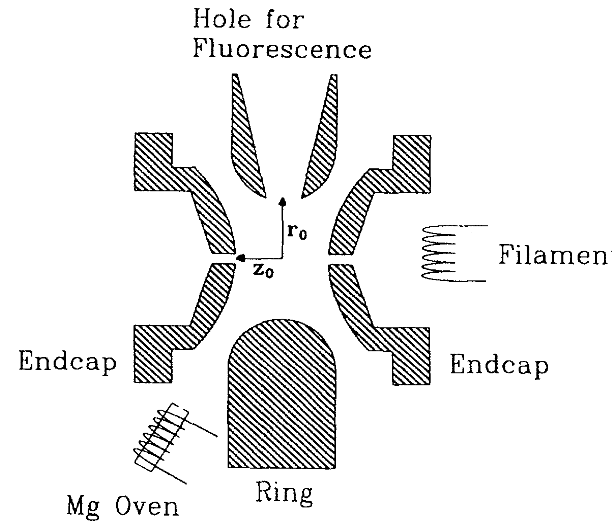
Trap practicalities

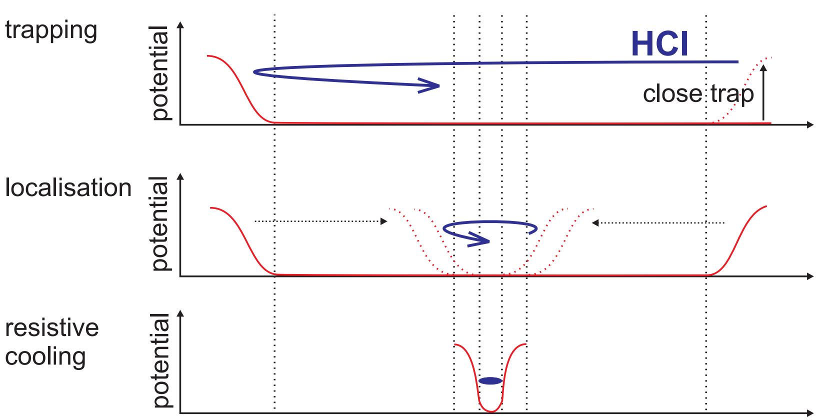


Loading of ions into the trap

Ions can be loaded into the trap in two ways:

- Create the ions inside the trap so ions are trapped instantly when created
  - Electron impact on a neutral atomic vapour
  - Ablation from a surface using a pulsed laser
  - Photoionisation of neutral atoms
- Transport the ions into the trap from elsewhere
  - Requires an external ion source
  - Trap potential on one electrode needs to be changed rapidly for efficient trapping
    - » Lower the potential to let ions in
    - » Raise it again before the ions have 'bounced' back



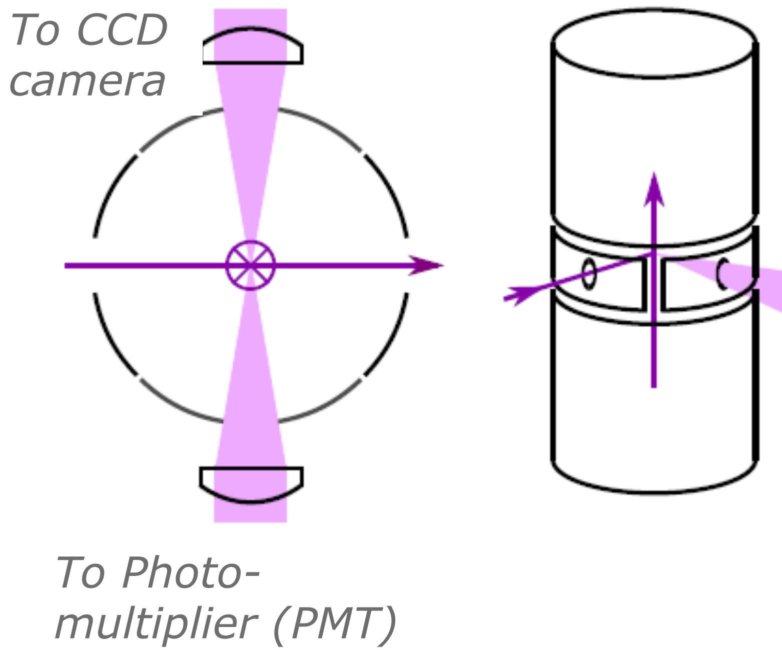


Trapping scheme for "SpecTrap"

Detection of ions

Three main ways to detect ions in a trap:

- Detect the light that they emit when excited by a laser
  - Sensitive and non-invasive
  - But needs atom with suitable transition
- Detect induced currents in external circuit
  - Sensitive at cryogenic temperatures
- Detect particles by ejecting them from the trap to a detector (e.g. MCP)
  - Destructive but works for all particles



See lectures by Joel Fajans

Cooling

Several ways to reduce the energy of trapped particles:

- Buffer gas cooling
  - Allow the ions to interact with an inert buffer gas at room temp.
  - Works for cyclotron and axial but not magnetron motion
- Radiative cooling
  - For electrons/positrons at high energy for the cyclotron motion
- Resistive cooling
  - Extracts energy from the ions when they induce currents in an external circuit (mainly axial and cyclotron)
  - can cool to a few K if apparatus is at liquid helium temperature
- Laser cooling
  - Very effective if suitable transition available
  - Can cool to ~1mK or lower
- Sympathetic cooling
  - Ions mixed with a laser-cooled species are cooled through collisions

See lectures by Caroline Champenois and Laurent Hilico

Summary of Lecture 1

- The Penning trap is an elegant device with many excellent features for experiments in physics
  - Especially precision measurements due to the extreme isolation
- It makes possible experiments that would otherwise not be possible
  - Especially those involving delicate quantum effects
- In the next lecture we will look at some of the different areas of physics where they are used.