Cooling atoms with lasers

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Trapped Charged Particles Winter school,
Les Houches january 2018

Outline

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1. The concept of the radiation pressure force

2. Beyond the concept : Motion induced effect on the atom-laser interaction

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2. Beyond the concept : Motion induced effect on the atom-laser interaction
3. The dark side of the force

the mechanical effect of light

To understand the interaction of light with atoms, one must consider atoms as system with quantised energy level.

- the lowest energy level is the ground state g and is stable (if the nucleus is stable!!)
- the higher energy levels are associated to excited states e_i, with a finite lifetime τ_{e_i}.
- when the energy difference between these levels is in the optical domain :

$$E_e - E_g = h\nu \qquad \lambda = \frac{c}{\nu} \simeq 400 \text{ nm} - 1\mu\text{m}$$

most excited states have a lifetime of the order of 10 ns

the mechanical effect of light

Absorption and emission of light by atoms :

- if the photon energy is close enough to the energy gap between levels g and e, absorption of the photon by the atom can occur.
- if the atom was initially in its ground state, it is excited to state e, where it remains for an *average* duration τ_e. To come back to the ground state, the atom needs to give back the received energy and it emits a photon,

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- these two emission processes result in different wave vector direction :
  - on random direction for spontaneous processes.
  - in the direction of the driving EM wave for stimulated processes.

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A photon carries energy **and** momentum $\hbar \mathbf{k}_L$

- there is a recoil induced by one photon absorption, $v_{rec} = \hbar k_L / m$.
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- if spontaneous emission is far most probable, the net gain over N cycles abs/em is $N \times v_{rec}$. This is the usual case for allowed optical transition.

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$$\omega_L - \mathbf{k}.\mathbf{v} = \omega_L^{at} = \omega_0 + \hbar\mathbf{k}_L^2/2m$$

- If $\omega_L < \omega_0$, the laser reaches the atomic resonance for atom moving against the laser beam and then $||\mathbf{p}'|| < ||\mathbf{p}||$.

orders of magnitude

- $\hbar^2 \mathbf{k}_L^2 / 2m = \hbar \omega_{rec}$ is the recoil energy, of the order of few 10 kHz
- for Doppler laser cooling, strong dipolar transitions are used (natural linewidth $\simeq$ 10 MHz)
- The recoil energy is often not taken into account in Doppler laser cooling and its condition reduces to

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- To optimise the efficiency and control the limit temperature, one must go back to the fundamental of atom-laser interaction.

Looking inside the radiation pressure force

- the force on a atom is $\Delta\mathbf{p}/\Delta t$:
  $\hbar\mathbf{k}_L \times$(number of photons scattered spontaneously per unit time)
- number of photons scattered per unit time$=\Gamma \times P_e=$
  (number of photons scattered per unit time when in the excited state $\Gamma = 1/\tau_e$)$\times$(probability to be in the excited state P_e)
- one needs to know how to control P_e :

$$\mathbf{F} = \hbar\mathbf{k}_L \times \Gamma \times P_e$$

the two-level atom in a laser electric field

- $|g\rangle, |e\rangle$ are the two involved atomic states, with energy defined by $E_e - E_g = \hbar\omega_0$ and $\omega_L \simeq \omega_0$.
- without any laser field, the hamiltonian of an atom at rest is

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- within the **dipolar approximation** ($\lambda_L \gg r_{at}$), the laser electric field couples to the transition dipole $d_{e.g.}$ by

$$V_{AL}(\mathbf{r}, t) = -\mathbf{d}_{eg}.\mathbf{E}_L(\mathbf{r}, t)$$

where $\mathbf{r}$ is the position of the atom center of mass.

- the transition is driven by the dipole operator polarised along the local electric field polarisation ϵ_L

$$\mathbf{d}_{eg} = d_{eg}\epsilon_L\left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

the two-level atom in a laser electric field

- in a single active electron atom

$$\mathbf{d}_{eg} = q_e \langle e|\mathbf{r}_e|g\rangle$$

- if the laser wave is a plane wave

$$\mathbf{E}_L(\mathbf{r}, t) = E_L \epsilon_L \cos(\omega t - \Phi(\mathbf{r}))$$

- the Rabi frequency scales the interaction strength

$$\hbar\Omega_1(\mathbf{r}) = -\mathbf{d}_{eg}.\epsilon_L E_L(\mathbf{r})$$

the time evolution

- the evolution of the internal degrees of freedom is known through the density matrix of the internal state ρ. It obeys the master equation

$$\frac{\partial}{\partial t}\rho = -\frac{\mathrm{i}}{\hbar}[H,\rho] + \mathcal{L}\rho \tag{3}$$

- the hamiltonian is $H = H_0 + V_{AL}$ and $\mathcal{L}\rho$ rules the non hamiltonian evolution induced by relaxation (spontaneous emission, collisions,...). $\mathcal{L}$ is a Lindbald operator.

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- the projection of the master equation on the basis $(|g\rangle, |e\rangle)$ forms the optical Bloch equations (OBE).
- the relative motion of the atom in the wave $\mathbf{r}(t)$ induces a modulation of the laser-atom interaction in

$$V_{AL}(\mathbf{r},t) = \hbar\Omega_1(\mathbf{r}(t))\left(|e\rangle\langle g| + |g\rangle\langle e|\right)\cos(\omega t - \Phi(\mathbf{r}(t)))$$

correct Eq.7 page 120

let's assume an atom at rest

- then $\mathbf{p} = \mathbf{0}$, $\Phi(\mathbf{r}) = \Phi_0$ and $\Omega_1(\mathbf{r}) = \Omega_1$.
- the OBE are

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i(\rho_{eg} - \rho_{ge})\Omega_1 \cos(\omega_L t - \Phi_0) - \gamma_p \rho_{ee} \quad (4)$$

$$\frac{\mathrm{d}\rho_{eg}}{\mathrm{d}t} = -i\omega_0 \rho_{eg} + i(\rho_{ee} - \rho_{gg})\Omega_1 \cos(\omega_L t - \Phi_0) - \gamma_d \rho_{eg} \quad (5)$$

  considering that $\rho_{gg} + \rho_{ee} = 1$ and that $\rho_{ge} = \rho_{eg}^*$, you know everything!

- if the only source of decoherence is spontaneous emission

$$\gamma_p = \Gamma \quad \text{and} \quad \gamma_d = \Gamma/2 \quad \text{with} \quad \Gamma = 1/\tau_e$$

correct Eq.8b and 11.b page 121

let's assume an atom at rest

- we focus on the envelop evolution and get ride of the rapid evolution

$$\rho_{eg} = \tilde{\rho}_{eg} \exp(-i(\omega_L t - \Phi_0))$$

- furthermore,we use the decomposition

$$\cos(\omega_L t - \Phi_0) = \frac{\exp(i(\omega_L t - \Phi_0)) + \exp(-i(\omega_L t - \Phi_0))}{2}$$

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- two kinds of terms driving the time evolution :

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i\Omega_1(\tilde{\rho}_{eg} e^{-i\omega_L t} - \tilde{\rho}_{ge} e^{i\omega_L t}) \frac{e^{i(\omega_L t - \Phi_0)} + e^{-i(\omega_L t - \Phi_0)}}{2} - \gamma_p \rho_{ee}$$

we neglect the fast oscillating terms $e^{\pm i2\omega_L t}$ and keep the slow envelop effect : the "secular approximation" also called the rotating wave approximation (RWA)

in the rotating wave approximation (RWA)

- in the RWA :

$$\frac{\mathrm{d}\rho_{ee}}{\mathrm{d}t} = i(\tilde{\rho}_{eg} - \tilde{\rho}_{ge})\Omega_1/2 - \gamma_p\rho_{ee} \quad (6)$$

$$\frac{\mathrm{d}\tilde{\rho}_{eg}}{\mathrm{d}t} = -i(\omega_0 - \omega_L)\tilde{\rho}_{eg} + i(\rho_{ee} - \rho_{gg})\Omega_1/2 - \gamma_d\tilde{\rho}_{eg} \quad (7)$$

- three relevant components: $Re(\tilde{\rho}_{ge})$, $Im(\tilde{\rho}_{ge})$, $(\rho_{ee} - \rho_{gg})$

$$\begin{aligned} u(t) &= Re(\tilde{\rho}_{ge}) \\ v(t) &= Im(\tilde{\rho}_{ge}) \\ w(t) &= (\rho_{ee} - \rho_{gg})/2 \\ &= \rho_{ee} - 1/2 \end{aligned}$$

$$\begin{aligned} \dot{u}(t) &= \Delta_L v(t) - \gamma_d u(t) \\ \dot{v}(t) &= -\Delta_L u(t) - \Omega_1 w(t) - \gamma_d v(t) \\ \dot{w}(t) &= \Omega_1 v(t) - \gamma_p(w(t) + 1/2) \end{aligned}$$

$$\Delta_L = \omega_L - \omega_0$$

The stationary solutions when $\gamma_p = \Gamma$ and $\gamma_d = \Gamma/2$

- they depend on a parameter s called the **saturation parameter** :

$$s = \frac{\Omega_1^2/2}{\Delta_L^2 + \Gamma^2/4}$$

- on resonance

$$s = s_0 = 2\Omega_1^2/\Gamma^2 = I/I_s$$

I_s is characteristic of the transition strength and is of the order of few $\mathrm{mW/cm^2}$ for dipolar transition in alkali (sodium, rubidium, cesium...) and alkali-earth ion (Be^+, Ca^+, Sr^+...)

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- assuming $\dot{u}(t) = \dot{v}(t) = \dot{w}(t) = 0$

$$u_{st} = \frac{\Delta_L}{2\Omega_1}\frac{s}{1+s} \qquad v_{st} = \frac{\Gamma}{2\Omega_1}\frac{s}{1+s} \qquad \rho_{ee} = w_{st} + 1/2 = \frac{1}{2}\frac{s}{1+s}$$

Come back to the radiation pressure on **an atom at rest**

- the force behaves like $P_e = \rho_{ee}$:

$$\mathbf{F} = \Gamma \times \rho_{ee} \times \hbar\mathbf{k} \qquad \rho_{ee} = \frac{1}{2}\frac{s}{1+s} = \frac{1}{2}\frac{\Omega_1^2/2}{\Omega_1^2/2 + \Delta_L^2 + \Gamma^2/4}$$

- is maximum when $\Delta = 0$ **BUT** remember the atom is not moving...
- the spectral width (FWHM) of the force is

$$\Delta\omega = \Gamma\sqrt{1 + \frac{2\Omega_1^2}{\Gamma^2}} = \Gamma\sqrt{1+s_0}$$

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- the internal time evolution is ruled by Ω_1, $\Gamma = 1/\tau_e$ and $\Delta_L = \omega_L - \omega_0$.
- one can show that whatever is the laser intensity, it takes $\tau_e = 1/\Gamma$ for the internal dynamics to reach its stationary state.
- the time T_{ext} it takes to drive the atom out of the resonance line is such that $k_L \Delta v(T_{ext}) = \Gamma/2$
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- the internal dynamics follows *instantaneously* the external dynamics if

$$T_{ext} \ll T_{int} \quad \text{which requires } \Gamma \gg 2\omega_{rec} = \frac{\hbar k_L^2}{m}$$

$\Rightarrow$ **the broad line condition**

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- if $k_L v \ll \Gamma$, the linear expansion of F for small v gives

$$\mathbf{F} = -m\alpha\mathbf{v} \quad \text{with} \quad \alpha = \omega_{rec} s \frac{-4\Gamma\Delta_L}{\Delta_L^2 + \Gamma^2/4} \qquad s = \frac{\Omega_1^2/2}{\Delta_L^2 + \Gamma^2/4}$$

Doppler cooling in the broad line limit

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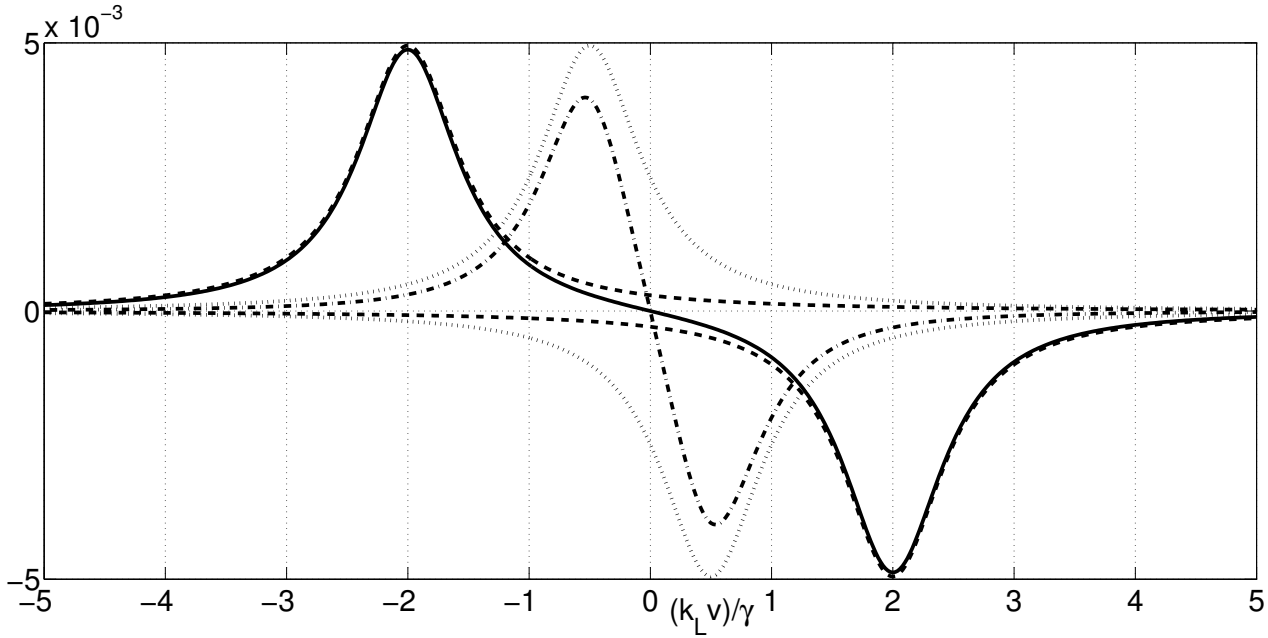
1D-Force in units of $(\hbar k_L \Gamma)$ vs velocity in units of $(k_L.v/\Gamma)$ for $s_0 = 0.01$



$\Delta_L = -2\Gamma$ (dashed and solid lines -for the resulting force)
$\Delta_L = -\Gamma/2$ (dotted and dash-dotted lines - for the resulting force).

Doppler cooling in the broad line limit

- the typical timescale for velocity damping is $1/\alpha$:

$$\tau_{DC} = \frac{1}{\omega_{rec} s_r} \frac{\Delta_L^2 + \Gamma^2/4}{4\Gamma|\Delta_L|}$$

- if Δ_L is too close to the resonance, this timescale tends to infinity.
- for $\Delta_L = -\Gamma/2$, $\tau_{DC} = 1/(4\omega_{rec} s_0)$
- for a typically low saturation parameter $s_0 = 1/10$, $\tau_{DC} = 2.5/\omega_{rec}$. For heavy alkalies, τ_{DC} is smaller than 1 ms (can be lower than 100 μs)
- in 2D, you can reduce the divergence of an atomic beam with laser perpendicular to the atomic beam.
- in 3D, you can cool atom in "optical molasses" with a velocity capture range v_c of the order of Γ/k_L

How cold can the atoms get?

- the norm $||\mathbf{v}^2||$ reaches a limit because of the fluctuation of the force, induced by spontaneous emission and absorption.
- it looks like brownian motion : the fluctuation of the force around its mean value is responsible for an increase of the momentum variance, linear in time.
- from spontaneous emission, the diffusion scales like

$$\Delta\mathbf{p}^2/\Delta t = (\hbar k_L)^2 \rho_{ee} \Gamma$$

- from the atom-laser interaction (for 1D, two beams)

$$\begin{aligned} \Delta\mathbf{p}^2/\Delta t &= (\hbar k_L)^2 \Delta^2 (N_+ - N_-)/\Delta t \\ &= (\hbar k_L)^2 < N_+ + N_- > /\Delta t = (\hbar k_L)^2 s_0 \Gamma \end{aligned}$$

- in 3D, $\Delta\mathbf{p}^2/\Delta t = 6(\hbar k_L)^2 s_0 \Gamma$ with s_0 defined for one beam.

How cold can the atoms get?

- by summing cooling and diffusion : $\Delta \mathbf{p}^2/\Delta t = 6(\hbar k_L)^2 s_0 \Gamma - 2\alpha \mathbf{p}^2$
- at equilibrium :

$$\left(\frac{\mathbf{p}^2}{2m}\right)_{eq} = \frac{3}{2} k_B T_{eq} = \frac{6(\hbar k_L)^2 s_0 \Gamma}{2m\alpha}$$

$$k_B T_{eq} = \frac{\hbar\Gamma}{4}\left(\frac{2|\Delta_L|}{\Gamma} + \frac{\Gamma}{2|\Delta_L|}\right)$$

- the smaller temperature is reached for $\Delta = -\Gamma/2$ and

$$k_B T_{min} = \hbar\Gamma/2$$

How cold can the atoms get?

$$k_B T_{min} = \hbar\Gamma/2$$

- ex: Rb ($\tau_e = 27$ ns), $T_{min} = 140\ \mu$K , for Na, $T_{min} = 240\ \mu$K
- the reached mean squared velocity v_D depends on the atomic mass :

$$v_D = \sqrt{\frac{\hbar\Gamma}{m}} = \sqrt{\frac{\hbar k_L}{m}\frac{\Gamma}{k_L}} = \sqrt{v_{rec} v_c}$$

  for sodium $v_{rec} = 0.03$ m/s and $v_c = 6$ m/s (capture range)
- all this makes sens if $v_D \ll v_c$, which implies $\hbar k_L^2/m = 2\omega_{rec} \ll \Gamma$: **the broad line condition!**

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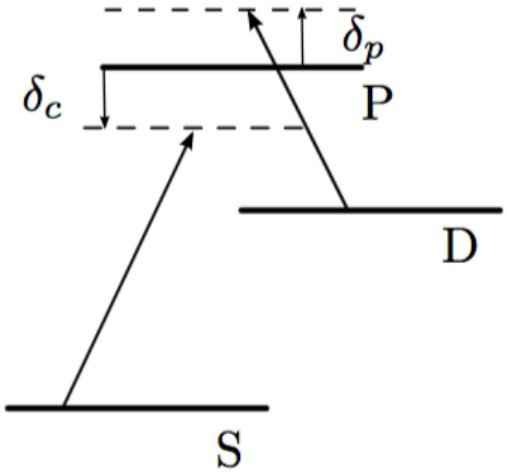
- sometimes two transitions are implied in 3 level systems like the heavy alkaline-earth ions (Ca^+, Sr^+, Ba^+)



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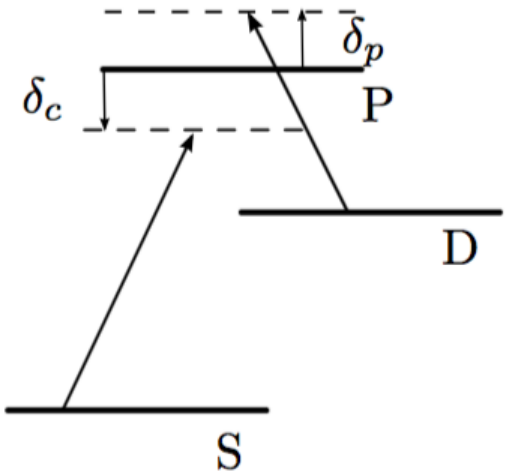
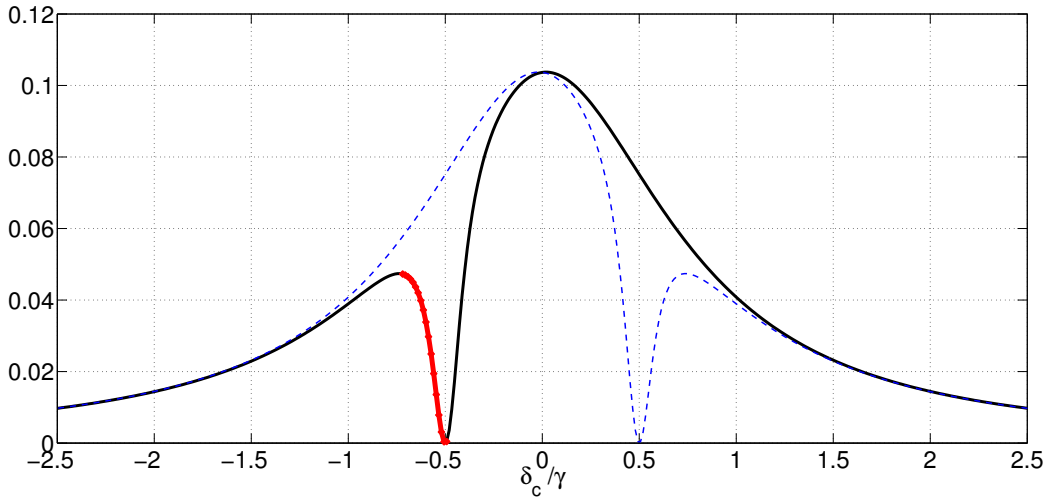
- an atom is rarely a two level system.
  It is OK for $J = 0 \rightarrow J = 1$ transition like in Ca,
  It is OK for alkali in 1D without polarisation mixing that gives ride to polarisation gradient.
- sometimes two transitions are implied in 3 level systems like the heavy alkaline-earth ions (Ca^+, Sr^+, Ba^+)





There is another force...Luke!

- going back to the Heisenberg picture where **r** and **p** are operator with a time evolution, this evolution is ruled by

$$\begin{aligned} \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} &= \frac{\mathbf{p}}{m} \\ \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} &= \mathbf{F} = -\langle \nabla V_{AL}(t) \rangle - \langle \nabla V_{AV} \rangle \end{aligned}$$

- the average force induced by coupling with the vacuum EM V_{AV} is null and, assuming the atomic wave packet is small compared to λ_L

$$\mathbf{F} = \langle \mathbf{d}_{eg}.\epsilon_L \rangle \nabla E_L(\mathbf{r}(t), t)$$

There is another force...Luke!

$$\mathbf{F} = \langle \mathbf{d}_{eg}.\epsilon_L \rangle \nabla E_L(\mathbf{r}(t), t)$$

- we now know the dipole through the time evolution of the internal degree of freedom

$$\begin{aligned} \langle \mathbf{d}_{eg}.\epsilon_L \rangle &= \mathbf{d}_{eg}.\epsilon_L(\rho_{eg} + \rho_{ge}) \\ &= 2\mathbf{d}_{eg}.\epsilon_L(u_{st}\cos(\omega_L t - \Phi(\mathbf{r})) - v_{st}\sin(\omega_L t - \Phi(\mathbf{r})) \end{aligned}$$

- u and v gives the dipole in phase and in quadrature (out of phase by $\pi/2$) with the electric field.
- the force is then

$$\mathbf{F} = -2\hbar\{u_{st}\cos(\omega_L ...) - v_{st}\sin(\omega_L ...)\}\nabla(\Omega_1(\mathbf{r})\cos(\omega_L ...))$$

There is another force...Luke!

$$\mathbf{F} = -2\hbar(u_{st}\cos(\omega_L t - \Phi(\mathbf{r})) - v_{st}\sin(\omega_L t - \Phi(\mathbf{r}))\nabla(\Omega_1(\mathbf{r})\cos(\omega_L t - \Phi(\mathbf{r}))$$

- after averaging over a time period of the electric field, two contributions to the force remain :

$$\mathbf{F} = -\hbar\Omega_1(\mathbf{r})\left(u_{st}\frac{\nabla\Omega_1(\mathbf{r})}{\Omega_1(\mathbf{r})} - v_{st}\nabla\Phi(\mathbf{r})\right)$$

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- in a plane travelling wave $\Phi(\mathbf{r}) = \mathbf{k}_L.\mathbf{r}$ and $\nabla\Omega_1(\mathbf{r}) = 0$:

$$\mathbf{F}_{RP} = \hbar\mathbf{k}_L\Omega_1 v_{st} = \hbar\mathbf{k}_L\Gamma\rho_{ee} = \hbar\mathbf{k}_L\frac{\Gamma}{2}\frac{s(\mathbf{r})}{1+s(\mathbf{r})}$$

we have here the radiation pressure force deduced we know already

The dipolar force

$$\mathbf{F}_{dip} = -\hbar\Delta_L \frac{s(\mathbf{r})}{1+s(\mathbf{r})} \frac{\nabla\Omega_1(\mathbf{r})}{\Omega_1(\mathbf{r})} = -\frac{\hbar\Delta_L}{2} \frac{\nabla s(\mathbf{r})}{1+s(\mathbf{r})}$$

- this force is conservative (no dissipation) : $\mathbf{F}_{dip} = -\nabla U_{dip}(\mathbf{r})$

$$U_{dip}(\mathbf{r}) = \frac{\hbar\Delta_L}{2} \log(1+s(\mathbf{r}))$$

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- depending on the sign of Δ_L, atoms are attracted or repealed by higher intensity.
- in practice, it is used with large detuning (to reduce radiation pressure), then for $\Delta_L \gg \Omega_1$

$$U_{dip}(\mathbf{r}) = \frac{\hbar\Omega_1^2(\mathbf{r})}{4\Delta_L}$$

- order of magnitude : few mK