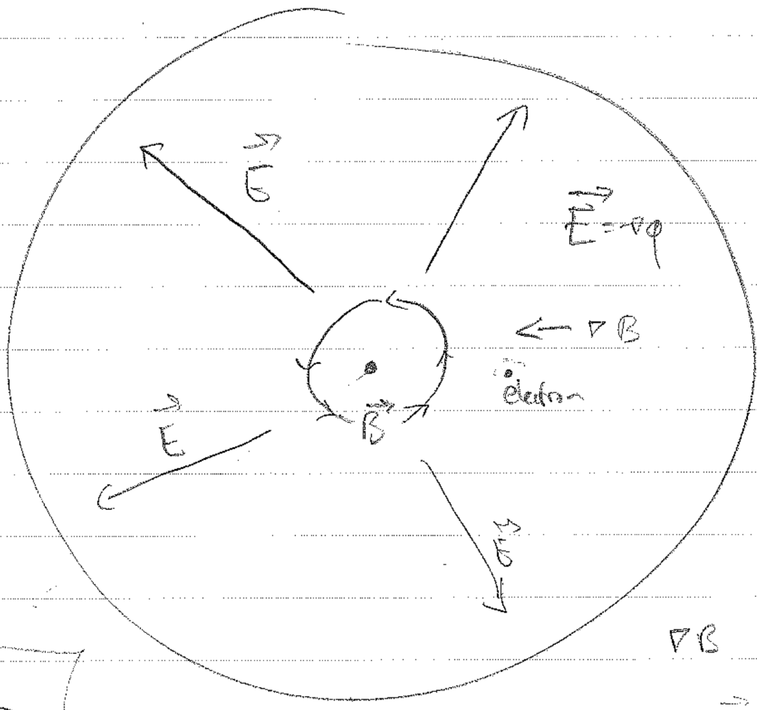
Solution to Problem 2

1) a)



$+\hat{\theta}$ direction

$\boxed{E = E_r \hat{r}}$

I ⊙

$I = I\hat{z}$
out of paper

$V_{\vec{E}} = \left|\frac{E}{B}\right| \hat{r} \times \hat{\theta} = \left|\frac{E}{B}\right| \hat{z}$ ⊙
out of paper

$V_{\nabla B}$
V_{RC} ⊗

∇B is in negative $\hat{r}$ dir.

$$\vec{v}_{\nabla B} = \tfrac{1}{2} m v_\perp^2 \frac{\vec{B} \times \nabla\vec{B}}{q\, B^3}$$

so direction is $\dfrac{\hat{\theta} \times (-\hat{r})}{-1} = -\hat{z}$

from neg. charge

$\vec{V}_{Rc}$ is in same direction as $v_{\nabla B}$ (vacuum)

b) We need to know $\vec{B}$, $\vec{E}$, ∇B, $\frac{1}{2}mv_\perp^2$, $\frac{1}{2}mv_\parallel^2$

$\vec{B}$ is easy: $2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r} =$

$$\frac{4\pi \cdot 10^{-7} \cdot 5 \cdot 10^{2}}{2\pi \cdot 10^{-2}} = \underline{10^{-2} \text{ Tesla}}$$

$\vec{E}$:

$$\varepsilon_0 \nabla \cdot E = \rho \Leftrightarrow \varepsilon_0 \int E \cdot dA = Q \Leftrightarrow$$

$$E\, \varepsilon_0 \cdot 2\pi r = \frac{Q}{\ell} \Leftrightarrow -\nabla\varphi = E = E(r) = \frac{Q}{\ell} \cdot \frac{1}{2\pi\varepsilon_0 r}\,\hat{r}$$

Integrate : $\varphi(b) - \varphi(r) = \frac{Q}{\ell}\frac{1}{2\pi\varepsilon_0} \cdot \int_r^b \frac{1}{r}\,dr$

so $\varphi(b) - \varphi(r) = \text{constant} \cdot \ln\left(\frac{b}{r}\right)$

$\varphi(b) = 0$ is given in the problem. So

$$-\varphi(a) = -\varphi_a = -460\,V = \text{const} \cdot \ln\left(\frac{b}{a}\right) \text{ so}$$

$$\text{const} = -460\,V / \ln\left(\frac{b}{a}\right)$$

$$\text{So } \varphi(r) = \frac{-460\,V}{\ln(b/a)} \cdot \left(-\ln\frac{b}{r}\right) = \frac{-460\,V}{\ln b/a} \cdot \ln\frac{r}{b}$$

$$= \frac{-460\,V}{\ln 100} \ln\left(\frac{r}{b}\right) = -100\,V \ln\left(\frac{r}{b}\right)$$

$$\text{so } E(r) = -\nabla\varphi(r) = +100\,V \cdot \frac{1}{r} \,;\; E(0.01\,m) = 10^4 \frac{V}{m}$$

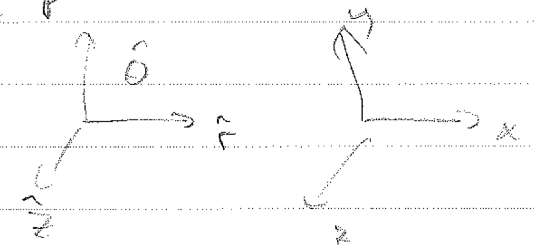
$$\text{So } v_E = \frac{E}{B} = \frac{10^4}{10^{-2}} = 10^6\ m/s$$

Now for $v_{\nabla B}$ and v_{Rc} :

We can calculate $v_\perp$ and $v_\parallel$ approximating B as straight and uniform - ok as long as

$$B/\nabla B \gg r_L \quad \text{and} \quad R_c \gg r_L$$

Let the local coordinates (x, y, z) point in the r, θ, z directions respectively, at the particle



The initial condition is $\vec{v} = 0$. Since there is no force in $\hat{\theta}$, $v_\parallel = v_\theta = 0$ for all times. In the perp. directions, particle will do gyration plus $E \times B$ drift. In order for the particle to have $\vec{v}(t=0)$, $v_\perp$ must equal v_E.
(Can also be derived more rigorously from the initial value problem)

$$\text{So } v_\perp = v_E = 10^6 \text{ m/s}$$

$$|v_{\nabla B}| = \frac{1}{2} m v_\perp^2 \frac{1}{eBr} = \frac{1}{2} \cdot 9.1 \cdot 10^{-31} \cdot 10^{12} \frac{1}{1.6 \cdot 10^{-19} \cdot 10^{-2} \cdot 10^{-2}}$$

$$= \underline{\underline{2.8 \cdot 10^4 \ \frac{m}{s}}}$$

$$|v_{R_c}| = \underline{\underline{0}} \quad \text{since } v_\parallel = 0$$