Solutions problem 3

Guiding center approximation is valid since $r_L \ll L$ and there is no time variation

$$F_{\parallel} = qE_{\parallel} - \mu \nabla_{\parallel} B$$

For a particle with $\mu = \frac{\frac{1}{2}mv_0^2}{B_0}$, the equilibrium point ($F_{\parallel}=0$) can be calculated as follows.

$$\nabla_{\parallel} B = \frac{2z}{z_0^2} B_0 \quad , \text{ so at } F_{\parallel}=0 \text{ we have}$$

$$qE_0 = \frac{\frac{1}{2}mv_0^2}{B_0} \frac{2z}{z_0^2} B_0$$

$$\Updownarrow$$

$$z = \frac{qE_0}{mv_0^2} z_0^2 \; ; \text{ for an electron, } q=-e, \, m=m_e$$

$$z = \frac{-eE_0}{mv_0^2} z_0^2 = z_{eq}$$

Let $\tilde{z} = z - z_{eq}$. Then $m_e \frac{dv_{\parallel}}{dt} = -eE_0 - \frac{1}{2}m_e v_0^2 \frac{2z}{z_0^2}$

can be rewritten in this form:

$$m_e \frac{d^2\tilde{z}}{dt^2} = -\frac{m_e v_0^2}{z_0^2}\tilde{z} \Leftrightarrow \frac{d^2}{dt^2}\tilde{z} = -\frac{v_0^2}{z_0^2}\tilde{z}$$

which is a harmonic oscillation with $\underline{\underline{\omega = \frac{v_0}{z_0} = \sqrt{\frac{2\mu B_0}{m_e z_0^2}}}}$

(Answers a))

b) Particle is at rest at $z=0$ at $t=0$ so the oscillation amplitude is $|0 - z_{eq}| = |z_{eq}| = \left|\frac{eE_0 z_0^2}{m_e v_0^2}\right|$

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Now to part c)

The equilibrium point that was calculated earlier is not the answer to this question. μ is different in this case; $\mu = \frac{1}{2}mv_0^2 / B$ (new equilib pt)

$$eE_0 = -\frac{\frac{1}{2}mv_0^2}{B(z)} \nabla_{\parallel} B(z) \quad (F_{\parallel} = 0)$$

$$\Updownarrow$$

$$eE_0 = -\frac{\frac{1}{2}mv_0^2}{B_0(1+z^2/z_0^2)} \cdot \frac{2z}{z_0^2} B_0$$

$$(z_0^2 + z^2)\, eE_0 = -mv_0^2 \cdot z$$

$$z^2 + \frac{mv_0^2}{eE_0} z + z_0^2 = 0$$

Let $z_E = \frac{mv_0^2}{eE_0}$

$$z^2 + z_E z + z_0^2 = 0$$

$$z = \left(-z_e \pm \sqrt{z_E^2 - 4z_0^2}\right)$$

If $z_E^2 > 4z_0^2$ there are two solutions

If $z_E^2 < 4z_0^2$ there are no solutions

If $|z_e| = |2z_0|$ there is one solution:

$$\underline{\underline{z = -z_E}} \quad (= -2z_0)$$