Problem 5

In principle there are two terms giving a force in the θ direction:

$$F_\theta = q v_r B_z (\hat{r} \times \hat{z}) + q v_z B_r (\hat{z} \times \hat{r})$$

v_r is the rate of change in Larmor radius - very small by comparison

$$F_\theta \approx q v_z B_r \hat{\theta} = q v_z \left(-\frac{1}{2} r_L \frac{\partial B_z}{\partial z}\right)$$

Leading to $\frac{\partial v_\perp}{\partial t} = \left|\frac{\partial v_\theta}{\partial t}\right| = +\frac{q}{m} v_z \frac{1}{2} r_L \frac{\partial B_z}{\partial z}$ F_θ

This is consistent with $\frac{d\mu}{dt} = 0$

$$\frac{d}{dt}(\mu B) = \frac{d}{dt}\left(\frac{1}{2} m v_\perp^2\right)$$

$$\frac{d\mu}{dt} B + \underbrace{v_\parallel \frac{\partial B}{\partial z}}_{\frac{dB}{dt}} \mu = m v_\perp \frac{dv_\perp}{dt} = m v_\perp \frac{q}{m} v_\parallel \frac{1}{2} \underbrace{r_L}_{\frac{mv_\perp}{qB}} \frac{\partial B_z}{\partial z}$$

$$\Updownarrow$$

$$\frac{d\mu}{dt} B + v_\parallel \frac{\partial B}{\partial z} \mu = v_\parallel \frac{\partial B}{\partial z} \underbrace{\frac{\frac{1}{2} m v_\perp^2}{B}}_{\mu}$$

$$\frac{d\mu}{dt} B = 0 \Leftrightarrow \frac{d\mu}{dt} = 0$$

QED