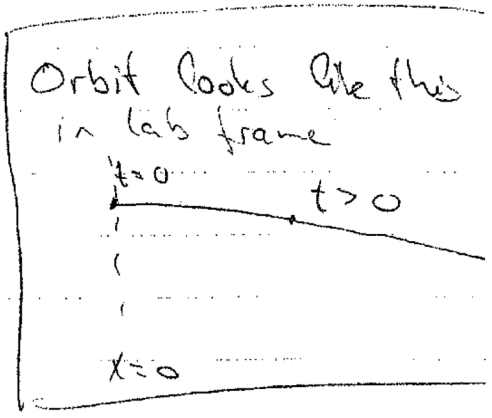
Problem 1

a) $\frac{d}{dt}(\vec{p}) = q(\vec{E} + \vec{v} \times \vec{B})$

x-component of this equation is:

$$\frac{d}{dt}(\gamma m_0 v_x) = q(E + v_y B)$$



If the particle is to come back to $x=0$, its x-velocity must begin to decrease, then reverse. For this to occur, it must feel a force qv_yB which exceeds qE in magnitude:

$$|qv_yB| > |qE| \Leftrightarrow |v_y||B| > |E| \Leftrightarrow |v_y| > \left|\frac{E}{B}\right|$$

But in our case, $\left|\frac{E}{B}\right| > c$ so this cannot happe Particle will continue to accelerate away from $x=c$ It never returns and there is no gyration.

b) Consistent with our observation, we guess that thee is a frame of reference in which $B'=0$ but $E' \neq 0$. By symmetry, we guess

$$\vec{v}_{frame} = \frac{\vec{E} \times \vec{B}}{E^2} c^2 \quad ; \text{note } |v_{frame}| = \frac{B}{E} c^2 < c \text{ so C}$$

$$\vec{B}' = \gamma\left(\vec{B} - \frac{\vec{v}}{c^2} \times \vec{E}\right) - \frac{\gamma^2}{\gamma+1} \frac{\vec{v}}{c}\left(\frac{\vec{v}}{c} \cdot \vec{B}\right)^{0} = \gamma(\vec{B} - \vec{B}) = 0 \quad \text{(It work}$$

$$\vec{E}' = \gamma\left(\vec{E} + \frac{(\vec{E} \times \vec{B}) \times \vec{B}}{E^2} c^2\right) - \frac{\gamma^2}{\gamma+1} \frac{\vec{v}}{c}\left(\frac{\vec{v}}{c} \cdot \vec{E}\right)^{0} = \gamma\left(1 + \frac{v^2}{c^2}\right)\vec{E} = \gamma^{-1}\vec{E}$$