Rotating Wall Technique

Part 1

Basic Physics Idea

Part 2

Result from various experiments

Part 3

Centrifugal separation

François Anderegg

UCSD

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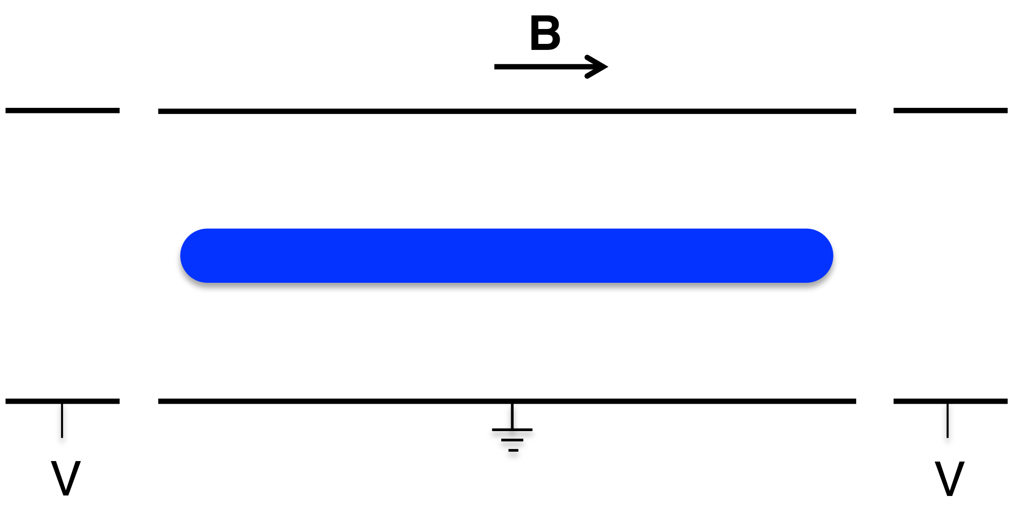
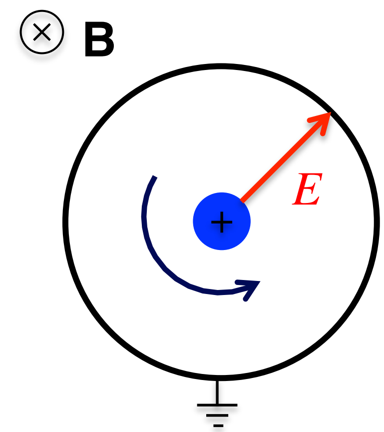




E x B rotates at

$$f_E(r) = \frac{E_r\, c}{B\, r\, 2\pi}$$

Electric field satisfies Gauss Law $\vec{\nabla} \cdot \vec{E} = 4\pi n q$

r direction cyl. coord. $$\frac{1}{r}\frac{\partial}{\partial r}(rE_r) = 4\pi n q$$

$$rE_r = 4\pi q \int n(r) r'\, dr'$$

"Top Hat" density $$E_r = \frac{4\pi q}{r}\, \frac{1}{2} r^2 n = 2\pi\, r\, n\, q$$

$$f_E = \frac{c\, n\, q}{B}$$

Constant density
=> **rigid rotor**

Angular momentum is conserved

$$P_\theta = \sum_j m v_{\theta j}\, r_j + \frac{q}{c} A_\theta r_j \qquad A_\theta = \frac{Br}{2}$$

Mechanical *Vector potential part*

$$P_\theta \cong \sum_j \frac{qB\, r_j^2}{2c}$$

For large magnetic field

$$\frac{dP_\theta}{dt} = 0 \quad \Longrightarrow \quad \sum_j r_j^2 = Const$$

Non-neutral plasmas are confined

Here theory assumed "perfect azimuthal symmetry"

Real world

Drags on spinning plasma

$P_\theta \neq Const$

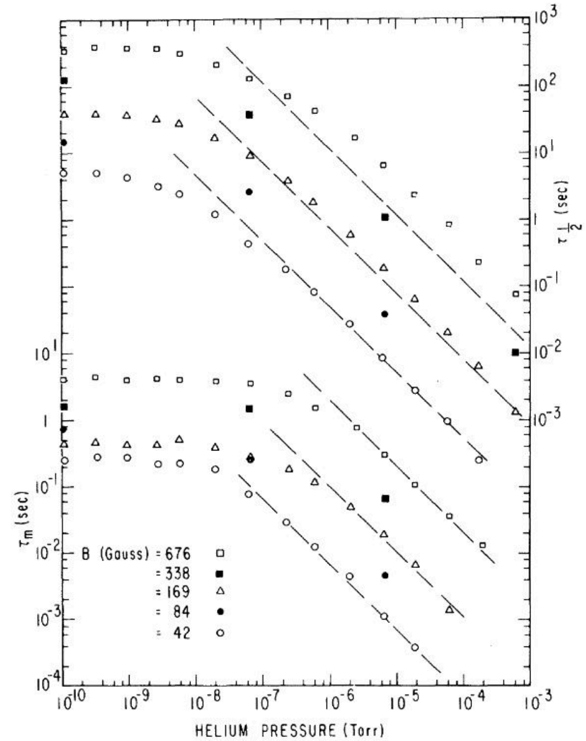
- **Residual gas**

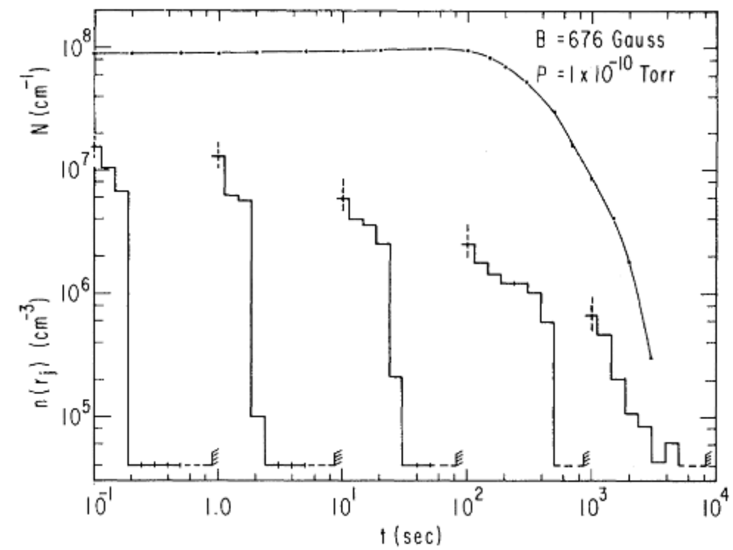
  10^{-10} Torr ~ 3×10^{6} atom / cm^{-3}

- **B field asymmetries electrodes imperfections**

Drags reduce P_θ

$$v_\theta \searrow \Rightarrow f_E \searrow \Rightarrow n \searrow \Rightarrow r_j \nearrow$$





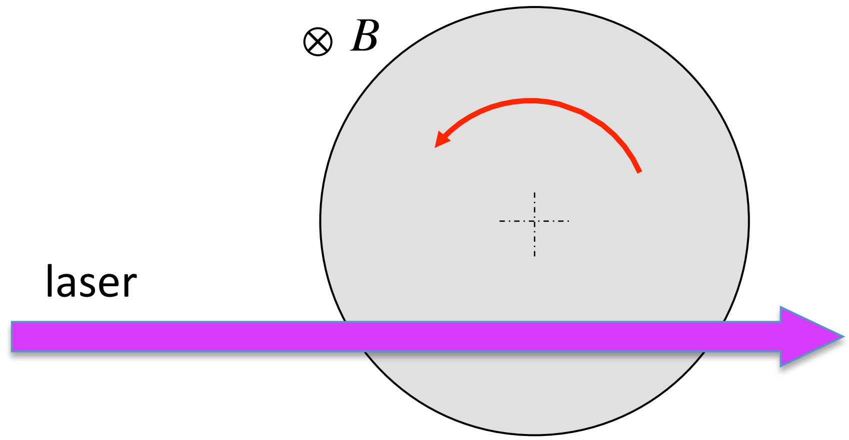
Phys. Rev. Lett. 44, 654 (1980)

Methods to add (remove) angular momentum

1) Sideband technique "*axialisation*"
Single particle technique *(not for plasma)*
Quadrupole drive at $\omega_z + \omega_m$ and damping at ω_z

Friday presentation: Richard Thompson

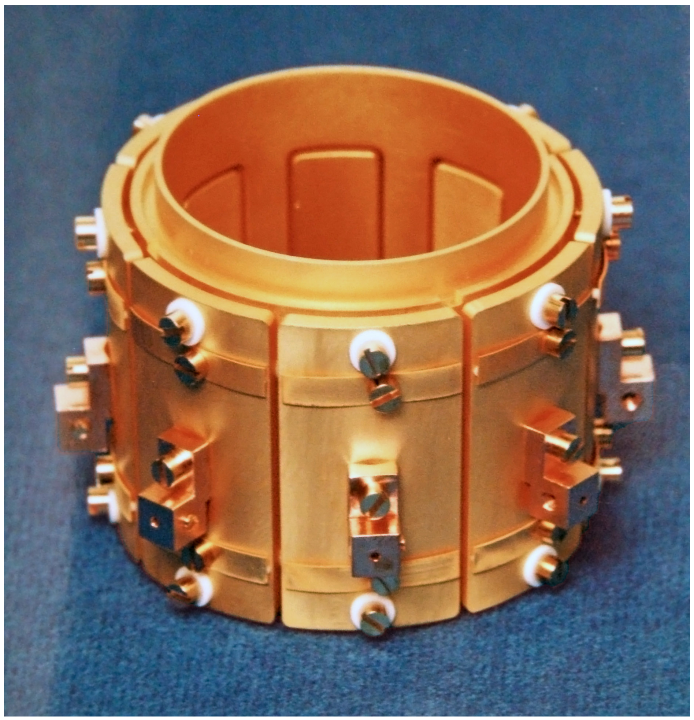
2) Radiation pressure from laser
Torque on plasma



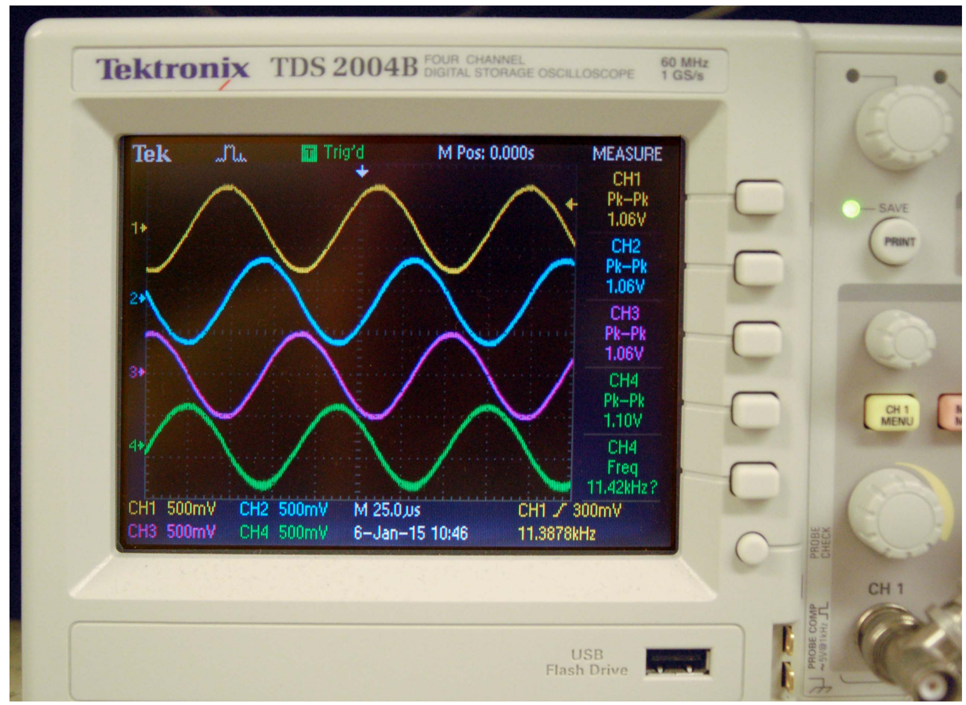
3) **Rotate the apparatus at the plasma rotation frequency.**
10kHz to MHz rate !!! *(not practical)*

Practical solution: **Azimuthally sectored electrodes**

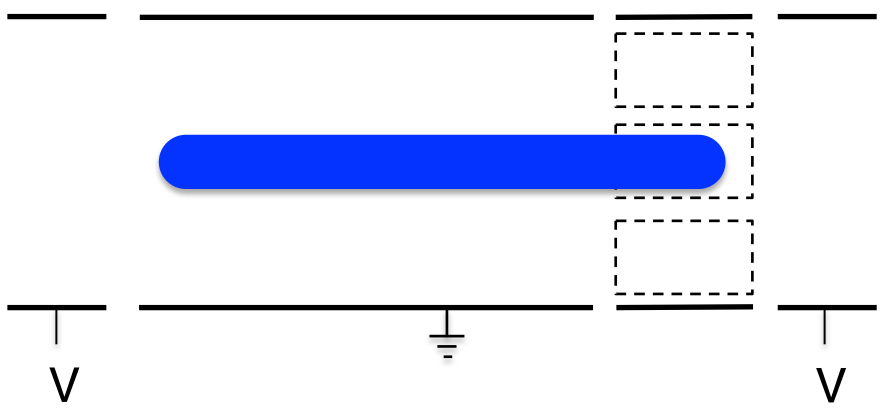
Solution applied a rotating electric field which “imitate” a rotating wall

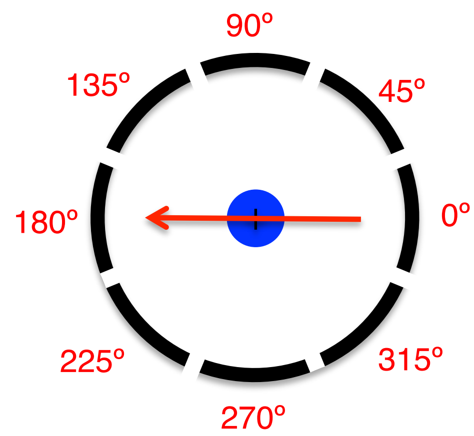


8 - sectors

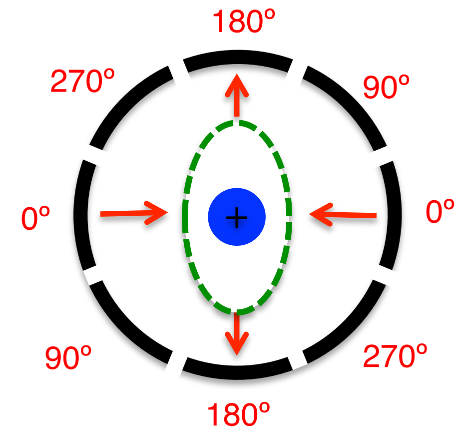


Sine waves applied to sector 1, 3, 5, 7
Traces of sector 2, 4, 6, 8 not shown.





Dipole drive
($m_\theta = 1$)



Quadrupole drive
($m_\theta = 2$)

Potential on a sector $$\phi_j(\theta,t) = A_{RW} \cos\left[m_\theta(\theta_j - 2\pi f_{RW} \cdot t)\right]$$

⚠ Spatial harmonics due to finite sector size

Example: 4 sector drive ($m_\theta = 2$) $A_{m_\theta=-2} / A_{m_\theta=2} = 1$ same amplitude !

8 sector drive ($m_\theta = 2$) $A_{m_\theta=-6} / A_{m_\theta=2} \cong 2/3$ 60% $m_\theta = 2$ ✔
40% $m_\theta = -6$ ✖

Torque balance

$$\tau \equiv \frac{d}{dt}\frac{-\langle r^2\rangle}{R_W^2}$$

$\tau_{RW} = \tau_{ambient}$ Steady state

$\tau_{RW} > \tau_{ambient}$ compression

$f_{RW} > f_E$

Thermodynamic argument
Phys. Plasmas ***5****, 2163 (1998)*

Common sense !

Rotating wall is doing work on plasma => heat plasma

At a minimum: $$2\pi f_{RW}\tau_{RW} = 2\pi f_E \tau_{ambient}$$

Unwanted spatial harmonics
Imperfection in RW drive

=> Heat

High temperature plasmas have less coupling to the rotating electric field

High temperature => less collisional => weaker coupling

Need cooling

Typical forms of cooling use for RW

Cyclotron radiation

Light particles: electron, positron

$$\text{radiation time} \equiv \frac{-T}{\dot{T}} = \frac{9mc^3}{8e^2\Omega_c^2} \cong \frac{4.\ \text{sec}}{B_{\text{Tesla}}^2}$$

Collision with gas

Heavy particles: ions

$$\dot{T} \cong \nu_{iN}(T - T_N)$$

$$\nu_{iN} = 0.025\ \text{s}^{-1} \left(\frac{P_N}{10^{-9}\,\text{Torr}}\right)$$ (Mg^+ on H_2)

Successful cooling of positrons using vibrational modes of large gas molecules

Danielson et al. *Rev. Mod. Phys.*, 2015

Laser cooling

Limited to a few ions *"cyclone transition"*

Today and Thursday presentation: Caroline Champenois

How to couple the Rotating Wall to the plasma

Plasma shields the applied electric field !
(Debye shielding)

Work in the vicinity of a plasma mode
Apply RW perturbation close to the end of plasma

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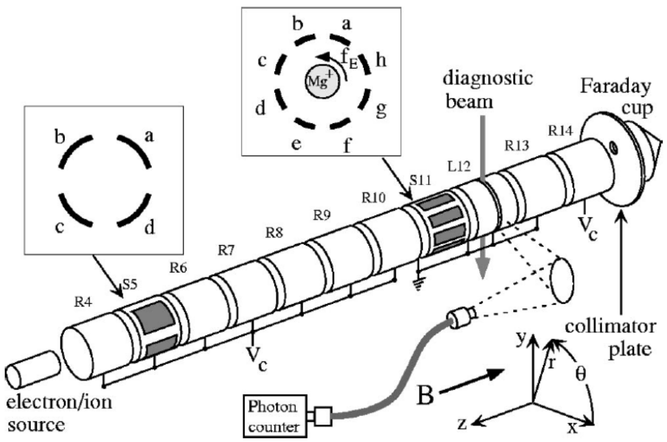
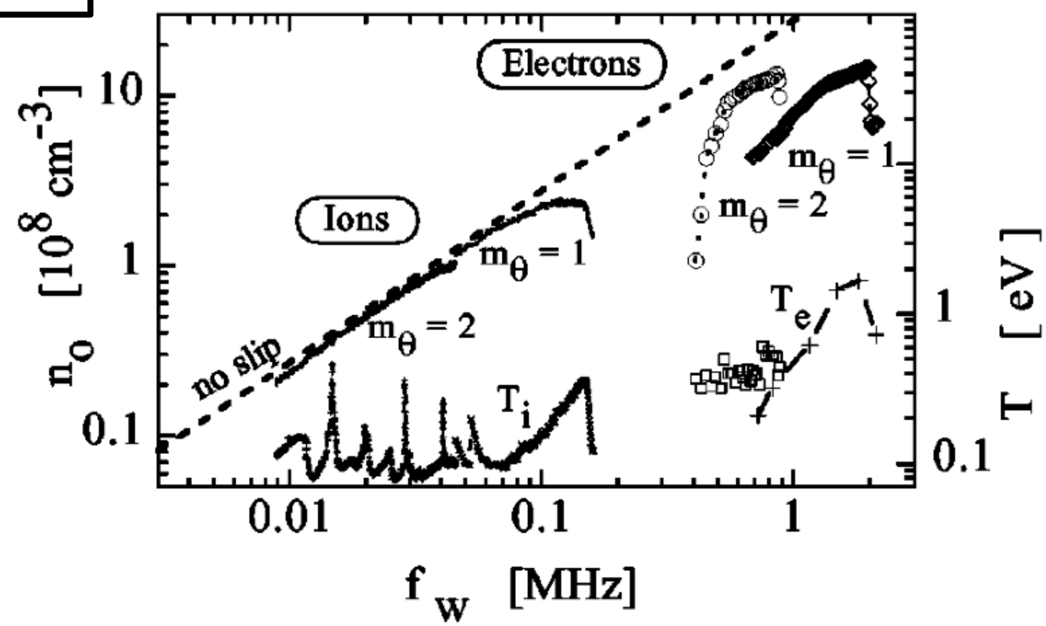
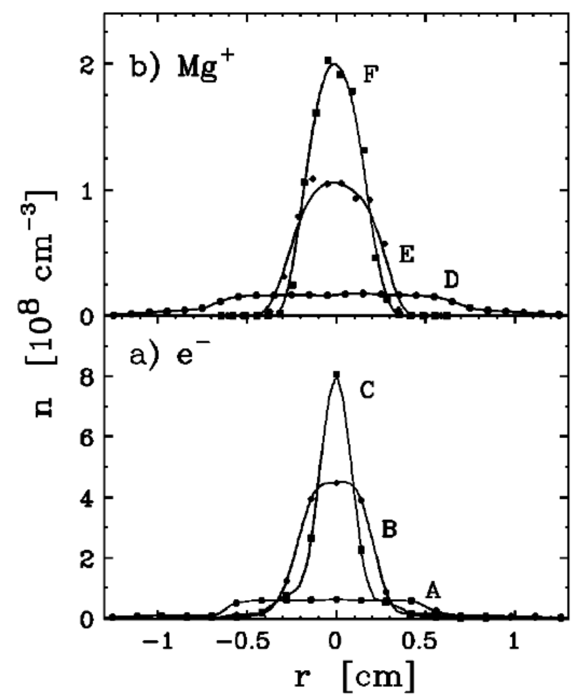
UCSD Mg^+ and e^- experiments

PRL **78**, 875 (1997); PRL **81**, 4875 (1998); POP **7**, 2776 (2000)



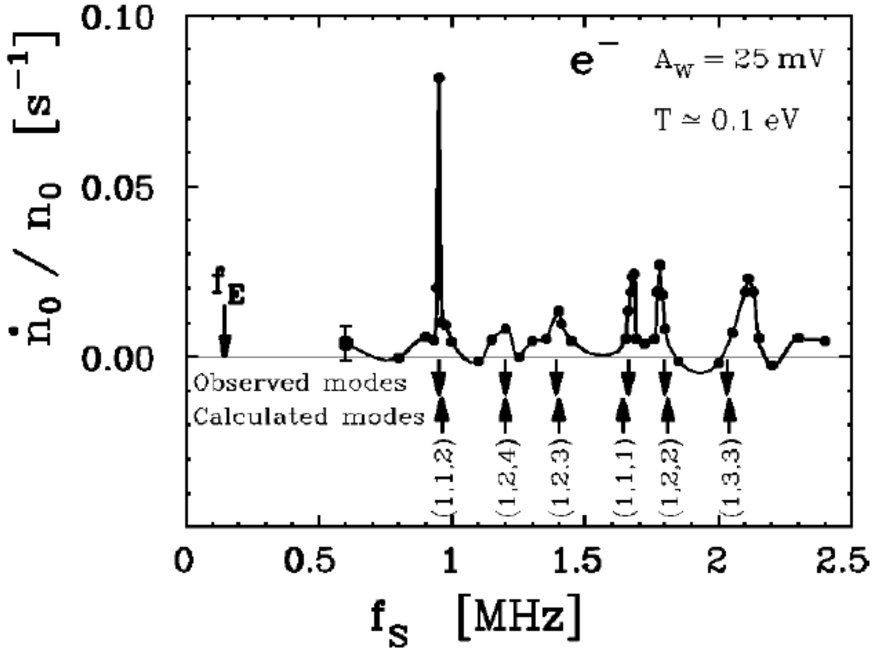




- 3×10^9 electrons or 10^9 Mg^+ ions
- e^- cyclotron cooling; weak Mg^+- neutral cooling (0.05 s^{-1})
- m_θ=1 or m_θ=2 rotating wall applied near ends of plasma
- significant compression observed with both e^- and Mg^+
- steady state confinement for weeks
- slip $f_{rw} > f_r$
- maximum compression density $n_0 \sim 10^9$ cm^{-3}

limited by $\left.\frac{\dot{n}_0}{n_0}\right)_{bkg} \propto n_0^2$ and heating

UCSD Mg^+ and e^- experiments



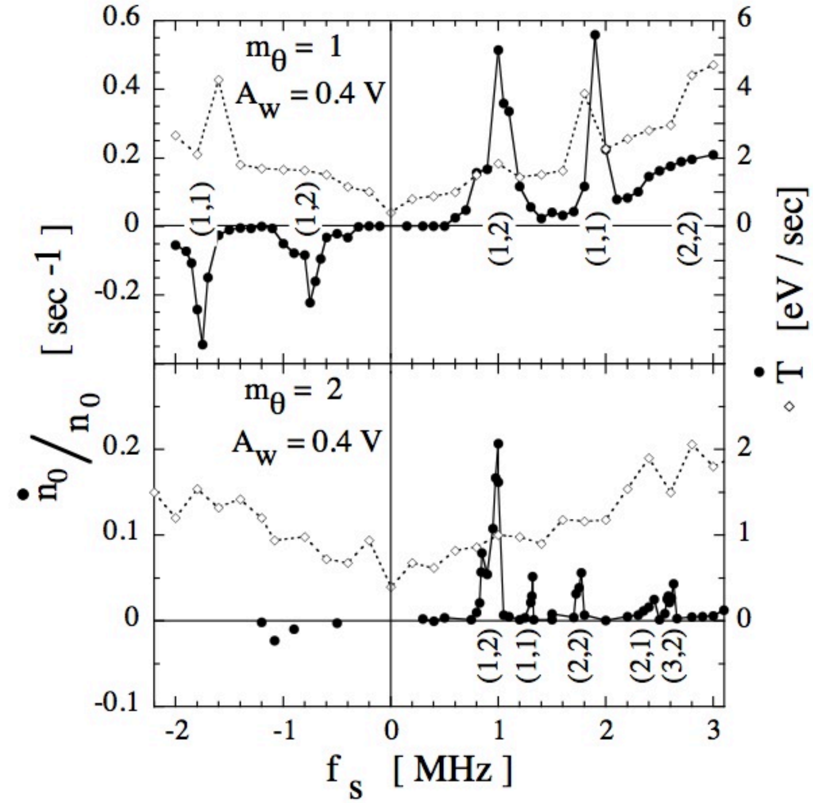


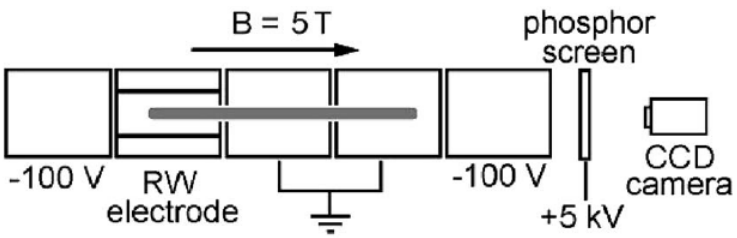
FIG. 5. Density compression rates and plasma heating for large amplitude $m_\theta = 1$ and $m_\theta = 2$ rotating drives. The compression peaks are associated with shifted (m_z, m_r) modes.

Strong torques applied by RW coupling to plasma modes (Trievelpiece-Gould)

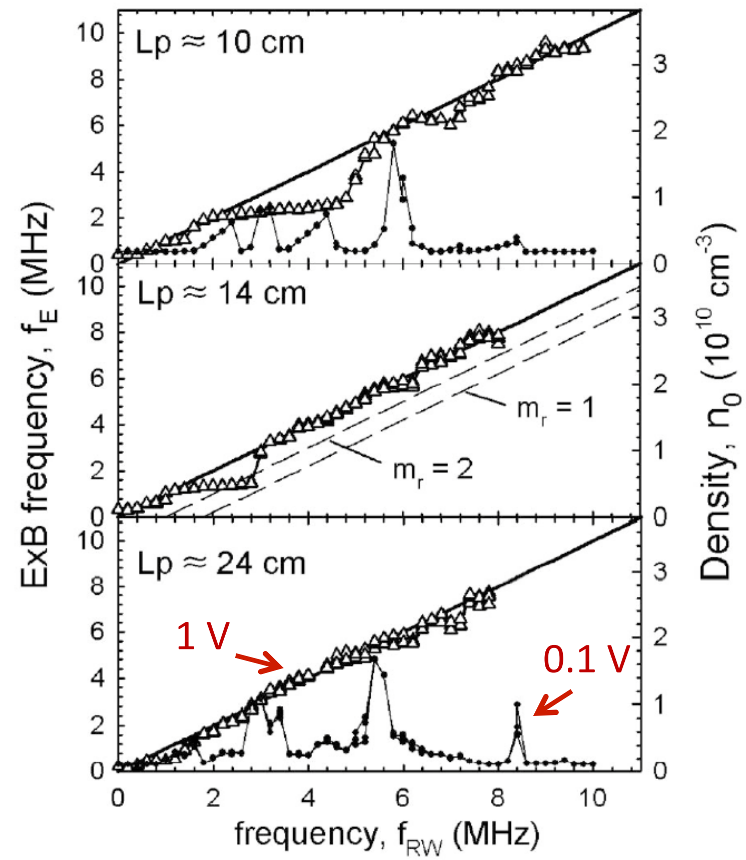
PRL **78**, 875 (1997); PRL **81**, 4875 (1998); POP **7**, 2776 (2000)

Danielson/Surko strong drive regime

PRL **94**, 035001 (2005); POP **13**, 055706 (2006); PRL **99**, 135005 (2007)



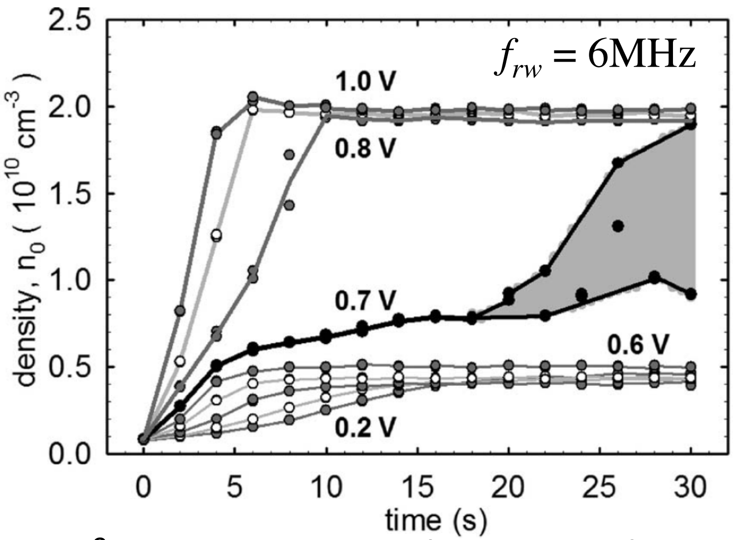
steady density vs applied RW frequency
weak wall (0.1 V) vs strong wall (1.0 V)



- ~ 10^9 e^-, B=5T, e^- cyclotron cooling τ_c~ 0.16 s
- m_θ=1 rotating wall applied near end of plasma
- weak wall (0.1 V) compression at distinct frequencies, consistent with coupling to modes
- strong wall (1 V) compression at all frequencies, no apparent slip $f_{rot} \cong f_{RW}$

Danielson/Surko strong drive regime

PRL **94**, 035001 (2005); POP **13**, 055706 (2006); PRL **99**, 135005 (2007)



steady density vs applied RW frequency
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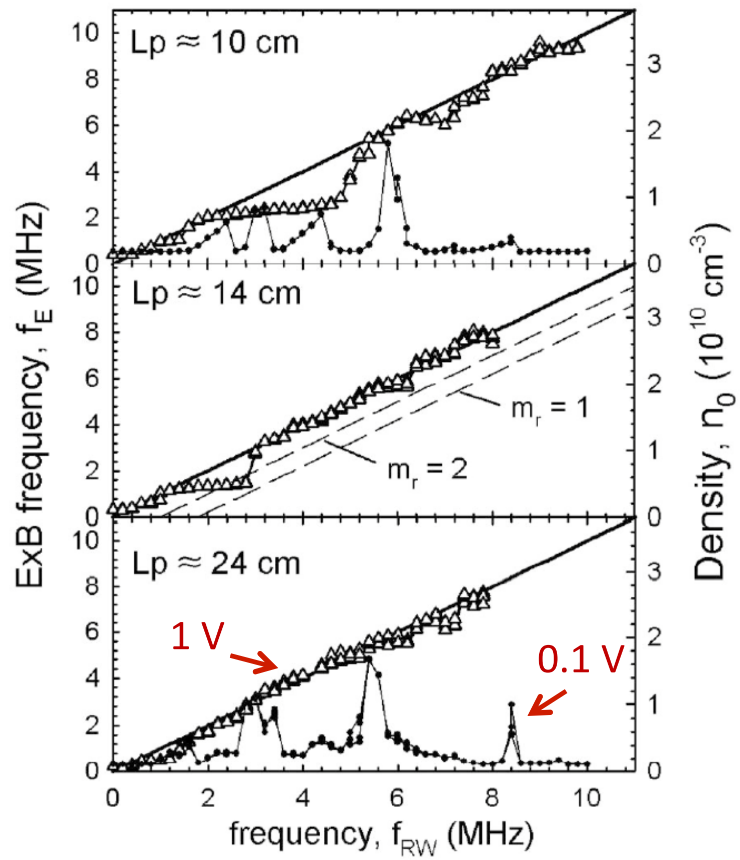
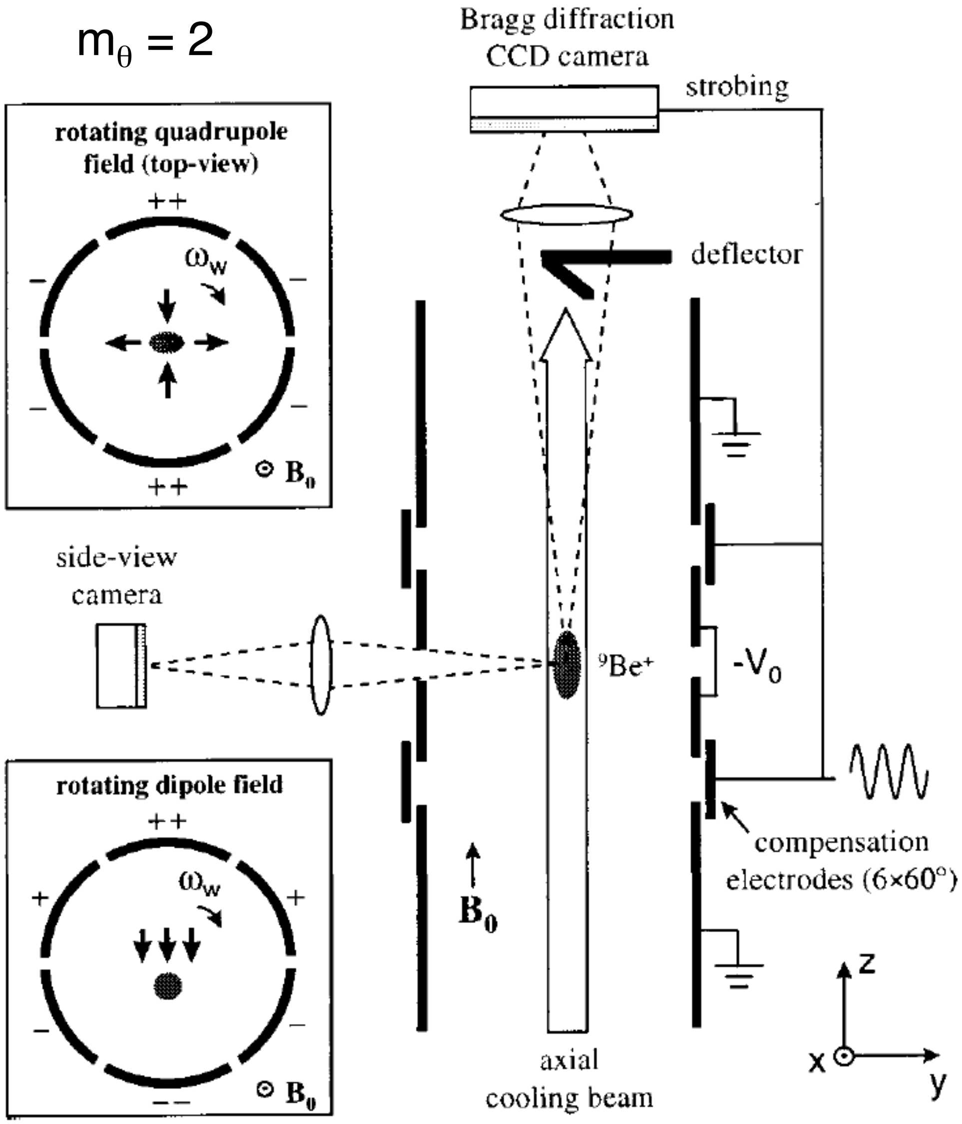


- ~ 10^9 e^-, B=5T, e^- cyclotron cooling τ_c~ 0.16 s
- m_θ=1 rotating wall applied near end of plasma
- weak wall (0.1 V) compression at distinct frequencies, consistent with coupling to modes
- strong wall (1 V) compression at all frequencies, no apparent slip $f_{rot} \cong f_{RW}$
- critical drive strength observed
- maximum density n_0 ~ 3 x10^9 cm^{-3}
- Strong drive regime characterized by $\left.\frac{\dot{n}_0}{n_0}\right)_{bkg}$ independant of n_0 and $\nu_{ee} > f_{bounce}$
- Increased cyclotron cooling, lower outward transport => enable access to strong drive regime

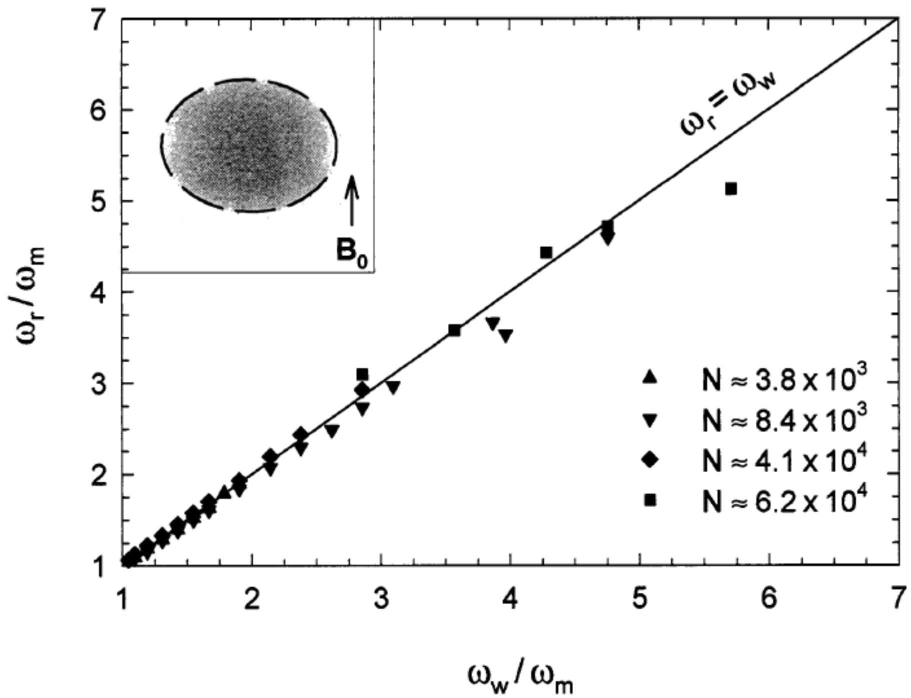
NIST phase-locked rotating wall

Huang et al., PRL **80**, 73 (1998); POP **5**, 1656 (1998)

$m_\theta = 2$



$m_\theta = 1$



- $10^2 < N < 10^6$ laser-cooled Be^+ ions
- T < 10 mK
- compression and no slip observed with both $m_\theta = 2$ and $m_\theta = 1$ walls
- rotating wall applied <u>uniformly</u> across axial extent of plasma !!

John Bollinger NIST

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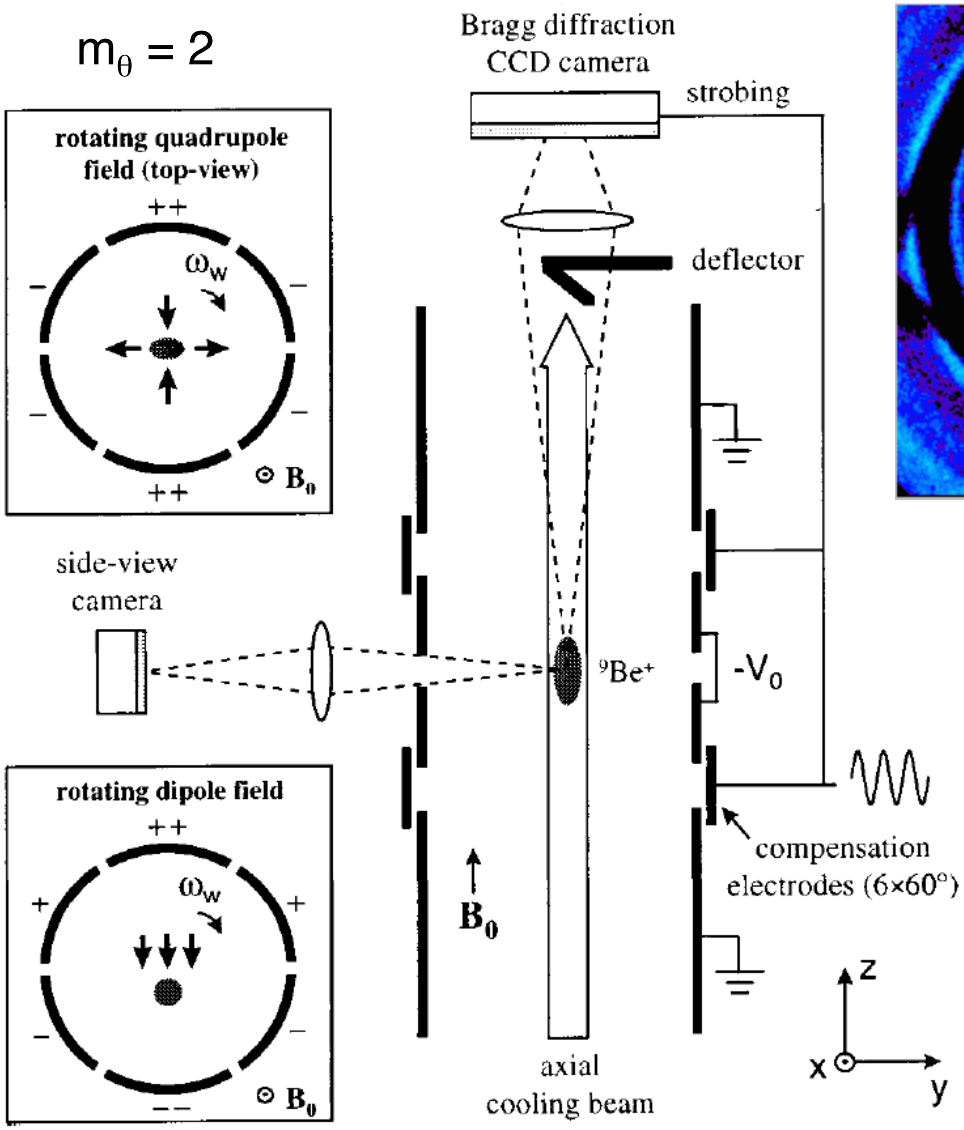
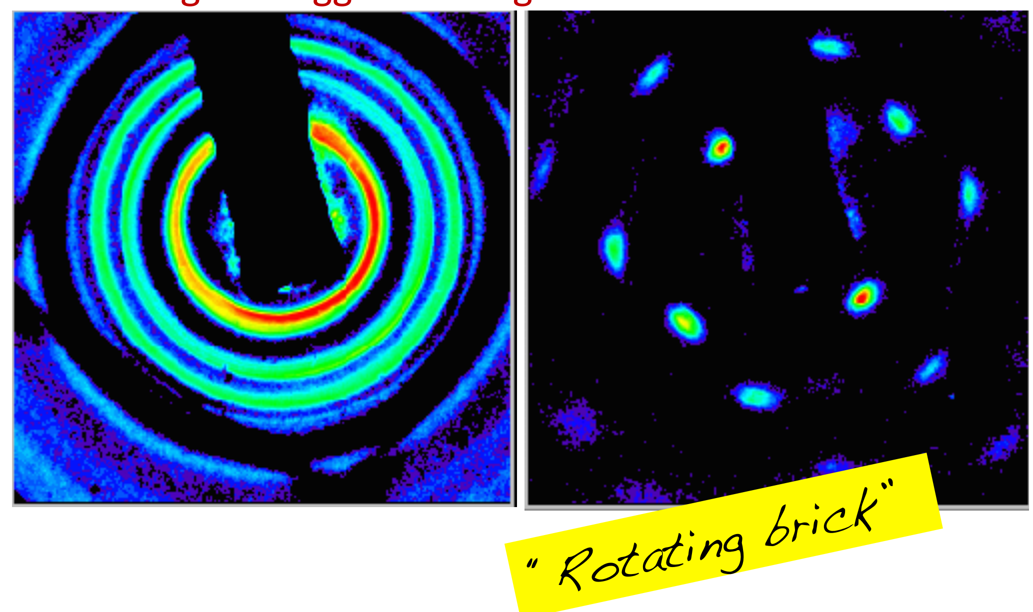


$m_\theta = 1$

time-averaged Bragg scattering

strobed



- $10^2 < N < 10^6$ laser-cooled Be^+ ions
- T < 10 mK
- compression and no slip observed with both $m_\theta = 2$ and $m_\theta = 1$ walls
- rotating wall applied <u>uniformly</u> across axial extent of plasma !!
- in steady state rotating wall and plasma (crystal) rotation are phase coherent

John Bollinger NIST

NIST phase-locked rotating wall

Huang et al., PRL **80**, 73 (1998); POP **5**, 1656 (1998)

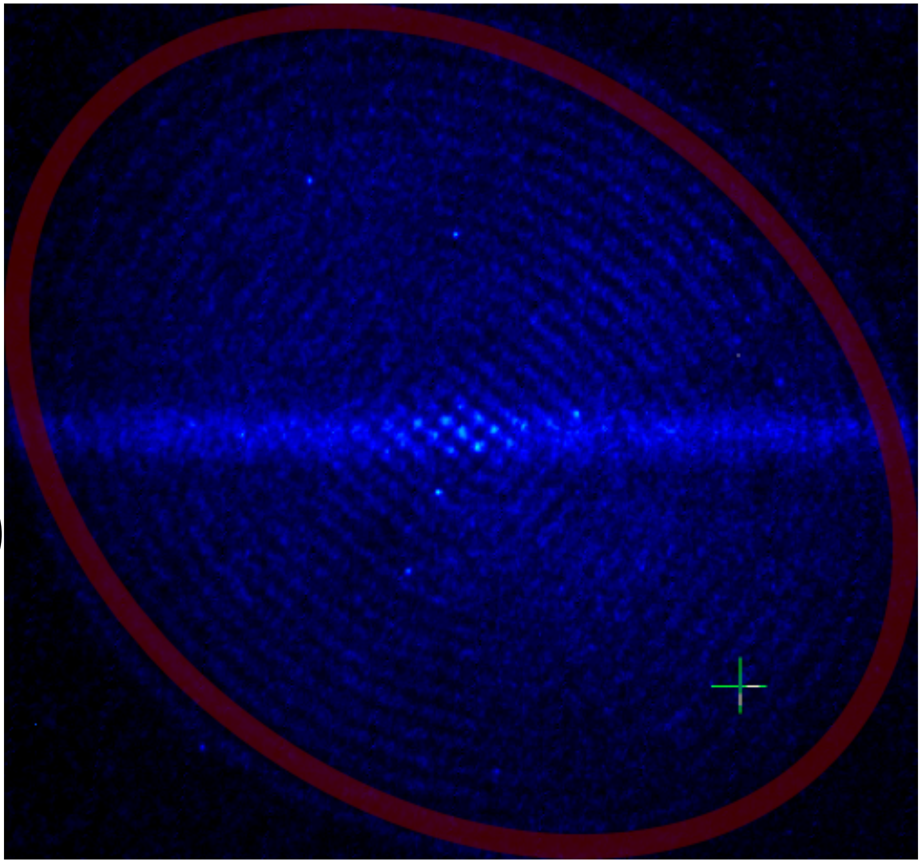
- Torque mechanism for $m_\theta = 2$ wall with crystalized plasma easy to understand
- $m_\theta = 2$ wall changes the potential in the rotating frame

$$q\varphi_{rot}(r,z) = \frac{m\omega_z^2}{2}\left(z^2 + \beta r^2\right) \qquad \beta \equiv \frac{\omega_r\left(\Omega_c - \omega_r\right)}{\omega_z^2} - \frac{1}{2}$$

$$q\phi_{rot}(x,y,z)\big|_{quad\ wall} = \frac{m\omega_z^2}{2}\left(z^2 + (\beta+\delta)x^2 + (\beta-\delta)y^2\right)$$

$$\delta \propto V_{wall}$$

- Plasma shape is a tri-axial ellipsoid;
  rotating boundary applies torque;
  shear forces in crystal transmit forces to interior



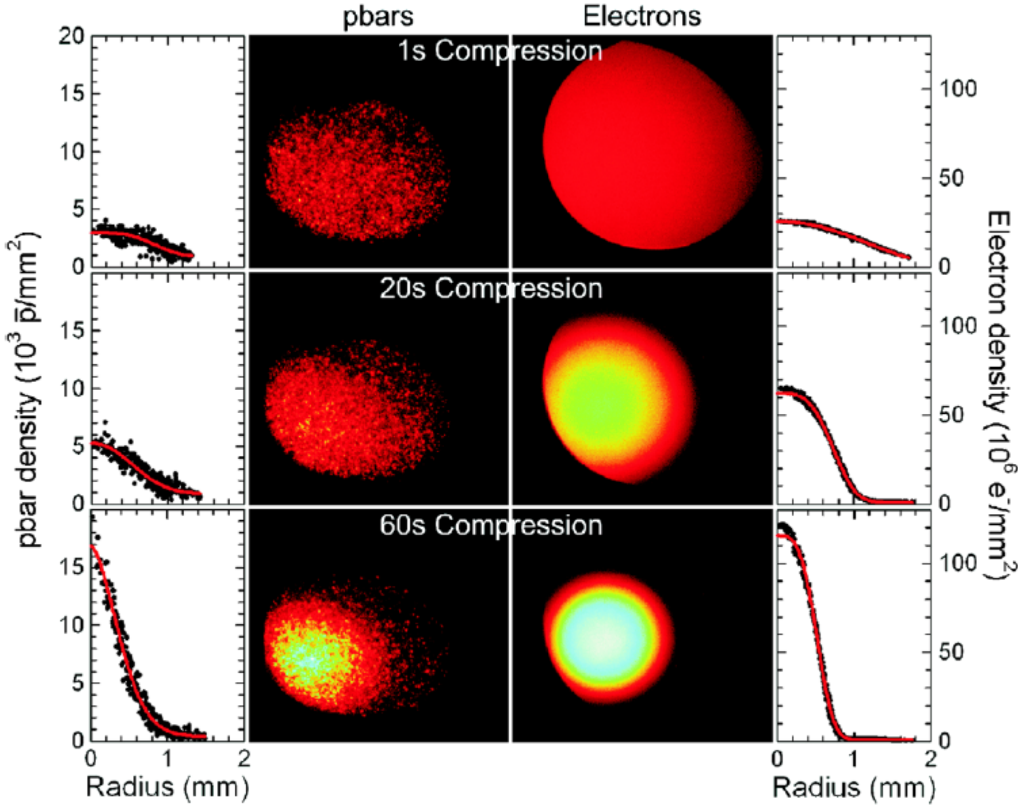
top-view image of a planar crystal showing distortion of the radial boundary with an m_θ=2

John Bollinger NIST

Other examples of rotating wall compression

- mixed e^-/pbar plasmas

From Andresen et al., PRL **100**, 203401 (2008)



- many other experiments…

Summary rotating wall

- Technique is very successful,
  allows essentially unlimited confinement time (weeks)

- Technique allows control of the plasma density

- The art of coupling to rotating perturbation to the plasma is still not understood completely.

Publications can be found at nnp.UCSD.edu

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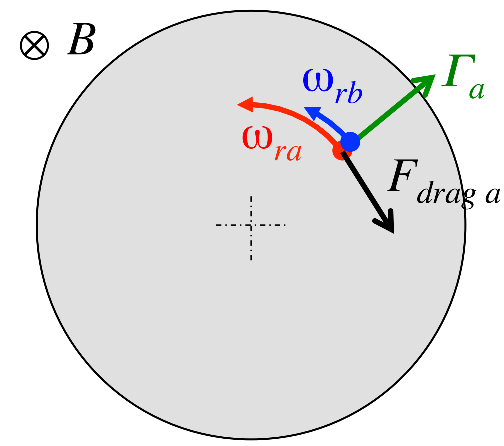
François Anderegg UCSD

Centrifugal separation



Radial force balance

$$0 = n_a \left[qE(r) + \frac{qB}{c}\omega_{ra} r + m_a \omega_{ra}^2 r \right] - \frac{\partial p_a}{\partial r}$$

Pressure of species a $\quad p_a = n_a T_a$

$$\omega_{ra} = \omega_E(r) + \frac{c}{qBn_a(r)\, r} \frac{\partial p_a}{\partial r} - \frac{\omega_{ra}^2}{\Omega_{ca}}$$

$$\omega_E(r) = \frac{E(r)\, c}{B\, r}$$

$$\Omega_{ca} = \frac{qB}{m_a c}$$

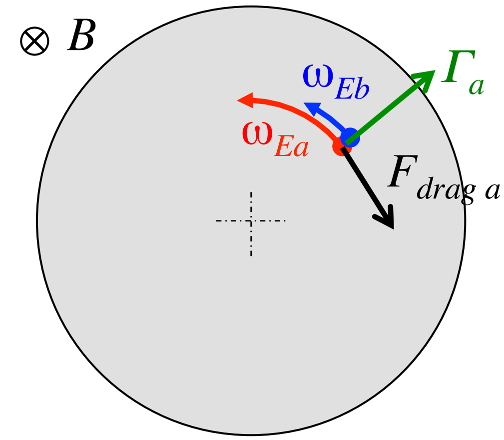
In general each species rotates at a different rate

$$F_{drag\ \mathrm{a}} = \sum_b \nu_{ab}\ m_a\ r\ (\omega_{ra} - \omega_{rb})$$

FxB drift => radial flux

$$\Gamma_{ra} = n_a \frac{c\ F_{drag\ \mathrm{a}}}{q\ B}$$

FxB drift => radial flux



$$\Gamma_{ra} = n_a \frac{c\ F_{drag\ \mathrm{a}}}{q\ B}$$

$$= \sum_b \frac{D_{ab}}{T_a} \left[\frac{\partial p_a}{\partial r} - \frac{n_a}{n_b} \frac{\partial p_b}{\partial r} + n_a (m_b \omega_{rb}^2 - m_a \omega_{ra}^2) r \right]$$

$$D_{ab} = \nu_{ab} r_{c_a}^2$$ Diffusion coefficient

When $\begin{cases} T_a(r) = T_b(r) = T \\ \omega_{ra}(r) = \omega_{rb}(r) = \omega_r \end{cases}$ Flux stops

Rate of centrifugal separation

$$Rate \sim \frac{D_{ab}}{R_p^2} \sim 0.1\mathrm{s}^{-1} \left(\frac{n_0}{10^7 \mathrm{cm}^{-3}} \right) \sqrt{\frac{\mu(\mathrm{amu})}{T(\mathrm{K})}} \frac{1}{(B(\mathrm{Tesla}) R_p(\mathrm{cm}))^2}$$

Using thermodynamics equilibrium properties

$$f_j = n_j \left(\frac{m_j}{2\pi kT} \right)^{3/2} \exp\left(\frac{-1}{kT} (H_j + \omega_r P_{\theta j}) \right)$$

$$n_j(r) = n_j(0) \exp\left[\frac{-q_j}{kT} \left(\phi(r) - \underbrace{\frac{m_j}{q_j} \frac{\omega_r^2 r^2}{2}} + \frac{B\omega_r r^2}{2c} \right) \right]$$

Only difference between species

Centrifugal separation important if $q\left| \frac{m_1}{q_1} - \frac{m_2}{q_2} \right| \omega_r^2 R_p^2 > kT$

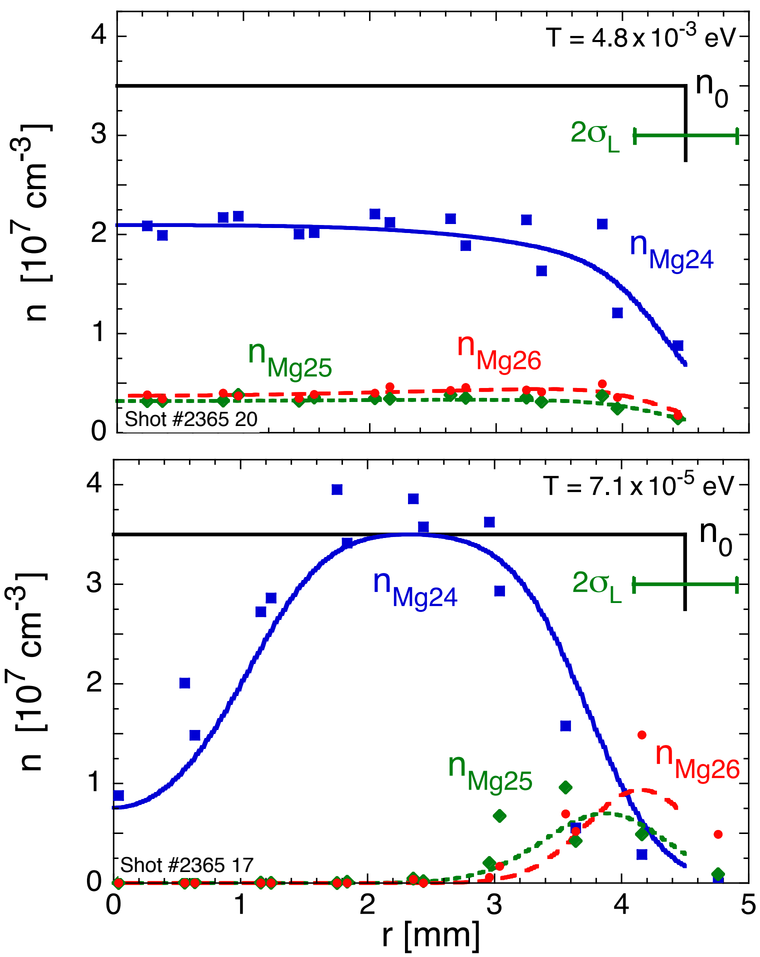
$$\left| m_1 - m_2 \right| \omega_r^2 R_p^2 > kT$$

Separation length $l_{sep} \equiv \frac{kT}{\left| m_1 - m_2 \right| \omega_r^2 R_p} < R_p$

Complete separation $l_{sep} \ll \lambda_D$

Centrifugal separation of a Magnesium plasma containing impurities

Affolter et al. Physics of Plasma (2015)



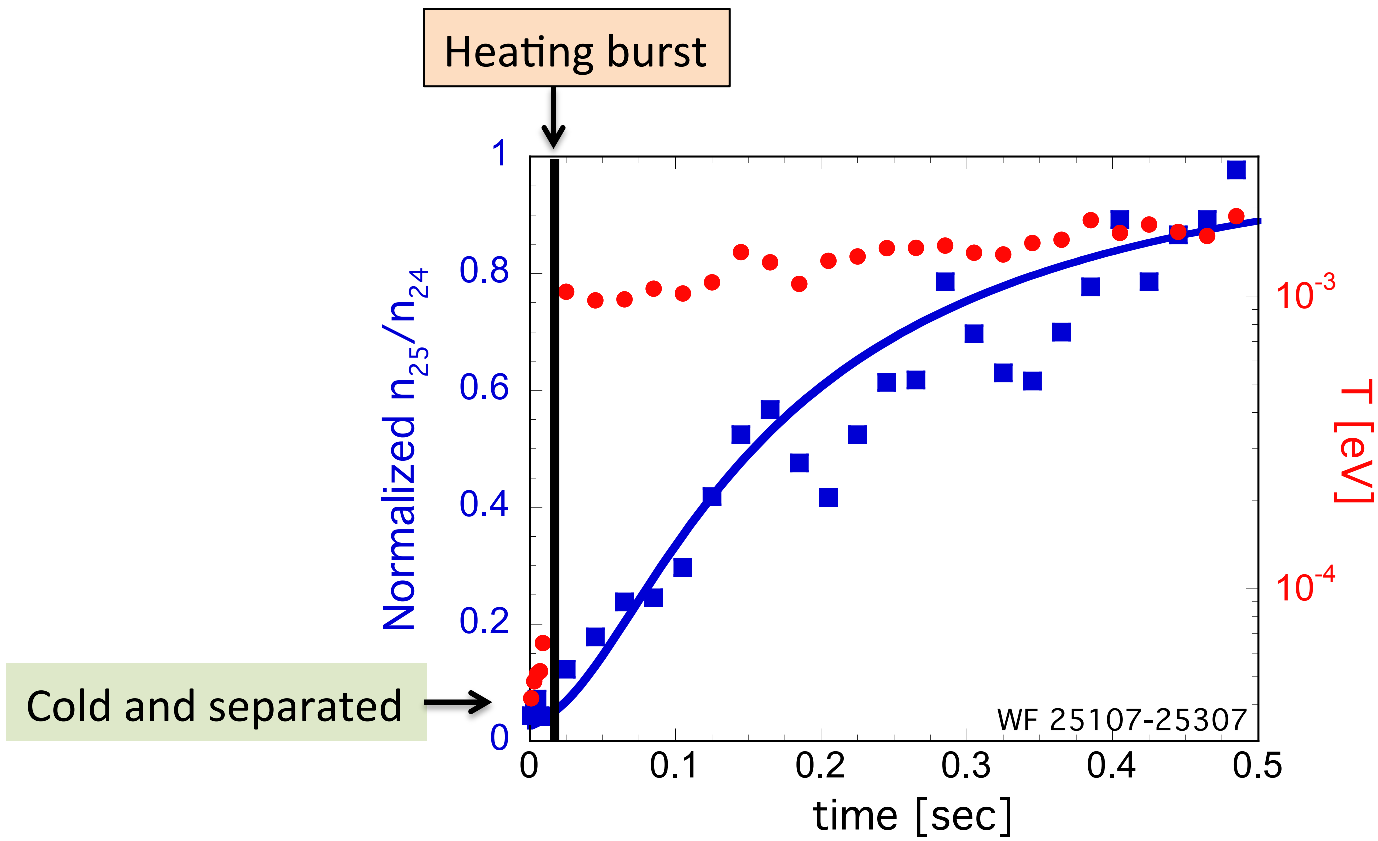
Warm density profile (mixed)

5 species
Mg_{24} Mg_{25} Mg_{26}, H_3O, O_2

Cold density profile (separated)

$$\frac{n_a(r)}{n_b(r)} = C_{ab} \exp\left[\frac{1}{2kT}(m_a - m_b)\ \omega_r r^2\right]$$

Re-mixing rate of a Magnesium plasma



$$Rate \sim \frac{D_{ab}}{R_p^2} \sim 0.1\text{s}^{-1}\left(\frac{n_0}{10^7\,\text{cm}^{-3}}\right)\sqrt{\frac{\mu(\text{amu})}{T(\text{K})}}\,\frac{1}{(B(\text{Tesla})R_p(\text{cm}))^2}$$

$$\Gamma_a = \sum_b \frac{D_{ab}}{T_a}\left[\frac{\partial p_a}{\partial r} - \frac{n_a}{n_b}\frac{\partial p_b}{\partial r} + n_a(m_b\omega_{rb}^2 - m_a\omega_{ra}^2)r\right]$$

$$D_{ab} = \nu_{ab} r_{c_a}^2$$

Summary centrifugal separation

Multispecies plasma are centrifugally separated if $\left|m_1 - m_2\right| \omega_r^2 R_p^2 > kT$

The separation length is $$l_{sep} \equiv \frac{kT}{\left|m_1 - m_2\right| \omega_r^2 R_p} < R_p$$

The separation complete when $l_{sep} \ll \lambda_D$

For plasma close to thermal equilibrium, the **equilibrium densities** and the **separation rate** (remixing rate) is correctly described by the present theory.

Publications can be found at nnp.UCSD.edu

Simple Tutorial Problems

1) Calculate the ExB rotation frequency of a single species, uniform density plasma, of density $10^7 cm^{-3}$ in a 3Tesla magnetic field.
   a) Consider an electron plasma
   b) Consider a Magnesium ion plasma

2) Estimate the minimum rotation frequency necessary to observe centrifugal separation of Magnesium 24 from Magnesium 25 for a temperature of 10K and a plasma radius of 5mm.