Problem set for Penning trap lectures

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Question 1

Starting from the Lorentz force on a charged particle in static electric (**E**) and magnetic (**B**) fields,

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

with an electrostatic potential

$$\phi(\mathbf{r}) = A(2z^2 - x^2 - y^2),$$

find the three oscillation frequencies (ω_z, ω'_c and ω_m) for a singly charged calcium ion in a magnetic field of 1 T and with A=1×10^5 Vm^{-2}.

Also find the frequencies for the case of an electron in the same fields.

Question 2

Consider two stationary calcium ions located on the z-axis of the trap in Question 1, at positions $+z_0$ and $-z_0$. By equating the confining force from the trap with the Coulomb repulsion, find the equilibrium value of z_0.

Question 3

Thinking of an ion in the above trap as a quantum mechanical simple harmonic oscillator, calculate the "width" of the ground state wave function. [If you can't remember the formula for this, you can estimate it by calculating the amplitude of the *classical* motion of an ion having the zero-point energy $\hbar\omega_z/2$.]

Comparing the results of Questions 2 and 3, do you think it is possible to observe effects due to the overlap of ion wavefunctions in a trap?

Question 4

In *Angels and Demons*, Dan Brown imagines a device that sounds similar to a Penning trap but stores enough liquid antihydrogen to destroy the Vatican. Imagine instead a real Penning trap (with a 1 T field) containing 1cm^3 of antiprotons at the maximum possible density (i.e. at the Brillouin limit). What is the number density of antiprotons and how much energy would be released if the antiprotons were allowed to annihilate?

Question 5

In our trap at Imperial College we have a modified cyclotron frequency of 650 kHz, an axial frequency of 200 kHz and a magnetron frequency of 50 kHz (approximately). The Doppler cooling limits are 1.0 mK, 0.5 mK and 0.04 mK respectively. What are the corresponding mean quantum numbers for the three motions in equilibrium?

Question 6

Consider a single ion of hydrogen-like uranium ($^{235}U^{91+}$) in a Penning trap. Find its three oscillation frequencies in a magnetic field of 4 T and with A=1×10^5 Vm^{-2} (see Question 1).

The ground state Lamb shift in uranium is 460 eV. By what fraction does this change the rest mass of the ion? (You may want to use the fact that the atomic unit of mass is equivalent to 911 keV.)

If it is desired to measure the ground state Lamb shift to ~1% by "weighing" the ion, what fractional precision is required for the cyclotron frequency, and what absolute frequency precision does this imply?