Problem set for Penning trap lectures

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Question 1

Starting from the Lorentz force on a charged particle in static electric (**E**) and magnetic (**B**) fields,

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

with an electrostatic potential

$$\phi(\mathbf{r}) = A(2z^2 - x^2 - y^2),$$

find the three oscillation frequencies (ω_z, ω'_c and ω_m) for a singly charged calcium ion in a magnetic field of 1 T and with A=1×10^5 Vm^{-2}.

Also find the frequencies for the case of an electron in the same fields.

Solution in the Trapped Charged Particles textbook

Question 2

Consider two stationary calcium ions located on the z-axis of the trap in Question 1, at positions $+z_0$ and $-z_0$. By equating the confining force from the trap with the Coulomb repulsion, find the equilibrium value of z_0.

Solution in the Trapped Charged Particles textbook

Question 3

Thinking of an ion in the above trap as a quantum mechanical simple harmonic oscillator, calculate the "width" of the ground state wave function. [If you can't remember the formula for this, you can estimate it by calculating the amplitude of the *classical* motion of an ion having the zero-point energy $\hbar\omega_z/2$.]

Comparing the results of Questions 2 and 3, do you think it is possible to observe effects due to the overlap of ion wavefunctions in a trap?

Solution in the Trapped Charged Particles textbook

Question 4

In *Angels and Demons*, Dan Brown imagines a device that sounds similar to a Penning trap but stores enough liquid antihydrogen to destroy the Vatican. Imagine instead a real Penning trap (with a 1 T field) containing 1cm^3 of antiprotons at the maximum possible density (i.e. at the Brillouin limit). What is the number density of antiprotons and how much energy would be released if the antiprotons were allowed to annihilate?

Solution in the Trapped Charged Particles textbook

Question 5

In our trap at Imperial College we have a modified cyclotron frequency of 650 kHz, an axial frequency of 200 kHz and a magnetron frequency of 50 kHz (approximately). The Doppler cooling limits are 1.0 mK, 0.5 mK and 0.04 mK respectively. What are the corresponding mean quantum numbers for the three motions in equilibrium?

Solution:

Using the equations from Lecture 1, slides 20, 21, 22, we find

Axial energy = $n_z h\nu_z$ = <PE> + <KE> = kT.
There is ½kT per degree of freedom.
Hence n_z = 52 for the given value of T.
(note that I have left out the ½ in the QM expression for energy as n>>1)

For cyclotron you have to use the different expressions for the total energy. It is the kinetic energy that's equal to ½kT. The potential energy has a different value (and is actually egative) so the full expression for the energy is
$n_c h\nu_c' = \frac{1}{2} kT[1 - \frac{1}{2} \nu_z^2 / \nu_c'^2]$ giving $n_c = \frac{1}{2} kT \times 0.95 / h\nu_c' = 31$

Similarly for the magnetron motion we find that (don't forget the minus sign)
$-n_m h\nu_m = \frac{1}{2} kT[1 - \frac{1}{2} \nu_z^2 / \nu_m^2]$ giving $n_c = \frac{1}{2} kT \times 0.95 / h\nu_m = 58$

Question 6

Consider a single ion of hydrogen-like uranium ($^{235}U^{91+}$) in a Penning trap. Find its three oscillation frequencies in a magnetic field of 4 T and with A=1×10^5 Vm^{-2} (see Question 1).

The ground state Lamb shift in uranium is 460 eV. By what fraction does this change the rest mass of the ion? (You may want to use the fact that the atomic unit of mass is equivalent to 911 keV.)

If it is desired to measure the ground state Lamb shift to ~1% by "weighing" the ion, what fractional precision is required for the cyclotron frequency, and what absolute frequency precision does this imply?

Solution:

Using the equations from the solution to Q1 we find that the axial frequency is 610 kHz, the modified cyclotron frequency is 24 MHz and the magnetron frequency is 8 kHz. So in this case the modified cyclotron frequency is only very slightly different from the true cyclotron frequency.

To measure the true cyclotron frequency you need to measure the modified cyclotron frequency with the highest precision as this contributes most to the quadrature sum of the three frequencies (remember the expression we gave in the lectures to calculate the cyclotron frequency even in the presence of perturbations).

The uranium nucleus has a rest mass energy of 235 × 911 MeV. The ratio of the Lamb shift (460 eV) to this is 2 × 10^{-9}.

To measure the Lamb shift to 1% you therefore need to measure the modified cyclotron frequency to a relative precision of 2 × 10^{-11}. Since the frequency is about 24 MHz, the absolute precision required is about 5 mHz.