Imperial College London

Penning Traps
Lecture 3

Richard Thompson
Quantum Optics and Laser Science Group
Department of Physics
Imperial College London

Outline of lecture 3

Lecture 1: Penning Trap Basics

Lecture 2: Review of Experiments

Lecture 3: Rotating Frame and Axialisation

7. The rotating frame
8. Cooling in the rotating frame
9. Axialisation
10. Conclusion

7. The Rotating Frame

- It turns out that much of the physics of the Penning trap is simplified if we move to a frame rotating at half the cyclotron frequency.
  - The *centripetal force* ($-m\omega^2\boldsymbol{r}$) gives rise to a force that gives a confining potential in this frame
  - This overcomes the negative radial potential in the lab frame leading to a net confining potential
  - The *Coriolis force* ($-2\boldsymbol{\omega}\wedge\boldsymbol{v}$) gives rise to a force perpendicular to the velocity of a particle that cancels out the force due to the magnetic field
  - Therefore in this frame there is effectively no magnetic field
- **As a result, the radial motion in this frame reduces to standard two-dimensional SHM**

Equations of motion

- The original radial equations of motion were

$$\ddot{x} + \omega_c \dot{y} - (\omega_z^2 / 2)x = 0$$

$$\ddot{y} - \omega_c \dot{x} - (\omega_z^2 / 2)y = 0$$

or $$\ddot{u} - i\omega_c \dot{u} - (\omega_z^2 / 2)u = 0$$ with $u=x+iy$

- The transformation to a frame rotating at $\omega_c/2$ is $u \rightarrow u(1-i\omega_c t/2)$
- This gives us

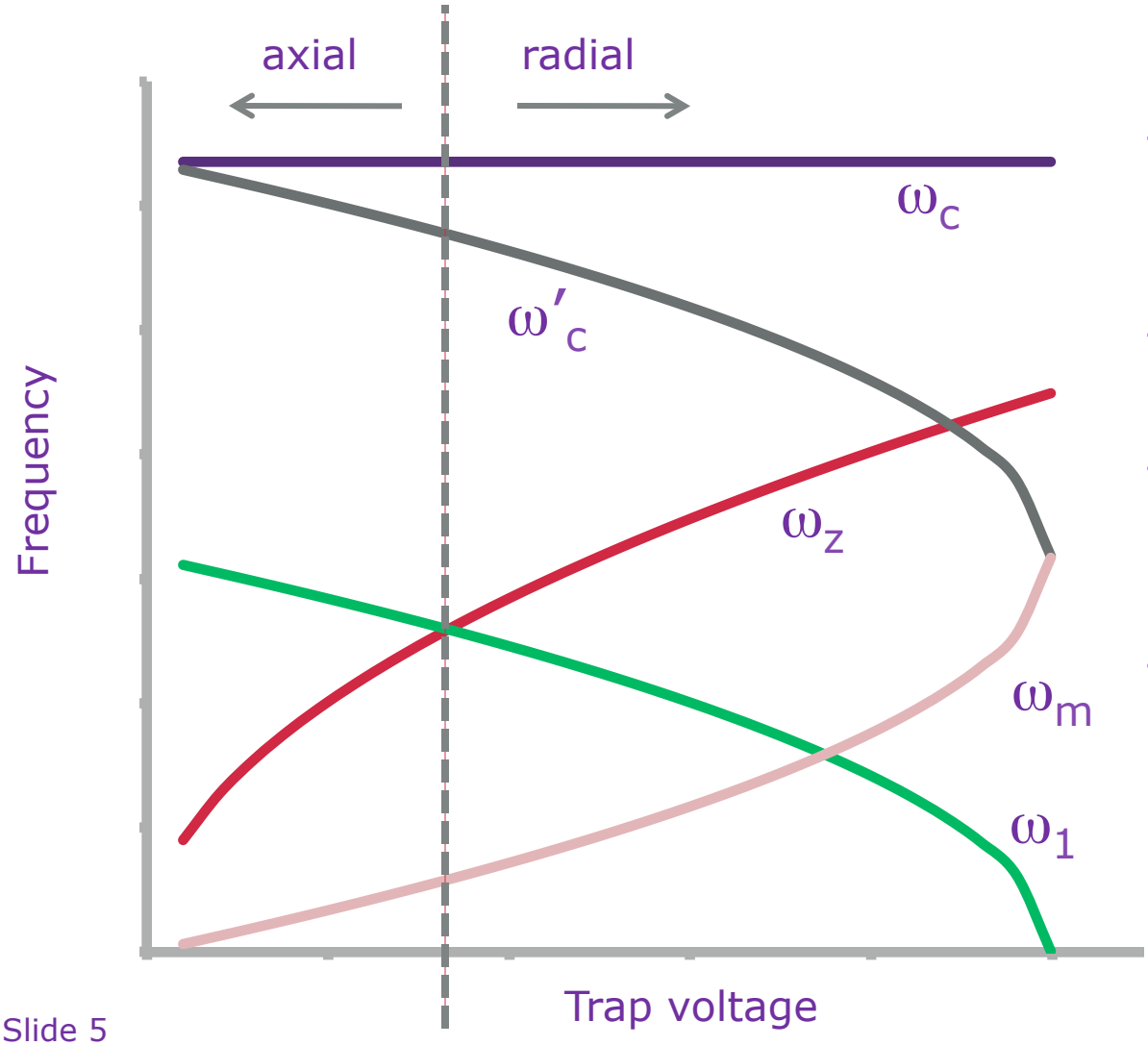
$$\ddot{u} + (\omega_c^2 / 4 - \omega_z^2 / 2)u = 0$$

or $$\ddot{u} + \omega_1^2 u = 0$$

with $$\omega_1^2 = \omega_c^2 / 4 - \omega_z^2 / 2$$

- This is just SHM in a 2D potential well at frequency ω_1
  - No magnetic field in this frame

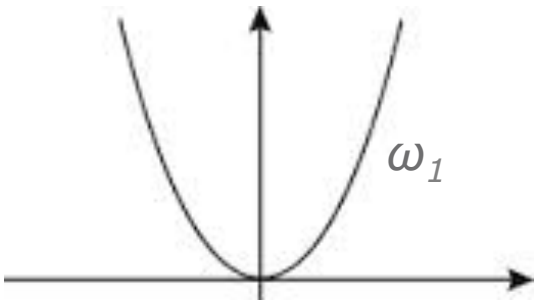
Oscillation frequencies for small crystals

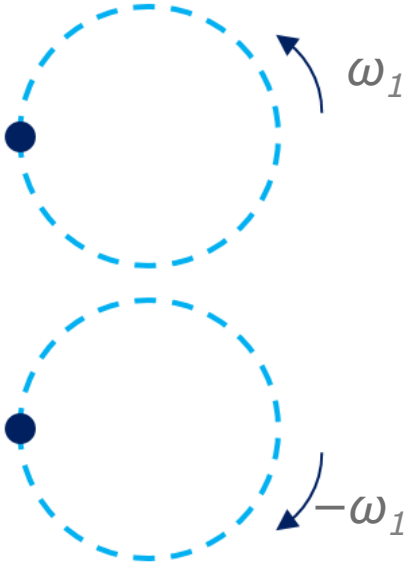


- ω_1 is the oscillation frequency in the rotating frame
- It reduces as the trap voltage is increased
- It becomes zero at the point where the trap becomes unstable
- The dotted line indicates where the trapping potential is spherical

Oscillation frequencies in the two frames

- In the frame rotating at $\omega_c/2$ the potential is a 2D harmonic potential well of frequency ω_1.
- We take the two normal modes to be
  - anti-clockwise rotation (ω_1)
  - clockwise rotation ($-\omega_1$)
- When these are transformed back to the lab frame they become
  - Cyclotron motion ($\omega_c/2+\omega_1$)
  - Magnetron motion ($\omega_c/2-\omega_1$)
  - Now **both are anti-clockwise**





How does the motion transform?

- The motion is much simpler to describe in the rotating frame (RF)
- Why do we usually end up with $r_m > r_c$?
  - A particle at rest in the lab frame will be orbiting at $-\omega_c/2$ in the rotating frame so it will have a larger magnetron component

Pure cyclotron

Pure magnetron

Equal cyclotron and magnetron

Cyclotron smaller than magnetron

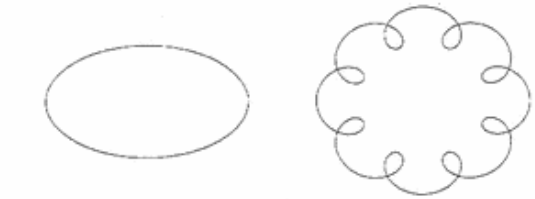
Cyclotron bigger than magnetron

Can we visualise the motion?

- Yes we can!
- But in order to do so we need a 2D simple harmonic potential: this represents the potential *in the rotating frame*
  - A wok makes an excellent model of a 2D potential well
  - A ball bearing is an excellent model of an ion
  - But the motion of a ball bearing in a wok is not very interesting
- We need to simulate the effect of the magnetic field
  - Then we can see what the motion is like *in the laboratory frame*

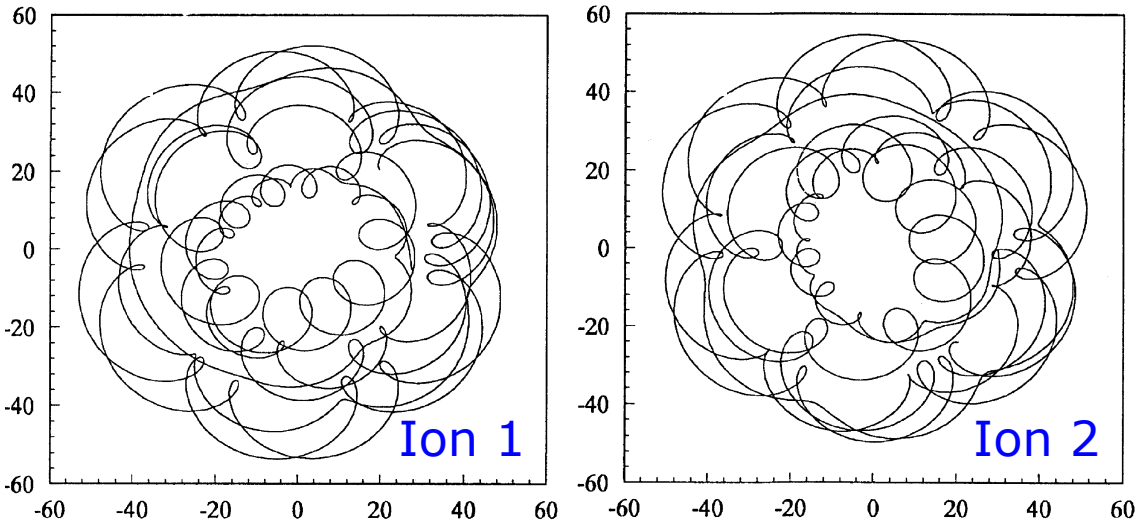
Can we visualise the motion?

- In order to simulate the effect of the magnetic field we need to view the 2D simple harmonic potential **from a rotating frame**
  - Solution: use a rotating camera



Centre of mass and relative motion of two ions

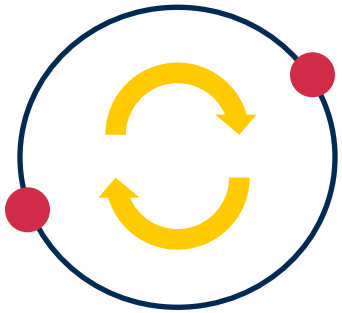
- The motion of two ions appears very complicated
- But actually it's very simple if you separate it into centre of mass and relative motions
- In the same way, things simplify when you observe the motion in the rotating frame!



Two ions in the rotating frame

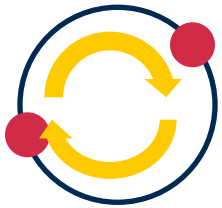
The rotation frequency depends on ion separation, due to the Coulomb interaction

- Large separation:
  - $\omega'_R \sim \omega_1$ in RF
  - $\omega_R \sim \omega_m$ in LAB



Quasi-independent particles

- Medium separation
  - $\omega'_R < \omega_1$ in RF
  - $\omega_R > \omega_m$ in LAB



Coulomb interaction slows rotation down

- Small separation:
  - Brillouin flow
  - $\omega'_R = 0$ in RF
  - $\omega_R = \omega_c/2$ in LAB

Minimum separation set by ω_1

Many ions in the rotating frame

- A plasma is easy to picture in a potential well and in the absence of the magnetic field
- If the plasma is not rotating in the rotating frame it is at Brillouin flow
  - Frequency of the radial potential is ω_1 so density is

$$n = m\omega_c^2 \varepsilon_0 / 2e^2$$

because $\omega_c^2 = 2\omega_1^2 + \omega_z^2$

- If it rotates at ω'_R in the rotating frame:

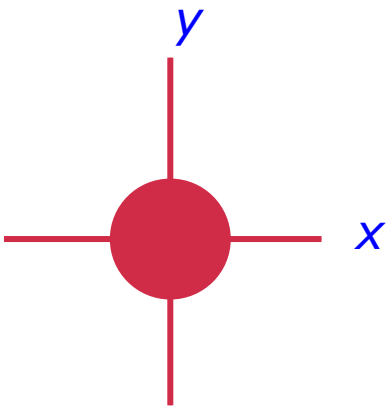
$$\omega_1^2 \Rightarrow \omega_1^2 - \omega'^2_R$$

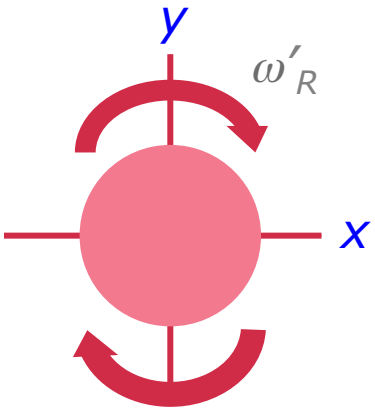
because the centrifugal force causes the density to drop

- Rearranging: $n = 2\varepsilon_0 m\omega_R(\omega_c - \omega_R)/e^2$

as we had before

- The aspect ratio of the ellipsoid also changes





8. Cooling in the rotating frame

- We will now look again at cooling in general in the Penning trap as viewed from the rotating frame
- We will find this gives us new insights into how cooling works
- Remember the rotating frame is the one rotating at $\omega_c/2$ relative to the Laboratory frame.
  - There is effectively no magnetic field in this frame and the trap oscillation frequency is ω_1

Effect of a buffer gas in the rotating frame

- A buffer gas gives a uniform damping force in the lab frame
- In the rotating frame this damping force is rotating at $-\omega_c/2$
  - This looks like a whirlpool
- It creates a torque that rotates a single ion in the negative sense at $-\omega_1$ and accelerates it outwards
  - This corresponds to increasing magnetron radius
  - This is why buffer gas damping increases the magnetron radius
- A **uniform cooling laser** beam also gives a uniform damping force in the lab frame

Equations of motion with damping

- Remember that the equation of motion for an ion in the rotating frame is

$$\ddot{u} + \omega_1^2 u = 0$$

- A uniform damping force adds a term $+\gamma\dot{u}$
- In the presence of a *rotating* damping force (rotating at angular frequency ω_0) the equation becomes

$$\ddot{u} + \gamma(\dot{u} - iu\omega_0) + \omega_1^2 u = 0$$

- And if we put in the trial solution $u=u_0\exp(i\omega t)$ we find

$$\omega = \pm\omega_1 + i\gamma(1 \pm \omega_0 / \omega_1)$$

- For positive damping the imaginary part must be positive for both solutions
  - If the damping in the Lab frame is *uniform*, $|\omega_0| = \omega_c/2$ which is greater than ω_1, so the damping is negative and the motion is **unstable**
  - If $\omega_0 < \omega_1$, the damping is always positive and the motion is **stable**
    - » We achieve this by offsetting the laser beam from the trap centre

Laser cooling with an offset beam

- A laser beam **offset from the centre of the trap** gives a rotating damping force
  - The laser gives angular momentum to the ion
  - The rotation speed of the damping (say ω_L) is proportional to the gradient of laser intensity
  - In the rotating frame this damping force is rotating at $\omega_0 = -\omega_c/2+\omega_L$
  - This still looks like a whirlpool but if it rotates between $-\omega_1$ and $+\omega_1$ it will cool the motion
- This is why an offset laser beam cools the magnetron radius
- In fact the rate of rotation of the damping force in the lab is given by:

$$\omega_L = \frac{Rate\ of\ change\ of\ scattering\ rate\ with\ ion\ position}{Rate\ of\ change\ of\ scattering\ rate\ with\ ion\ velocity}$$

Laser cooling of ion cloud

- For a cloud of ions forming a plasma, the rotating damping force will drag the plasma till they both rotate at the same frequency $\omega_R = \omega_L$
- This rotation frequency is determined by the laser beam parameters as before
- We can therefore control the density and shape of the plasma by changing the laser parameters
- Remember the rotation frequency (in the lab) and the density are related by

$$n = 2\varepsilon_0 m\omega_R(\omega_c - \omega_R)/e^2$$

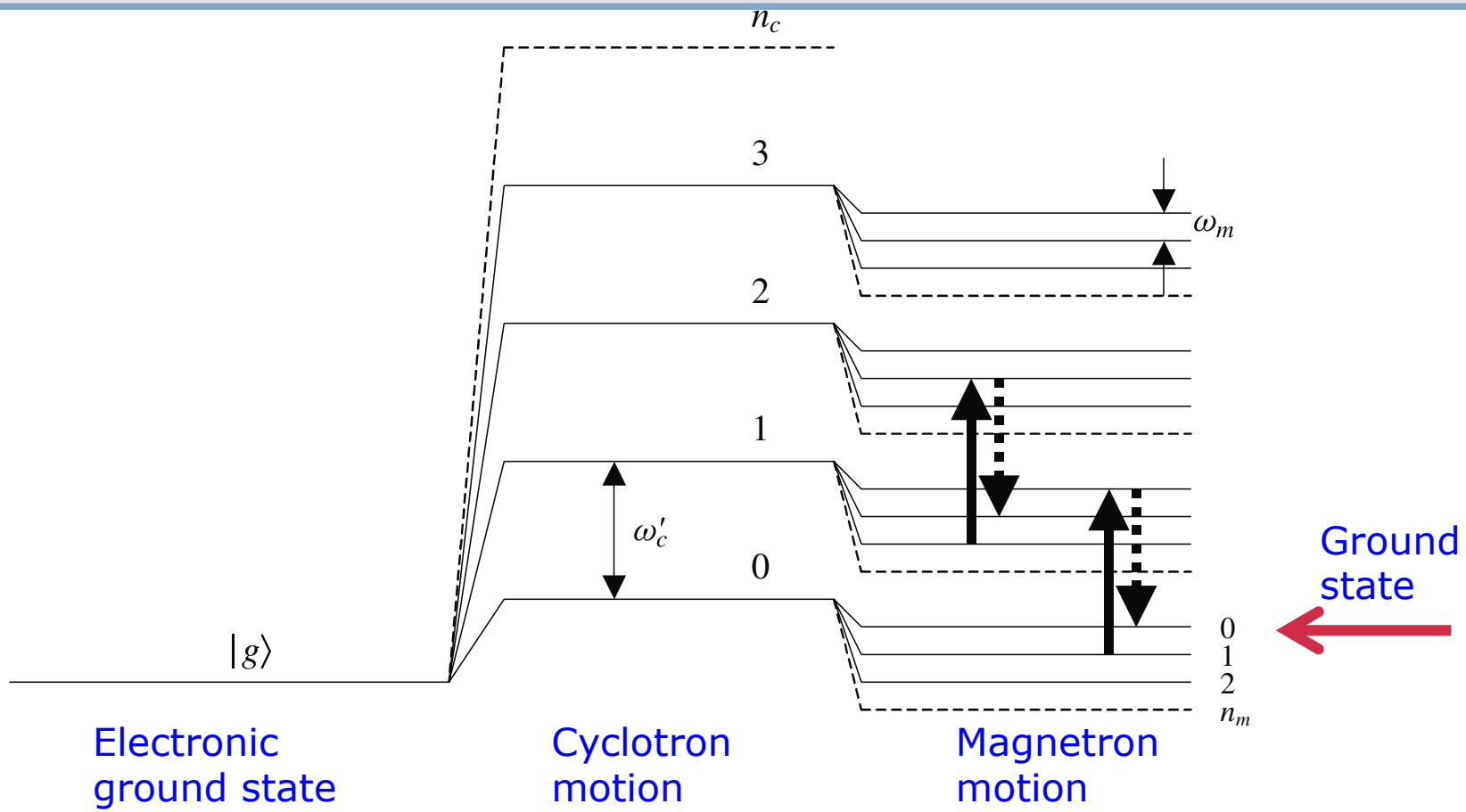
9. Axialisation

- Axialisation is a technique for cooling the magnetron motion
- It requires two things:
  - Coupling of the magnetron motion to another motion in the trap
  - A damping mechanism for the second motion
- When set up properly, it results in cooling of both motions at the same time
- BEWARE: It also goes by other names:
  - Sideband cooling [not to be confused with optical sideband cooling]
  - Magnetron centering

Coupling of motions

- In general two oscillators are coupled by excitation at their *difference frequency* to exchange energy
  - e.g. a laser driving a transition between two atomic states
- In the Penning trap we can couple the magnetron and cyclotron motions by excitation at their *sum frequency*
  - This is because of the negative energy of the magnetron motion
  - The sum frequency is just the cyclotron frequency $\omega_c = \omega'_c + \omega_m$
- Classical equations of motion show that a radial quadrupole field is required
- We can also think of it in terms of quantum mechanical levels:
  - Excitation at ω_c drives n_c to n_c+1 and n_m to n_m-1

Quantum mechanical picture



Excitation at $\omega_c = \omega_c' + \omega_m$ by axialization drive

Cooling of cyclotron motion via laser cooling

Damping with the axialisation technique

Damping can be provided by a number of means:

- ***Buffer gas***
  - Used in mass spectrometry experiments
  - Especially Fourier Transform ICR (ion cyclotron resonance)
  - Gives a well controlled damping force on all particles
- ***Resistive cooling***
  - Used widely in cryogenic environments
- ***Laser cooling***
  - Ions are much better localised when axialisation applied
  - Laser beam can be directed through trap centre as offset is no longer required
  - But the damping force is only applied to ions located in the laser beam and this is not ideal
- Note that the magnetron can also be coupled to the *axial motion* using excitation at $\omega_z - \omega_m$ (needs different field symmetry)

Effect of the coupling

- The coupling causes energy exchange at a rate δ between the two modes of motion
- If the modes are damped at rates γ_c (>0) and γ_m (<0) then

$$\dot{r}_c = \delta r_m - \gamma_c r_c, \qquad \dot{r}_m = -\delta r_c - \gamma_m r_m,$$

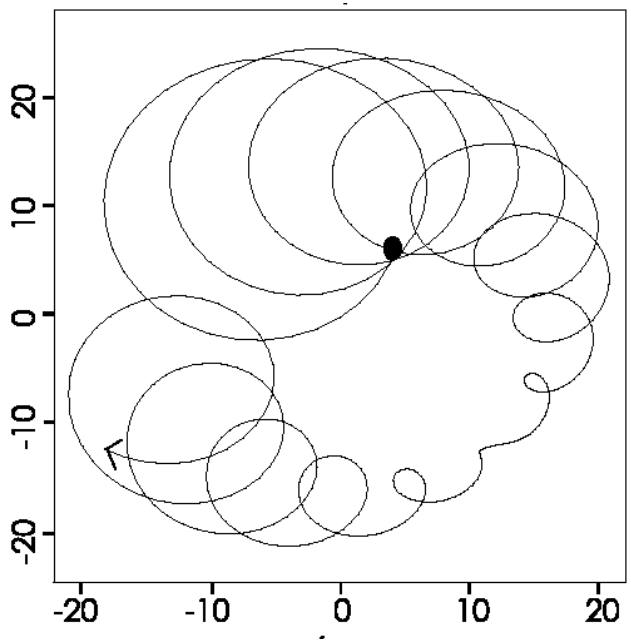
where $\delta \approx eV_{ax} / mR^2$ with V_{ax} << trap voltage

- The condition for axialisation to work is

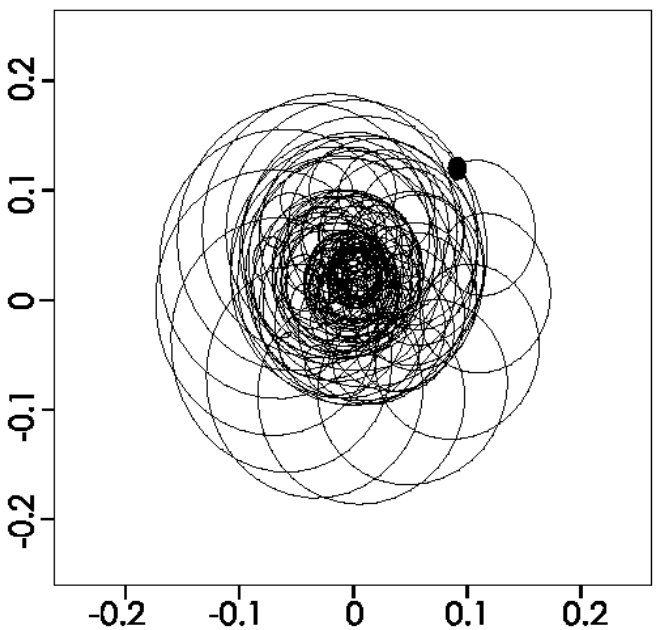
$$\delta^2 > -\gamma_c \gamma_m$$

Simulation of axialisation

- With coupling alone, the orbital energy exchanges between magnetron and cyclotron motion
- With damping as well, the amplitude of both motions decreases



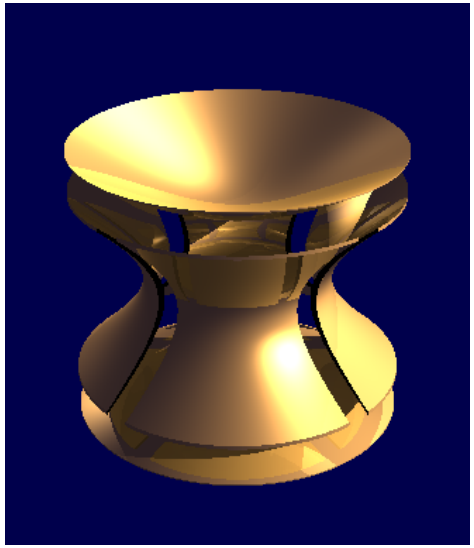
No damping

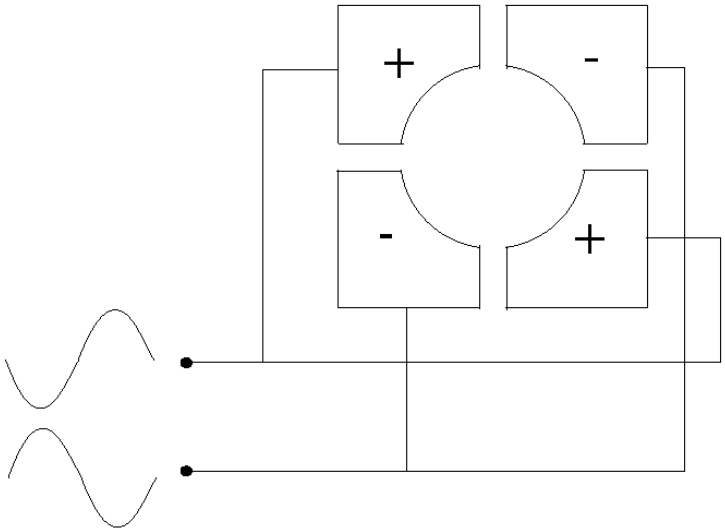


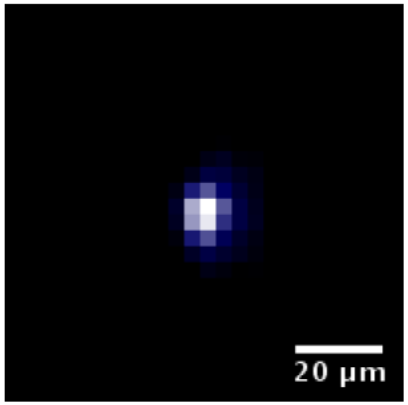
With damping

How do we apply the field?

- With axialisation we apply a radial quadrupole field at ω_c
  - We need four segments (minimum) to apply a radial quadrupole field
  - e.g. by splitting the ring electrode into 4 segments
  - This allowed us to get our first well localised single ion images

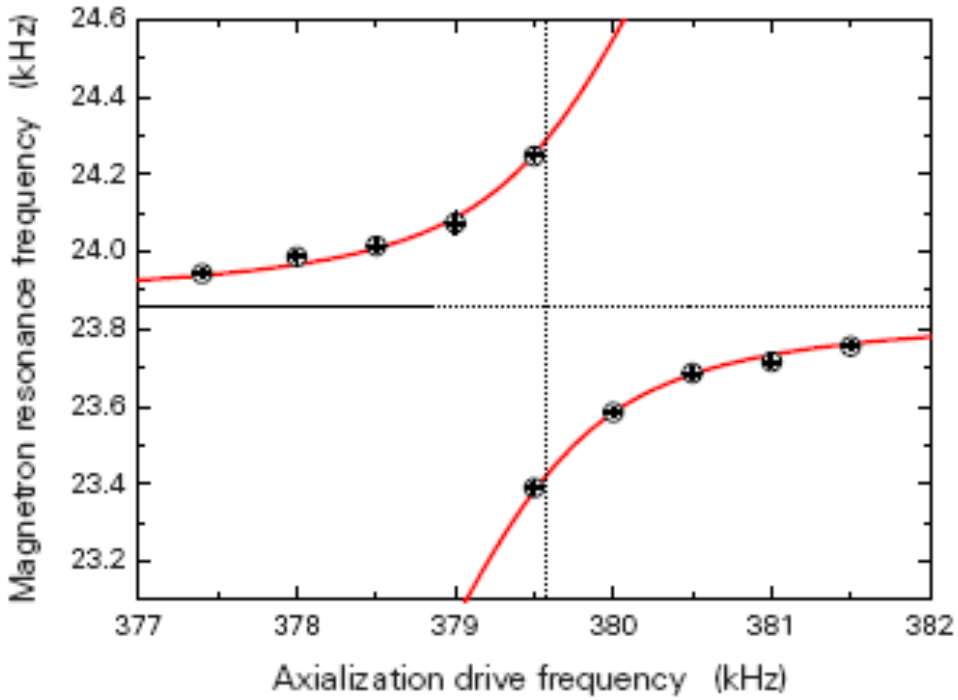


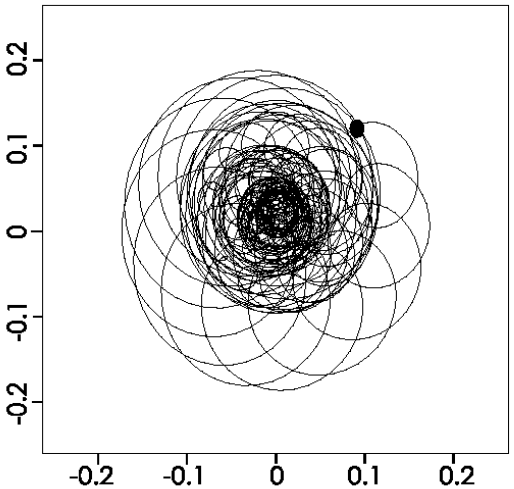




(d) Single ion

Results of axialisation experiments



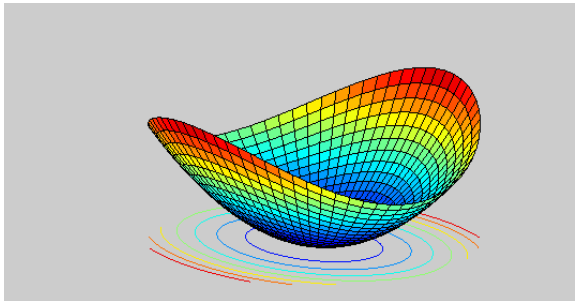


Effect of quadrupole drive with cyclotron damping

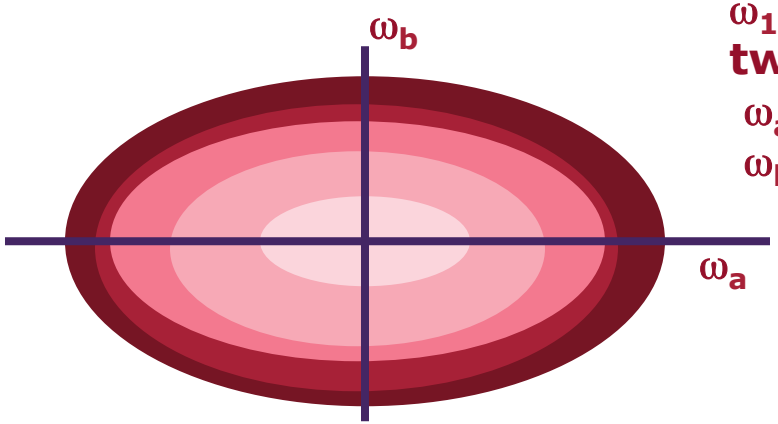
- Equivalent to two coupled and damped simple harmonic oscillators
- There are therefore two 'normal modes' when the axialisation drive is close to resonance
  - We see an avoided crossing with a gap equal to the coupling strength

Rotating frame picture

- For axialisation we apply an oscillating quadrupole field at ω_c.
- This can be decomposed into two counter-rotating quadrupoles at frequency $\omega_c/2$
  - One of them is therefore stationary in the rotating frame
  - It "squeezes" the potential
  - The potential in this frame is no longer cylindrically symmetrical
  - The normal modes are now **linear oscillations** parallel and perpendicular to the axis of the "squeeze"
    - » These frequencies are slightly different
  - If the initial condition is circular motion in one direction this sets **both** normal modes in motion and this gives beats between them
  - The particle oscillates between clockwise (cyclotron) and counterclockwise (magnetron) rotation



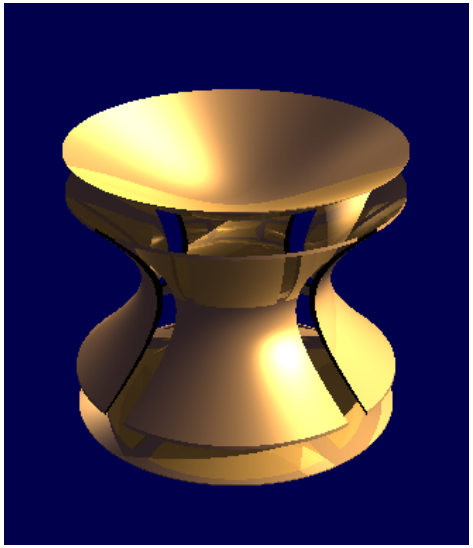
Use of oscillating field to force alignment



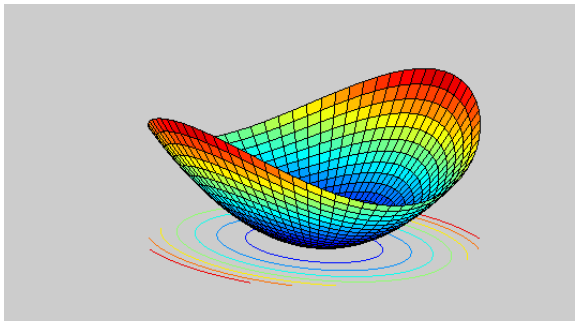
ω_1 splits into two modes:

$\omega_a < \omega_1$

$\omega_b > \omega_1$



- With a small number of cold ions this can be used to force the particles to line up along the "soft" axis (ω_a)

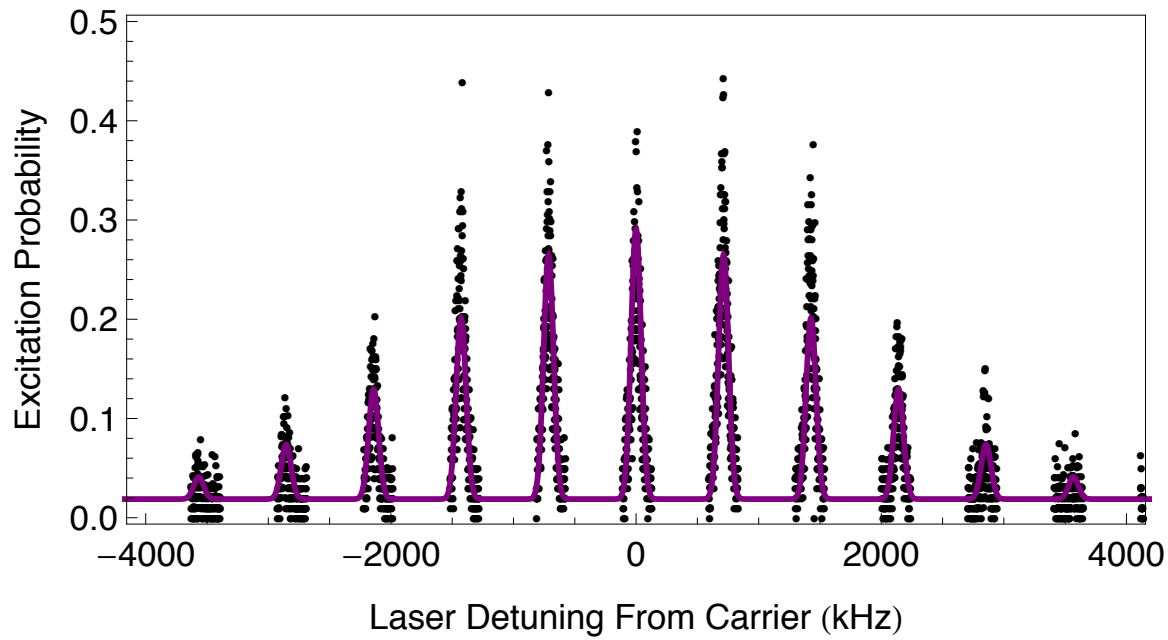


Relation to the Rotating Wall

- Axialisation is the application of an *oscillating quadrupole at* ω_c
  - It can be decomposed into rotating quadrupoles at $\omega_c/2$
  - In general axialisation is used as a *resonant* process in a single (or few) particle system to couple the centre of mass frequencies
- The Rotating Wall is the application of a *rotating quadrupole* at some frequency ω_R
  - It's used to force a plasma of many particles to rotate at ω_R
  - (it is often also used with a rotating *dipole*)
- If $\omega_R = \omega_c/2$ then then we have Brillouin flow and the techniques are (nearly) equivalent

Radial spectrum at low potential

- The (fast) cyclotron motion gives rise to sidebands
- The ~4 MHz FWHM corresponds to a cyclotron temperature of ~7 mK
- Each cyclotron sideband has structure due to the magnetron motion
  - but individual sidebands are not resolved here



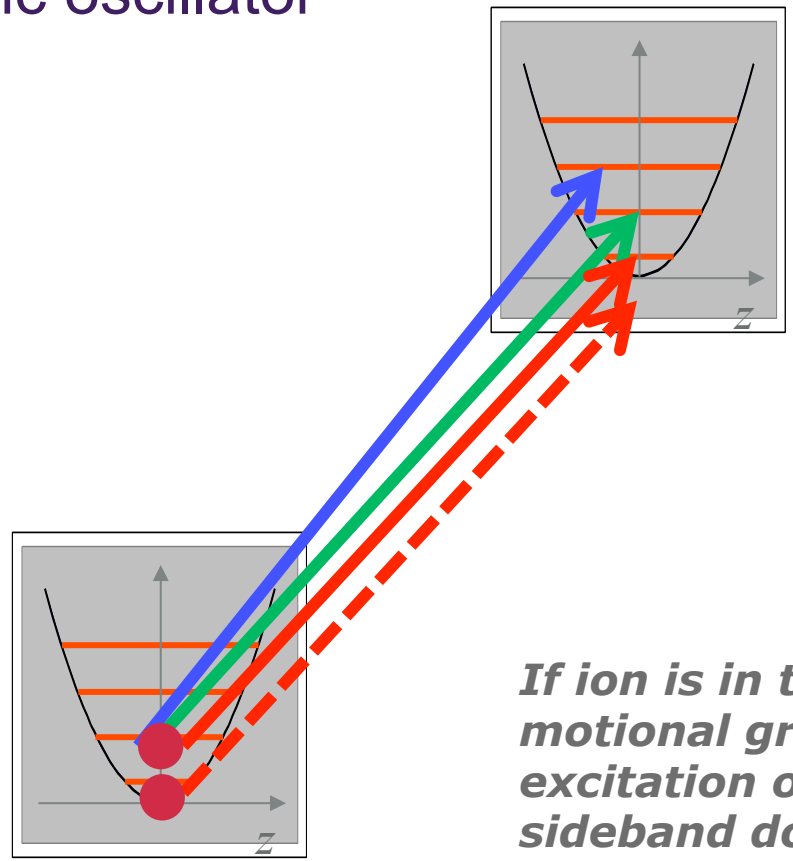
The narrow width of the magnetron structure demonstrates that its "temperature" is very low (~40 μK)

See Mavadia et al
*Phys. Rev. A **89**, 032502*

Driving Carrier and Sideband Transitions

A single ion in a trap is an example of a QM simple harmonic oscillator

- Carrier
- Blue Sideband
- Red Sideband



|e>

|g>

What if the ion is already in the motional ground state?

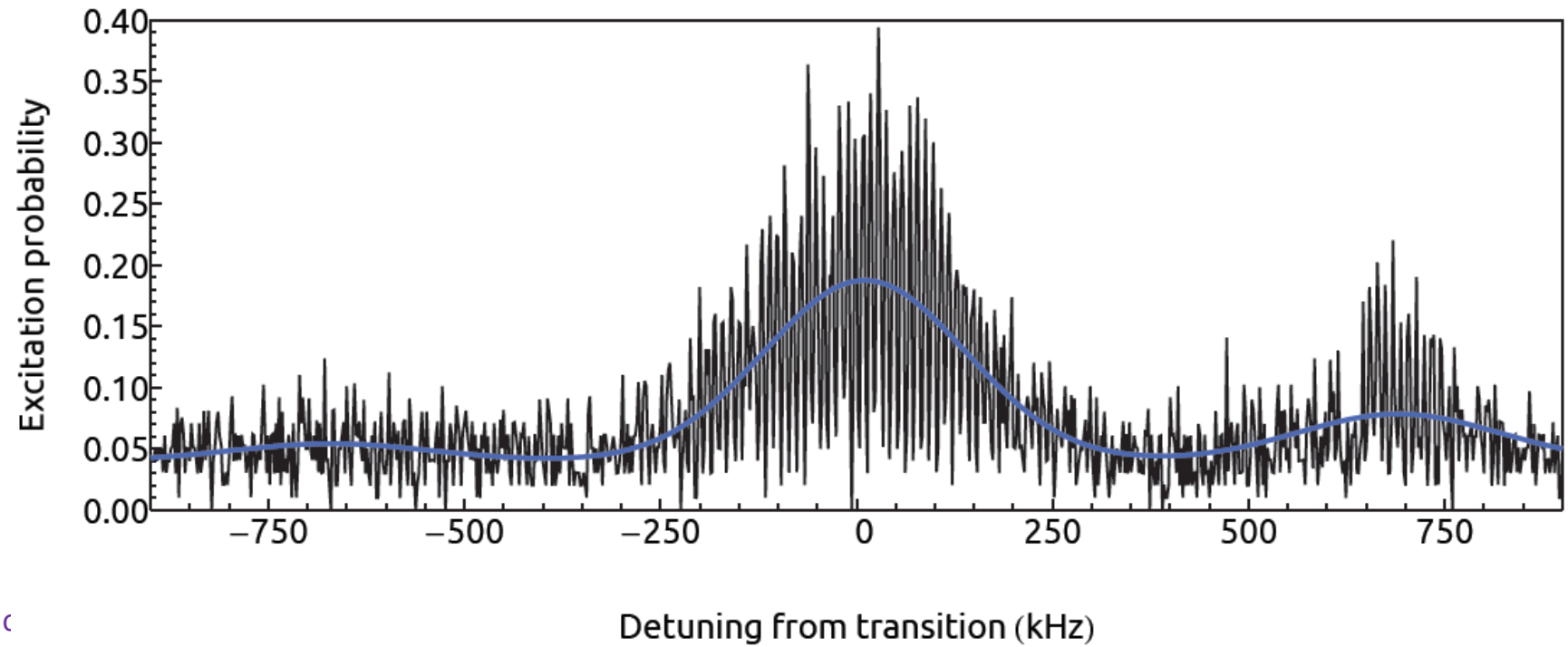
If ion is in the motional ground state excitation on the red sideband does nothing!

Problems for radial sideband cooling

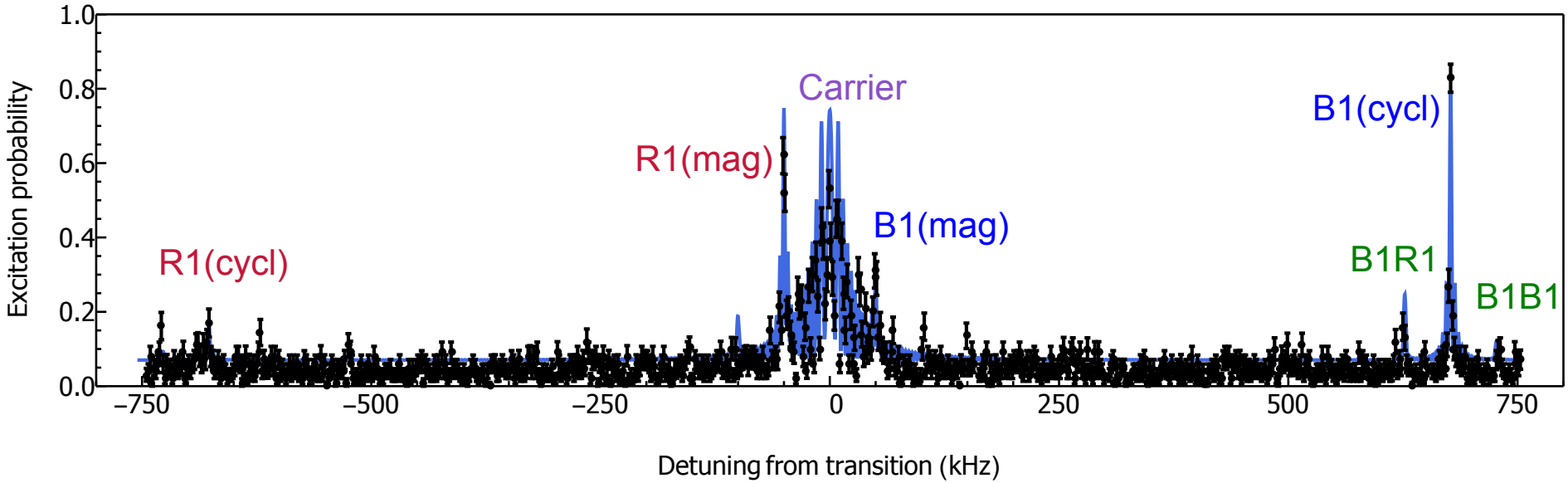
- Need to cool two modes at the same time
  - We have gained experience of this with ion crystals
- The magnetron sidebands are unresolved
  - Increase trap voltage to raise magnetron frequency
- The magnetron energy is negative
  - Cool on the ***blue*** sidebands of magnetron motion, not ***red***
- The initial quantum number of magnetron motion is very large (n up to 1000 in some cases after Doppler cooling)
  - Use the axialisation technique to couple to cyclotron motion

Radial cooling – first results

- The cyclotron motion can be cooled by driving its first red sideband
  - The spectrum shows that the cyclotron motion is close to the ground state



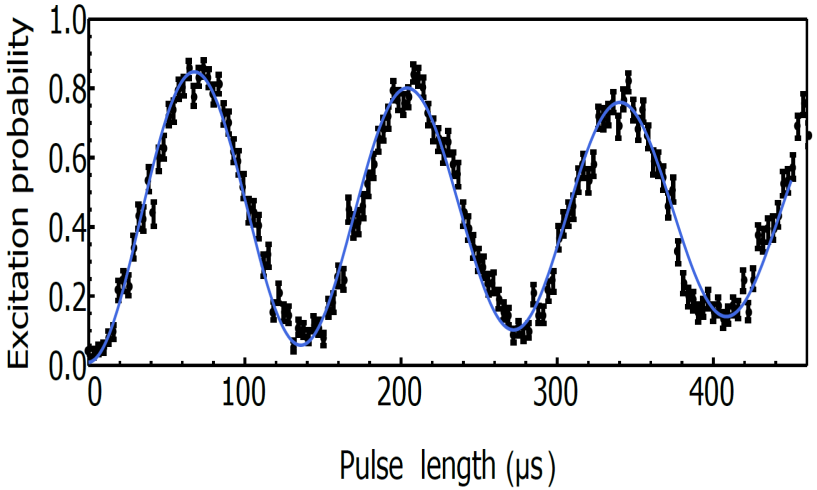
Sideband cooled radial spectrum



- This only works with the use of strong axialisation
- The carrier is very strong to bring out the other sidebands
- The asymmetry in cyclotron sidebands indicates n_c=0.07±0.03
- The (reversed) asymmetry in the magnetron sidebands indicates n_m=0.40±0.06
- Weak second-order sidebands can also be seen

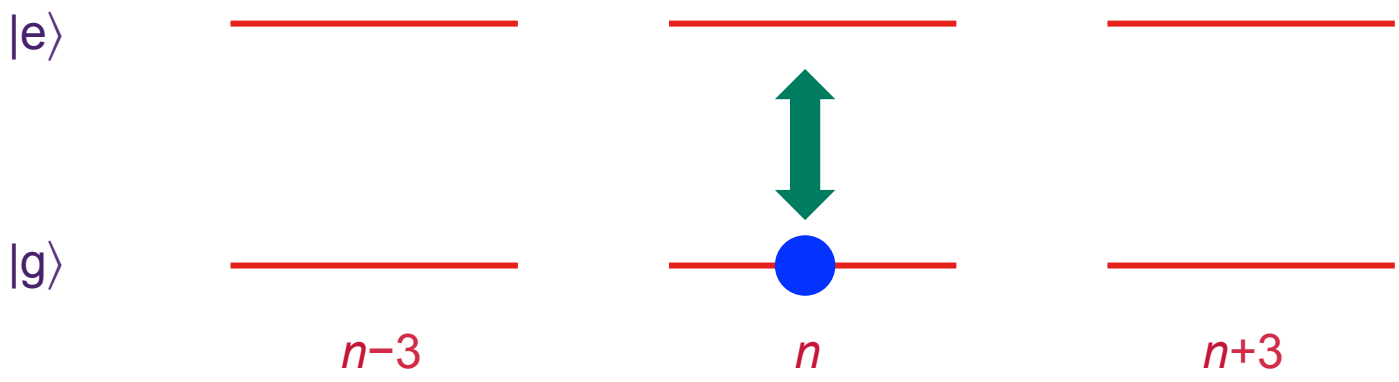
Sideband heating

- Sideband cooling takes us from high *n* towards *n=0*
- By driving on the *blue* sideband instead of the *red* sideband, we get sideband heating
- The result is that we can prepare the system in a narrow range of *n* (up to *n*~100 or higher) in a controlled way
- We observe coherent behaviour after this heating



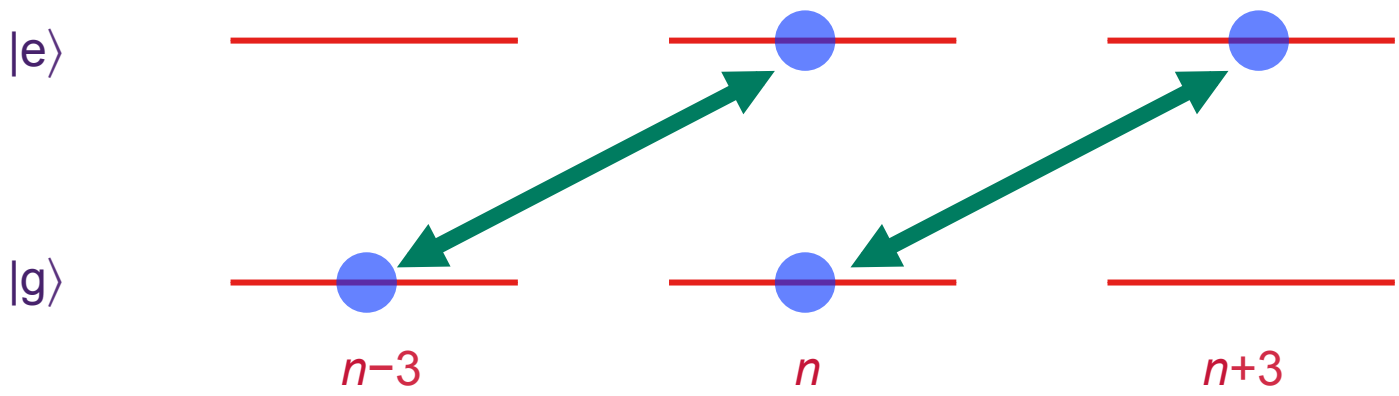
Rabi oscillations on 4th red sideband around *n*=280

Preparation of superposition of high-*n* states



- A π/2 carrier pulse creates a coherent superposition of $|g,n\rangle$ and $|e,n\rangle$

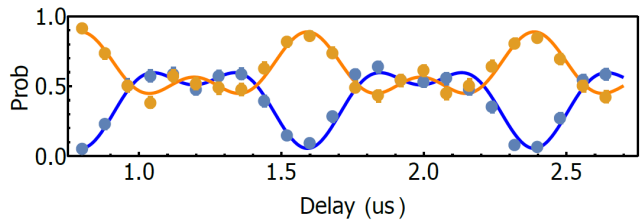
Preparation of superposition of high-*n* states



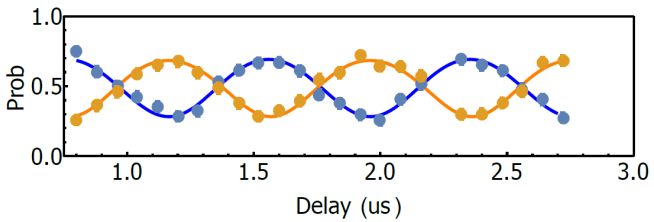
- A π/2 carrier pulse creates a coherent superposition of $|g,n\rangle$ and $|e,n\rangle$
- A π/2 B3 pulse then creates a coherent superposition of $|g,n\rangle$, $|g,n-3\rangle$, $|e,n\rangle$ and $|e,n+3\rangle$
- Period of free evolution T
- Probe the coherence with a second pair of pulses on B3 and carrier (with variable phases)

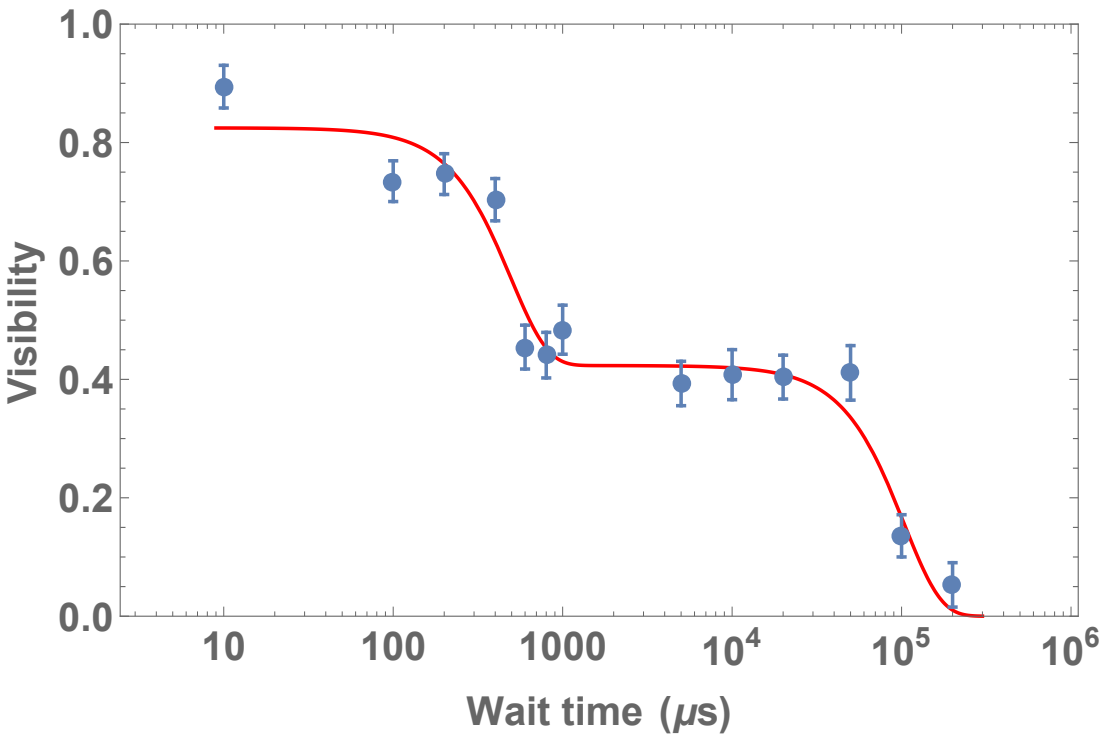
Coherence measurements

10 μs



10 ms





- At small *T* we see fringe visibility ~1
- After 1 ms the optical coherence is lost and the visibility drops to ~0.5
- Motional coherence is preserved out to ~100 ms for Δn=3
- *Again we see that the Penning trap is a well-controlled system with unique properties*

Conclusion

- Penning traps are really good for a wide variety of experiments in different fields of physics

- **Thanks for listening!**