Cooling **trapped** atoms with lasers

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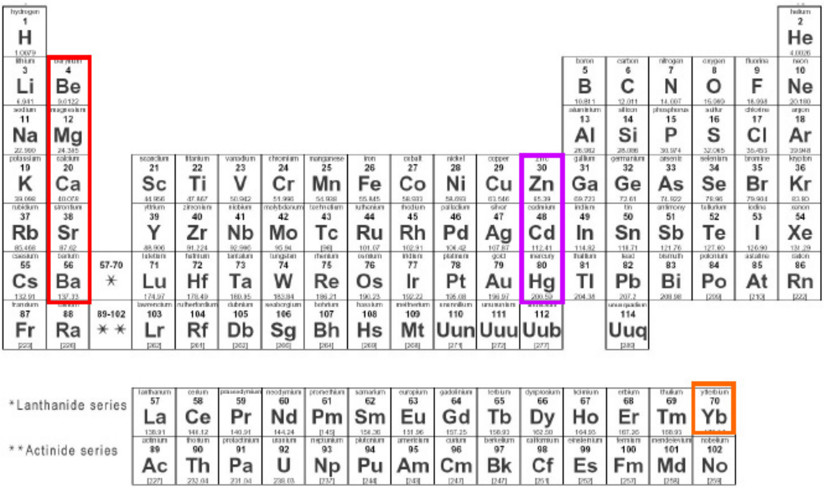
Trapped Charged Particles Winter School,
Les Houches january 2018

Outline

1. what's new with a bounded motion?
2. the strong binding regime
3. Resolved sideband cooling
4. ...and beyond

What ions are concerned?



motion induced effects in atom-laser interaction

- are induced by the interaction modulation

$$V_{AL}(\mathbf{r},t) = -\mathbf{d}_{eg}.E_L(\mathbf{r})\epsilon_L \cos(\omega t - \Phi(\mathbf{r}))$$

- role of the reduction of the motion amplitude first demonstrated in 1952 and called the Lamb-Dicke effect

PHYSICAL REVIEW VOLUME 89, NUMBER 2 JANUARY 15, 1953

The Effect of Collisions upon the Doppler Width of Spectral Lines

R. H. DICKE
Palmer Physical Laboratory, Princeton University, Princeton, New Jersey
(Received September 17, 1952)

Quantum mechanically the Doppler effect results from the recoil momentum changing the translational energy of the radiating atom. The assumption that the recoil momentum is given to the radiating atom is shown to be incorrect if collisions are taking place. If the collisions do not cause broadening by affecting the internal state of the radiator, they result in a substantial narrowing of the Doppler broadened line.

- if $\Delta r \ll \lambda_L$, the motion has no effect...

what fails from our previous analysis???

- the broad line picture is not true anymore : the relevant timescale T_{ext} for atom external dynamics is shorter than $\tau_e = 1/\Gamma$.

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- the broad line picture is not true anymore : the relevant timescale T_{ext} for atom external dynamics is shorter than $\tau_e = 1/\Gamma$.
- we have to keep the position dependance in the atom-laser interaction.
- let's assume the atom oscillates along the laser propagation direction along Ox

$$x(t) \simeq X\cos(\omega t)$$

$x(t) = X\cos(\omega_x t)$ in the classical limit

1. the motion induced phase modulation of the atom-laser interaction is periodic :

$$e^{ik_L x(t)} = e^{ik_L X\cos\omega_x t} = \mathcal{J}_0(k_L X) + i\mathcal{J}_1(k_L X)e^{\pm i\omega_x t} - \mathcal{J}_2(k_L X)e^{\pm i2\omega_x t}\ldots$$

   $\mathcal{J}_n$ are Bessel functions and $k_L X$ is the modulation index
2. from the atom point of view, it is like the laser emits several (coherent) lines with angular frequencies ω_L, $\omega_L \pm \omega_x$, $\omega_L \pm 2\omega_x \ldots$.

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3. central band (ω_L) : $\Omega_L \mathcal{J}_0(k_L X)$,
   first sidebands ($\omega_L \pm \omega_x$) : $\Omega_L \mathcal{J}_1(k_L X)$,
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4. in the low saturation case

$$P_e(\Delta) = \sum_{m=-\infty}^{m=+\infty} \frac{1}{2} \frac{\Omega_L^2 \mathcal{J}_m^2(k_L X)}{\Omega_L^2 \mathcal{J}_m^2(k_L X) + 2(\Delta - m\omega)^2 + \Gamma^2/2}$$

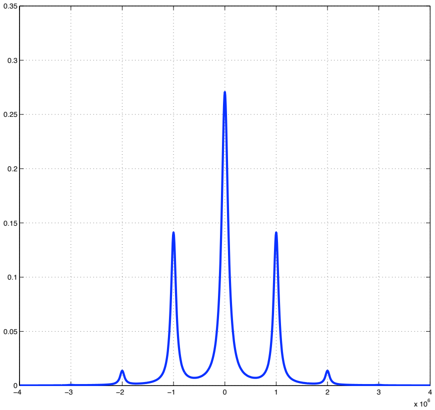
$x(t) = X\cos(\omega_x t)$ in the classical limit

$$k_L X = 1$$

$$\omega/2\pi = 1\mathrm{MHz}$$

$$\Omega_L = \Gamma$$

$$\Gamma/2\pi = 10^5\ \mathrm{Hz}$$



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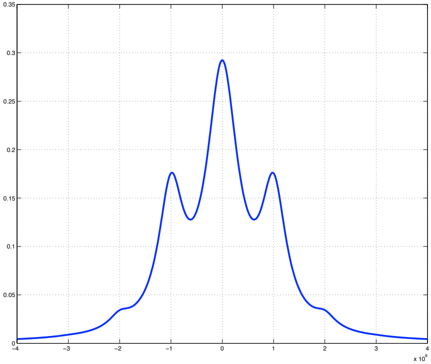

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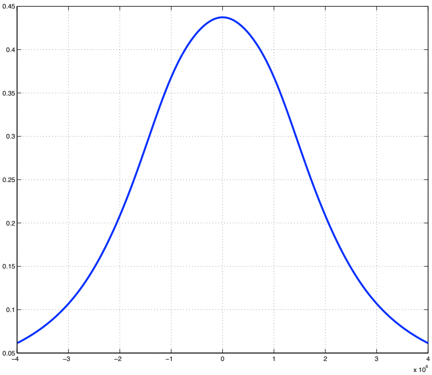

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- for excitation on a broad transition ($\Gamma > \omega_x$), the oscillating nature of the motion is not very relevant to describe the spectrum.
- indeed, it can be shown that for $\Gamma \gg \omega_x$, the ions can be treated as free (regime often called the "weak binding regime", which fills the broad line condition).

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- for $\Gamma \ll \omega_x$ and a low temperature, this classical description of the motion is not sufficient to explain the observed spectrum.

a more realistic case

In a realistic experiment, the laser does not propagate along a symmetry axis of the trap. In a RF trap, the motion can include non negligible μmotion and even excess μmotion. Then

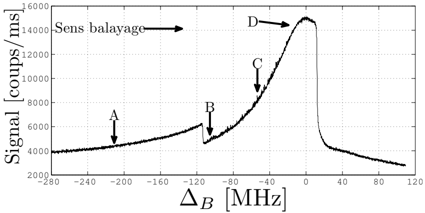
$$\mathbf{k}_L.\mathbf{r} = k_x(X\cos\omega_x t+\epsilon_x)(1+\frac{q_x}{2}\cos\Omega t)+k_z(Z\cos\omega_z t+\epsilon_z)(1+\frac{q_z}{2}\cos\Omega t)$$

and the same spectral decomposition in Bessel function can be done with several frequencies, as long as these frequencies are not in a rational ratio.

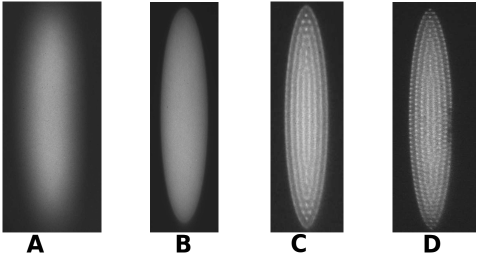
Doppler laser cooling of a large sample of Ca^+

Fluorescence spectra while scanning the cooling laser

Fluorescence sent to a PM



Picture of the cloud along a spectrum



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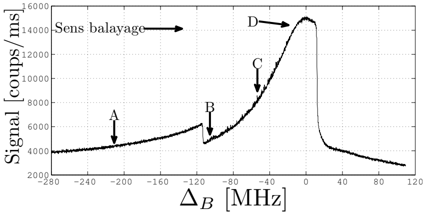

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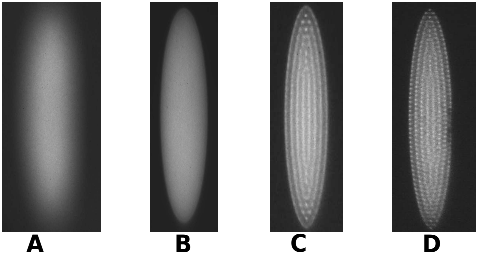
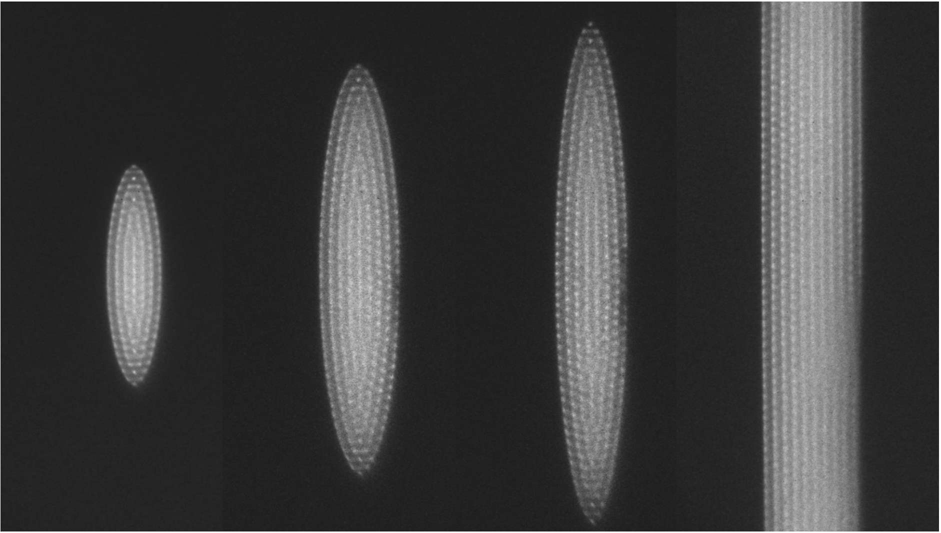


the abrupt transition is the signature of the competition between laser cooling and RF heating

R. Blümel *et al* Nature, (1988)
L. Hornaeker, M. Drewsen PRA **66**, 013412 (2002)

In a quadrupole trap, from liquid to crystal:



Being in the strong binding regime

There are two ways to reach this regime :

- choose an ion with a dipole forbidden transition. In several cases, it is an electric quadrupolar transition ($S \rightarrow D$), with natural linewidth ≤ 1 Hz. This is the case for Ca^+, $Sr^{\ +}$, Ba^+, Ra^+, Yb^+, Hg^+. Let's also mention experiments with In^+, making use of an intercombination line.

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- use Raman coupling by two lasers between two sub-levels (Zeeman or hyperfine) of the ground state and an extra excited state. The system behaves like a two-level system with infinite lifetimes.
  This is commonly used with Be^+, Mg^+, Zn^+, Cd^+.

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$$\overline{n} = \frac{e^{-\hbar\omega_x/k_B T_D}}{1 - e^{-\hbar\omega_x/k_B T_D}} = \frac{e^{-2\omega_x/\Gamma_{DA}}}{1 - e^{-2\omega_x/\Gamma_{DA}}}$$

In the limit where $\omega_x \ll \Gamma_{DA}/2$ (DA for "dipole allowed"),

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- for low temperature, a quantum description of the motion must be included in the Hamiltonian

Summary of useful equations for H.O:

$$\hat{H}_0^m = \frac{\hat{p}^2}{2m} + \frac{m\omega_x^2}{2}\hat{x}^2 \Rightarrow \hat{H}_0^m = \hbar\omega_x(\hat{a}^\dagger\hat{a} + \frac{1}{2})$$

$$\begin{aligned} \hat{x} &= \sqrt{\frac{\hbar}{2m\omega_x}}(\hat{a}^\dagger + \hat{a}) = x_0(\hat{a}^\dagger + \hat{a}) \\ \hat{p} &= i\sqrt{\frac{m\hbar\omega_x}{2}}(\hat{a}^\dagger - \hat{a}) \end{aligned} \tag{1}$$

$$[\hat{a}, \hat{a}^\dagger] = \hat{1} \qquad \hat{N} = \hat{a}^\dagger\hat{a}$$

$$\begin{aligned} \hat{a}^\dagger|n\rangle &= \sqrt{n+1}|n+1\rangle \\ \hat{a}|n\rangle &= \sqrt{n}|n-1\rangle \end{aligned} \tag{2}$$

time evolution

- the "*clean*" way to get rid of the free evolution and keep the evolution induced by atom-laser coupling and further do the RWA is to use the Interaction Picture (half way between Schrödinger and Heisenberg)

$$\hat{V}^I = \hat{\mathcal{U}}_0^\dagger(t)\hat{V}\hat{\mathcal{U}}_0(t) \qquad \hat{\mathcal{U}}_0(t) = \exp(-i\hat{H}_0 t/\hbar)$$

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- if the motion is not quantised $\hat{H}_0 = \frac{\hbar\omega_0}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$

$$\hat{V}^I_{AL} = \frac{\hbar\Omega_1}{2}\left(|e\rangle\langle g|e^{-i\Delta_L t}e^{i(k_L x+\psi)} + |g\rangle\langle e|e^{i\Delta_L t}e^{-i(k_L x+\psi)}\right)$$

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- when the motion is also quantised, the self Hamiltonian of the system is $\hat{H}_0 + \hat{H}_0^m$ (with eigenstates $|g\rangle|n\rangle, |e\rangle|m\rangle$):

$$\hat{V}^I_{AL} = \frac{\hbar\Omega_1}{2}\left(|e\rangle\langle g|e^{-i(\Delta_L t-\psi)}e^{i\hat{H}_0^m t/\hbar}e^{ik_L\hat{x}}e^{-i\hat{H}_0^m t/\hbar} + H.c.\right)$$

time evolution

- by using the relation

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-1/2[\hat{A},\hat{B}]} \quad \text{if} \quad [\hat{A},[\hat{A},\hat{B}]] = [\hat{B},[\hat{A},\hat{B}]] = 0$$

and by using $\exp(\hat{A}) = \hat{1} + \hat{A} + \hat{A}^2/2 + \hat{A}^3/6 + \ldots$ one can demonstrate these useful relations to move to the interaction picture:

$$\hat{a}^n e^{-i\omega t \hat{a}^\dagger \hat{a}} = e^{-i\omega t \hat{a}^\dagger \hat{a}} e^{-in\omega t} \hat{a}^n \quad (3)$$

$$\hat{a}^{\dagger n} e^{-i\omega t \hat{a}^\dagger \hat{a}} = e^{-i\omega t \hat{a}^\dagger \hat{a}} e^{in\omega t} \hat{a}^{\dagger n} \quad (4)$$

$$e^{i\eta(\hat{a}+\hat{a}^\dagger)} = e^{i\eta\hat{a}} e^{i\eta\hat{a}^\dagger} e^{-\eta/2} \quad (5)$$

- and then, finally:

$$\hat{V}^I_{AL} = \frac{\hbar\Omega_1}{2}\left(|e\rangle\langle g| e^{-i(\Delta_L t-\psi)} \exp\left(i\eta\hat{a}^\dagger e^{i\omega_x t} + i\eta\hat{a} e^{-i\omega_x t}\right) + H.c.\right)$$

with $\eta = k_L x_0$, the Lamb-Dicke parameter (x_0 is the size of the ground vibrational wave-packet).

motion and internal state are coupled in the transition

- For each transition modifying the vibrational state, an effective Rabi frequency can be defined by

$$\Omega_{n,n+p} = \Omega_L |\langle n+p| e^{i\eta(\hat{a}+\hat{a}^\dagger)} |n\rangle|$$

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- We can gain insight by making a Taylor expansion of $\hat{V}^I_{AL}$ with η. Up to the second order in η:

$$\begin{array}{ll}
n \rightarrow n & \Omega n, n = \Omega_L(1-\eta^2 n) \\
n \rightarrow n+1 & \Omega n, n+1 = \Omega_L \eta \sqrt{n+1} \\
n \rightarrow n-1 & \Omega n, n-1 = \Omega_L \eta \sqrt{n} \\
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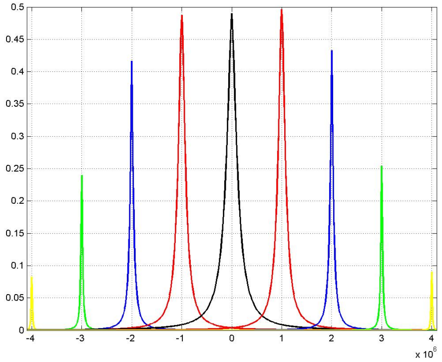
- many experiments implying high precision spectroscopy (QI, metrology...) requires to reach this regime where the Doppler spectra is reduced to the main carrier (central) band and small first order sideband.

the quantum limit

thermal equilibrium

$\Omega/2\pi = 1$ kHz,
$\Gamma_L/2\pi = 10$ kHz
$^{40}Ca^+$,$\lambda_L = 730$ nm
$\omega/2\pi = 1$ MHz :
$\eta_0 = 0.094$
$T = 2.5$ mK
$\overline{n} = 51.6$



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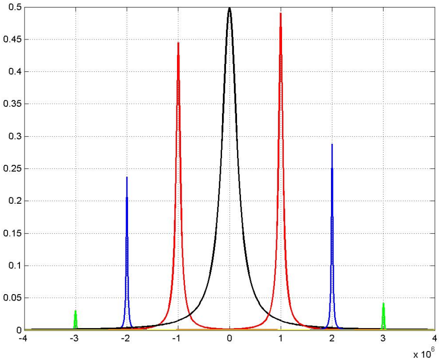

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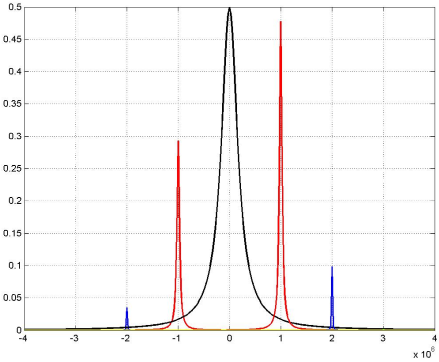

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$$\eta = \frac{2\pi}{\lambda_L}\sqrt{\frac{\hbar}{2m\omega_x}}$$

- building trap with large ω_x is very important as it makes η smaller.
- for a given temperature, the averaged occupation number $\bar{n}$ is smaller

$$\bar{n} = \exp(-\hbar\omega_x/k_B T)/(1 - \exp(-\hbar\omega_x/k_B T))$$

In the Lamb-Dicke regime $\exp(ik_L x) \simeq 1 + ik_L x$

$$\hat{V}^I = \frac{\hbar\Omega_1}{2}|e\rangle\langle g| \left(\hat{1}e^{-i(\Delta_L t-\psi)} + i\eta\hat{a}^\dagger e^{-i(\Delta_L-\omega_x)t-\psi} + i\eta\hat{a}e^{-i(\Delta_L+\omega_x)t-\psi} + H.$$

the laser interaction reduced to three components:

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n \to n & \qquad \Omega n, n = \Omega_1 \\
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\end{aligned}$$

This is very close to the Jaynes-Cummings hamiltonian (but here, there is the anti-Jaynes-Cumming terms!)
You can treat each line as an independent line for an atom at rest.

a global view

- If the recoil frequency $\omega_{rec} = \hbar k_L^2/2m$ and $\omega_{x,y,z} \ll \Gamma$, then the usual Doppler cooling process works and the semi-classical treatment can be applied.

$$\begin{array}{ccccccc} \Gamma_{DF} & < & \omega_{rec} & < & \omega_{x,y,z} & < & \Gamma_{DA} \\ \simeq 1\text{ Hz} & & \simeq 10\text{ kHz} & & \simeq 1\text{ MHz} & & \simeq 10\text{ MHz} \end{array}$$

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- Contrary to neutral atom Doppler cooling, a single direction of propagation is enough to cool trapped ions, as long as this direction has projection on all the symmetry axis of the trap.

Resolved sideband cooling, 1st exp in 1989 NIST, Hg+

$$\hat{V}^I = \frac{\hbar\Omega_1}{2}|e\rangle\langle g| \left(\hat{1}e^{-i(\Delta_L t-\psi)} + i\eta\hat{a}^\dagger e^{-i(\Delta_L-\omega_x)t-\psi} + i\eta\hat{a}e^{-i(\Delta_L+\omega_x)t-\psi} + H.\right.$$

- the laser detuning aims at the first red sideband to take away one phonon on each photon absorption

$$\Omega n, n-1 = \Omega_1\eta\sqrt{n}$$

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- You increase the scattering rate (1 photon/s is not enough!) : $\Gamma_{DF} \rightarrow \Gamma_e \ll \omega_x$

How efficient is it? (S. Stenholm, in 1986)

- you consider the saturation is low and that the probability to be in the excited state you aim is

$$P_e = \frac{\Omega^2}{\Gamma_e^2}\frac{\Gamma_e^2}{4\Delta^2+\Gamma_e^2} = \frac{\Omega^2}{\Gamma_e^2}W(\Delta)$$

- excitation on the central band has the strength $\Omega = \Omega_1$
  excitation on the first red sideband has the strength $\Omega = \Omega_1\eta\sqrt{n}$
- spontaneous emission also suffer from the change in vibration quantum number : the spontaneous scattering rate on $n \to n+1$ is

$$\Gamma_e\eta_e^2(n+1)$$

How efficient is it?

$R^{\pm}$ is the rate for transition from $|g,n\rangle$ to $|g,n\pm 1\rangle$.

$$
\begin{aligned}
R^{+} &= W(\delta)\frac{\Omega_L^2}{\Gamma_e^2}\times\Gamma_e\eta_e^2(n+1) + W(\delta-\omega_x)\frac{\Omega_L^2\eta^2(n+1)}{\Gamma_e^2}\times\Gamma_e \quad (6)\\
&= \frac{\Omega_L^2}{\Gamma_e}\left(\eta_e^2 W(\delta)+\eta^2 W(\delta-\omega_x)\right)(n+1) = A^{+}(n+1). \quad (7)\\
R^{-} &= \frac{\Omega_L^2}{\Gamma_e}\left(\eta_e^2 W(\delta)+\eta^2 W(\delta+\omega_x)\right) n = A^{-} n. \quad (8)
\end{aligned}
$$

the time evolution of the averaged vibrational number is

$$
\begin{aligned}
\frac{\mathrm{d}\overline{n}}{\mathrm{d}t} &= R^{+}(+1)+R^{-}(-1) = A^{+}(\overline{n}+1) - A^{-}\overline{n} \quad (9)\\
&= (A^{+}-A^{-})\overline{n} + A^{+}. \quad (10)
\end{aligned}
$$

If $\delta = -\omega_x$ and $\omega_x^2 \gg \Gamma_e^2$

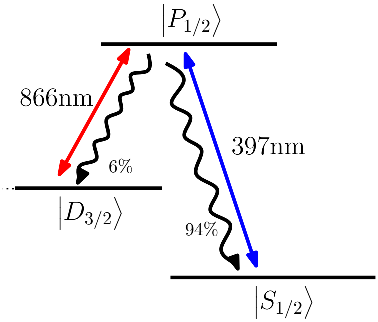
$$
\overline{n_f} \simeq \frac{A^{+}}{A^{-}} = \frac{\Gamma_e^2}{4\omega_x^2}\left(\frac{1}{4}+\frac{\eta_e^2}{\eta^2}\right) \ll 1
$$

two ions on the axis of a linear RF trap

$$
\begin{array}{lll}
\text{c.o.m}_x & \omega_{com} & = \omega_x \\
\text{stretching}_x & \omega_{str} & = \sqrt{3}\omega_{com} \\
\text{c.o.m}_y & & = \omega_y \\
\text{c.o.m}_z & & = \omega_z \\
x - y \text{ rocking} & & = \sqrt{\omega_y^2 - \omega_x^2} \\
x - z \text{ rocking} & & = \sqrt{\omega_z^2 - \omega_x^2}
\end{array}
$$

for N ions, $3N$ vibrational bands to cool....

An alternative to resolved SB cooling : EIT cooling



- Δ_B : detuning on the blue laser (397 nm)

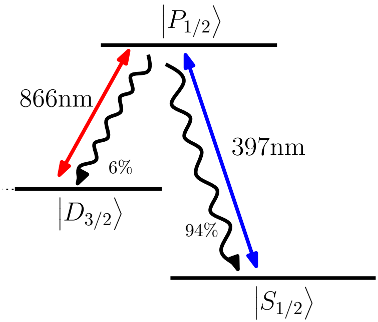
$$\Delta_B = \omega_{LB} - \omega_{PS}$$

- Δ_R : detuning on the red laser (866 nm)

$$\Delta_R = \omega_{LR} - \omega_{PD}$$

the linear combination of the 2 stable/metastable states $|\psi_{NC}\rangle$ is such that $\hat{H}_{AL}|\psi_{NC}\rangle = 0$

An alternative to resolved SB cooling : EIT cooling



$$
\begin{array}{ccc}
|P\rangle & |S\rangle & |D\rangle \\
\end{array}
\begin{pmatrix}
-\Delta_B & \frac{\Omega_B}{2} & \frac{\Omega_R}{2} \\
\frac{\Omega_B}{2} & 0 & 0 \\
\frac{\Omega_R}{2} & 0 & \Delta_R - \Delta_B
\end{pmatrix}
$$

$$
|\psi_{NC}\rangle = \frac{-\Omega_R |S\rangle + \Omega_B |D\rangle}{\overline{\Omega}}
$$

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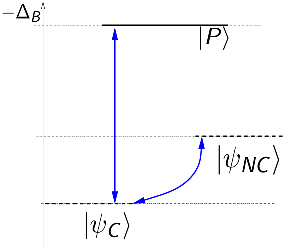

$$|\psi_{NC}\rangle = \frac{-\Omega_R\,|S\rangle + \Omega_B\,|D\rangle}{\overline{\Omega}}$$

$$|\psi_C\rangle = \frac{+\Omega_R\,|S\rangle + \Omega_B\,|D\rangle}{\overline{\Omega}}$$

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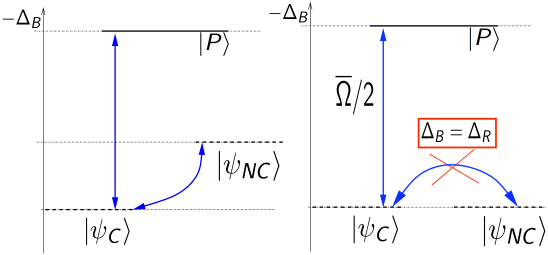

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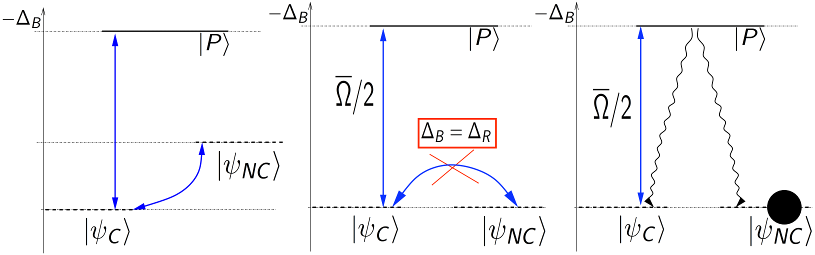

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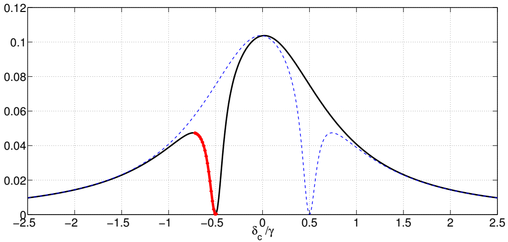
Coherent Population Trapping (CPT) in $|\psi_{NC}\rangle$

an alternative to resolved SB cooling : EIT cooling

Population in the P excited state vs $\delta_c/\gamma \equiv \Delta_B/\Gamma_P$.



$\tau_P = 7$ ns and the branching ratio to D is $1/15$ (Ca^+ typical parameter).
$\Omega_B = \Gamma_P/2$, $\Omega_R = \Gamma_P/5$.
Blue curve : $\delta_p \equiv \Delta_R = +\gamma/2$. Black curve : $\delta_p \equiv \Delta_R = -\Gamma/2$.

Quantised motion with EIT (Morigi *et al.* (2000))

- with $\delta_p = \delta_c$ (or $\Delta_B = \Delta_R$), the $n \to n$ central band is canceled
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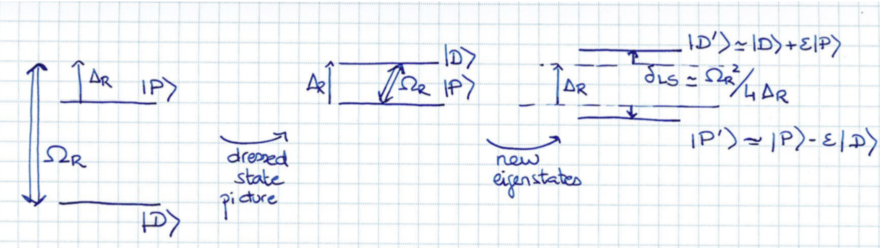

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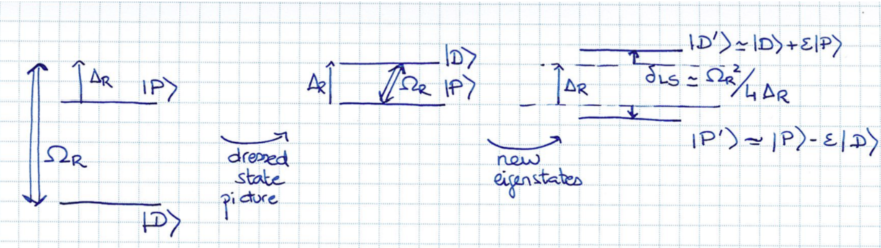
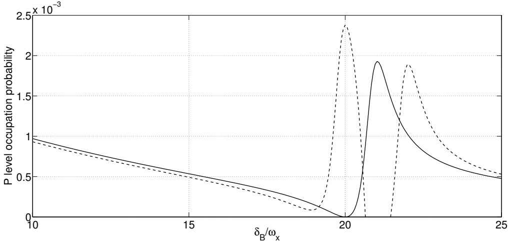


- the cooling transition is driven under saturation :
  for $\Delta_B = \Delta_R + \delta_{\rm LS}$, it drives $n \to n$
  for $\Delta_B = \Delta_R + \delta_{\rm LS} \pm \omega_x$, it drives $n \to n \pm 1$

Quantised motion with EIT

- the pump transition is tuned to have $\delta_{\rm LS} = \omega_x$
- then, for $\Delta_B = \Delta_R$, $n \to n$ is canceled **and** $n \to n-1$ is on resonance.

Semi-classic calculation for motion $X\cos(\omega_x t)$ with $k_c X = 0.2$



$$\delta_p = +20\omega_x \text{ and } \delta_{\rm LS} = \omega_x.$$

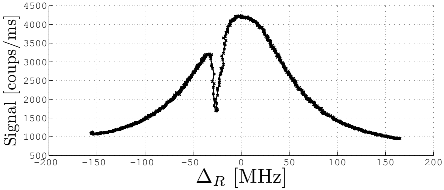
dark line : total probability to be in P state, dominated by the central band
dashed line : the ± 1 sideband contributions expanded by a factor of 50.

From the idea to realisation (Roos *et al.* (2000))

- The efficiency relies on a complete cancellation of the $n \to n$ transitions, obtained by the dark line.

In practice, CPT results in a grey line because of the relative phase drift of the two involved lasers.

$|\psi_{NC}\rangle \propto (-\Omega_R |S\rangle + \Omega_B |D\rangle)$



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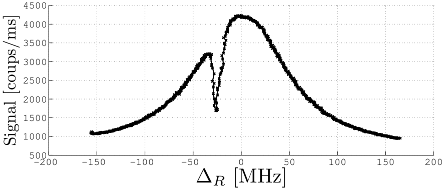

- Experimental demonstration of EIT cooling is based on two Zeeman sub-levels of the ground state.
- Different polarisation to address different Zeeman sub-levels.

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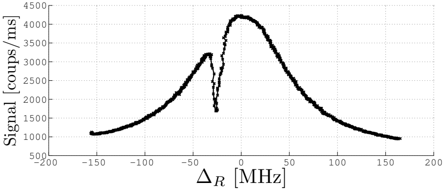

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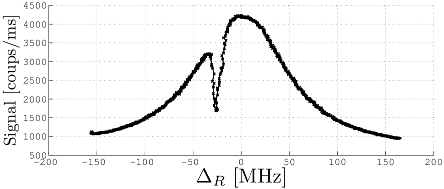

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- Different polarisation to address different Zeeman sub-levels.
- Any default in the polarisation may lead to a $n \rightarrow n$ or $n \rightarrow n + 1$ transition.
- The main advantage is that several vibrational bands can be addressed in the width of the EIT line.

From the idea to realisation...again (Roos *et al.* (2016))

PHYSICAL REVIEW A **93**, 053401 (2016)

Electromagnetically-induced-transparency ground-state cooling of long ion strings

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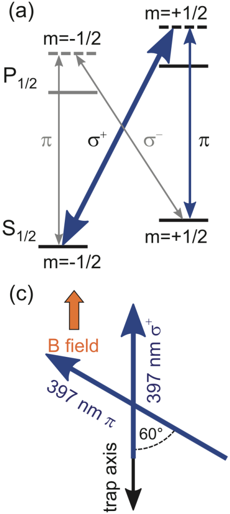
Michael Brownnutt,[2,†] Rainer Blatt,[1,2] and Christian F. Roos[1,2,‡]
[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, Technikerstraße 21a, 6020 Innsbruck, Austria*
[2]*Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*
(Received 18 March 2016; published 2 May 2016)

Electromagnetically-induced-transparency (EIT) cooling is a ground-state cooling technique for trapped particles. EIT offers a broader cooling range in frequency space compared to more established methods. In this work, we experimentally investigate EIT cooling in strings of trapped atomic ions. In strings of up to 18 ions, we demonstrate simultaneous ground-state cooling of all radial modes in under 1 ms. This is a particularly important capability in view of emerging quantum simulation experiments with large numbers of trapped ions.

From the idea to realisation...again (Roos *et al.* (2016))



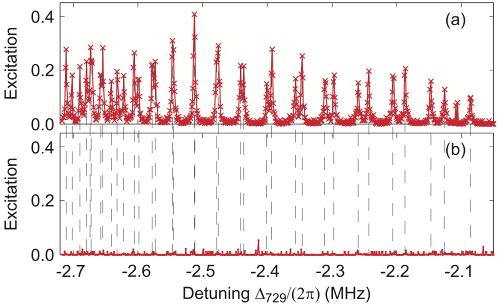
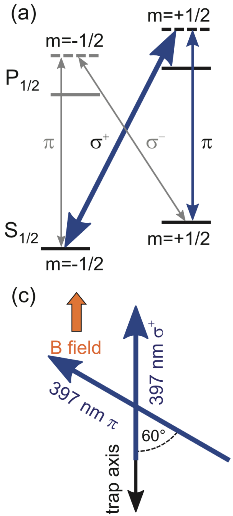


FIG. 5. (a) Red sideband radial mode spectrum of an 18-ion crystal after Doppler cooling. (b) Red sideband radial mode spectrum after Doppler cooling followed by 1 ms of EIT cooling with a σ^+ induced light shift set to 2.3 MHz. Dashed lines connecting (a) and (b) indicate the mode frequencies.

From the idea to realisation...again (Roos *et al.* (2016))



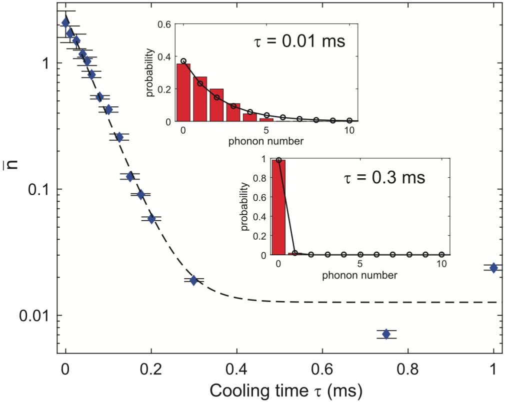


FIG. 7. EIT cooling dynamics. The radial mode at 2.08 MHz of an 18-ion crystal in a linear trap was EIT cooled with a σ^+ induced light shift of 2.3 MHz for varying cooling duration. A RAP pulse

Another alternative to resolved side-band cooling?

PRL 119, 043001 (2017) PHYSICAL REVIEW LETTERS week ending 28 JULY 2017

3D Sisyphus Cooling of Trapped Ions

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Department of Physics, Simon Fraser University, Burnaby, British Columbia, V5A 1S6, Canada

(Received 1 March 2016; revised manuscript received 11 May 2017; published 25 July 2017)

Using a laser polarization gradient, we realize 3D Sisyphus cooling of $^{171}Yb^+$ ions confined in and near the Lamb-Dicke regime in a linear Paul trap. The cooling rate and final mean motional energy of a single ion are characterized as a function of laser intensity and compared to semiclassical and quantum simulations. Sisyphus cooling is also applied to a linear string of four ions to obtain a mean energy of 1–3 quanta for all vibrational modes, an approximately order of magnitude reduction below Doppler cooled energies. This is used to enable subsequent, efficient sideband laser cooling.