Entanglement in trapped-ion experiments

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Innsbruck, Austria

Les Houches, January 19, 2018

Entanglement in trapped-ion experiments

- What is entanglement?
- Why are we interested in entanglement?
- Entanglement as a QIP resource
- Decoherence in ion trap experiments
- Classical vs quantum correlations
- Quantum state tomography

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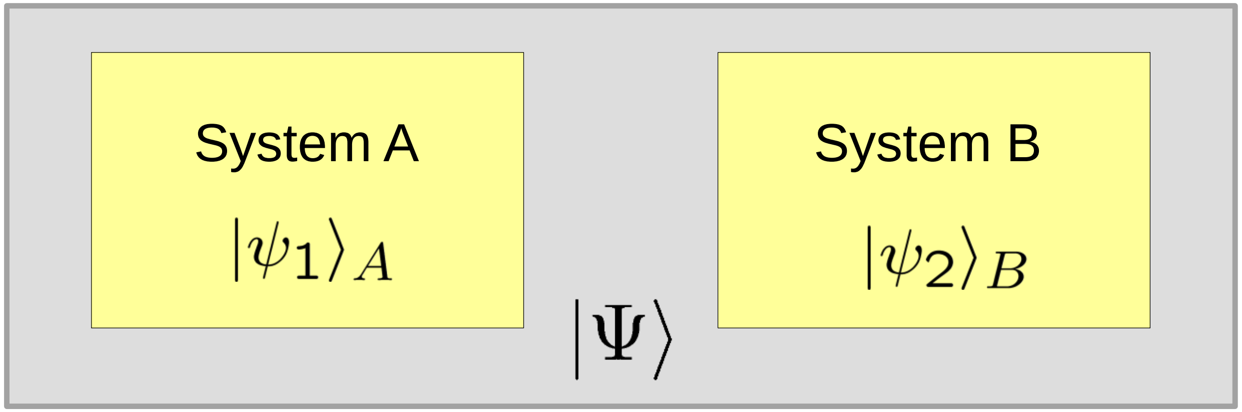
Entanglement of pure quantum states



Joint state

$$|\Psi\rangle \; = |\psi_1\rangle_A \otimes |\psi_2\rangle_B = |\psi_1\rangle|\psi_2\rangle$$

<u>product</u> or <u>separable</u> state

$$|\Psi_2\rangle = |\phi_1\rangle|\phi_2\rangle$$

Superposition principle

$$|\Psi\rangle \; = \alpha|\psi_1\rangle|\psi_2\rangle + \beta|\phi_1\rangle|\phi_2\rangle$$

<u>entangled</u> state

(unless α=0 or β=0 or $|\phi_1\rangle = |\psi_1\rangle$ or $|\phi_2\rangle = |\psi_2\rangle$)

A state $|\Psi\rangle$ is entangled if it cannot be written as a product state.

Pure quantum states: separable and entangled states

Examples:

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

maximally entangled

$$|\Psi_2\rangle = \frac{1}{2}(|00\rangle| + |01\rangle - |10\rangle + |11\rangle)$$

maximally entangled

$$= |0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) - |1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

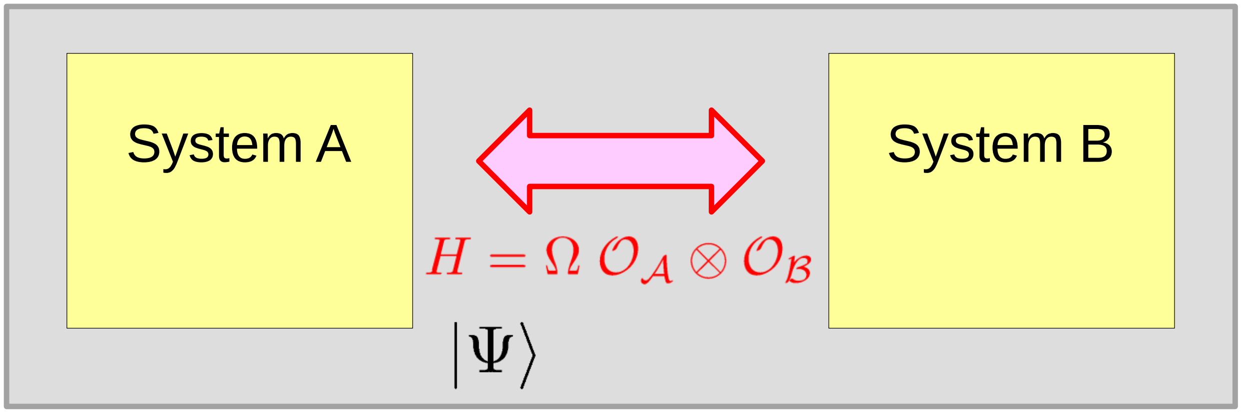
$$|\Psi_3\rangle = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle + |11\rangle)$$

separable

$$= \frac{1}{2}(|0\rangle - |1\rangle) \otimes (|0\rangle - |1\rangle)$$

Why are we interested in entanglement?

In isolated composite quantum systems,
entanglement is the rule, not the exception



A and B are brought into contact at time t and interact shortly:

$$|\Psi(t)\rangle = |\psi_A\rangle \otimes |\psi_B\rangle \longrightarrow |\Psi(t+\Delta\tau)\rangle = e^{i\Delta\tau H}|\psi_A\rangle \otimes |\psi_B\rangle$$

$$\approx (\mathbb{I} + i\Delta\tau H)|\psi_A\rangle \otimes |\psi_B\rangle$$

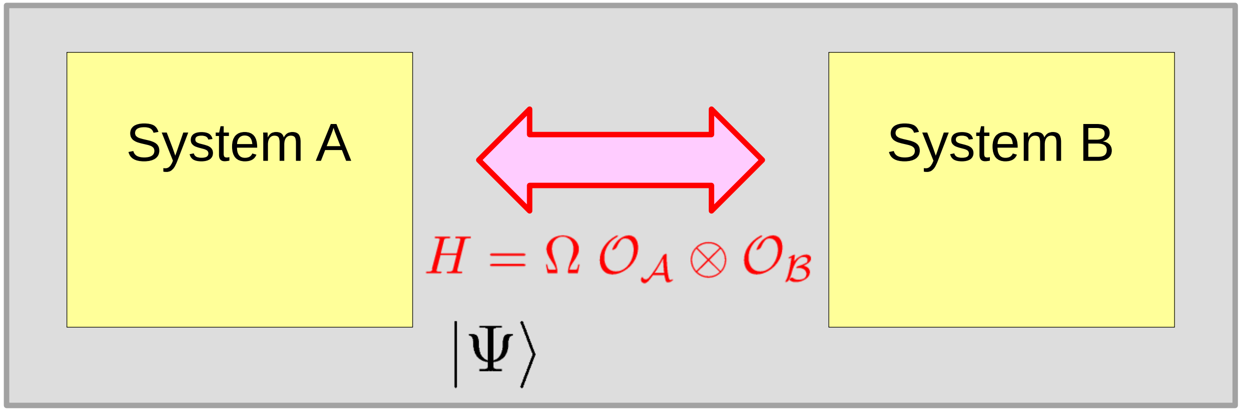
$$= |\psi_A\rangle \otimes |\psi_B\rangle + \mathcal{O}_{\mathcal{A}}|\psi_{\mathcal{A}}\rangle \otimes \mathcal{O}_{\mathcal{B}}|\psi_{\mathcal{B}}\rangle$$

unentangled

almost certainly entangled

Why are we interested in entanglement?

In isolated composite quantum systems, entanglement is the rule, not the exception

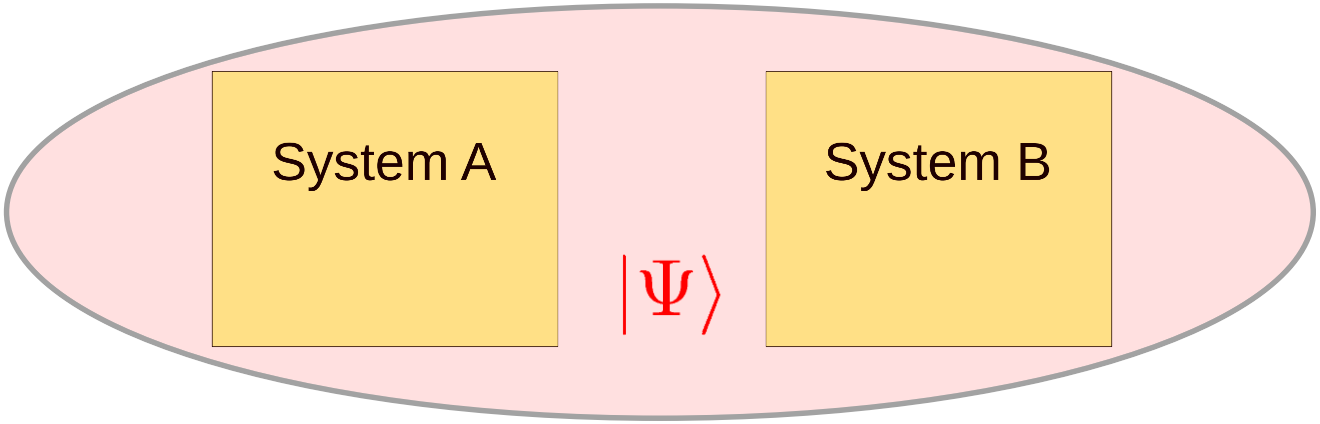


Because of the interaction

- the state of system A influences the dynamics of B and vice versa
- the systems exchange information in terms of quantum correlations

Why are we interested in entanglement?

Entanglement is a characteristic trait of quantum physics.



Bell inequalities:

There are entangled quantum states for which measurement of correlations in different measurement bases cannot be explained by models of hidden variables.

Why are we interested in entanglement?

Entanglement is a resource for
quantum information processing protocols

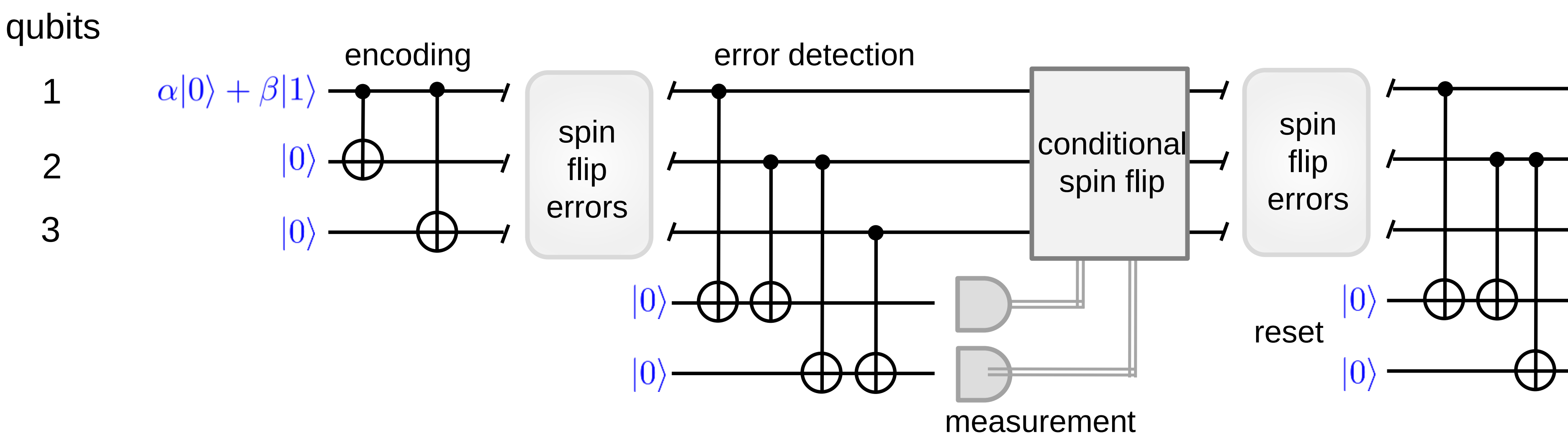
Applications:

- Quantum teleportation
  Transfer of quantum information in a quantum network
- Quantum cryptography
  Shared EPR pairs can be used for secure communication
- Quantum error correction
  Entanglement enables quantum error correction protocols
- …

Error correction example: correcting spin flip errors

qubit			statistical mixture of	probability:
1	$\alpha\|0\rangle + \beta\|1\rangle$	spin flip errors	$\alpha\|0\rangle + \beta\|1\rangle$	$1-p$
			$\alpha\|1\rangle + \beta\|0\rangle$	p

Quantum error correction of spin flip errors



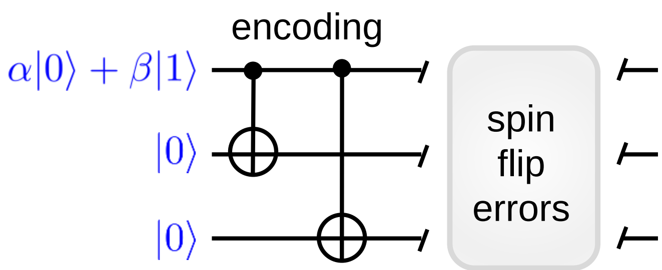
Quantum error correction of spin flip errors

qubits

1

2

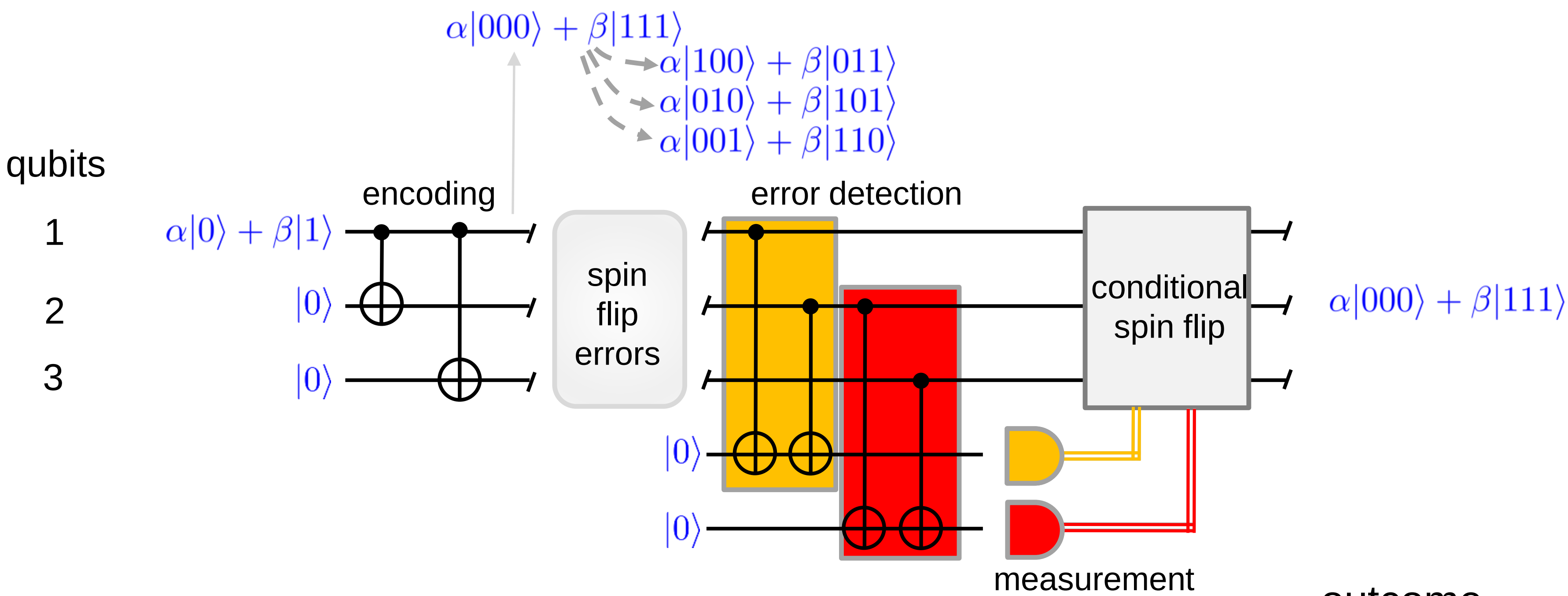
3



statistical mixture of	probability:
$\alpha\|000\rangle + \beta\|111\rangle$	$(1-p)^3 \approx 1-3p$
$\alpha\|100\rangle + \beta\|011\rangle$	$p(1-p)^2 \approx p$
$\alpha\|010\rangle + \beta\|101\rangle$	$p(1-p)^2 \approx p$
$\alpha\|001\rangle + \beta\|110\rangle$	$p(1-p)^2 \approx p$
$\cdots$	

assuming $p \ll 1$

Quantum error correction of spin flip errors



	outcome
qubits 1 and 2 in the same state:	0
qubits 2 and 3 in the same state:	0

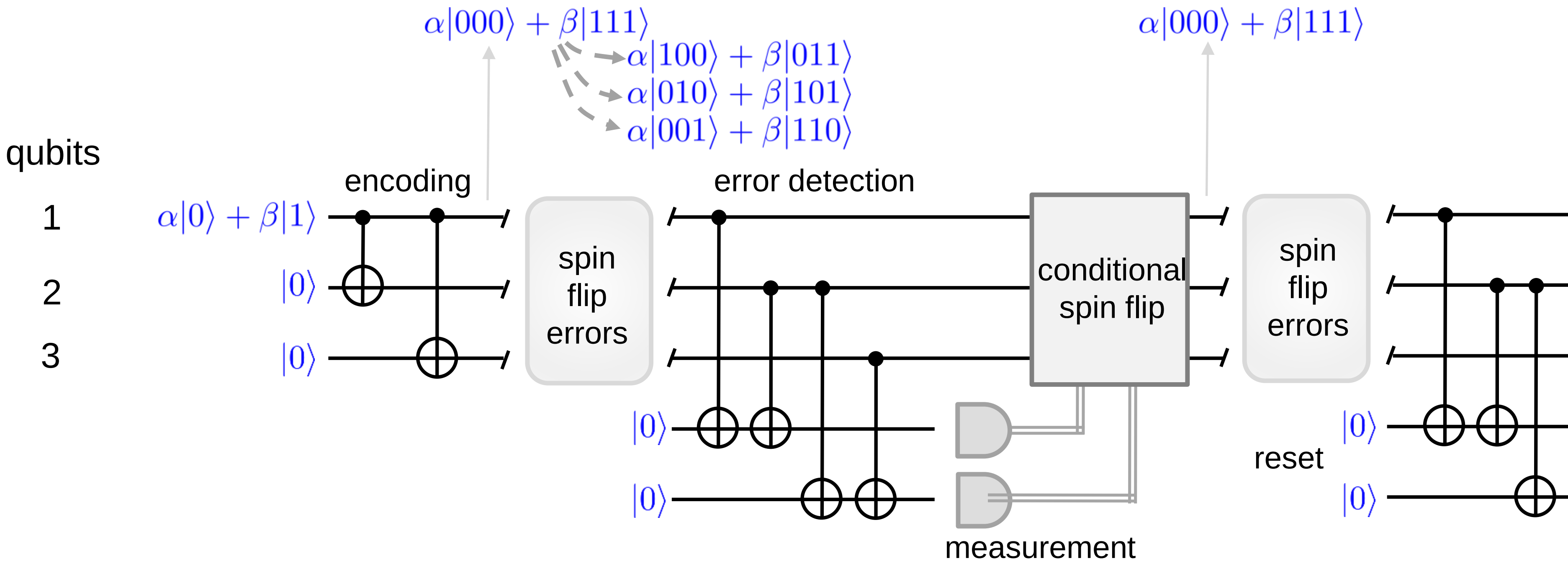
Outcome		Action
00	$\longrightarrow$	none
10	$\longrightarrow$	flip qubit 1
01	$\longrightarrow$	flip qubit 3
11	$\longrightarrow$	flip qubit 2

Result: we recover $\alpha|000\rangle + \beta|111\rangle$

with probability $(1-p)^3 + 3p(1-p)^2 \approx 1 - 3p^2$

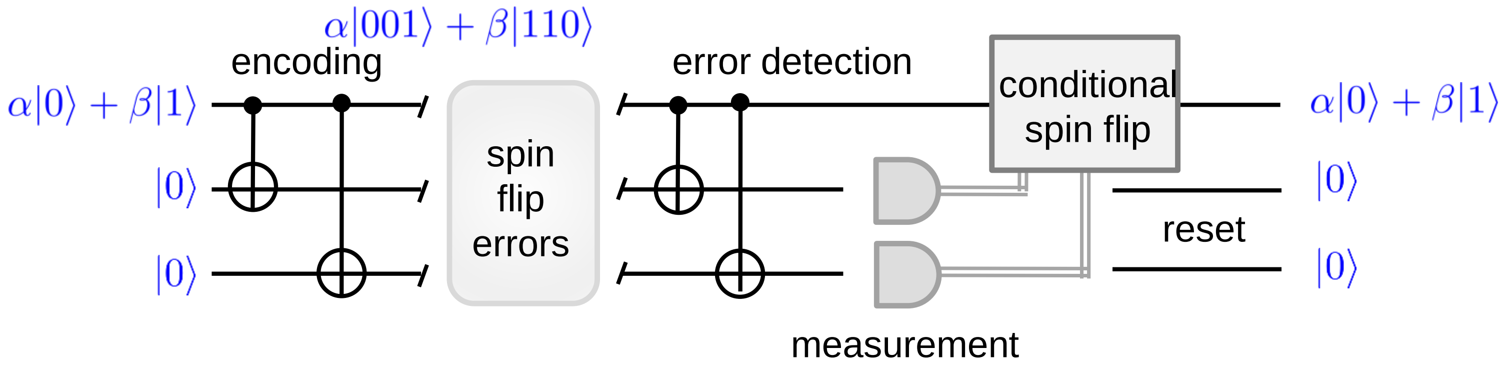
If $p < 0.5$, the protocols reduces the error rate.

Quantum error correction of spin flip errors



Experimental quantum error correction

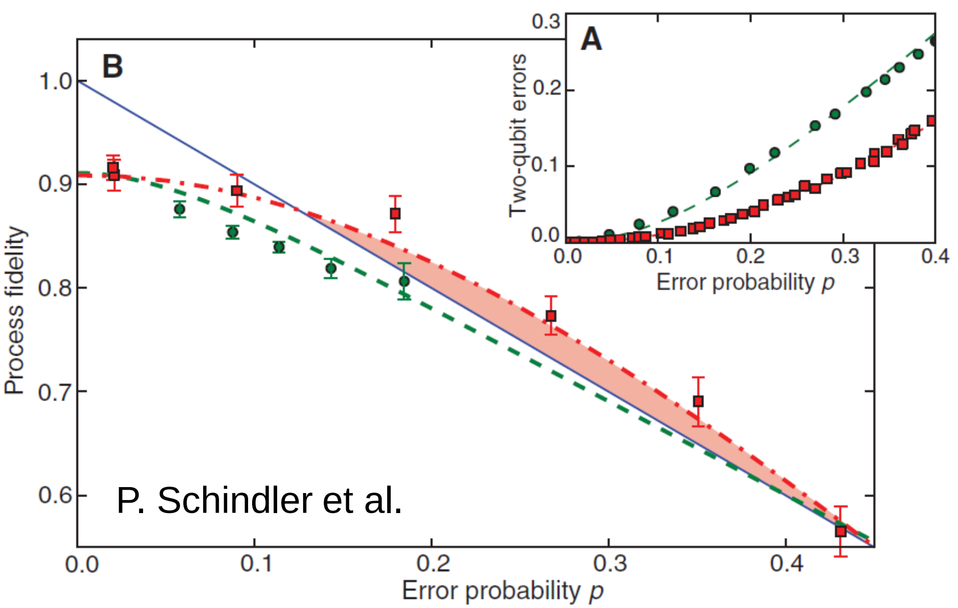
Three-qubit demonstrations:



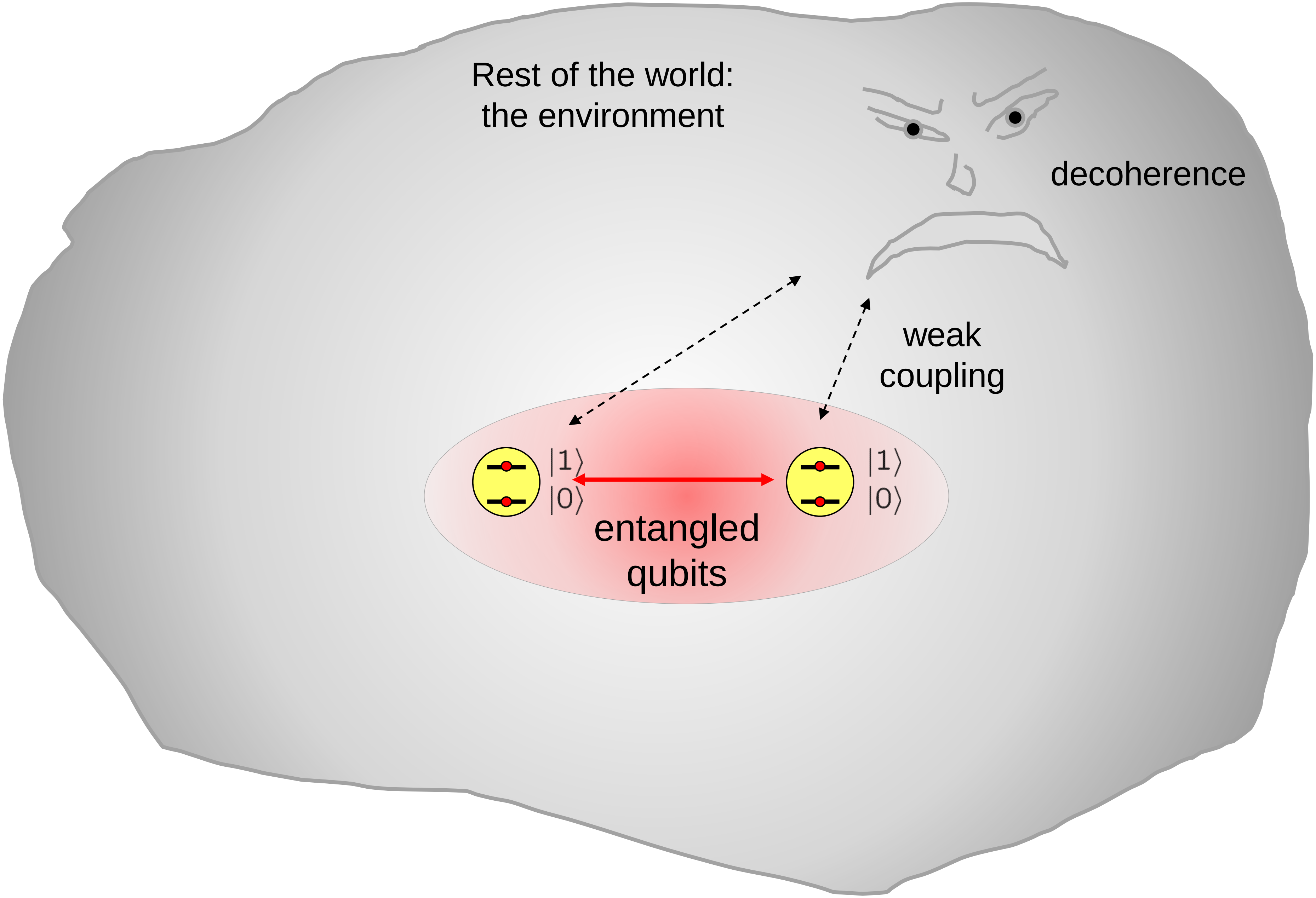
J. Chiaverini *et al.*, Nature **432**, 602 (2004)

P. Schindler *et al.*, Science **332**, 1059 (2011)

... and further more advanced demonstrations



Entanglement's enemy: decoherence



Decoherence in trapped ion experiments

<u>Motional decoherence</u>

- Dephasing by fluctuating trap voltages
- Heating of ion motion by fluctuating electric fields

<u>Electronic state decoherence</u>

- Spontaneous decay of metastable state (optical qubit)
- Spontaneous decay of excited state mediating the Raman coupling during gate operations (hyperfine qubit)
- Magnetic field noise
- Laser phase noise of qubit phase reference (optical qubit)
- Path length fluctuations of laser driving the qubit transition
- Laser intensity noise + beam pointing of laser coupling qubit states
- ...

Decoherence in trapped ion experiments

Gate times for one- and two-qubit gates: $\tau \sim$ 10-100 μs

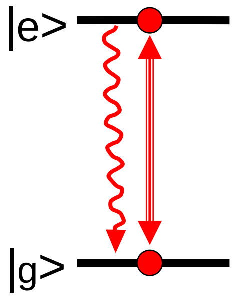
Motional decoherence

Dephasing by fluctuating trap voltages $\tau \sim$ 100 ms

$$\alpha|0\rangle + \beta|1\rangle \rightarrow |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$$

Heating of ion motion by fluctuating electric fields $\tau \sim$ 1-100 ms

Decoherence of optical qubits



- Spontaneous decay of metastable state — τ_{sp} ~ 1 s

  irrelevant in most experiments

- Magnetic field noise — τ_B ~ 1-100 ms

$$\Delta\nu = \mu_B(g_e - g_g)\Delta B/\hbar$$

  ('clock states' have much longer coherence times)

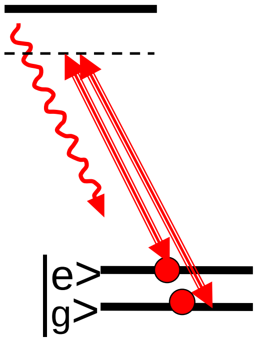
- Frequency noise of phase reference — τ_L ~ 1-100 ms

$$\Psi(0) = \alpha|g\rangle + \beta|e\rangle$$

$$\longrightarrow \Psi(t) = \alpha|g\rangle + \beta e^{-i\omega_0 t}|e\rangle$$

  requires ultrastable laser !

Decoherence of hyperfine qubits



- Spontaneous decay of excited state mediating the Raman coupling — τ_{sp} ~ 1-10 ms

  requires large detuning -> high laser power!

- Magnetic field noise — τ_B ~ 1-100 ms

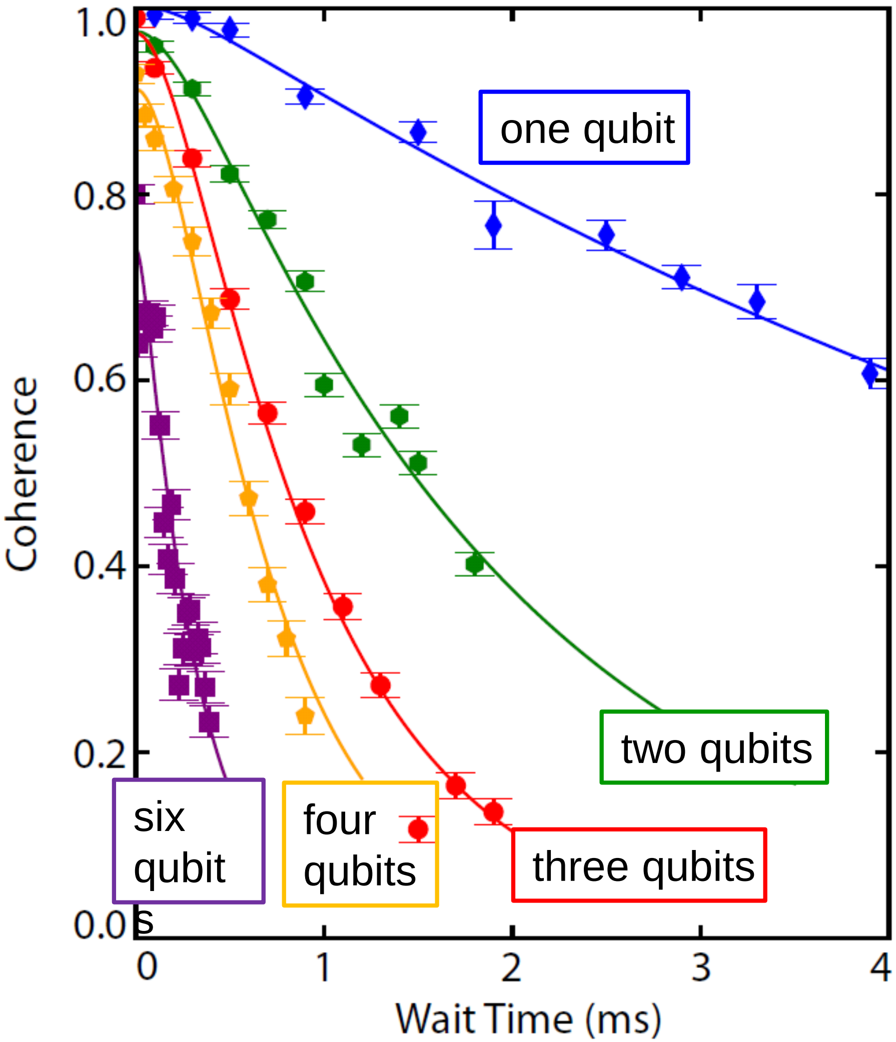
$$\Delta\nu = \mu_B(g_e - g_g)\Delta B/\hbar$$

  ('clock states' have much longer coherence times)

- Frequency noise of phase reference — negligible

GHZ-states: Coherence of large-scale entanglement

T. Monz et al., Phys. Rev. Lett. **106**, 130506 (2011)



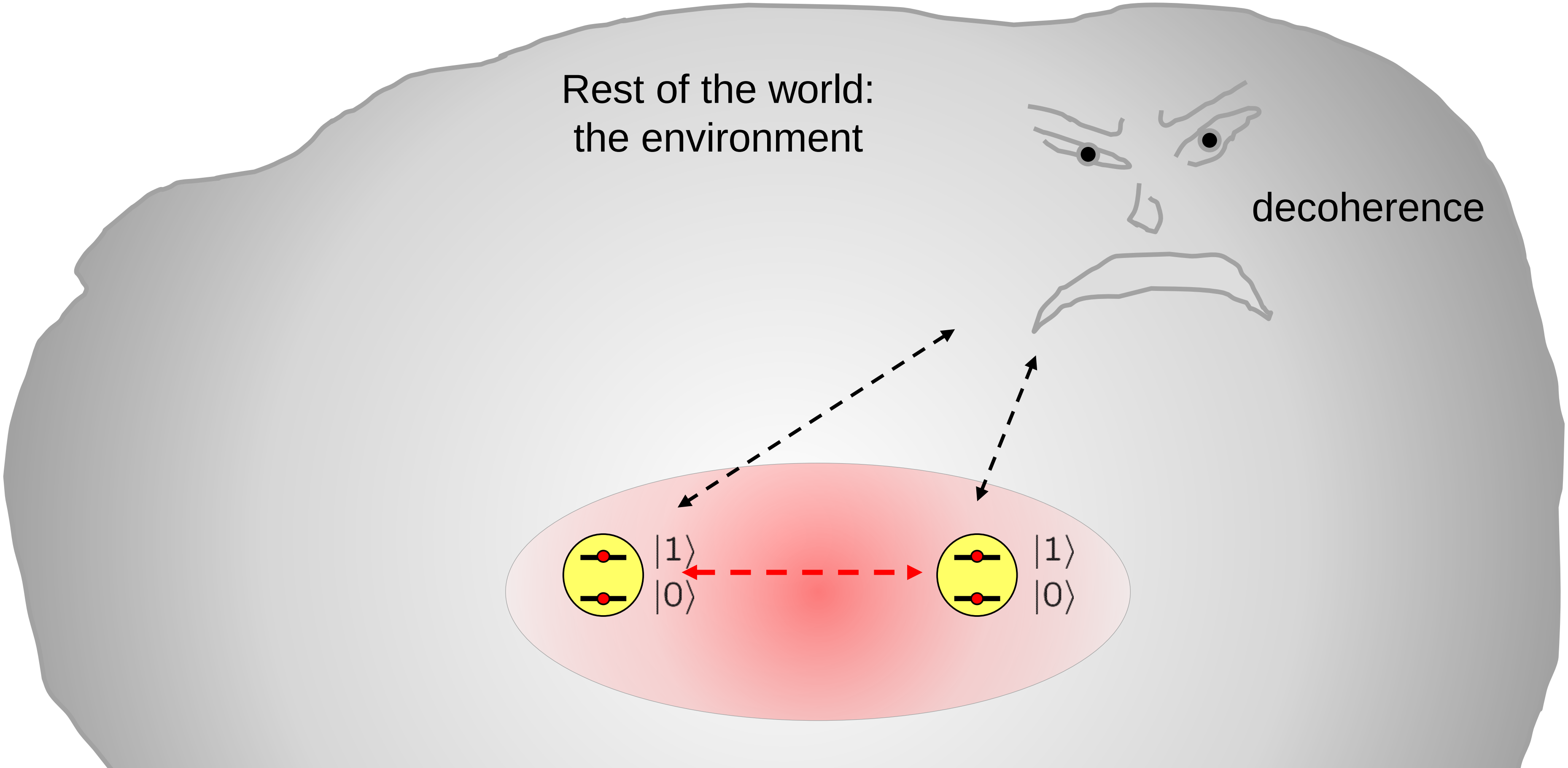
$$\tau_{coh} \propto \frac{1}{N}$$

or even

$$\tau_{coh} \propto \frac{1}{N^2}$$

N = number of qubits

Classical vs quantum correlations



Coupling to the environment

- can create classical correlations
- turns quantum correlations into classical correlations.

Classical vs quantum correlations

Example: Comparison of two different states

$$\Psi_- = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

$|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$

50% 50%

Both state have the same expectation values when measured in z-basis:

$$\langle \sigma_z^{(i)} \rangle = 0$$
$$\langle \sigma_z^{(1)} \sigma_z^{(2)} \rangle = -1$$

Only measurements along x or y reveal the difference:

$$\langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle = -1 \qquad \langle \sigma_x^{(1)} \sigma_x^{(2)} \rangle = 0$$
$$\langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle = -1 \qquad \langle \sigma_y^{(1)} \sigma_y^{(2)} \rangle = 0$$

Mixed quantum states

Density matrix formalism:

Pure states:

$$|\psi\rangle \longrightarrow \rho = |\psi\rangle\langle\psi|$$

Example:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$\rho = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} (\alpha^*, \beta^*) = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Mixed states:

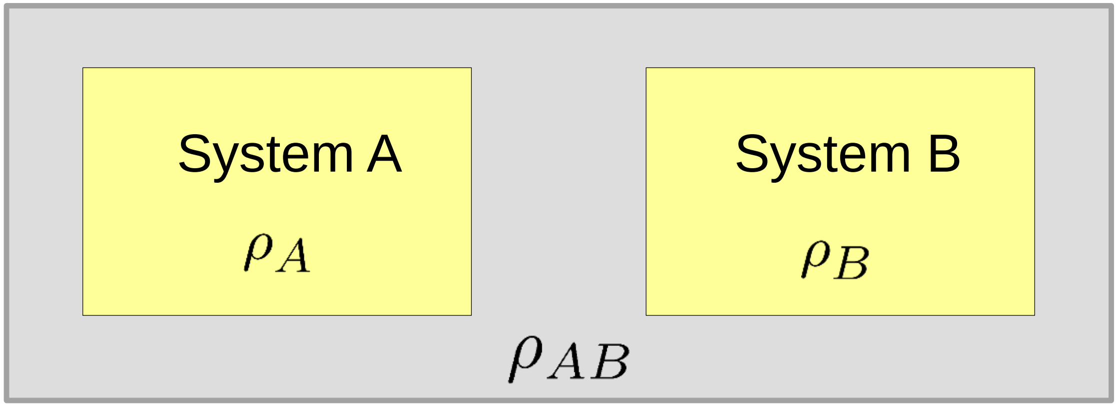
$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

with $\quad p_i \geq 0, \sum_i p_i = 1$

$$\rho = \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

$$= |\alpha|^2 |\downarrow\rangle\langle\downarrow| + |\beta|^2 |\uparrow\rangle\langle\downarrow|$$

Quantum states of composite systems



Example :
classically correlated state

$$\rho = \begin{pmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 \end{pmatrix}$$

$$= \frac{1}{2}(|\downarrow\downarrow\rangle\langle\downarrow\downarrow| + |\uparrow\uparrow\rangle\langle\downarrow\downarrow|)$$

Separable pure states

$$\rho_{AB} = \rho_A \otimes \rho_B$$

Separable mixed states

$$\rho_{AB} = \sum_i p_i \, \rho_{A,i} \otimes \rho_{B,i}$$

The state ρ_{AB} is <u>entangled</u> if it cannot be written as a separable state.

Detecting entanglement

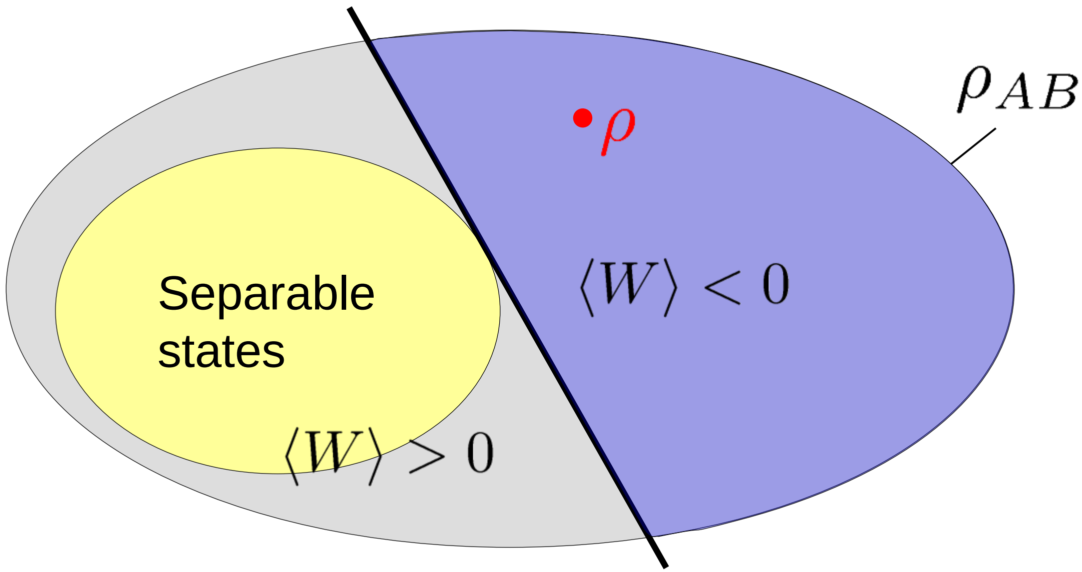
Deciding whether a state is entangled or not is a hard task.

Entanglement detection techniques:

- **Positive partial trace (PPT) cryterion**: check whether a density matrix after partial transposition has negative eigenvalues.
- **Entanglement witnesses**: Particular observables that have negative expectation value for some entangled states, but are positve for separable states.
- **Entanglement measures (for example: concurrence)**: Nonlinear functions of the density matrix that are zero for separable mixed states and positive for entangled states.
  Entanglement measures quantify entanglement but can be hard to calculate even if the density matrix is known; for two qubits, closed expressions exist.

Detecting entanglement by witness operators

Quantum states of a composite quantum system



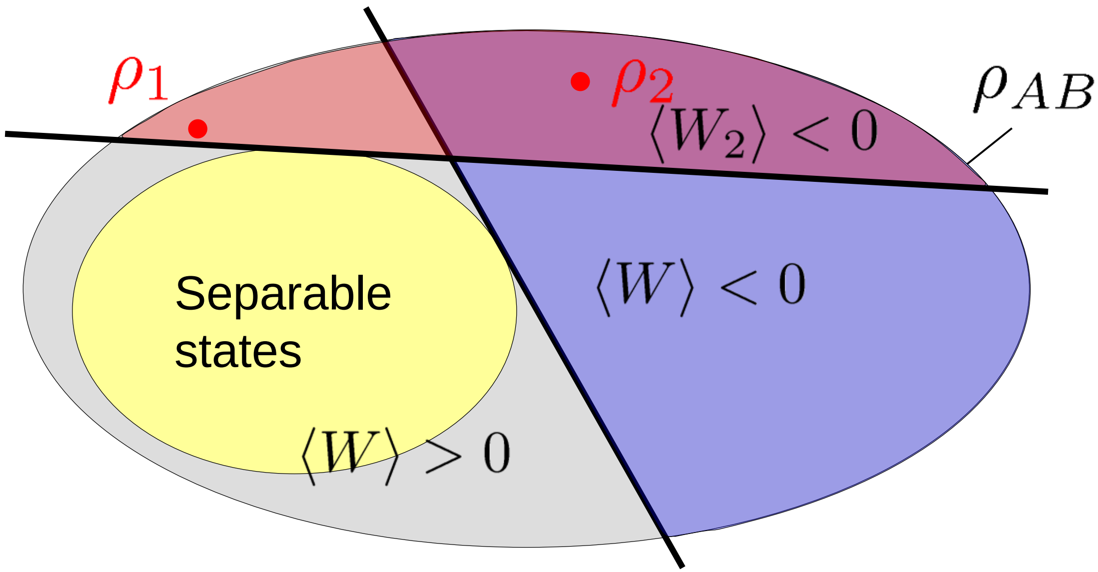
<u>Witness:</u> An observable W whose expectation value is positive for all separable states

$$\langle W \rangle = \mathrm{Tr}(\rho W) \geq 0$$

If we measure $\langle W \rangle < 0$ in an experiment, we can conclude that the state is entangled.

Detecting entanglement by witness operators

Quantum states of a composite quantum system



- Not every entangled state is detected by a witness $\longrightarrow \rho_1$

$$\mathrm{Tr}(\rho_1 W) \geq 0$$

- An entangled state can be detected by more than one witness $\longrightarrow \rho_2$

$$\mathrm{Tr}(\rho_2 W) < 0 \quad \mathrm{Tr}(\rho_2 W_2) < 0$$

Entanglement witnesses

Examples:

- A witness detecting the entangled state $|\psi\rangle$ can be constructed by setting

$$W = \alpha \mathbb{I} - |\psi\rangle\langle\psi|$$

$$\alpha = \max_{\phi \text{ separable}} |\langle\phi|\psi\rangle|^2$$

- The entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)$ is detected by the witness

$$W = 2\,\mathbb{I} - (\sigma_x^{(1)}\sigma_{x-z}^{(2)} + \sigma_x^{(1)}\sigma_{x+z}^{(2)} + \sigma_z^{(1)}\sigma_{x-z}^{(2)} - \sigma_z^{(1)}\sigma_{x+z}^{(2)})$$

$$\langle\psi|\sigma_x^{(1)}\sigma_{x-z}^{(2)}|\psi\rangle + \langle\psi|\sigma_x^{(1)}\sigma_{x+z}^{(2)}|\psi\rangle + \langle\psi|\sigma_z^{(1)}\sigma_{x-z}^{(2)}|\psi\rangle - \langle\psi|\sigma_z^{(1)}\sigma_{x+z}^{(2)})|\psi\rangle = 2\sqrt{2}$$

Bell inequalities of Clauser-Horne-Shimony-Holt type are entanglement witnesses !

Quantum state analysis

Experiment:
prepares
quantum states $\longrightarrow$ ρ_{exp}

How to characterize ρ_{exp} ?

- measure overlap with the state we hope to prepare
- demonstrate that the state is entangled

Can we do more ? **<u>Quantum state tomography</u>**

Yes ! We can determine ρ_{exp} . Based on the reconstruction,
we can calculate the result of any expectation value we are interested in.

Reconstruction of the density matrix

Representation of ρ as a sum of orthogonal observables A_i :

$$\rho = \sum_i \lambda_i A_i \text{ with } Tr(A_i A_j) = \delta_{ij}$$

ρ is completely detemined by the expectation values $<A_i>$:

$$\langle A_j \rangle = Tr(\rho A_j) = \sum_i \lambda_i Tr(A_i A_j) = \lambda_j$$

For a two-ion system : $A_i \in \{\sigma_i^{(1)} \otimes \sigma_j^{(2)}, \sigma_i \in \{I, \sigma_x, \sigma_y, \sigma_z\}\}$

$\longrightarrow$ Joint measurements of all spin components $\sigma_i^{(1)} \otimes \sigma_j^{(2)}$

$$\rho_R = \sum_{i=1}^{16} \langle A_i \rangle A_i$$

Bell state analysis

Measurement of $\langle\sigma_z\rangle$:

$$\langle\sigma_z\rangle = \rho_{\uparrow\uparrow} - \rho_{\downarrow\downarrow}$$

Measurement of $\langle\sigma_x\rangle$, $\langle\sigma_y\rangle$:

Rotation of the Bloch sphere prior to state measurement:

$$\langle\sigma_z\rangle_{U\rho U^{-1}} = Tr(\sigma_z U\rho U^{-1})$$

$$= Tr(\underbrace{U^{-1}\sigma_z U}_{\stackrel{!}{=}\,\sigma_x}\rho)$$

Measurement time: 40 s

prepare Bell state
no rotation
measure

200 repetitions

$\langle\sigma_z^{(1)}\rangle, \langle\sigma_z^{(2)}\rangle, \langle\sigma_z^{(1)}\sigma_z^{(2)}\rangle$

prepare Bell state
ion #1, y - rotation
ion #2, identity
measure

200 repetitions

$\langle\sigma_x^{(1)}\rangle, \langle\sigma_z^{(2)}\rangle, \langle\sigma_x^{(1)}\sigma_z^{(2)}\rangle$

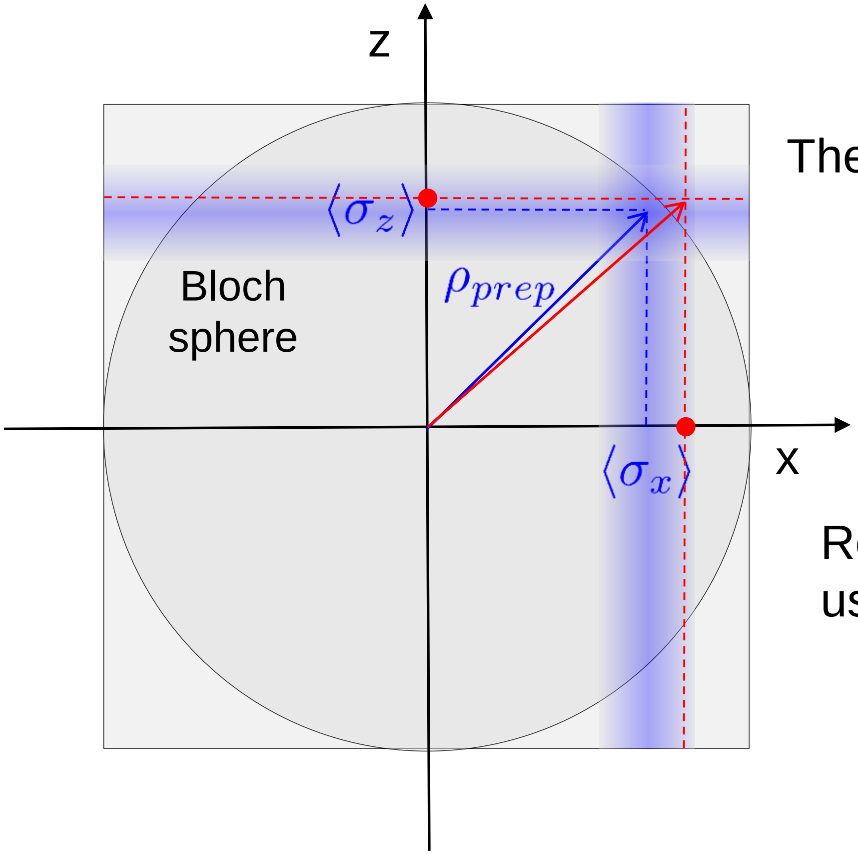
9 different settings

prepare Bell state
ion #1, x - rotation
ion #2, x - rotation
measure

200 repetitions

$\langle\sigma_y^{(1)}\rangle, \langle\sigma_y^{(2)}\rangle, \langle\sigma_y^{(1)}\sigma_y^{(2)}\rangle$

Example: Tomography of a qubit



The experimental procedure prepares the state ρ_{prep}

$$\rho_{prep} = \frac{1}{2}(I + \langle\sigma_x\rangle\sigma_x + \langle\sigma_y\rangle\sigma_y + \langle\sigma_z\rangle\sigma_z)$$

Reconstruction by estimation of $\langle\sigma_x\rangle, \langle\sigma_y\rangle, \langle\sigma_z\rangle$ using a finite number of copies of the state:

$$s_z = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow}\ , \ s_x = \ldots,\ s_y = \ldots$$

$$\rho_{tomo} = \frac{1}{2}(I + s_x\sigma_x + s_y\sigma_y + s_z\sigma_z) \neq \rho_{prep}$$

ρ_{tomo} might not be within the Bloch sphere !

DISASTER !!!

Maximum likelihood estimation

Is $\rho_R = \sum_i \langle A_i \rangle \, A_i$ positive semidefinite ?

… not necessarily:

with a finite number of measurements, we can only estimate expectation values

Maximum likelihood estimation: (Hradil '97, Banaszek '99)

In N experiments,the quantum state is projected onto the outcomes $|y_j\rangle$.

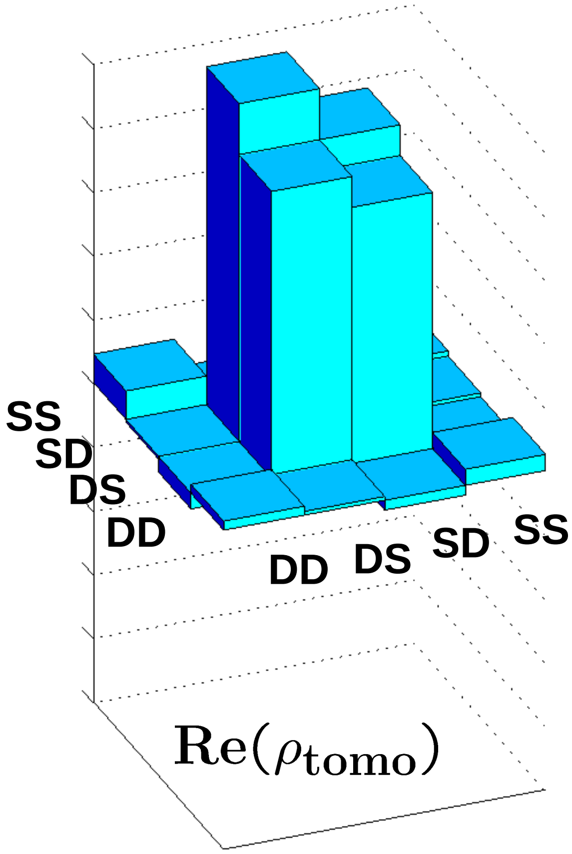
f_j : relative frequency of the outcome $|y_j\rangle$

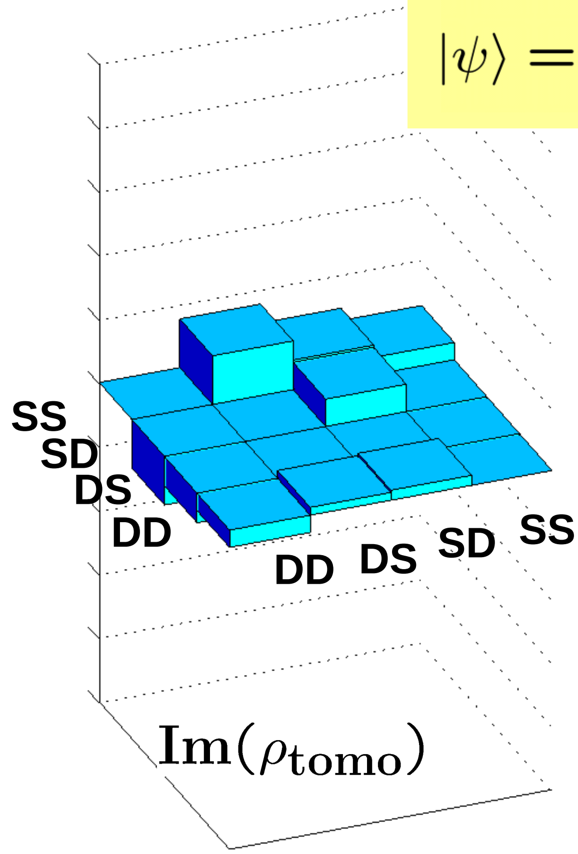
On the set of density matrices ρ, look for the one that maximizes

$$\mathcal{L}(\rho) = \prod_j \langle y_j|\rho|y_j\rangle^{N f_j}$$

$$\boxed{\text{Maximize } L(\rho) = \textstyle\sum_j f_j \log\langle y_j|\rho|y_j\rangle}$$

Bell state reconstruction with maximum likelihood estimation



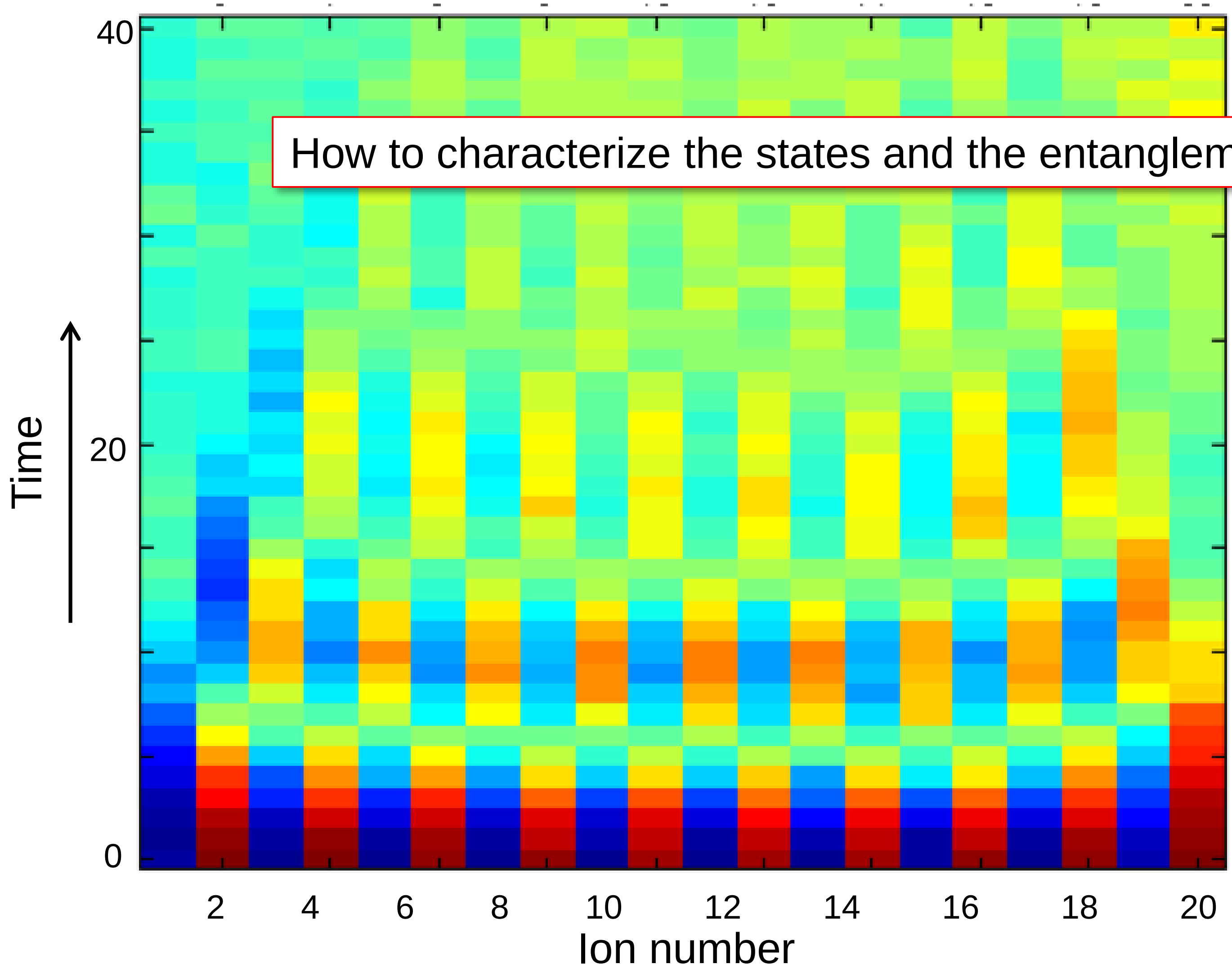


$$|\psi\rangle = \frac{1}{2}(|SD\rangle + |DS\rangle)$$

- State fidelity: $\langle\psi|\rho_{tomo}|\psi\rangle = 0.91$
- Violation of a Bell inequality: $\langle\rho_x^{(1)}\rho_{x-z}^{(2)}\rangle + \langle\rho_x^{(1)}\rho_{x+z}^{(2)}\rangle + \langle\rho_z^{(1)}\rho_{x-z}^{(2)}\rangle - \langle\rho_z^{(1)}\rho_{x+z}^{(2)}\rangle = 2.52(6) > 2$
- Entanglement of formation: $E(\rho_{tomo}) = 0.79$

20-ion magnetization dynamics

N/2 excitation subspace: number of states growths exponentially with N



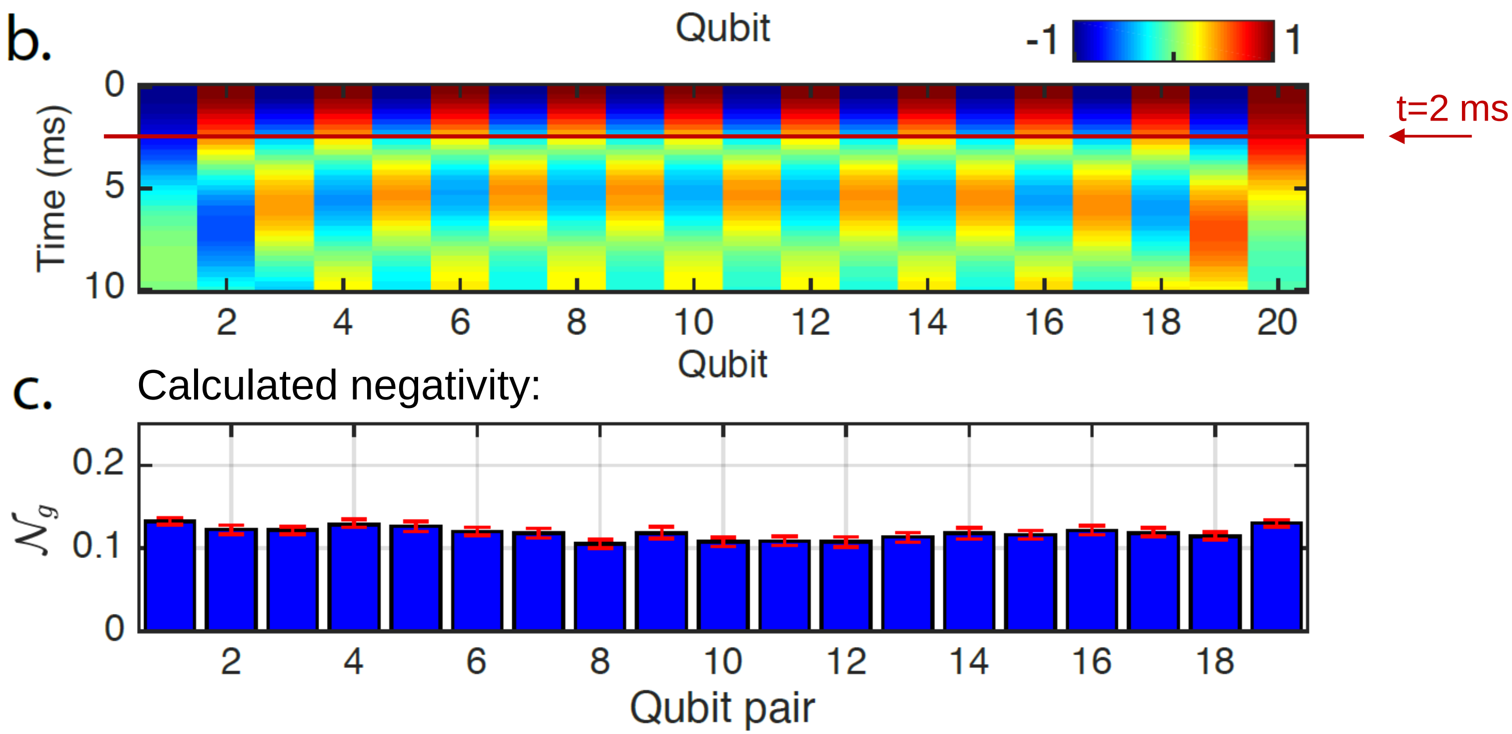
How to characterize the states and the entanglement created?

Entanglement detection in multi-ion experiments

Local characterization of the beginning of entanglement spreading

Option 1: Quantum state tomography to reconstruct the density matrix of subsystems

Reconstruction of all density matrices of nearest-neighbour pairs

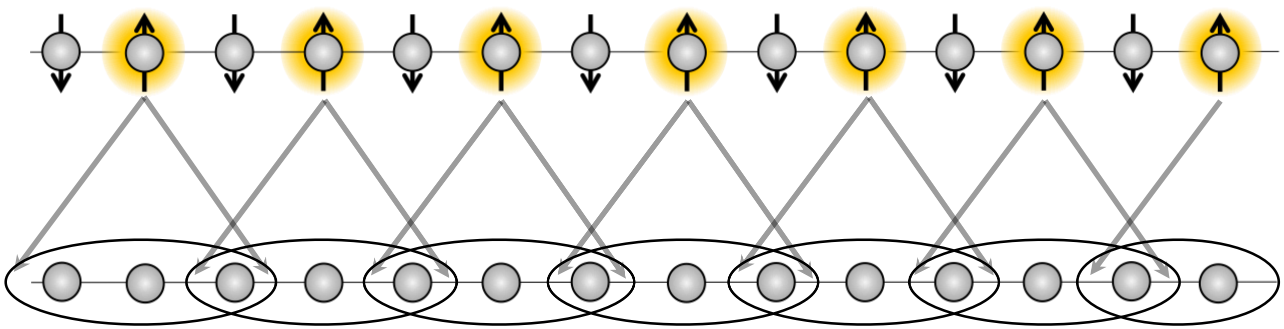


→ After 2 ms of time evolution, all ions are entangled with their neighbours

N. Friis et al., arXiv:1711.11092

Entanglement detection in multi-ion experiments

Local characterization of the beginning of entanglement spreading



Option 2: Measure all correlation functions between groups of neighbouring ions (pairs, triplets,...) and try to build up a global representation of the quantum state using a suitable parametrization of the state

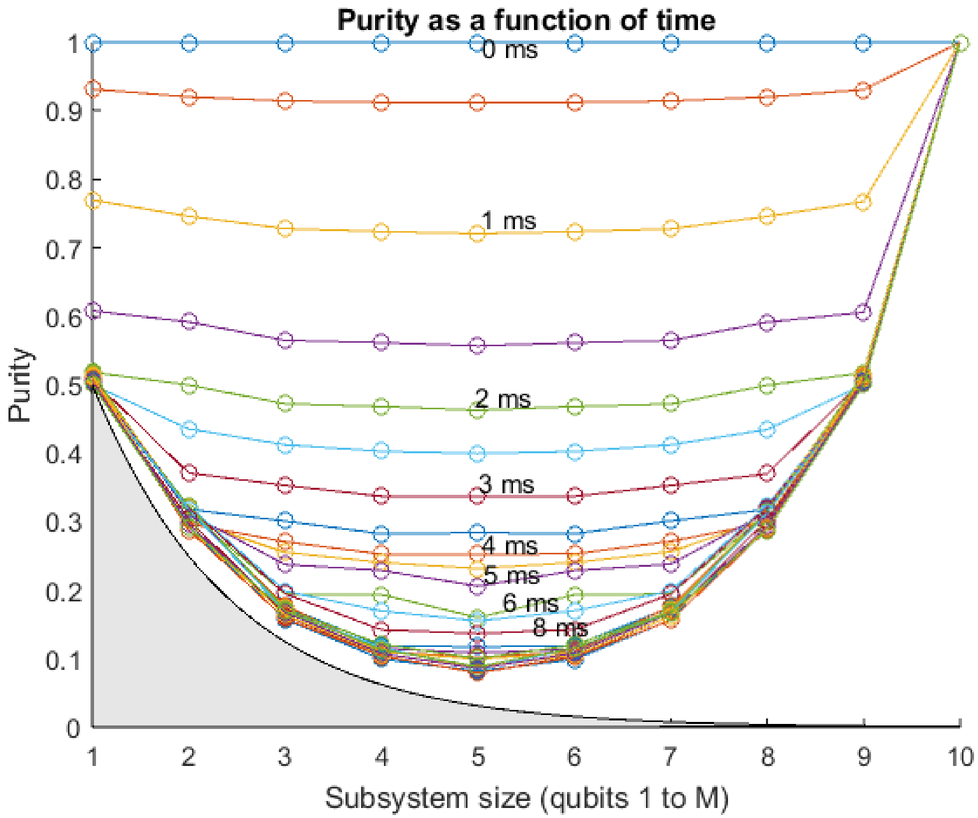
→ Matrix product state tomography

B. Lanyon, C. Maier et al, Nature Physics **13**, 1158 (2017)

Entanglement detection in multi-ion experiments

Option 3: Compare the purity of density matrices describing subsystems to the purity of the overall density matrix

Entanglement will make subsystems less pure than the bigger system.



Numerical simulation (!)
of 10-qubit system subjected
to long-range Ising interaction

First experimental results: see poster by Tiffany Brydges

The quantum way of processing information

USA TODAY

WEDNESDAY, JULY 14, 1999

COVER STORY

Beyond the PC: Atomic QC

Quantum computers could be a billion times faster than Pentium III

By Kevin Maney
USA TODAY

Around 2030 or so, the computer on your desk might be filled with liquid instead of transistors and chips. It would be a quantum computer. It wouldn't operate on anything so mundane as physical laws. It would employ quantum mechanics, which quickly gets into things such as teleportation and alternate universes and is, by all accounts, the weirdest stuff known to man.

This quantum computer would be a data rocket. It probably would do calcula-

...a quantum computer. It wouldn‘t operate on anything so mundane as physical laws. It would employ quantum mechanics, which quickly gets into things such as teleportation and alternate universes and is, by all accounts, the weirdest stuff known to man.