

Transverse phase-space measurements in MICE Step IV

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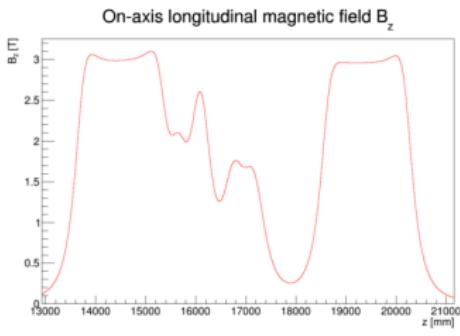
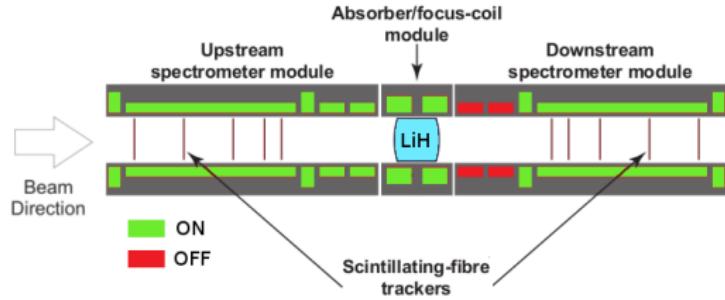


Step IV 2016/04 data

- Test case is run **setting 1.2**, solenoid mode

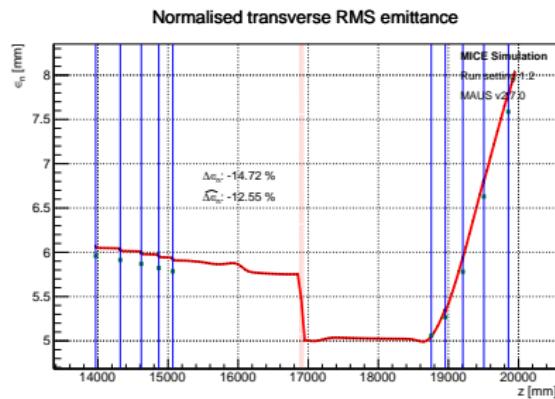
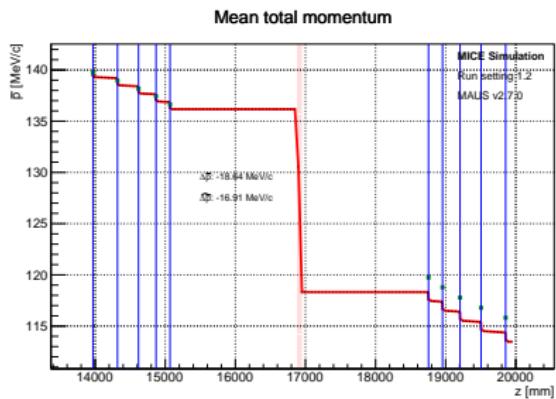
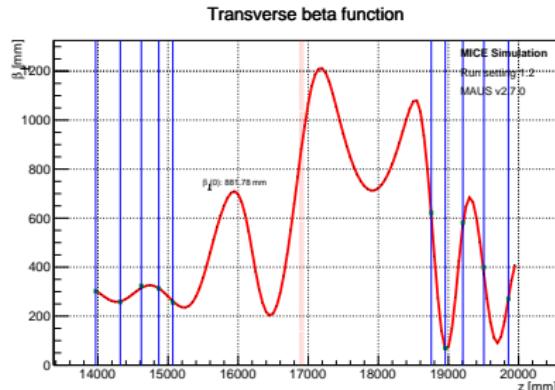
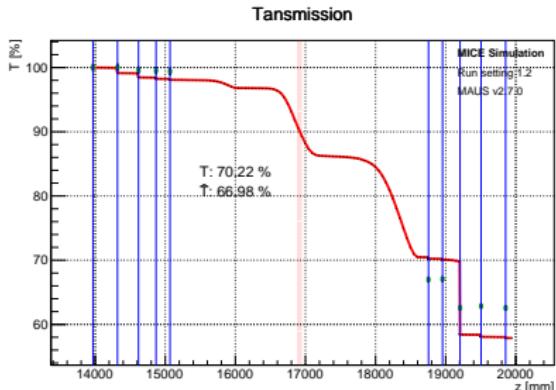
	ECE _U	M2 _U	M1 _U	FC	M1 _D	M2 _D	ECE _D
M2 _D	205.87	171.91	211.73	57.90	0	0	205.86

- 10 mm** input normalised emittance, **140 MeV/c** nominal momentum
- 65 mm **LiH** absorber
- 462978 TOF2 triggers, **58206 after selection**
(TKU+TKD, muon cut, “nonsense” cut, $p_z \in [135, 145]$)
- 100k muons ideal simulation



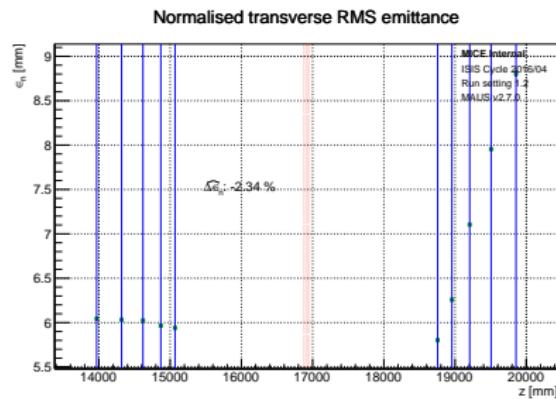
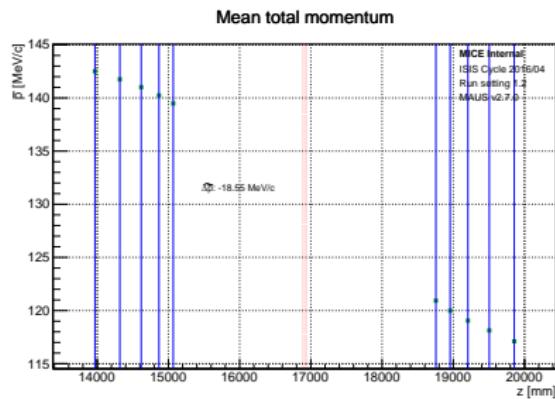
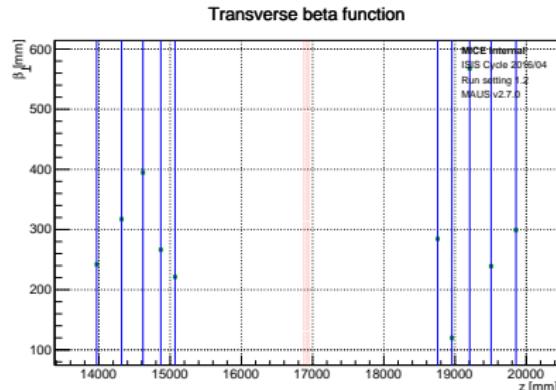
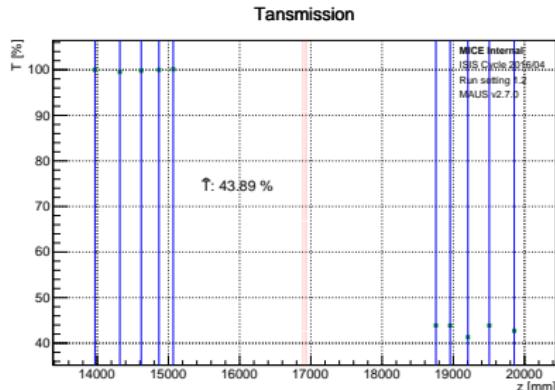
Setting 1.2, selected particles

Simulation



Setting 1.2, selected particles

Data



Challenges with the 2016/04 data

Due to the loss of M1D and the absence of M2D in the 2016/04 data:

- Low transmission through the cooling channel
- Beam nonlinearities, particularly in TKD

Dealing with low transmission:

- Select a **narrow beam** upstream that is efficiently transported, good option to study the material but unlikely to yield cooling
- Focus on the **core density** increase by means of fractional emittance of phase-space volume estimation rather than 4D RMS emittance

Dealing with nonlinearites:

- Focus on the **linear core** rather than include the tails
- Use **non-parametric density estimation** techniques, calculate the volume of phase-space probability contours

Transverse single-particle amplitude

One way to select a sample is to use **transverse amplitude** defined as:

$$A_{\perp} \equiv \epsilon_n u_i^T \Sigma^{-1} u_i \quad (1)$$

with Σ the covariance matrix and u_i the phase-space column vector of the i^{th} particle. The u_i are centred so that $u_{i,\alpha} = \alpha_i - \langle \alpha \rangle$. For a gaussian beam, the amplitudes are distributed as a χ^2 distribution:

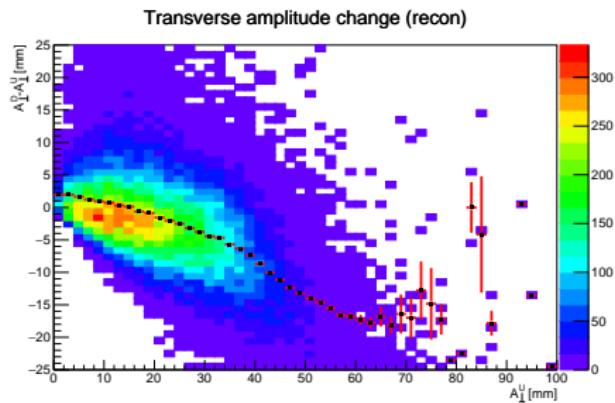
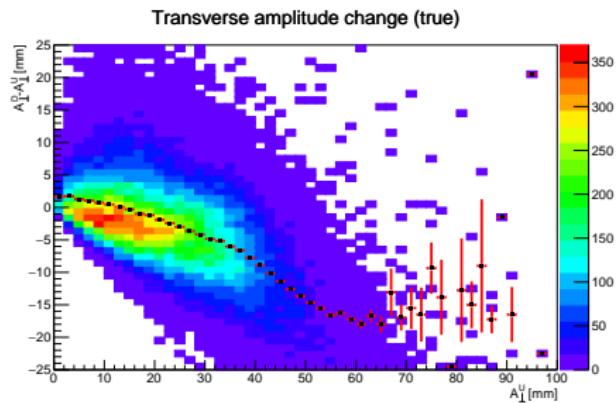
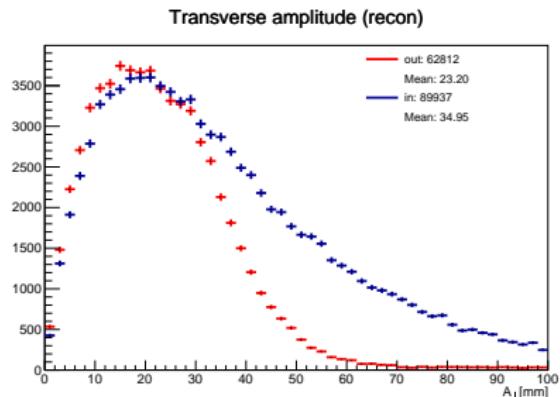
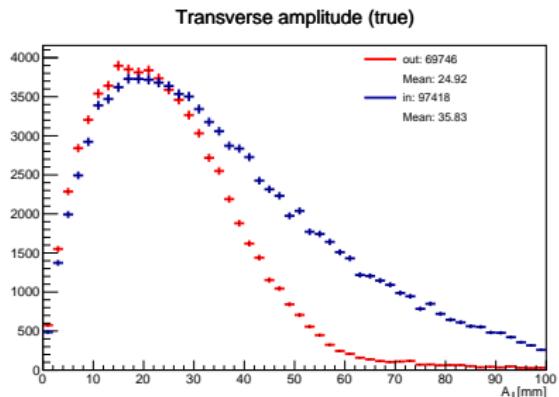
$$A_{\perp} \sim \epsilon_n \chi_4^2 \quad (2)$$

with 4 degrees of freedom. Its mean is $\langle A_{\perp} \rangle = 4\epsilon_n$.

The amplitude gives a definition of a weight to select **any given fraction α of the beam**, rejecting the tails if need be.

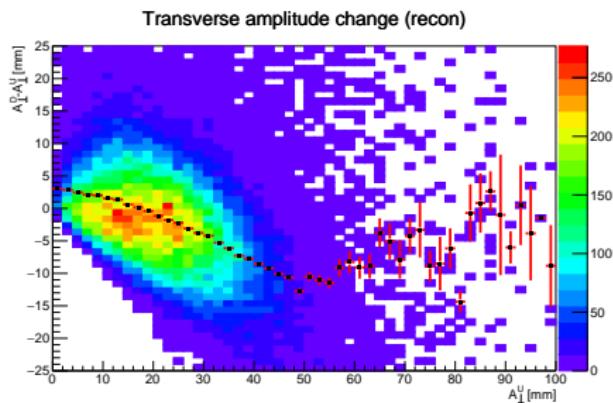
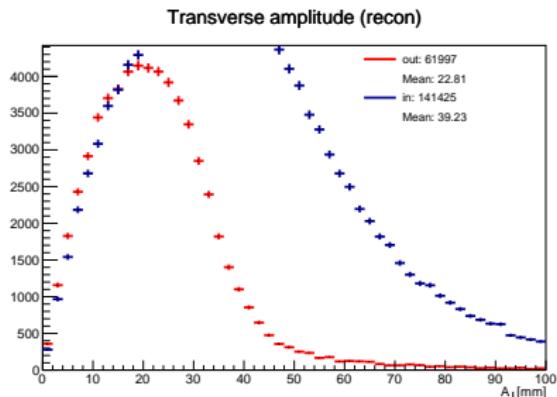
Transverse amplitudes, all

Simulation



Transverse amplitudes, all

Data

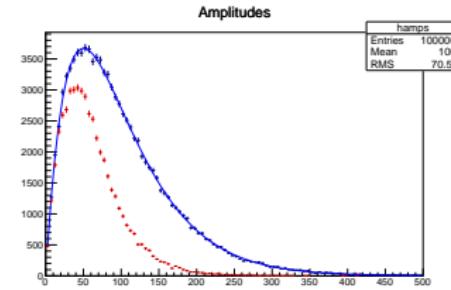
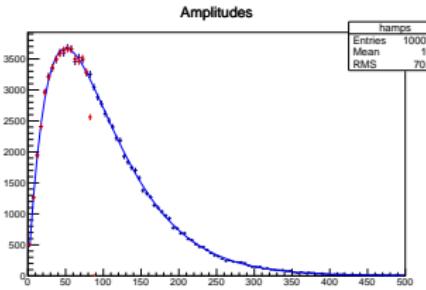


Using amplitudes to produce a subsample

The method to produce an α -subsample is as follows:

- ① calculate the amplitudes, A_{\perp}^i , $i = 1, \dots, N$, of every particle;
- ② find a limit A_{\perp}^{α} so that the sample of all particles that verify $A_{\perp} < A_{\perp}^{\alpha}$ represents a fraction α of the entire population;
- ③ re-evaluate the covariance matrix Σ on the reduced sample;
- ④ repeat 1., 2. and 3. until we get convergence on the sample.

- The RMS emittance of the subsample is the **subsample emittance**
- The volume occupied by the subsample **fractional emittance**
- Must select the **same amount of particles** up and downstream



Consequence of a subsample selection

Selecting a sample amplitude-wise out of the χ_4^2 distribution is equivalent to **truncating** the original distribution. It does **not** artificially increase the core density. For a fractional α , the cut-off is

$$\gamma(2, L/2\epsilon_n) = \alpha \quad \rightarrow \quad L/2\epsilon_n = -W\left(\frac{\alpha - 1}{e}\right) - 1, \quad (3)$$

with $W(\cdot)$, the product log function. This means that the expected subsample fractional change is

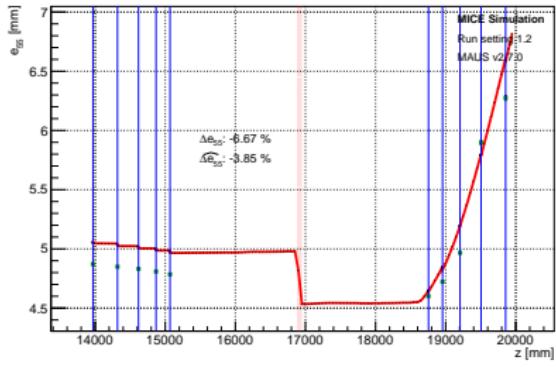
$$\begin{aligned} \frac{e_\alpha^o - e_\alpha^i}{e_\alpha^i} &= \frac{\langle A_\perp^o \rangle_T - \langle A_\perp^i \rangle_T}{\langle A_\perp^i \rangle_T} \\ &= \frac{\epsilon_n^o}{\epsilon_n^i} \frac{1 + (1 + 2W(\beta) - 2(W(\beta) + 1)^2) e^{-W(\beta)}}{1 + (1 + 2W(\beta) - 2(W(\beta) + 1)^2) e^{-W(\beta)} - 1} \\ &= \boxed{\frac{\epsilon_n^o - \epsilon_n^i}{\epsilon_n^i}}. \end{aligned} \quad (4)$$

Even if nonlinear, asymptotically we have $\Delta\epsilon_n/\epsilon_n^i = \lim_{\alpha \rightarrow 0} \frac{\Delta e_\alpha}{e_\alpha^i}$

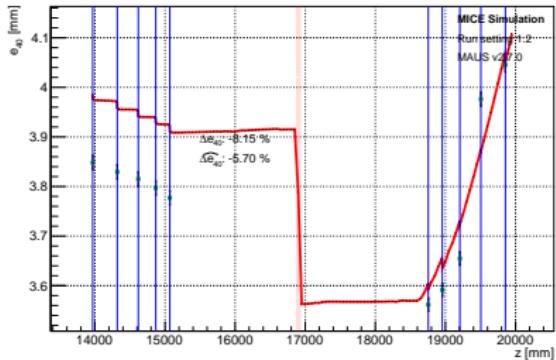
Subsample emittances

Simulation

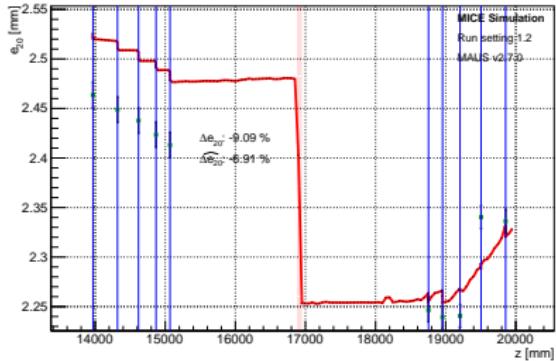
Subsample emittance (55%)



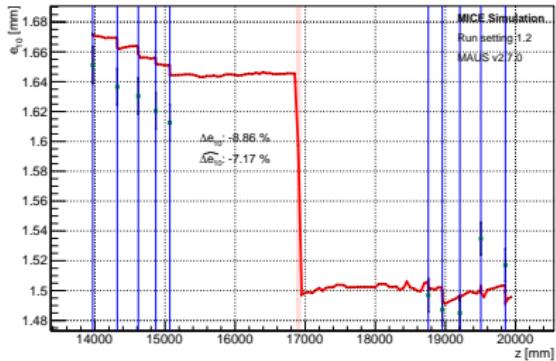
Subsample emittance (40%)



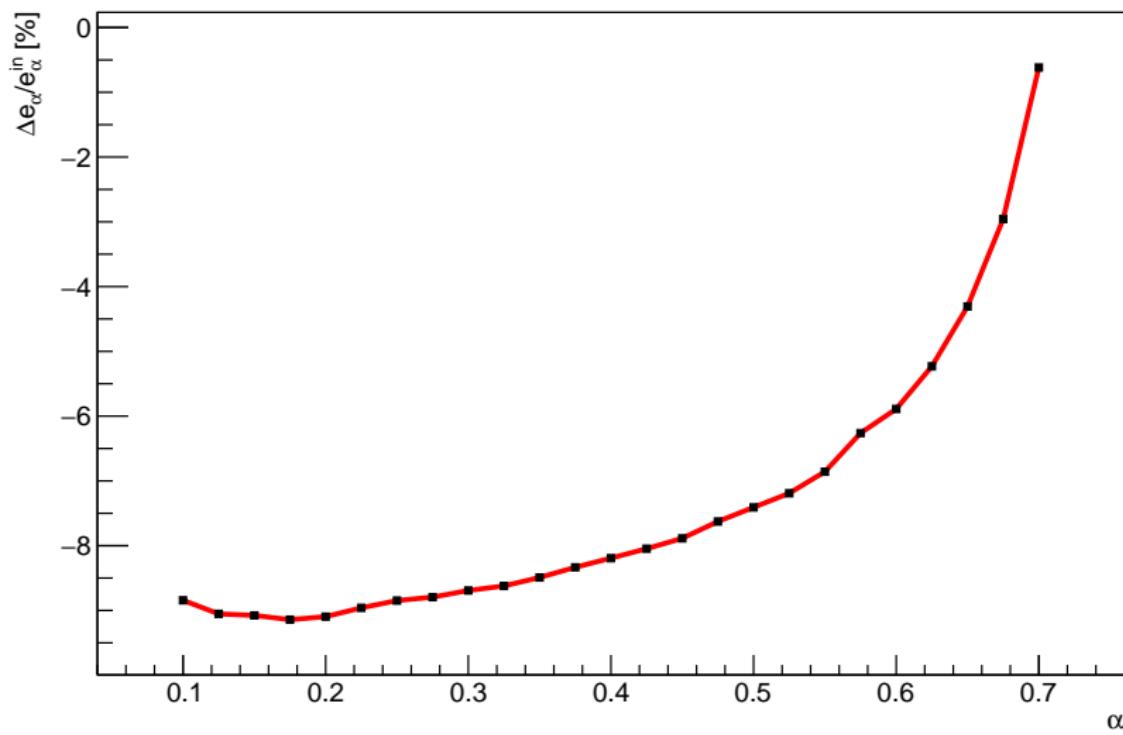
Subsample emittance (20%)



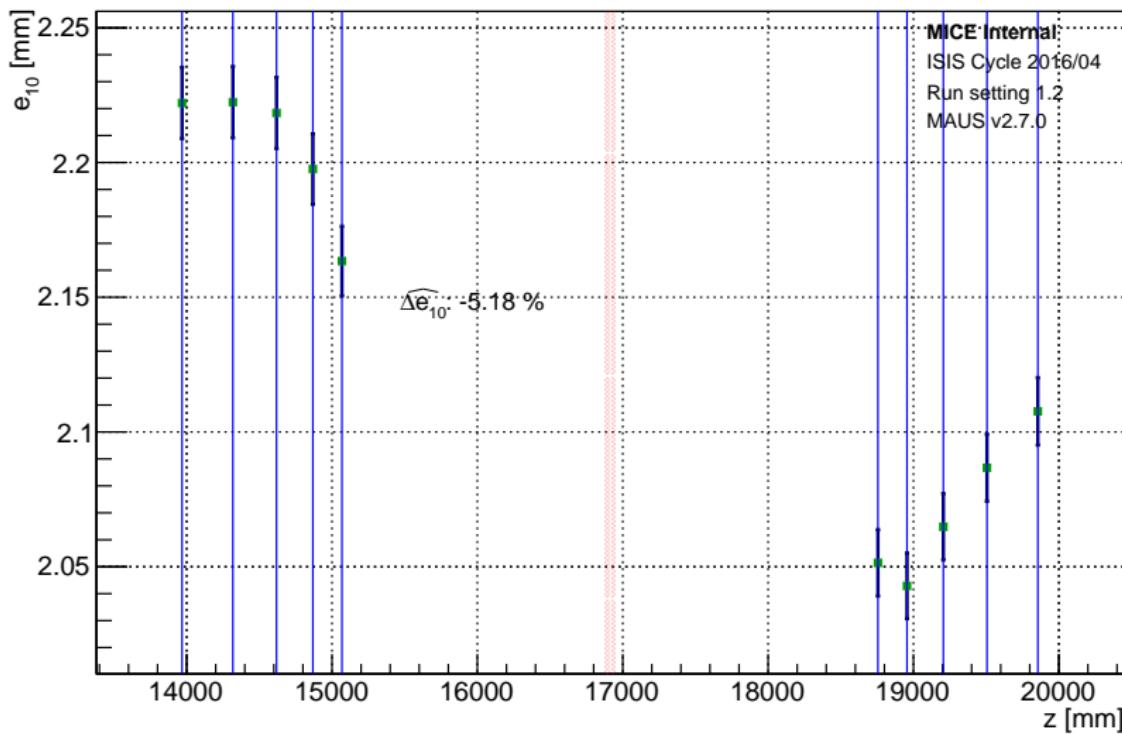
Subsample emittance (10%)



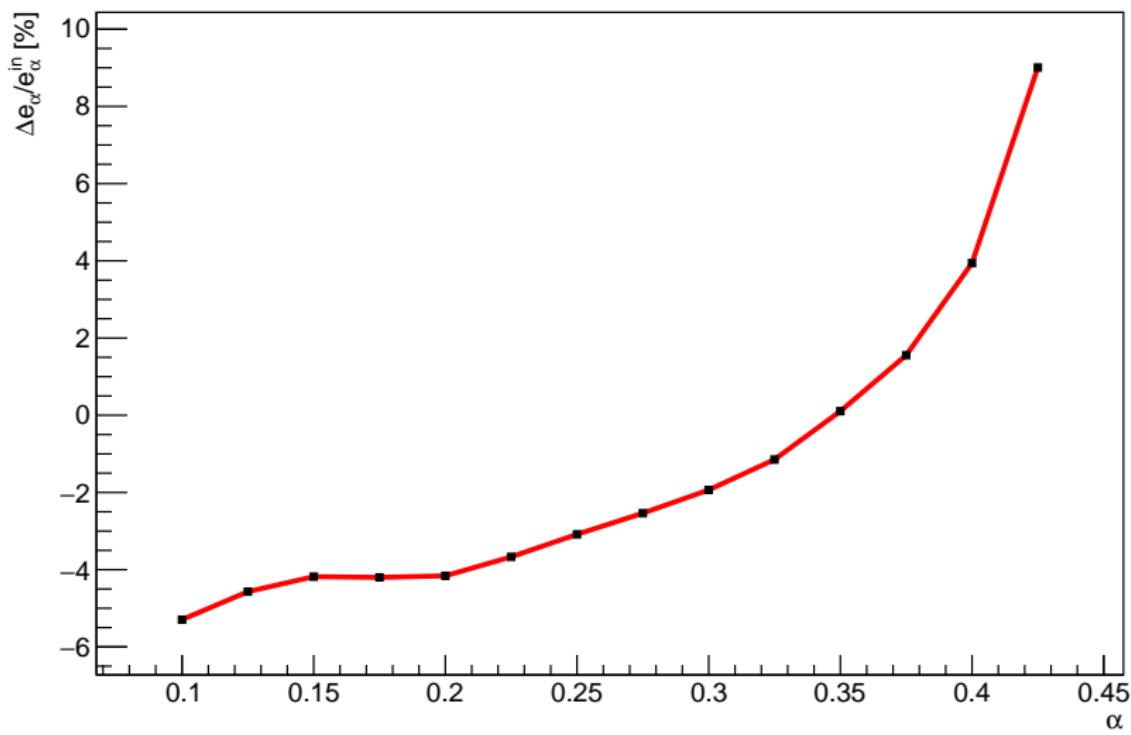
Subsample emittance change



Subsample emittance (10%)

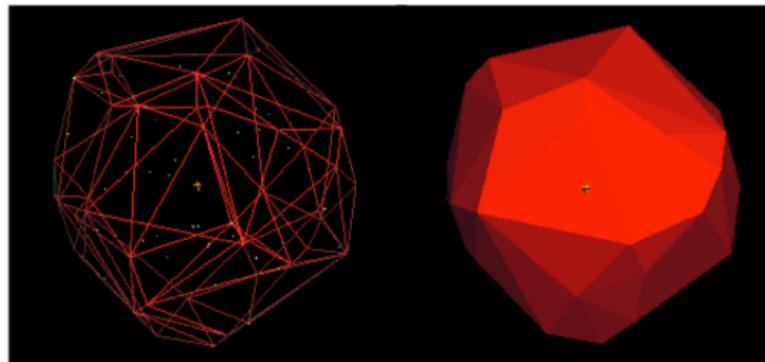


Subsample emittance change



Phase-space volume calculation

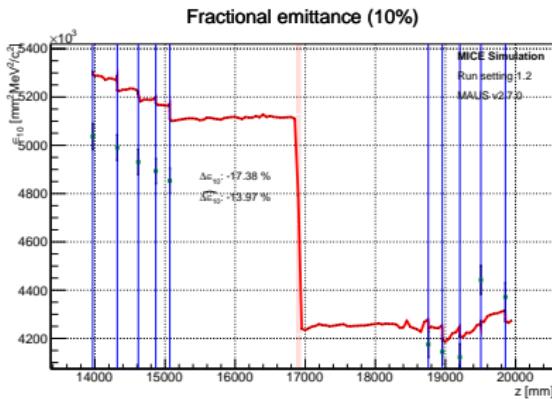
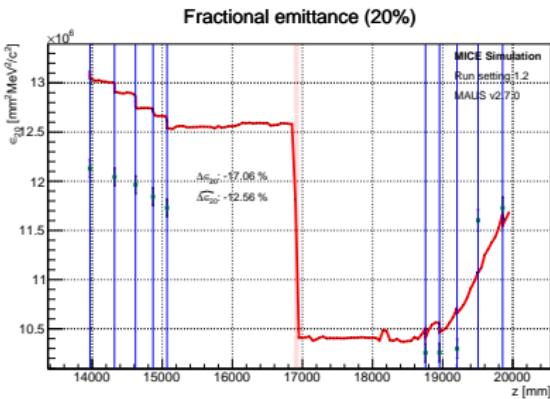
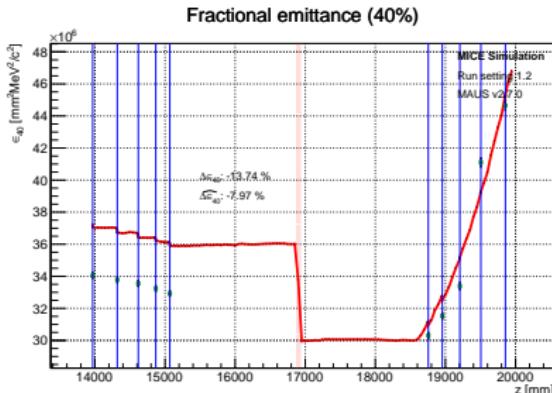
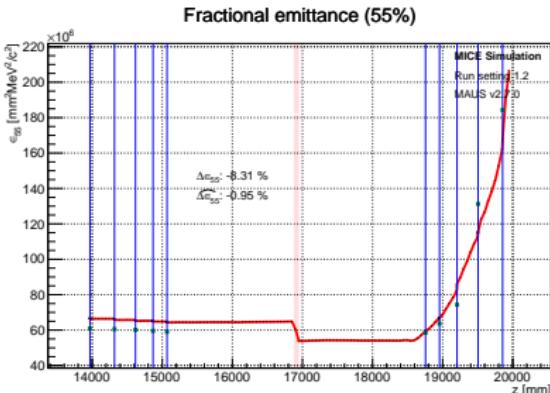
An alternative method to using the subsample emittance e_α as an FOM is to use the volume occupied by the selected particles instead. This can be done at any dimension by calculating the volume of the **convex hull** of the subset of points.



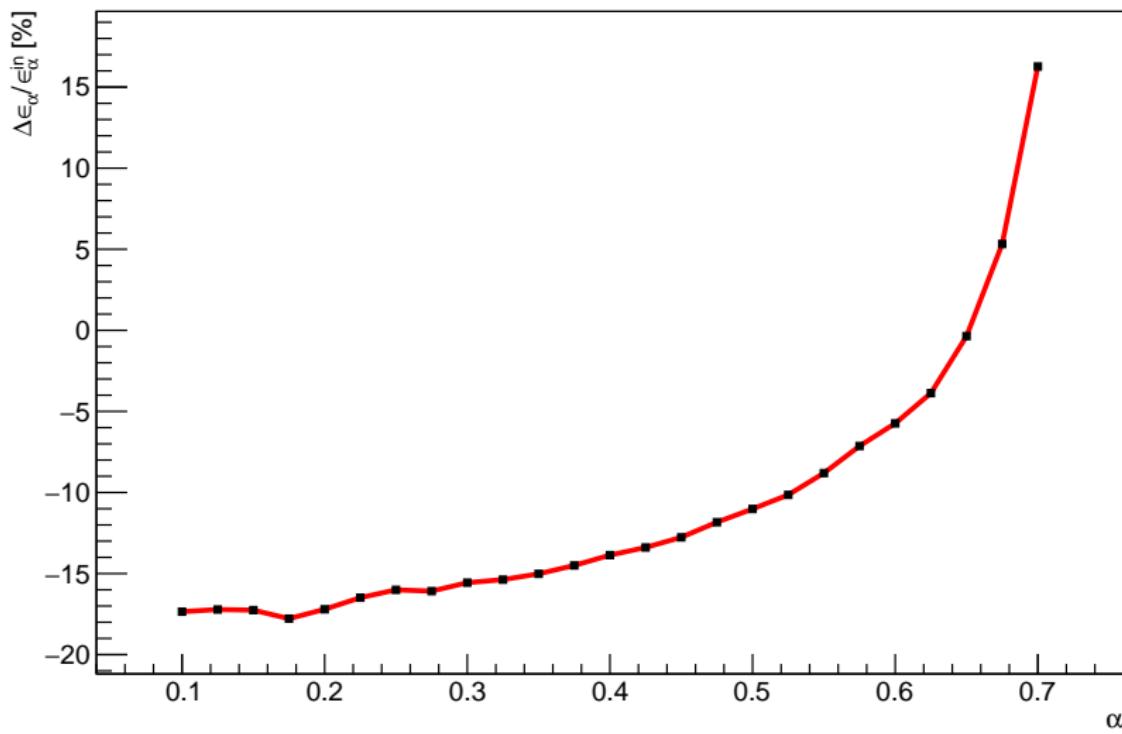
The convex hull or convex envelope of a set X of points in the Euclidean space is the smallest convex set that contains X . For instance, when X is a bounded subset of the plane, the convex hull may be visualized as the shape enclosed by a rubber band stretched around X .

Fractional emittances

Simulation



Fractional emittance change



Probability content of an n -RMS ellipsoid

An n -RMS ellipsoid is an ensemble of points in n dimensions that satisfy $\mathbf{x}^T \Sigma^{-1} \mathbf{x} = 1$. The integral of the corresponding n -Gaussian over the entire ellipse \mathcal{E}

$$p(\mathbf{x} \in \mathcal{E}) = \frac{1}{\pi^{\frac{n}{2}-1}} \gamma(n/2, R^2/2\sigma^2) \prod_{i=1}^{n-2} \sqrt{\pi} \frac{\Gamma(\frac{n-i}{2})}{\Gamma(\frac{n-i+1}{2})} \quad (5)$$

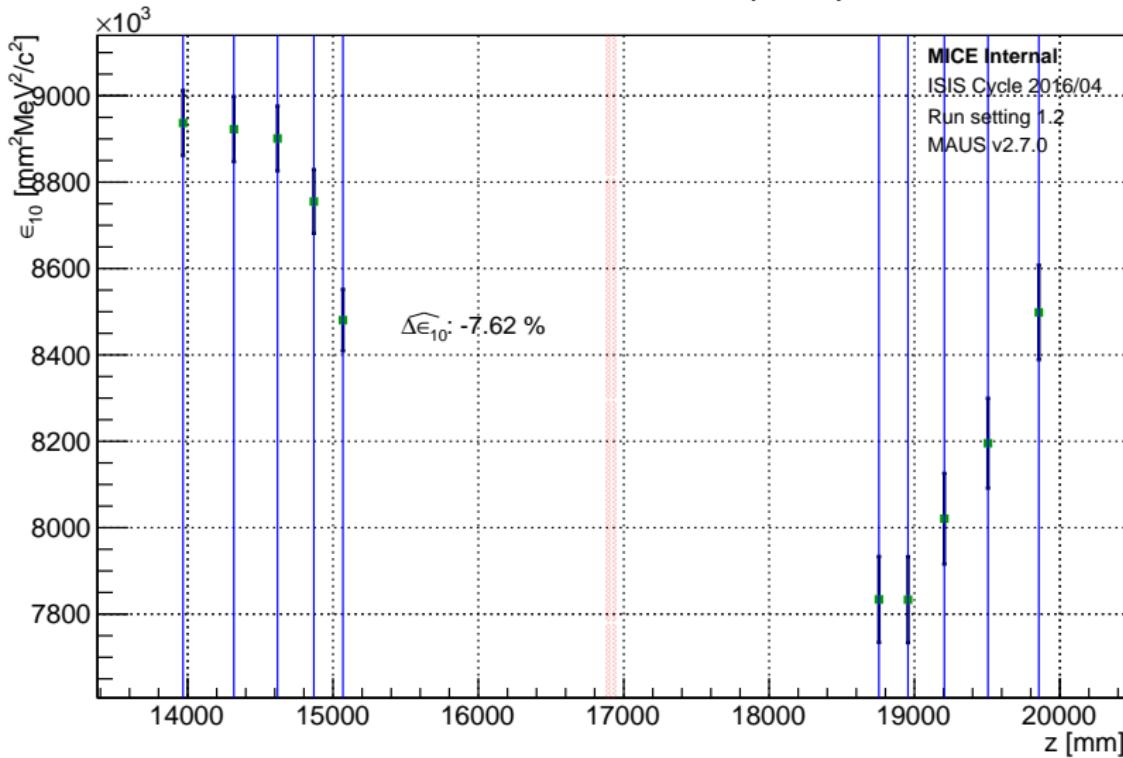
This yields radically different probability contents at different dimensions

n	1	2	3	4
V_n	$2(m\epsilon_n)^{1/2}$	$\pi m\epsilon_n$	$\frac{4}{3}\pi(m\epsilon_n)^{3/2}$	$\frac{1}{2}\pi^2(m\epsilon_n)^2$
$p(\mathbf{x} \in \mathcal{E})$	68.27 % $\text{erf}(1/\sqrt{2})$	39.35 % $1 - \frac{1}{\sqrt{e}}$	19.87 % $\text{erf}(1/\sqrt{2}) - \sqrt{\frac{2}{\pi e}}$	10.25 % $\frac{\pi+4}{2\pi} \left(1 - \frac{3}{2\sqrt{e}}\right)$

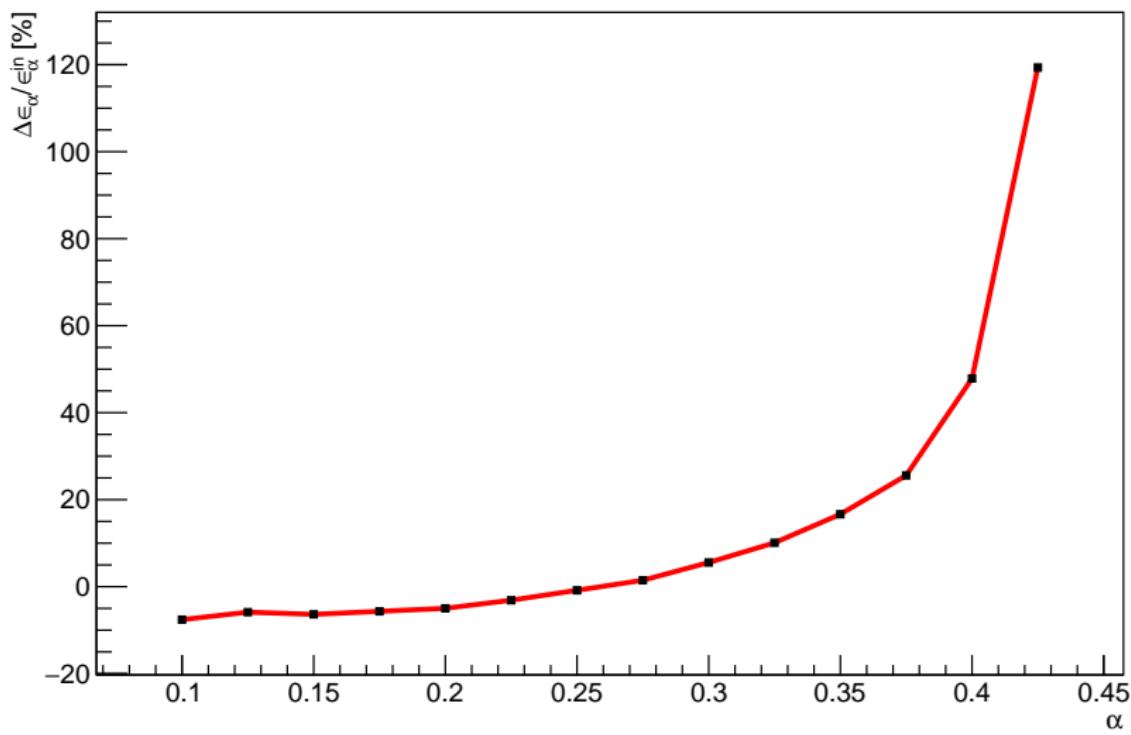
Given a volume measurement, the RMS emittance of a Gaussian beam is

$$\epsilon_n = \sqrt{\frac{2V_n}{\pi^2 m^2}} \rightarrow \begin{cases} \epsilon_i \simeq 9.62 \text{ mm} \\ \epsilon_o \simeq 8.78 \text{ mm} \end{cases} \rightarrow \boxed{\frac{\Delta\epsilon}{\epsilon_i} \simeq 8.7 \%!} \quad (6)$$

Fractional emittance (10%)



Fractional emittance change



Voronoi tessellation

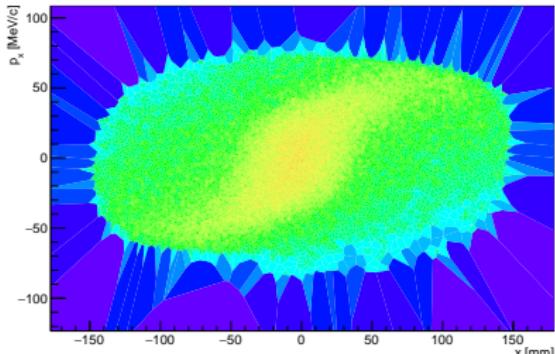
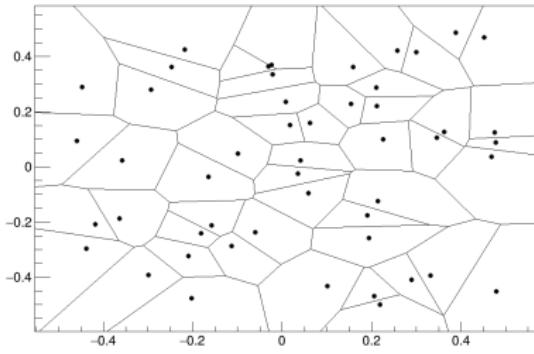
The general idea when constructing a phase space volume is to make the particle selection **from most dense to least dense** zones of the space, e.g. the low amplitude part of the space is more dense.

Voronoi tessellation pieces the space out into **regions** around each vertex that are closer to them than any other vertex:

$$R_k = \{x \in X \mid d(x, P_k) \leq d(x, P_j) \text{ for all } j \neq k\} \quad (7)$$

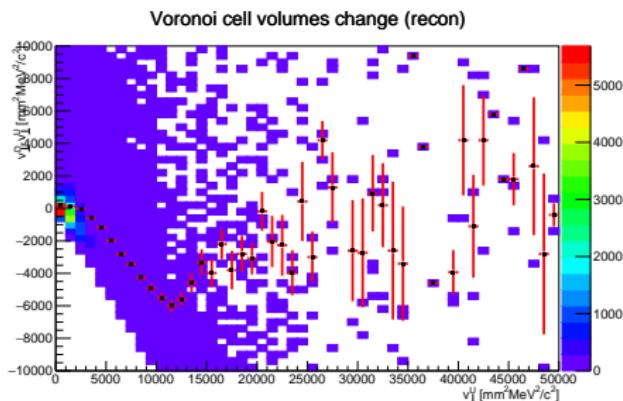
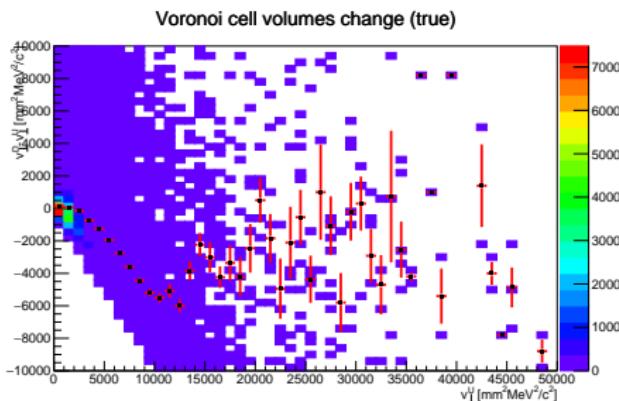
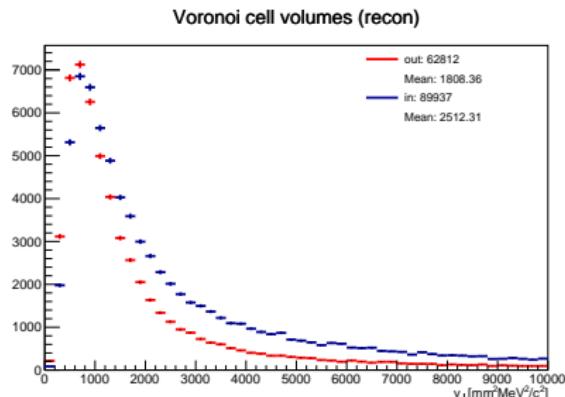
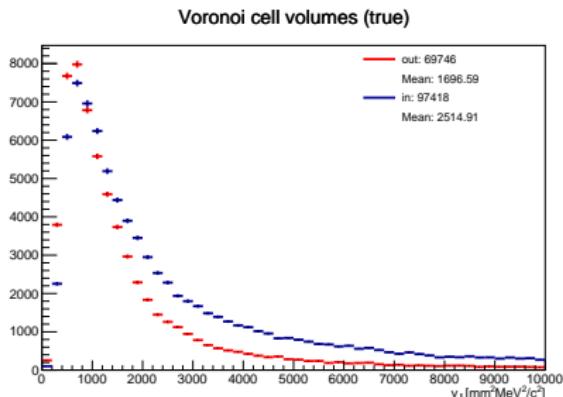
The smaller the region, the denser the space.

Voronoi tessellation



Voronoi volumes

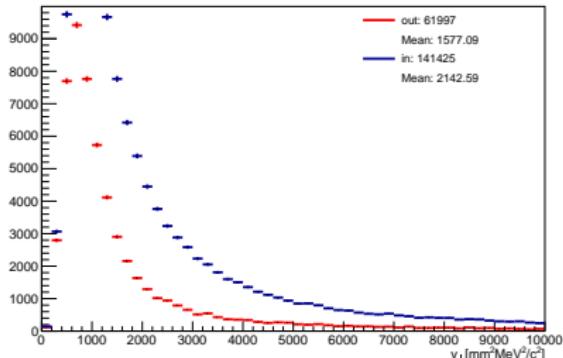
Simulation



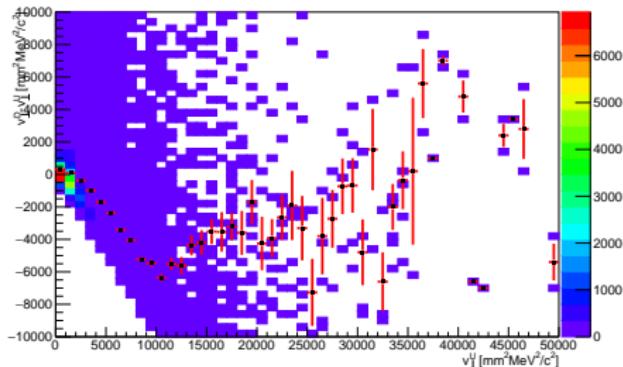
Voronoi volumes

Data

Voronoi cell volumes (recon)



Voronoi cell volumes change (recon)



Optimal binning to estimate the PDF

A histogram is the most traditional way to estimate density. Given M equally sized bins centred in the \vec{x}_i of individual contents N_i , the probability density function can be expressed as

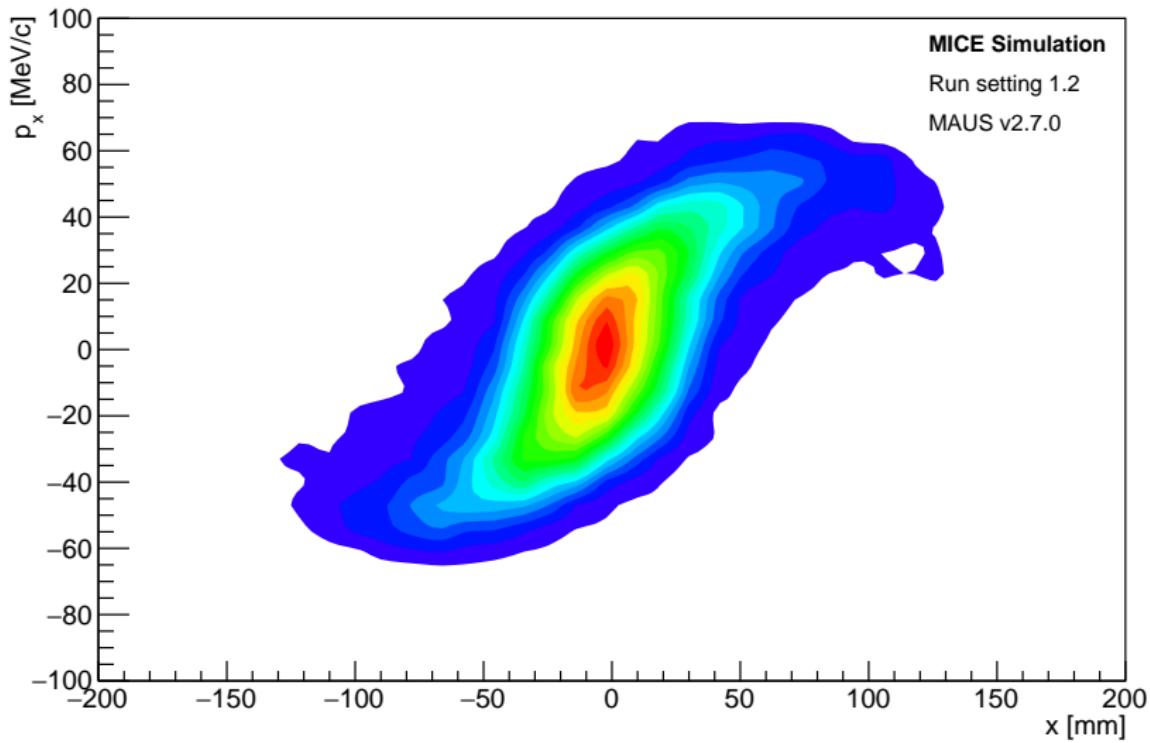
$$\hat{f}(\vec{x}) = \sum_{i=1}^M \frac{N_i}{\Delta \sum_{j=1}^n N_j} \chi_{B_i}(\vec{x}), \quad (8)$$

with the B_i the n -orthotope of volume Δ centred in \vec{x}_i .

How does one optimize the binning M ? A *jackknife* likelihood was proposed by D.W. Hogg defined as the performance of the histogram at predicting its own entries:

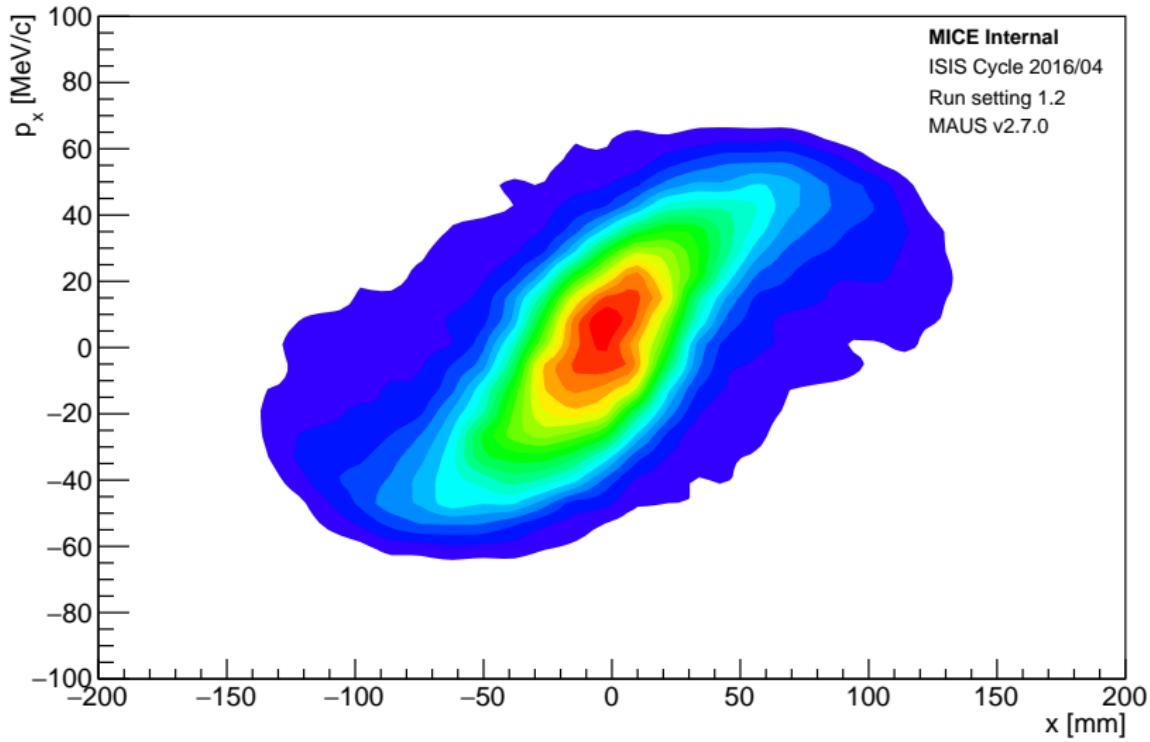
$$\ln L = \sum_{i=1}^N N_i \ln \left(\frac{N_i + \alpha - 1}{\Delta \left(\sum_{j=1}^N [N_j + \alpha] - 1 \right)} \right) \quad (9)$$

→ Maximizing this log likelihood yields the **most predictive binning**



TKD station 5 density

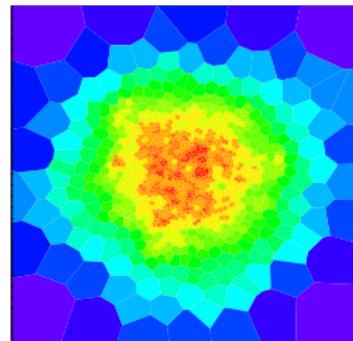
Data



Other non-parametric density estimations

Other methods **implemented** and being compared

- Penalized Centroidal Voronoi Tesselation (PCVT)
 - VT = **unbiased** but large variance, CVT = biased but small variance, PCVT takes the good in each to estimate density
 - Alternative: **bootstrapping** (PBATDE)



- k Nearest Neighbours (kNN)
- Local Reachability Density (LRD)
- Delaunary Tesselation Field Estimation (DTFE)

Conclusions

Two main challenges in the MICE Step IV 2016/04 data

- Low transmission, must **remove** the consequent **bias**
 - Nonlinearities, must have a **robust** phase-space volume measurement
- The full transmitted 4D RMS emittance is a poor estimator

An amplitude-wise fraction of the sample is **asymptotically unbiased**

- Take the **same amount of particles** up and downstream
- Evaluate the emittance change as a function of α
- Asymptotically converges towards the true amount of cooling

To get a faster, more linear convergence, use non-parametric statistics

- Does not rely on the convexity assumption
- Should ideally have a **flat response** to α
- **Many** options implemented and under investigation

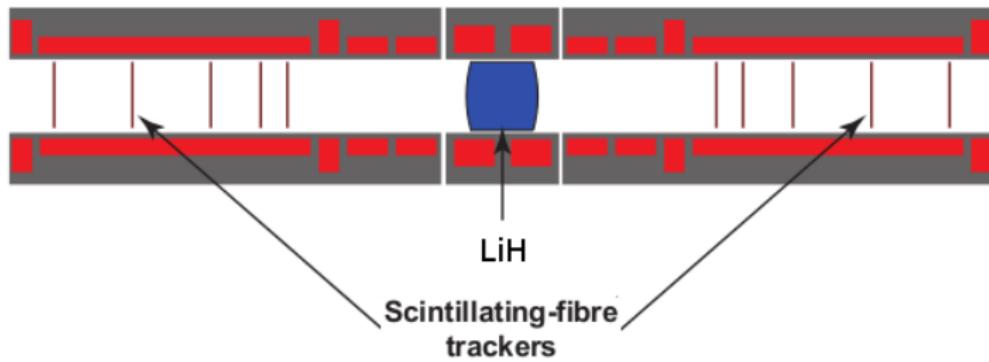
Back up slides

Geometries

In first approximation, a simplified geometry was used

- Two trackers in, 5 stations/tracker, 3 planes/station, full geometry
- A simple 65 mm-thick, 225 mm in radius cylinder of LiH (or not)
- Field maps generated in MAUS from the cooling channel currents
- Fixed emittance input beam at 13800 m (just before TKUS5)
- No momentum spread in the beam

The simulations were also run with the full MAUS geometry and the same input beam, it did not have any significant effect on the measurements.



Normalised RMS emittance

4D normalised RMS emittance:

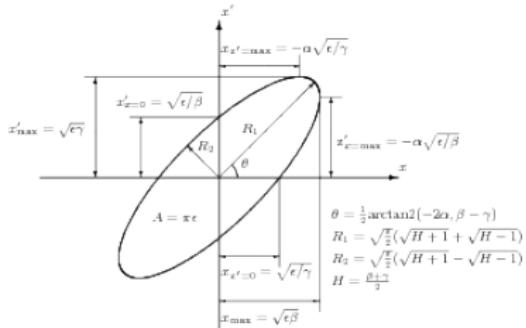
$$\epsilon_n = \frac{1}{m} \sqrt[4]{D} \quad (10)$$

with D the determinant of the covariance matrix defined by

$$D = \det \begin{bmatrix} V_{xx} & V_{xp_x} & V_{xy} & V_{xpy} \\ V_{p_x x} & V_{p_x p_x} & V_{p_x y} & V_{p_x p_y} \\ V_{yx} & V_{yp_x} & V_{yy} & V_{ypy} \\ V_{p_y x} & V_{p_y p_x} & V_{p_y y} & V_{p_y p_y} \end{bmatrix} \quad (11)$$

with $V_{\alpha\beta}$ the covariance of α and β defined as

$$V_{\alpha\beta} = \frac{1}{N} \sum_{i=1}^N (\alpha_i - \langle \alpha \rangle)(\beta_i - \langle \beta \rangle) = \langle \alpha \beta \rangle - \langle \alpha \rangle \langle \beta \rangle, \quad (12)$$



Limitations of the RMS definition

By its definition, the RMS **only** represents the **phase space volume** exactly if it is Normal distributed ($V_{RMS} = V$, $\frac{1}{(2\pi)^{\frac{d}{2}} |V|^{\frac{1}{2}}} \exp(x^T \Sigma^{-1} x)$):

- It is strongly affected by outliers (Large tail gives divergent $E(X^2)$)
 - It smears any small underlying structures
- Chromatic effects reduce cooling performance

For a multivariate Gaussian, the volume $m\epsilon_n$ does corresponds to the smallest volume containing a certain definite fraction α of the total particle sample.

After non-linear effect, the RMS emittance no longer represents the smallest volume containing a fraction α of the sample. It is necessary to use alternate figures-of-merit to get **rid of the tail effects**

We need to give a **weight** to individual particles

Alternative definition of amplitude

For some mean momentum $\langle p_z \rangle$, optical quantities are defined as

$$\begin{aligned}\alpha_{\perp} &= -\frac{\sigma_{xp_x} + \sigma_{yp_y}}{2m\epsilon_n}, & \beta_{\perp} &= \langle p_z \rangle \frac{\sigma_{xx} + \sigma_{yy}}{2m\epsilon_n} \\ \gamma_{\perp} &= \frac{1}{\langle p_z \rangle} \frac{\sigma_{xp_x p_x} + \sigma_{yp_y p_y}}{2m\epsilon_n}, & \beta_{\perp} \kappa - \mathcal{L} &= \frac{\sigma_{xp_y} - \sigma_{yp_x}}{2m\epsilon_n}\end{aligned}\quad (13)$$

For a centred phase space vector \mathbf{u} of coordinates $(u_x, u_{p_x}, u_y, u_{p_y})$, its associated transverse amplitude reads

$$\begin{aligned}A_{\perp} &\equiv \frac{1}{m\langle p_z \rangle} \beta_{\perp} (u_{p_x}^2 + u_{p_y}^2) + \langle p_z \rangle \gamma_{\perp} (u_x^2 + u_y^2) \\ &\quad + 2\alpha_{\perp} (u_x u_{p_x} + u_y u_{p_y}) + 2(\beta_{\perp} \kappa - \mathcal{L}) (u_x u_{p_y} - u_y u_{p_x}).\end{aligned}\quad (14)$$