Pulsar glitch dynamics in general relativity

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Sourie, Novak, Oertel & Chamel
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Plan of the presentation

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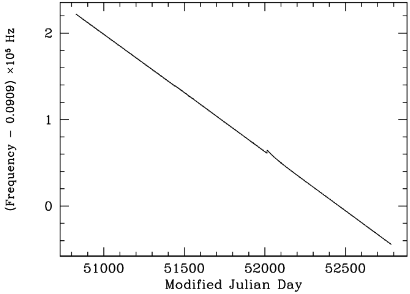
Introduction

Pulsar glitches

1RXS J1708−4009



Kaspi & Gavriil, *ApJ*, 2003

- Angular momentum loss through emission of electromagnetic waves

⇒ slowing down of the pulsar with

$$\dot{P} \sim 10^{-21}\text{–}10^{-10}$$

tiny changes in this slowing down = glitches

Pulsar glitches

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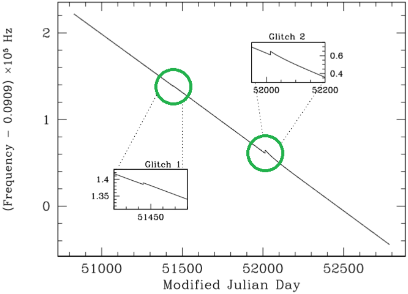

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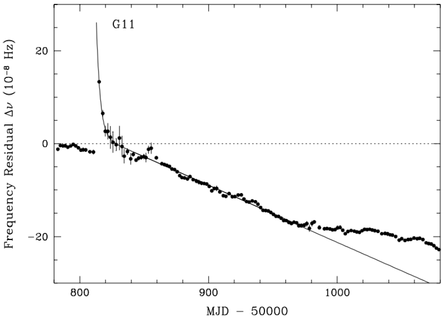
Pulsar glitch observations



Wong, Backer & Lyne, *ApJ*, 2001

- glitch **amplitude** are low:

$$\Delta\Omega/\Omega \sim 10^{-11} - 10^{-5}$$

- **rise time** is quite short :

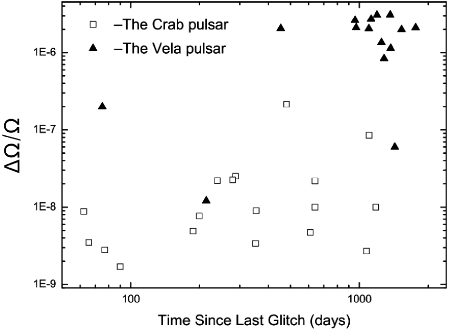
$$\boxed{\tau_{\rm r} < 30 \text{ s}} \quad \leftarrow\text{-- Vela}$$

- exponential **relaxation** during several days, up to months.

⇒ glitches are driven by **internal processes**

Different glitch types

Wang et al., *Ap&SS*, 2012



Giant glitches

- quasi-periodic
- narrow amplitude distribution

Standard glitches

- randomly spaced in time
- various amplitudes

Different glitch models

⇒moment of inertia reduction, with crustquakes
⇒transfer of angular momentum between two components, with superfluidity

Different glitch types

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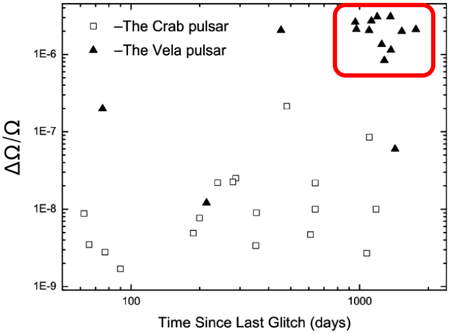

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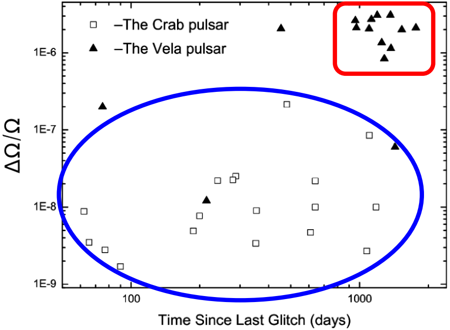

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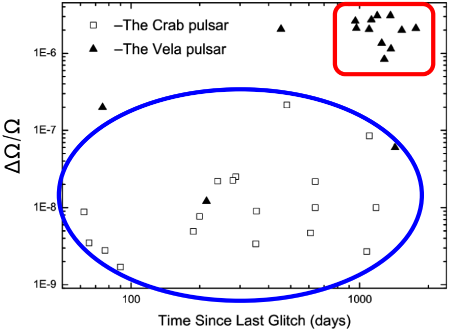

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Hypotheses

- General relativity to describe gravity
- Need to describe rotation $\Rightarrow$ axisymmetry
- Glitch time-scale $\gg$ hydro time-scale $\Rightarrow$ stationarity

In adapted coordinates, the metric depends only on (r, θ) and can take the form (quasi-isotropic gauge):

$$ds^2 = -N^2 dt^2 + A^2 \left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta \, (d\varphi - \omega\, dt)^2 ,$$

with the requirement of circularity condition for matter:

- no meridional (e.g. convective) currents,
- no mixed poloidal/toroidal magnetic field.

This is quite different from the Schwarzschild gauge used for the TOV system, in spherical symmetry.

Einstein equations

In quasi-isotropic gauge (+maximal slicing), Einstein equations turn into a system of four coupled non-linear elliptic PDEs:

- $\Delta N = \sigma_1$,
- $\Delta \omega = \sigma_2$,
- $\Delta (NB) = \sigma_3$,
- $\Delta (NA) = \sigma_4$.

Each σ_i contains terms involving matter and non-linear metric terms of non-compact support (Bonazzola et al. 1993).
$\Rightarrow$Contrary to spherical symmetry no matching to any known vacuum solution is possible (no Birkhoff theorem).
$\Rightarrow$Only boundary condition at $r \to \infty$: flat metric.

Numerical solution obtained using spectral methods (Grandclément & Novak 2009) and the LORENE library (http://lorene.obspm.fr).

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Two-fluid rotating models

Sourie et al. 2016

Two-fluid approach

Anderson & Itoh 1975

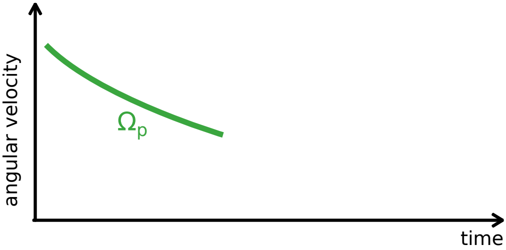
Two-fluid model

- charged particles :

  $\Omega_p = \Omega \leftrightarrow$ pulsar

- superfluid neutrons :

  $\Omega_n \gtrsim \Omega_p$



- Superfluid vortices can pin into the crust nuclei
- When a critical threshold is reached in terms of $\delta\Omega = \Omega_n - \Omega_p$, some vortices unpin and can freely move in radial direction

$\Rightarrow$ Transfer of angular momentum between both fluids and glitch

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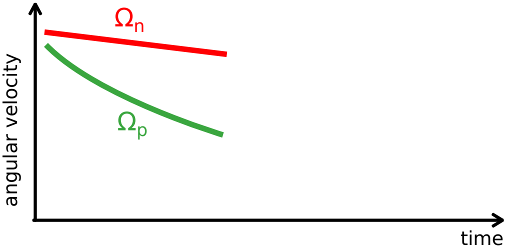

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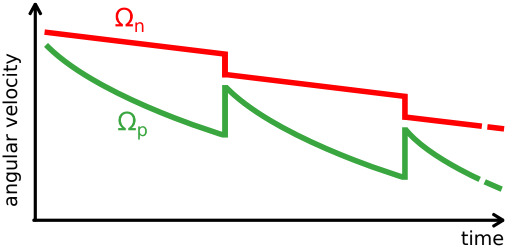

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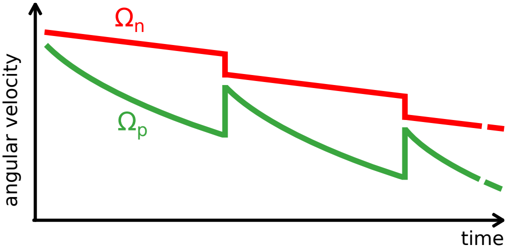

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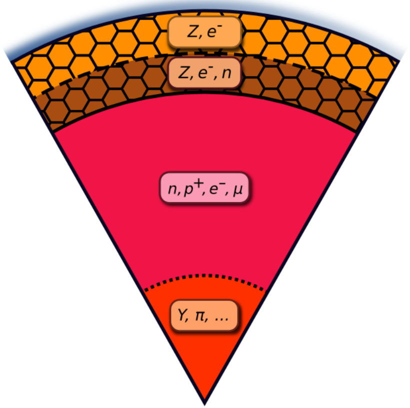
Hypotheses

Prix, Novak & Comer 2005



Equilibrium configurations:

- uniform composition : n, p, e^-
  $\rightsquigarrow$ crust is neglected
- rigid rotation :
  $\rightsquigarrow$ Ω_n and Ω_p = const.
- stationary and axisymmetric spacetime + isolated star.
- $T \ll T_F$, and no magnetic field.
- dissipation effects are neglected.

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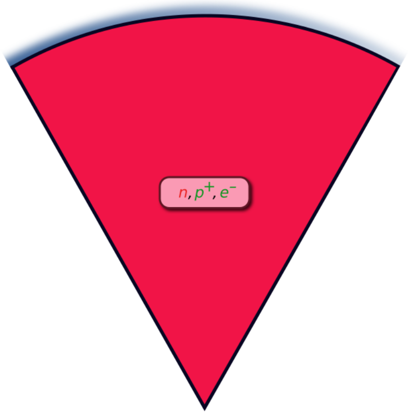

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TWO-FLUID RELATIVISTIC HYDRODYNAMICS

Carter 1989; Carter & Langlois 1998; Langlois, Sedrakian & Carter 1998

System made of two perfect fluids :

- superfluid neutrons $\rightarrow n_{\rm n}^{\alpha} = n_{\rm n} u_{\rm n}^{\alpha}$,
- protons & electrons $\rightarrow n_{\rm p}^{\alpha} = n_{\rm p} u_{\rm p}^{\alpha}$.

Energy-momentum tensor

1 fluid : $T_{\alpha\beta} = (\mathcal{E} + P)\, u_\alpha u_\beta + P g_{\alpha\beta}$

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2 fluids : $T_{\alpha\beta} = n_{{\rm n}\alpha} p^{\rm n}_{\beta} + n_{{\rm p}\alpha} p^{\rm p}_{\beta} + \Psi g_{\alpha\beta}$

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$\hookrightarrow$ conjugate momenta

$$\begin{cases} p^{\rm n}_{\alpha} & \propto u^{\rm n}_{\alpha} \\ p^{\rm p}_{\alpha} & \propto u^{\rm p}_{\alpha} \end{cases}$$

without entrainment

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Entrainment matrix:

$$\begin{cases} p^{\rm n}_{\alpha} \propto u^{\rm n}_{\alpha} \\ p^{\rm p}_{\alpha} \propto u^{\rm p}_{\alpha} \end{cases} \longrightarrow \begin{cases} p^{\rm n}_{\alpha} = \mathcal{K}^{\rm nn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm np} n^{\rm p}_{\alpha} \\ p^{\rm p}_{\alpha} = \mathcal{K}^{\rm pn} n^{\rm n}_{\alpha} + \mathcal{K}^{\rm pp} n^{\rm p}_{\alpha} \end{cases}$$

without entrainment

$\Rightarrow$ entrainment effect

New, realistic equations of state

$$\mathcal{E}\left(n_{\rm n}, n_{\rm p}, \Delta^2\right) \longleftrightarrow \Psi\left(\mu^{\rm n}, \mu^{\rm p}, \Delta^2\right)$$

Relativistic mean field model :

nucleon - nucleon interactions ⇔ effective meson exchange

	DDH Typel & Wolter (1999)	**DDHδ** Avancini et al. (2009)	exp. constraints Oertel et al. (2017)	[units]
n_0	0.153	0.153	0.158 ± 0.005	[fm^{-3}]
B_{sat}	16.3	16.3	15.9 ± 0.3	[MeV]
K	240	240	240 ± 40	[MeV]
J	32.0	25.1	31.7 ± 3.2	[MeV]
L	55	44	58.7 ± 28.1	[MeV]
$M_{\rm G}^{\max,0}$	2.08	2.16	$\gtrsim 2$	[$\mathrm{M}_\odot$]

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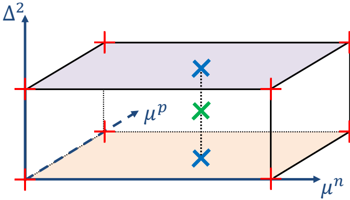
⇢ taking into account entrainment



interpolation of 3-parameter EoS tables in LORENE, following Swesty (1996)

With entrainment

$$p^\alpha_X = \mathcal{K}^{XX} n_X u^\alpha_X + \mathcal{K}^{XY} n_Y u^\alpha_Y$$

Dynamic effective mass:

$$\boxed{p^i_X = \tilde{m}_X \; u^i_X} \qquad i \in \{1,2,3\}$$

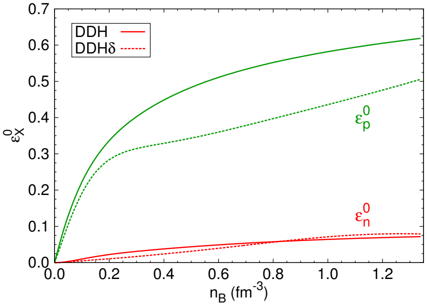
in the fluid-Y rest-frame

In the $u^i_Y = 0$ frame:

relativity

$$\tilde{m}_X = \mu^X \times \left(1 - \varepsilon_X\right)$$

entrainment



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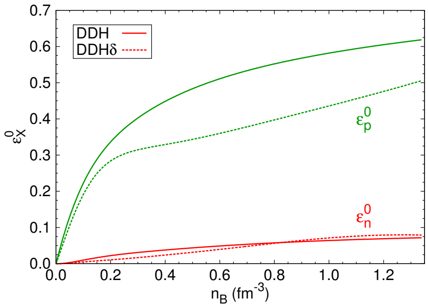

WITH ENTRAINMENT

$$p_X^\alpha = \mathcal{K}^{XX} n_X u_X^\alpha + \mathcal{K}^{XY} n_Y u_Y^\alpha$$

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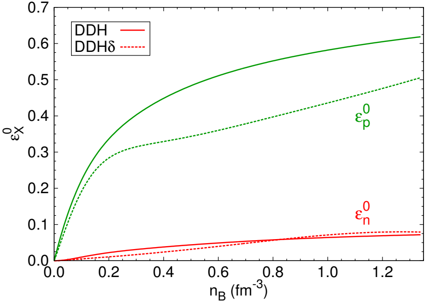


Numerical results

entrainment vs. Lense-Thirring

The (Komar) angular momentum J_X is such that

$$\mathrm{d}J_X = I_{XX}\ \mathrm{d}\Omega_X + I_{XY}\ \mathrm{d}\Omega_Y$$

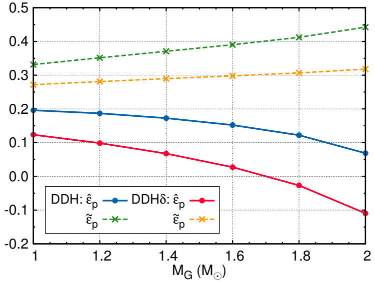
Total coupling coefficient $\hat{\varepsilon}_X = I_{XY} / (I_{XX} + I_{XY})$ depends

ENTRAINMENT

- due to strong interaction between nucleons
- measured with the global entrainment coefficient $\tilde{\varepsilon}$ (integration of ε over the star)

Lens-Thirring effect

- due to GR dragging of inertial frames by each fluid
- measured with the metric term $g_{t\varphi}$



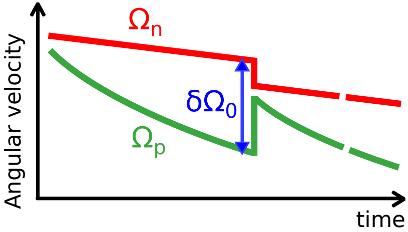
Glitch models

Rise phase

Sourie et al. 2017

Mutual friction

No external torque $\Rightarrow$exchange of angular momentum between neutrons and protons through mutual friction torque $\Gamma_{\rm mf}$



From Langlois *et al.* (1998), with straight vortices parallel to the rotation axis: interplay between

- Magnus force due to neutron fluid
- drag force caused by charged particles

$$\Gamma_{\rm mf} = -\int \frac{\mathcal{R}}{1+\mathcal{R}^2}\Gamma_{\rm n} n_{\rm n} \varpi_{\rm n} \chi_{\perp}^2 \, {\rm d}\Sigma \times (\Omega_{\rm n} - \Omega_{\rm p}) = -\bar{\mathcal{B}} \times 2 \hat{I}_{\rm n} \Omega_{\rm n} \zeta \times \delta\Omega$$

Rise time

Evolution equations:

$$\begin{cases} \dot{J}_{\rm n} = +\,\Gamma_{\rm mf}, \\ \dot{J}_{\rm p} = -\,\Gamma_{\rm mf}. \end{cases} \dashrightarrow \quad \frac{\delta\dot{\Omega}}{\delta\Omega} = -\frac{\hat{I}\hat{I}_{\rm n}}{I_{\rm nn}I_{\rm pp} - I_{\rm np}^2} \times 2\bar{\mathcal{B}}\zeta\Omega_{\rm n}$$

⇒Analytic approximation:

$$\delta\Omega(t) = \delta\Omega_0 \times \exp\left(-\frac{t}{\tau_{\rm r}}\right)$$

$$\tau_{\rm r} = \frac{\hat{I}_{\rm p}}{\hat{I}} \times \frac{1 - \hat{\varepsilon}_{\rm p} - \hat{\varepsilon}_{\rm n}}{2\zeta\bar{\mathcal{B}}\Omega_{\rm n}}$$

⇒Numerical modeling:

$\Omega_{\rm n}(t)$ and $\Omega_{\rm p}(t)$ are determined by integration from $\Omega_{\rm n,0} > \Omega_{\rm p,0}$

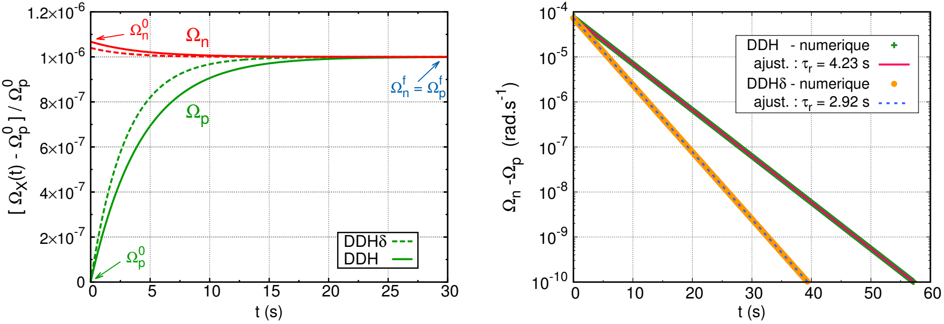
PARAMETERS

$M_G, \Omega, \Delta\Omega/\Omega, {\rm EoS}, \bar{\mathcal{B}}$

Time evolution

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega_{\rm n}^{f} = \Omega_{\rm p}^{f} = 2\pi \times 11.19\ \mathrm{Hz},$$

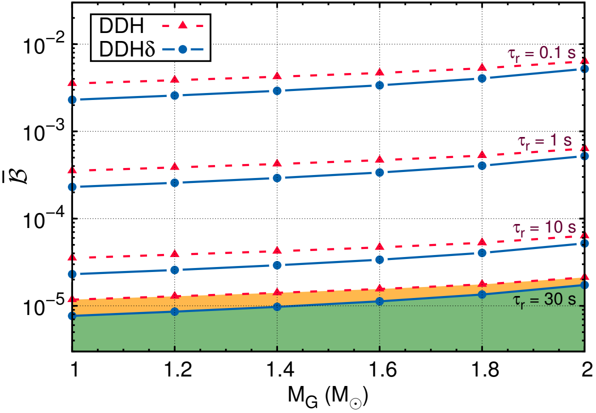
$$M_{\rm G} = 1.4\ \mathrm{M}_{\odot}\ \&\ \bar{\mathcal{B}} = 10^{-4}$$



--→ Rise times can be estimated with high accuracy without time integration, using only equilibrium models.

Vela pulsar

$\Delta\Omega/\Omega = 10^{-6}$, $\Omega_{\rm n}^f = \Omega_{\rm p}^f = 2\pi \times 11.19$ Hz



- $\bar{\mathcal{B}} \nearrow \Longrightarrow \tau_{\rm r} \searrow$
- Constraints on $\bar{\mathcal{B}}$:

  $\tau_{\rm r} < 30$ s $\Rightarrow \bar{\mathcal{B}} > 10^{-5}$
- $\bar{\mathcal{B}} < 1/2 \rightsquigarrow \tau_{\rm r} > 0.6$ ms

↪ a glitch event is a quasi-stationary process, from the hydro viewpoint

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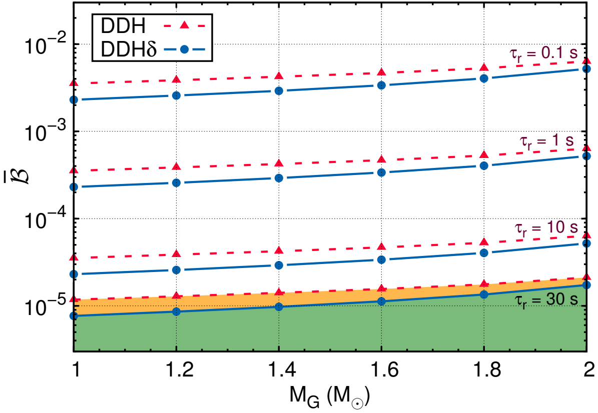

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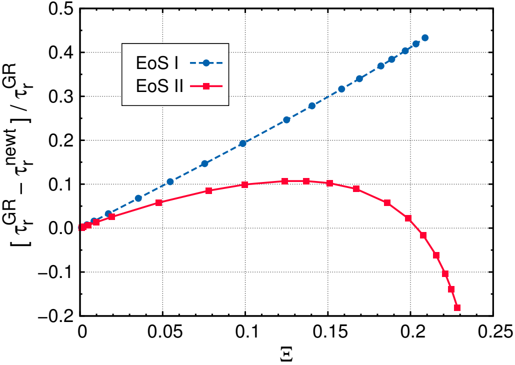
Influence of general relativity

Comparison using two analytic EoSs from Prix *et al.* (2005)



- Compactness defined as

$$\boxed{\Xi = \frac{G\,M_G}{R_{circ}c^2}}$$

  with $\Xi_{NS} \simeq 0.2 - 0.3$
- depends on Ω, too.

$\Rightarrow$impact of general relativity on glitch dynamics can be quite strong!

Conclusions – perspectives

- Precise models of rotating neutron stars in GR
- Realistic EoS for 2 fluids, including entrainment
- Quasi-stationary approach, with analytic formula for rise time
- Additional coupling between fluids due to Lens-Thirring effect
- Strong overall influence of GR on glitch rise time

For the future:

- Looking for accurate data to constrain rise time
- Local modeling of glitch unpinning and movement
- Taking into account crust in global models?

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