

Modification of Compact Star Observables due to Quantum Fluctuations

An Application of Functional Renormalization Group Method for Superdense Nuclear Matter

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References:

[arXiv:1604.01717](https://arxiv.org/abs/1604.01717) [hep-th], *Eur. Phys. J. C* (2015) **75**: 2, PoS(EPS-HEP2015)369

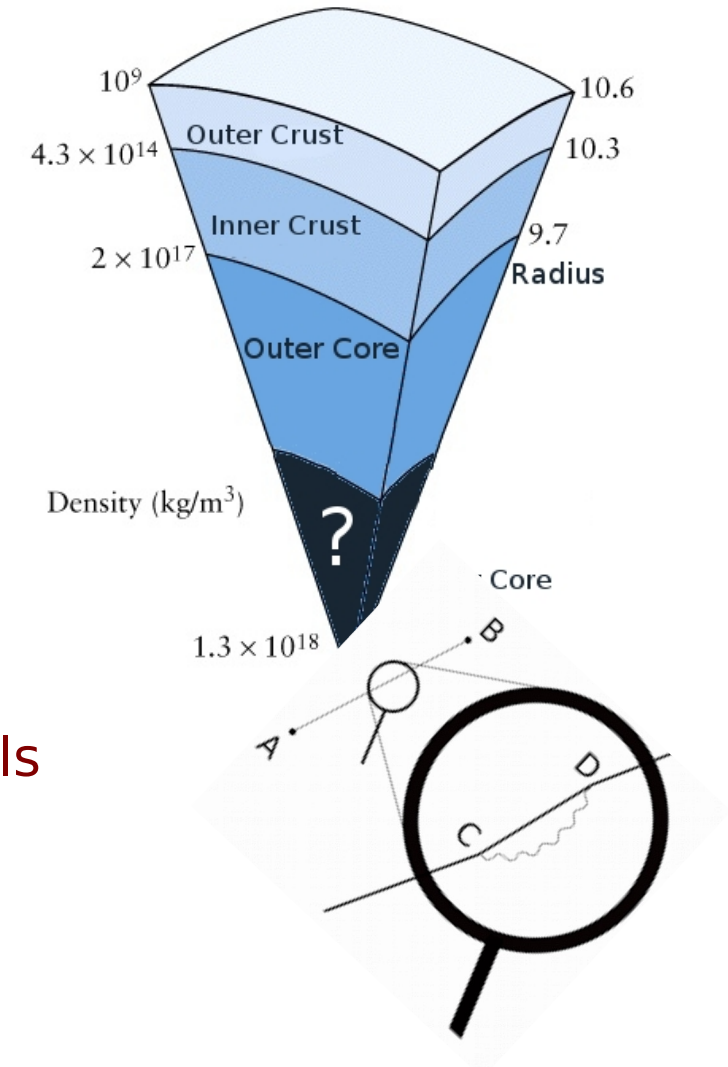
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MPCSRG 2017, Yerevan, Armenia, 18th-21st September 2017



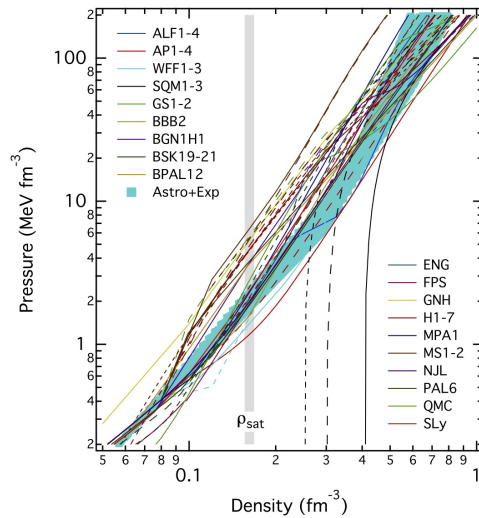
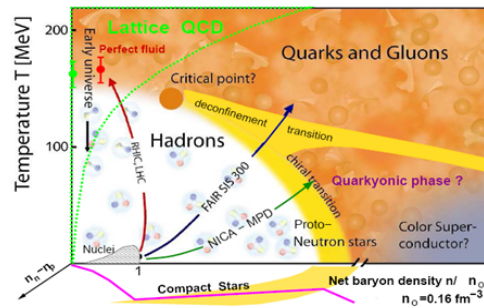
Outline

- Motivation
 - Introduction to FRG method
- Ansatz for the effective action:
 - Fermi gas model at finite temperature with a Yukawa coupling
- Solving Wetterich equation for finite chemical potential
 - Local polynomial approximation
 - Wetterich equation at zero temperature
 - Solution techniques
- Results and comparison of the FRG results to other models
 - Phase structure and Equation of State at different approximations
 - Comparing Equation of State and compact star observables

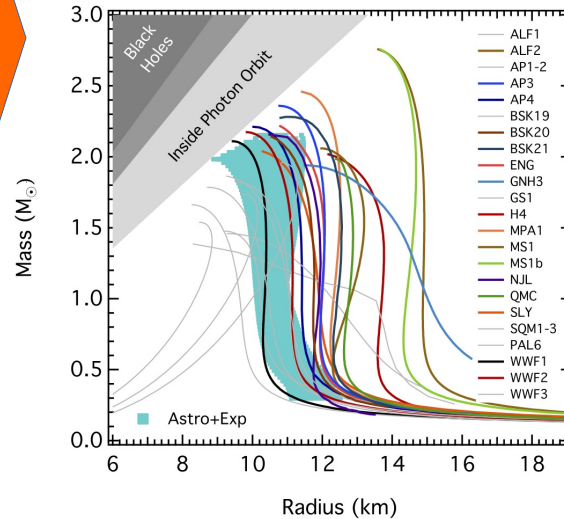
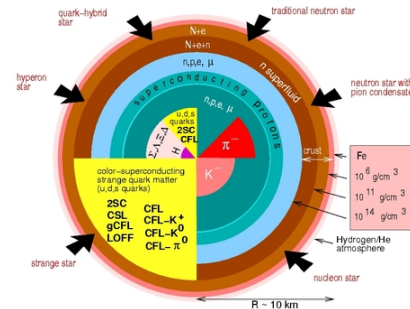


NewCompStar Motivation

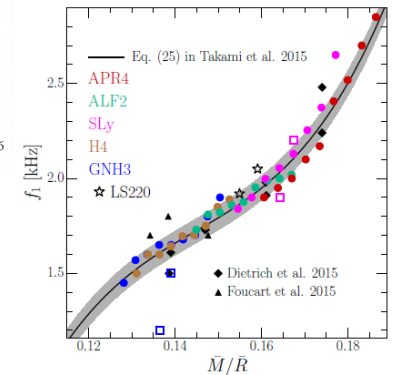
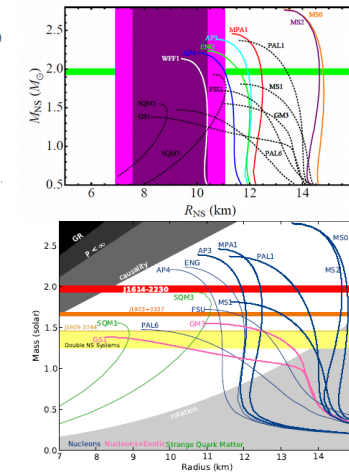
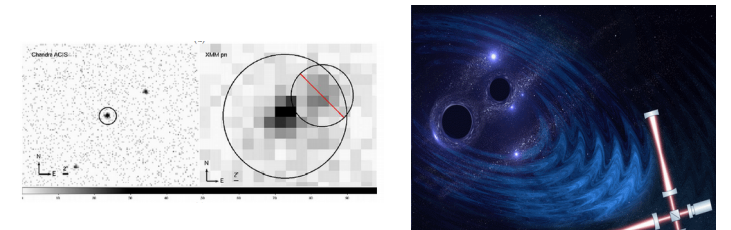
EoS
from exp & theory



Application in
compact stars

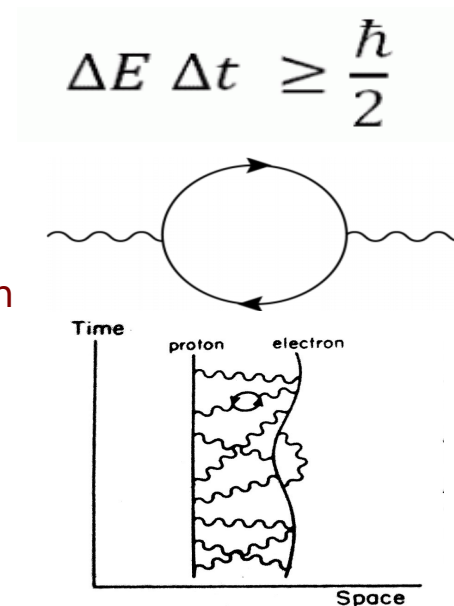
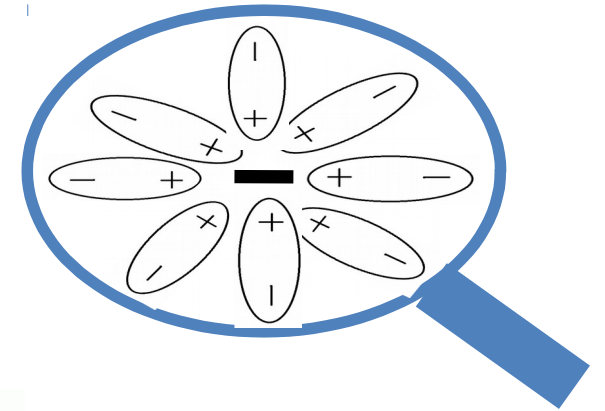


Constraints by
astrophysical observations



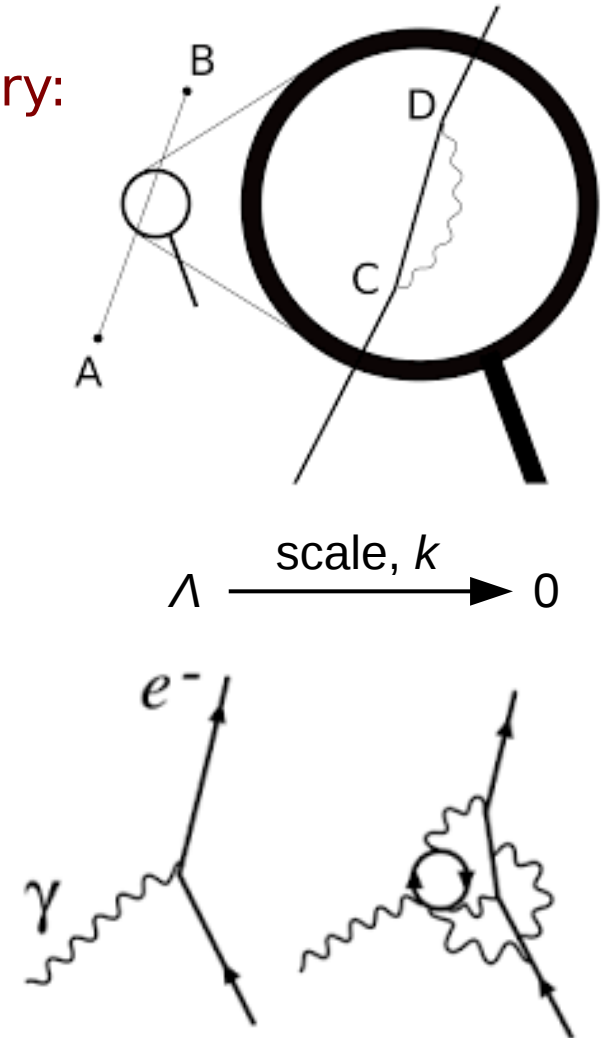
Motivation for FRG

- **Observation:** Considering a point charge, which polarizes the medium seems like point charge with a modified charge.
- **Basic idea:** Due to the interaction, the measurable (effective) properties differs from the bare quantities.
- **Quantum corrections:**
 - Heisenberg uncertainty
high-energy reaction for a short time is allowed
 - Pair production & annihilation
bosonic propagator is modified due to the pair production
 - Self-interaction
Interaction is a sum of many tiny- and self interaction



Motivation for FRG

- It is hard to get effective action for an interacting field theory:
e.g.: EoS for superdense cold matter ($T \rightarrow 0$ and finite μ)
- Taking into account quantum fluctuations using a scale, k
 - Classical action, $S = \Gamma_{k \rightarrow \Lambda}$ in the UV limit, $k \rightarrow \Lambda$
 - Quantum action, $\Gamma = \Gamma_{k \rightarrow 0}$ in the IR limit, $k \rightarrow 0$
- FRG Method
 - Smooth transition from macroscopic to microscopic
 - RG method for QFT
 - Non-perturbative description
 - Not depends on coupling
 - **BUT: Technically it is NOT simple**



Functional Renormalization Group (FRG)

- ▶ FRG is a general non-perturbative method to determine the effective action of a system.
- ▶ **Scale dependent effective action (k scale parameter)**

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \text{STr} \left[\frac{\partial_k R_k}{\Gamma_k^{(2)} + R_k} \right]$$

Wetterich
equation

$k=\Lambda$
Classical action



Integration

$k=0$
Quantum
fluctuations
included

Functional Renormalization Group (FRG)

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Wetterich
equation

- ▶ **Ansatz** for the integration,
 - not need to be perturbative
 - scale-dependent coupling

$$\Gamma_k = \sum_{l=1}^{l=N} \frac{g_l(k)}{l!} \hat{O}_l$$

Functional Renormalization Group (FRG)

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Wetterich
equation

- ▶ **Regulator**
 - Determines the modes present on scale, k
 - Physics is regulator independent

Ansatz: Interacting Fermi-gas model

Ansatz for the effective action:

$$\Gamma_k[\varphi, \psi] = \int d^4x \left[\bar{\psi} (i\partial\!\!\!/ - g\varphi) \psi + \frac{1}{2} (\partial_\mu\varphi)^2 - U_k(\varphi) \right]$$

Fermions : $m=0$, **Yukawa-coupling** generates mass

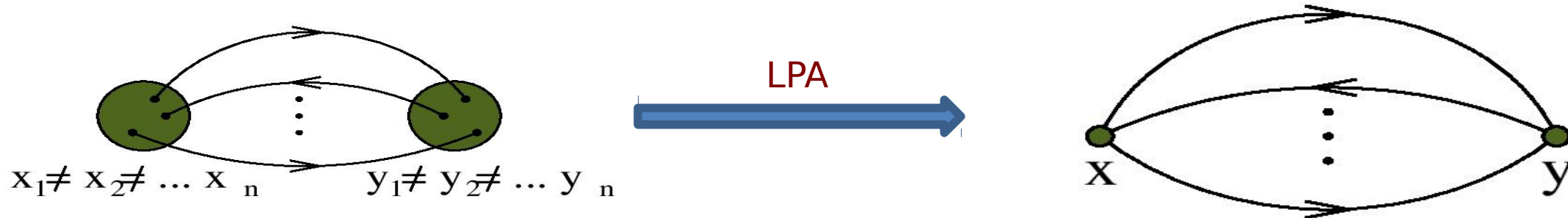
Bosons: the **potential** contains self interaction terms

We study the scale dependence of the potential only!!

Local Potential Approximation (LPA)

What does the ansatz exactly mean?

LPA is based on the assumption that the contribution of these two diagrams are close. (*momentum dependence of the vertices is suppressed*)



This implies the following ansatz for the effective action:

$$\Gamma_k [\psi] = \int d^4x \left[\frac{1}{2} \psi_i K_{k,ij} \psi_j + U_k (\psi) \right]$$

Interacting Fermi-gas at finite temperature

Ansatz for the effective action:

$$\Gamma_k[\varphi, \psi] = \int d^4x \left[\bar{\psi} (i\partial - g\varphi) \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - U_k(\varphi) \right]$$



Wetterich -equation

$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\underbrace{\frac{1 + 2n_B(\omega_B)}{\omega_B}}_{\text{Bosonic part}} + 4 \underbrace{\frac{-1 + n_F(\omega_F - \mu) + n_F(\omega_F + \mu)}{\omega_F}}_{\text{Fermionic part}} \right]$$

Bosonic part

Fermionic part

$$U_\Lambda(\varphi) = \frac{m_0^2}{2} \varphi^2 + \frac{\lambda_0}{24} \varphi^4$$

$$\omega_F^2 = k^2 + g^2 \varphi^2$$

$$\omega_B^2 = k^2 + \partial_\varphi^2 U$$

$$n_{B/F}(\omega) = \frac{1}{1 \mp e^{-\beta\omega}}$$

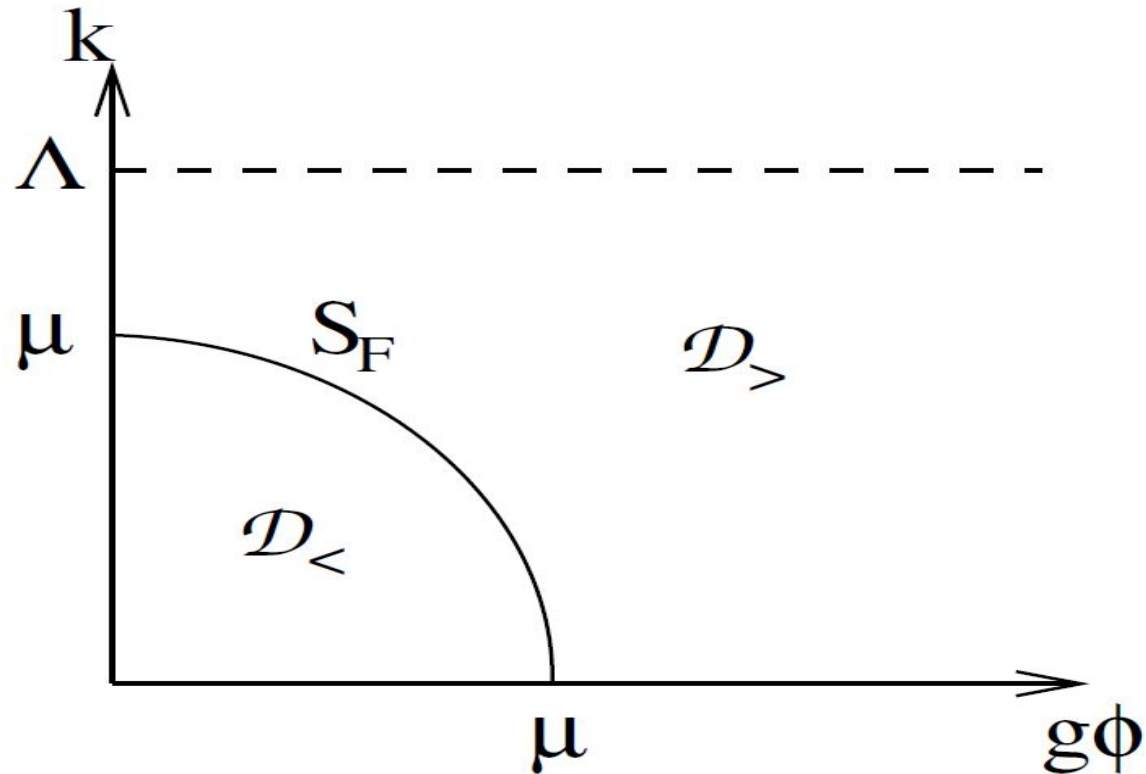
Interacting Fermi-gas at zero temperature

$$T=0, \mu \neq 0$$



$$n_F(\omega) \rightarrow \Theta(-\omega)$$

We have two equations for the two values of the step function each valid on different domain



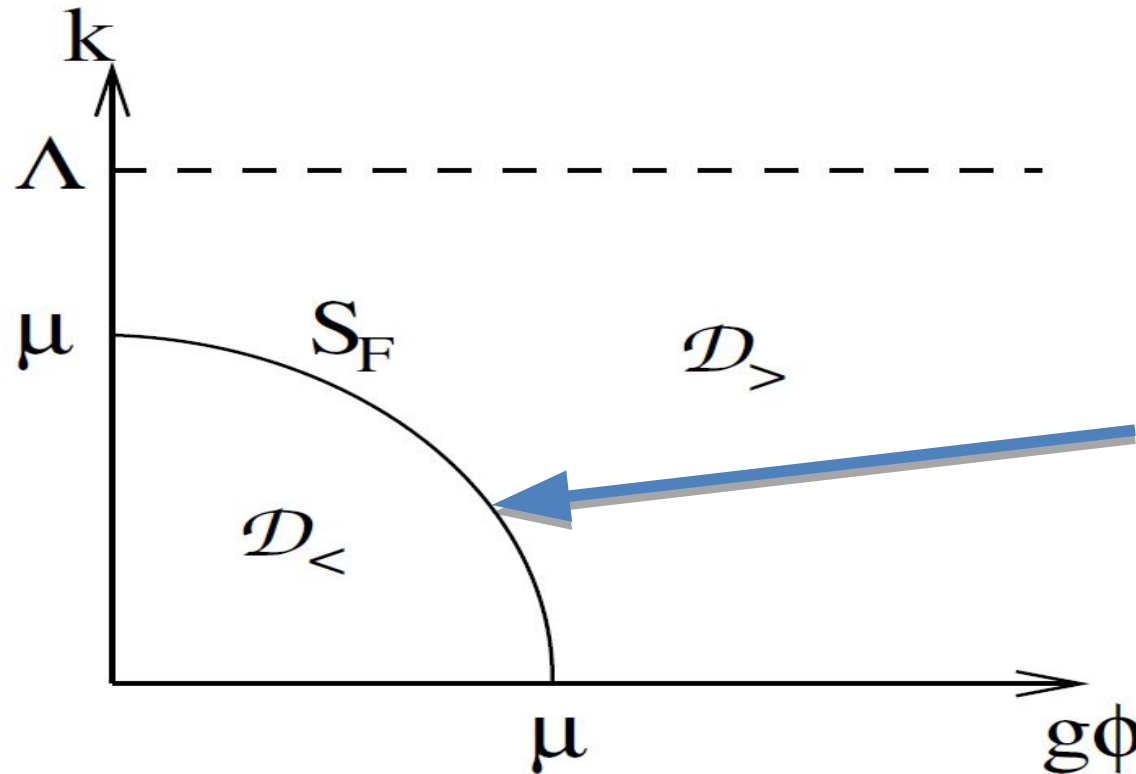
Interacting Fermi-gas at zero temperature

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$$k_F = \sqrt{\mu^2 - g^2 \phi^2},$$

Fermi-surface

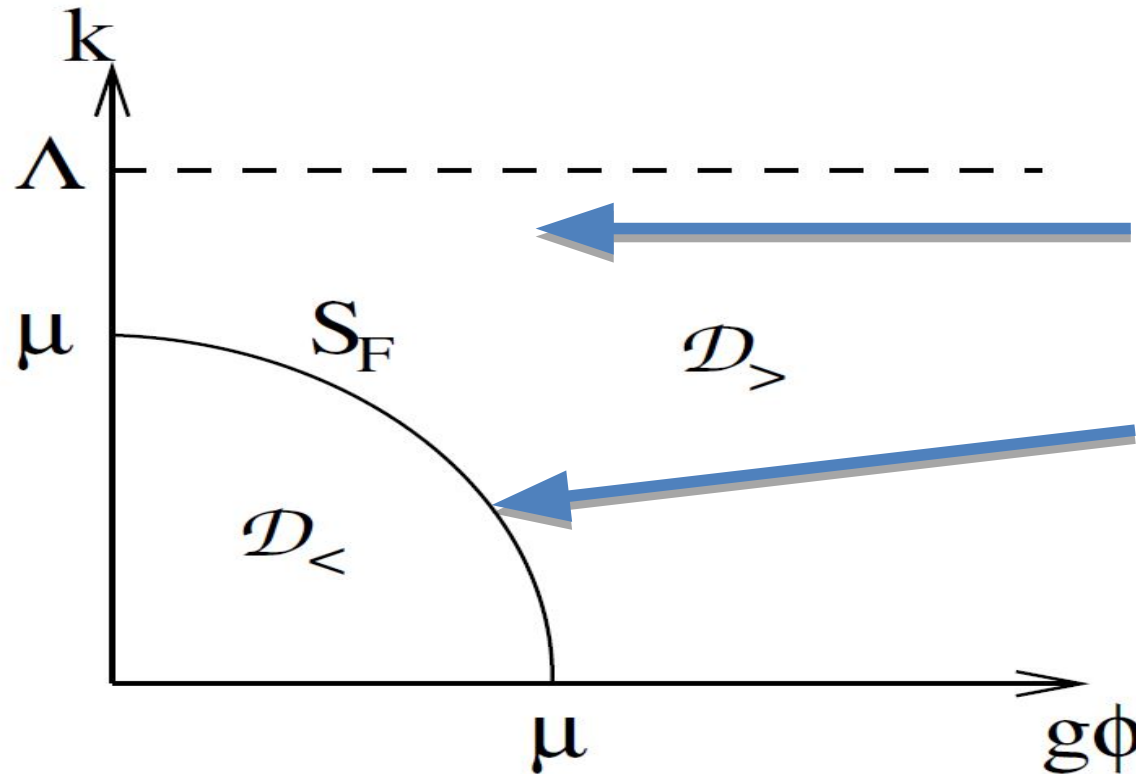
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$$\partial_k U_k = \frac{k^4}{12\pi^2} \left[\frac{1}{\omega_B} - \frac{4}{\omega_F} \right]$$

$$k_F = \sqrt{\mu^2 - g^2 \varphi^2},$$

Fermi-surface

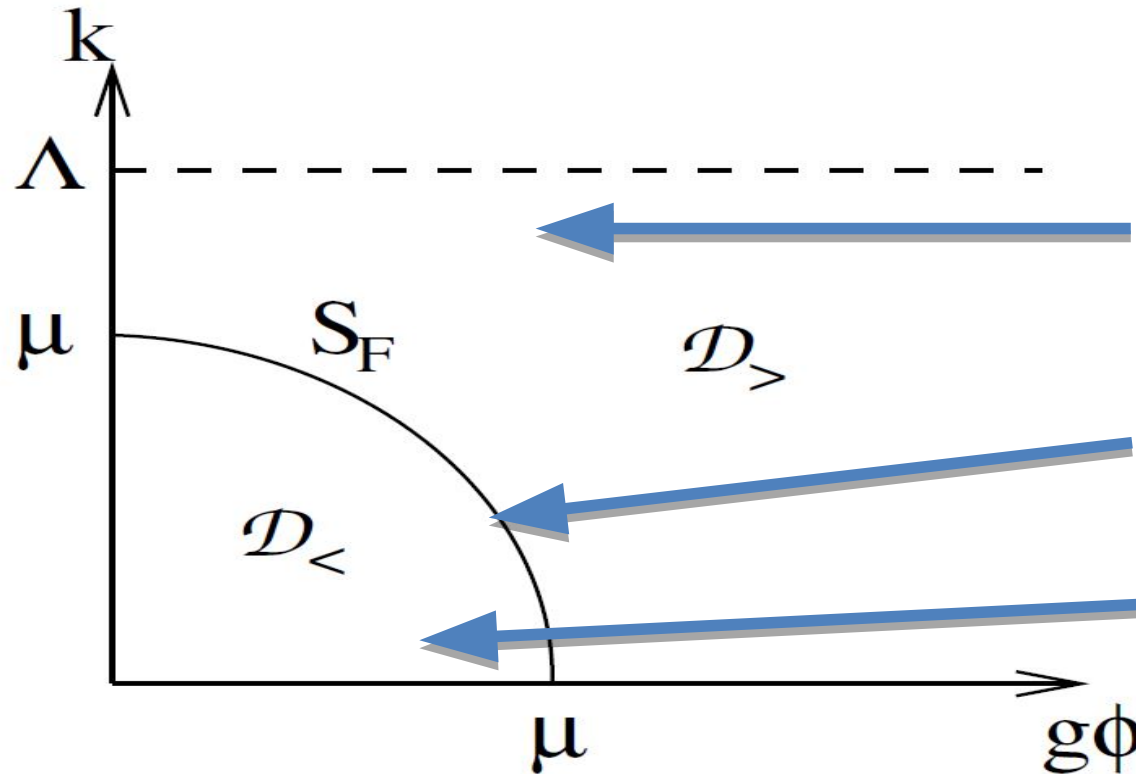
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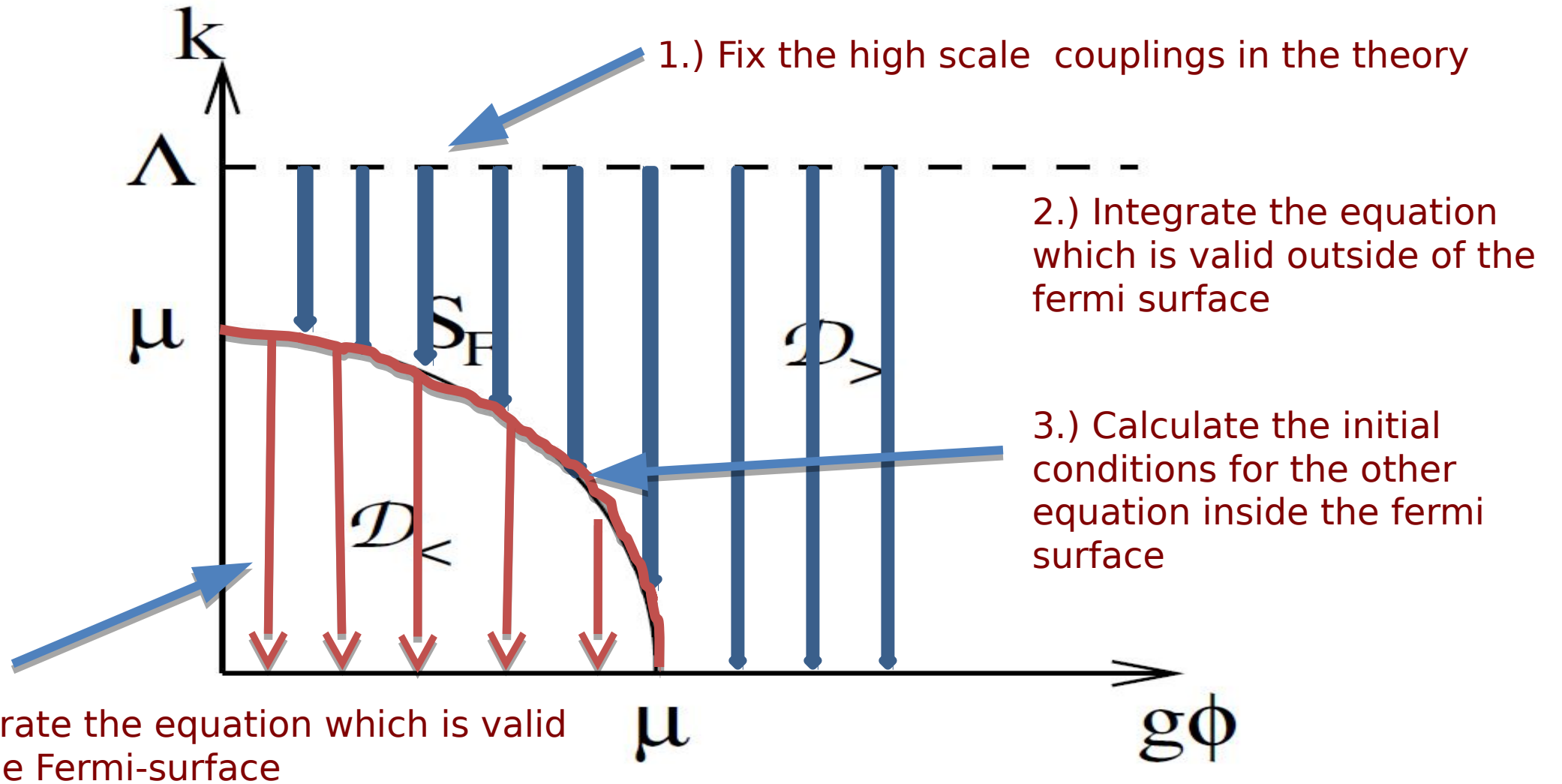
$$k_F = \sqrt{\mu^2 - g^2 \varphi^2},$$

$$\partial_k U_k = \frac{k^4}{12\pi^2} \frac{1}{\omega_B}$$

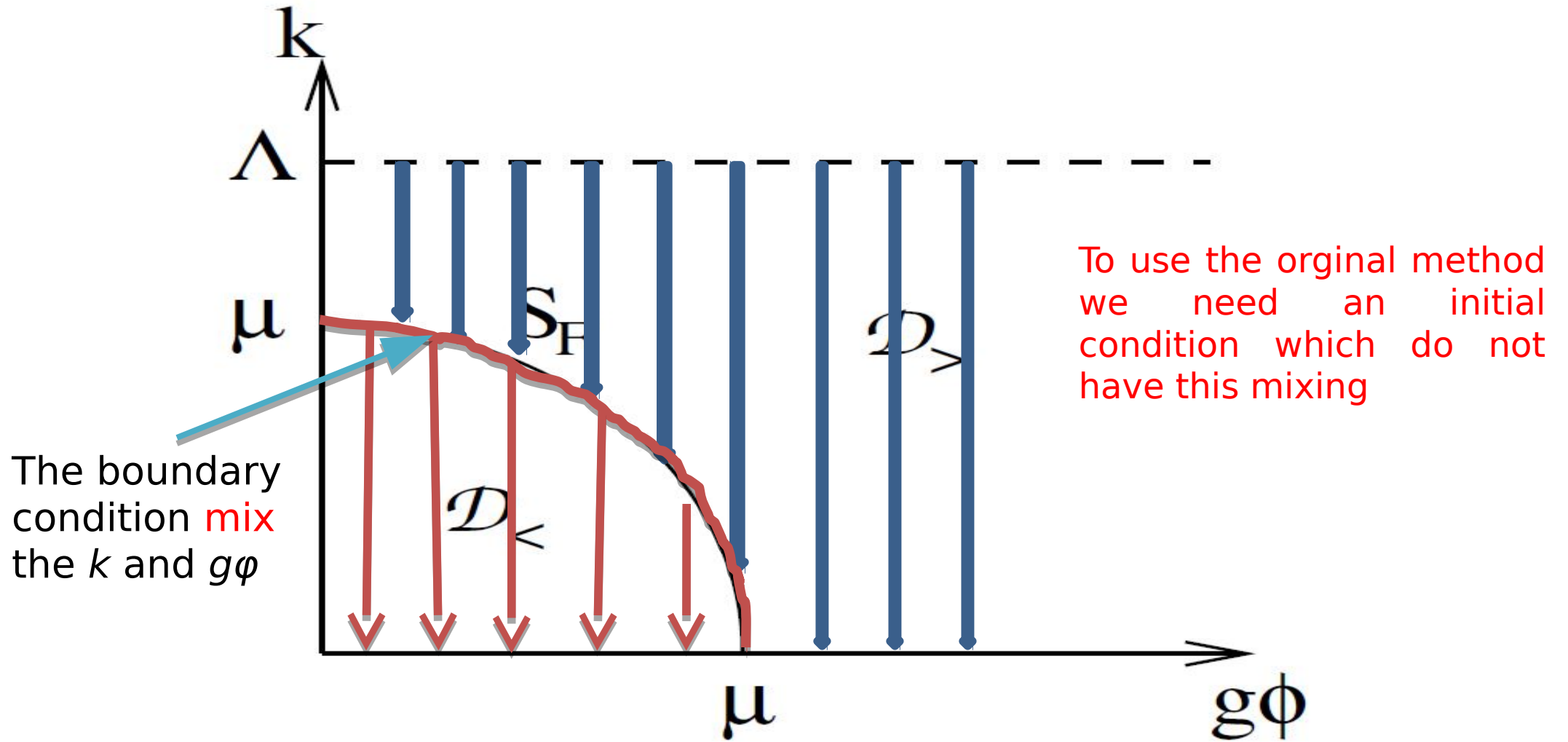
Fermi-surface

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel

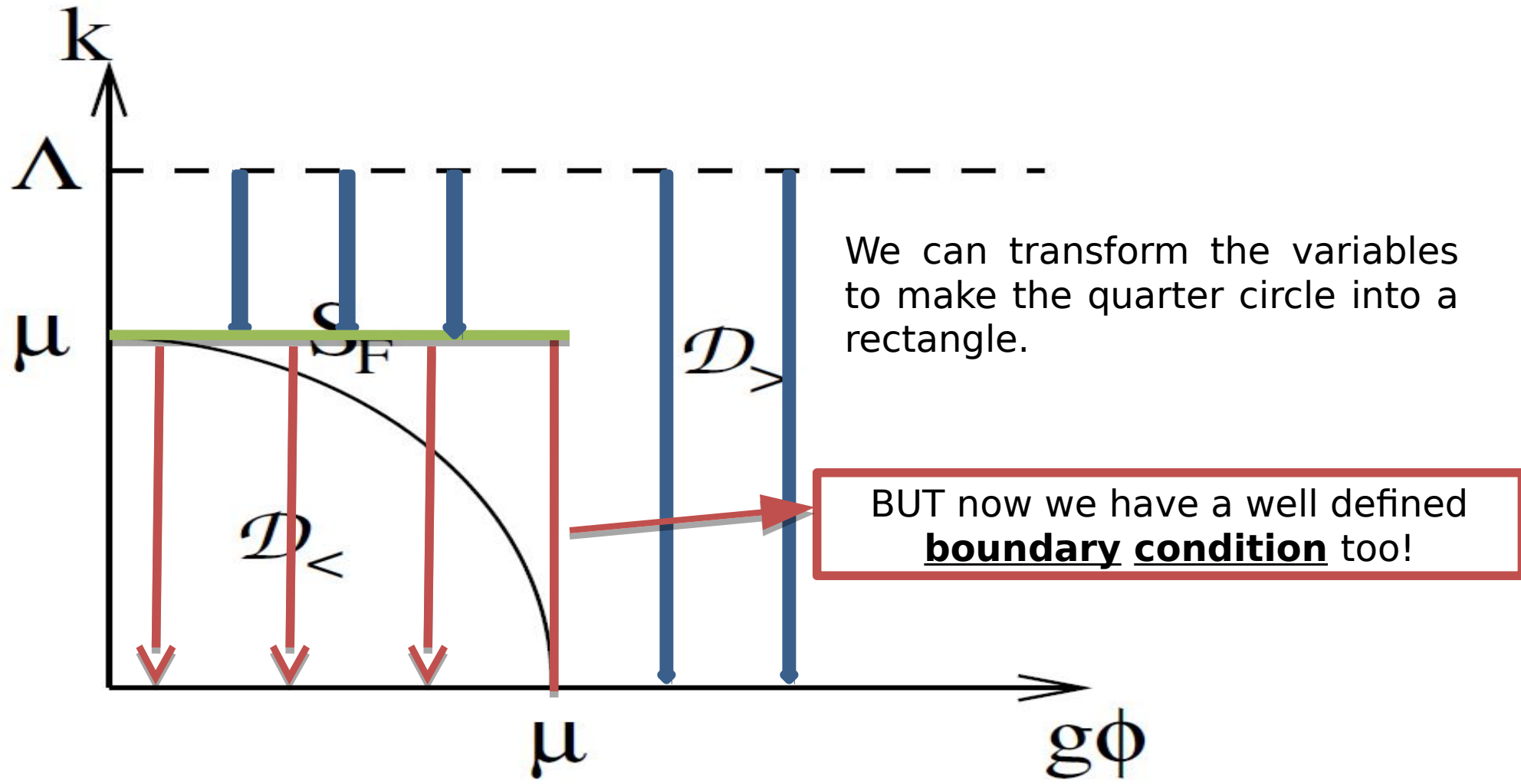
Integration of the Wetterich-equation



BUT...



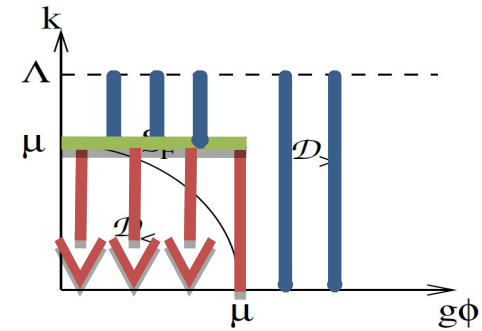
Solution: Need to transform the variables



Solution: Circle \rightarrow Rectangle transformation

- ▶ Coordinate transformation is required with: $(k, \varphi) \mapsto (x, y)$
 - mapping the Fermi-surface to rectangle
 - Keep the symmetries of the diff. eq.
 - Circle-rectangle transformation:

$$x = \varphi_F(k), \quad y = \frac{\varphi}{x}$$



- ▶ Transformation of the potential:

$$\tilde{U}(x, y) = V_0(x) + \tilde{u}(x, y)$$

with boundary condition at the Fermi-surface, V_0

- ▶ Transformed Wetterich-eq: $x\partial_x \tilde{u} = -xV_0' + y\partial_y \tilde{u} - \frac{g^2(kx)^3}{12\pi^2} \frac{1}{\sqrt{(kx)^2 + \partial_y^2 \tilde{u}}}$,
- ▶ and the new boundary conditions: $\tilde{u}(x = 0, y) = \tilde{u}(x, y = \pm 1) = 0$.

Solution of transformed Wetterich by an orthogonal system

- ▶ Solution is expanded in an orthogonal basis to accommodate the strict boundary condition in the transformed area

$$\tilde{u}(x, y) = \sum_{n=0}^{\infty} c_n(x) h_n(y) \quad h_n(1) = 0 \quad \int_0^1 dy h_n(y) h_m(y) = \delta_{nm}$$

- ▶ The square root in the Wetterich-equation is also expanded:

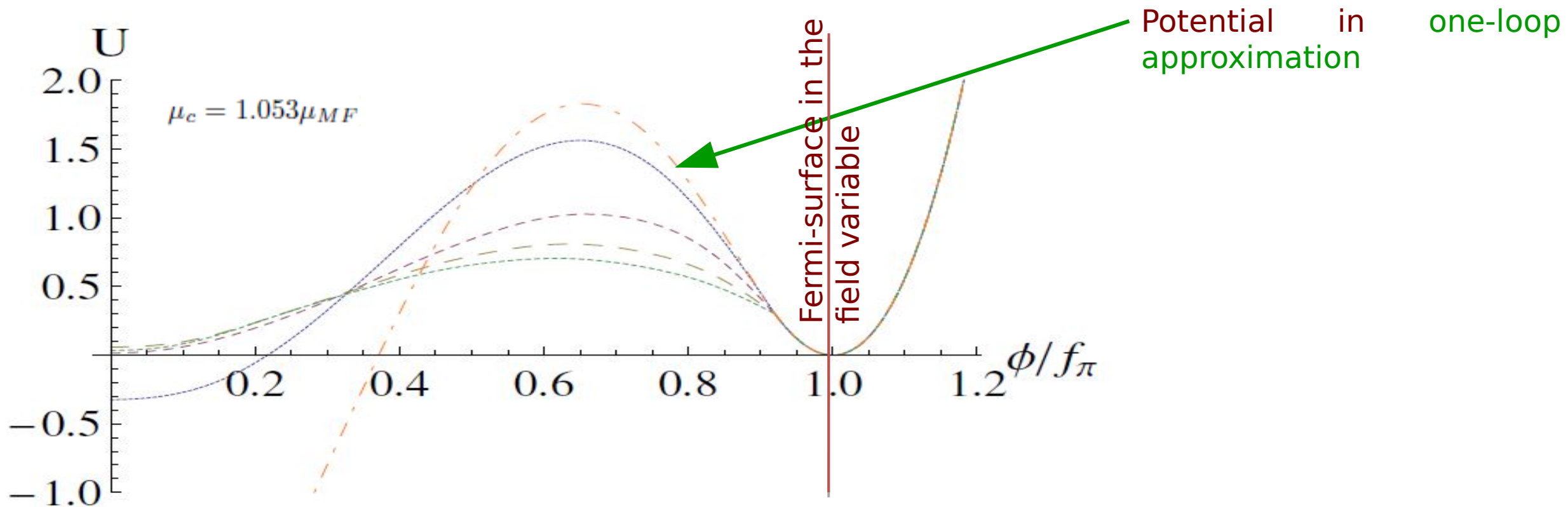
$$x c'_n(x) = \int_0^1 dy h_n(y) \left[-x V'_0 + y \partial_y \tilde{u} - \frac{g^2 (kx)^3}{12\pi^2} \underbrace{\sum_{p=0}^{\infty} \binom{-1/2}{p} \frac{(\partial_y^2 \tilde{u} - M^2)^p}{\omega^{2p+1}}}_{\text{Expanded square root}} \right]$$

Where: $\omega^2 = (kx)^2 + M^2$

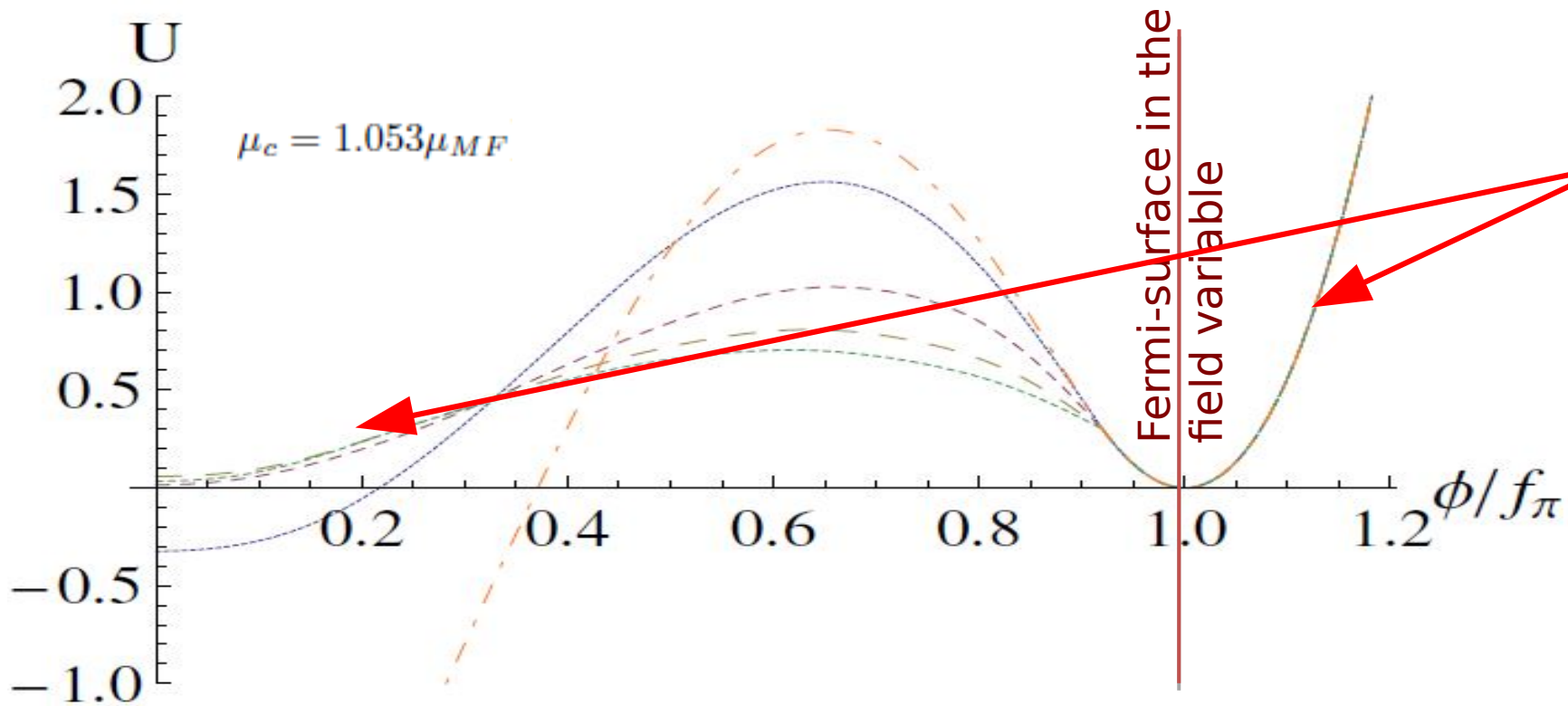
Expanded square root

We use harmonic base: $h_n(y) = \sqrt{2} \cos q_n y, \quad q_n = (2n + 1) \frac{\pi}{2}$

Result: The Effective Potential & Comparison



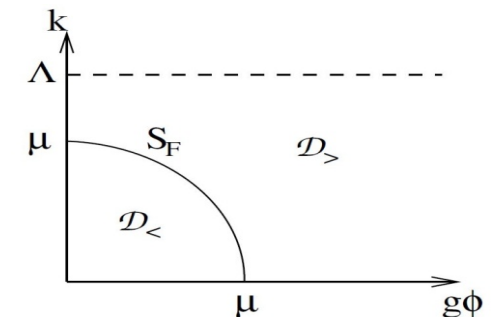
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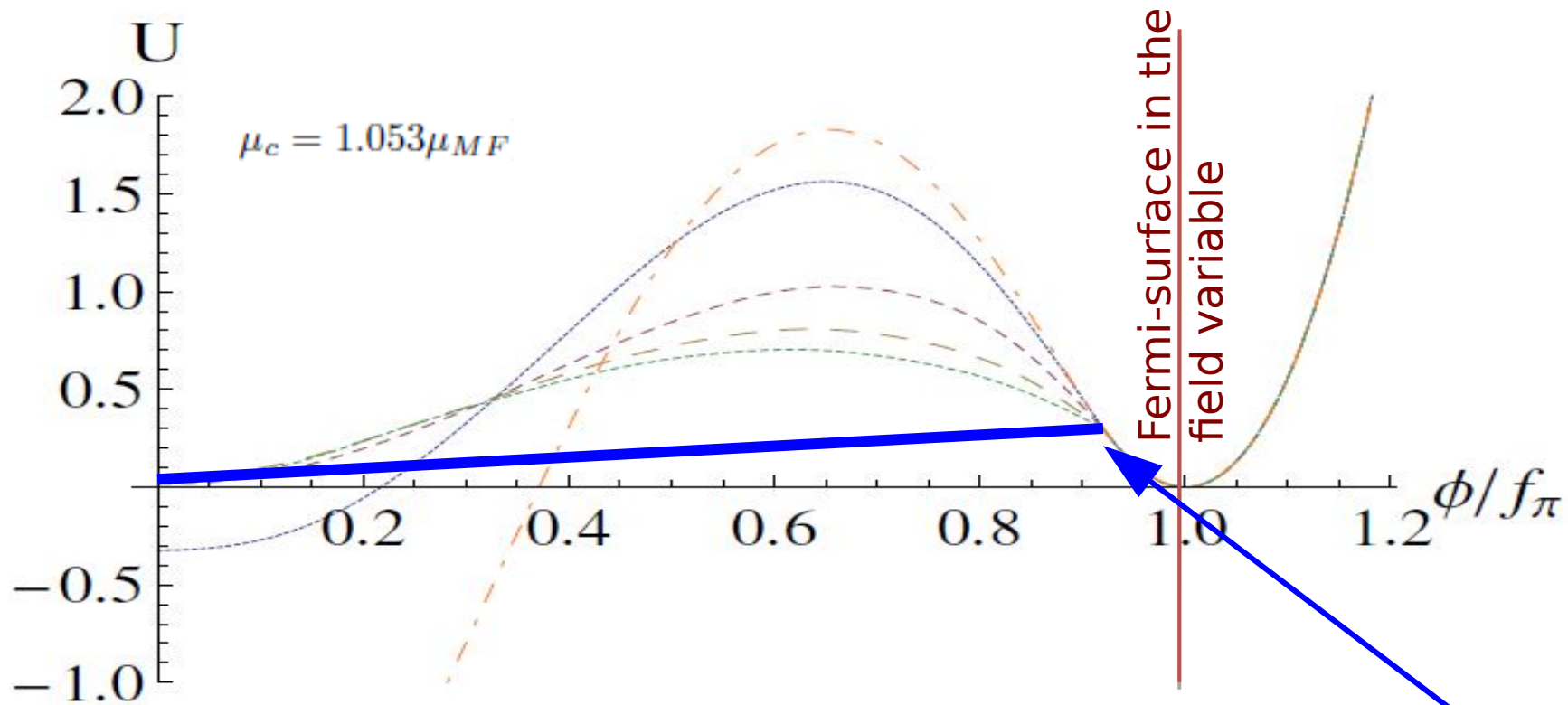
Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is **convex** → **coarse grained action**

Solution changes only below Fermi-surface, since switch to another equation



Result: The Effective Potential & Comparison

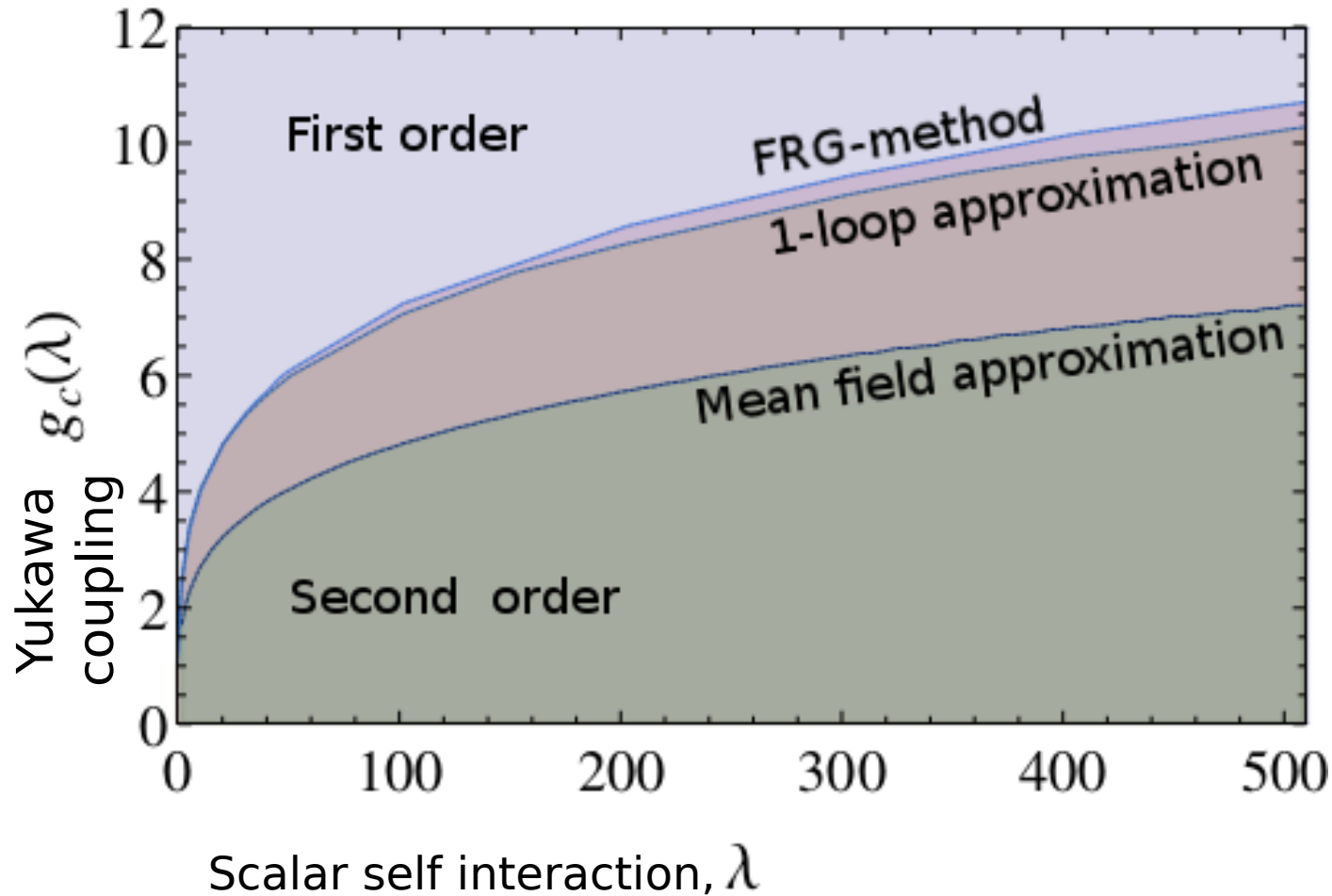


Potential in one-loop approximation

Higher orders of the Taylor-expansion for the square root converge fast where the potential is **convex** → **coarse grained action**

In the **concave** part of the potential solution is slowly converges to a straight line, because the free energy (effective potential) must be convex from thermodynamical reasons → **Maxwell construction**

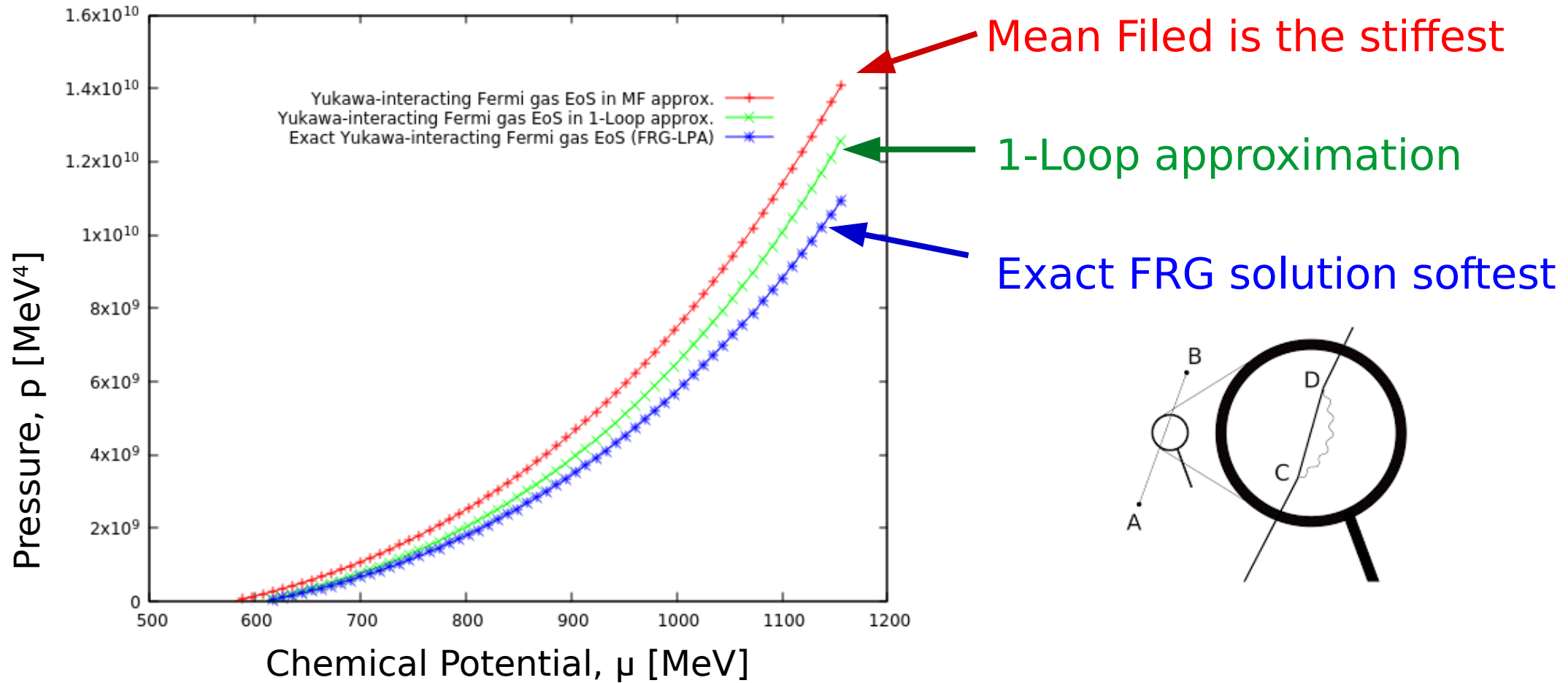
Result: Phase structure of interacting Fermi gas model



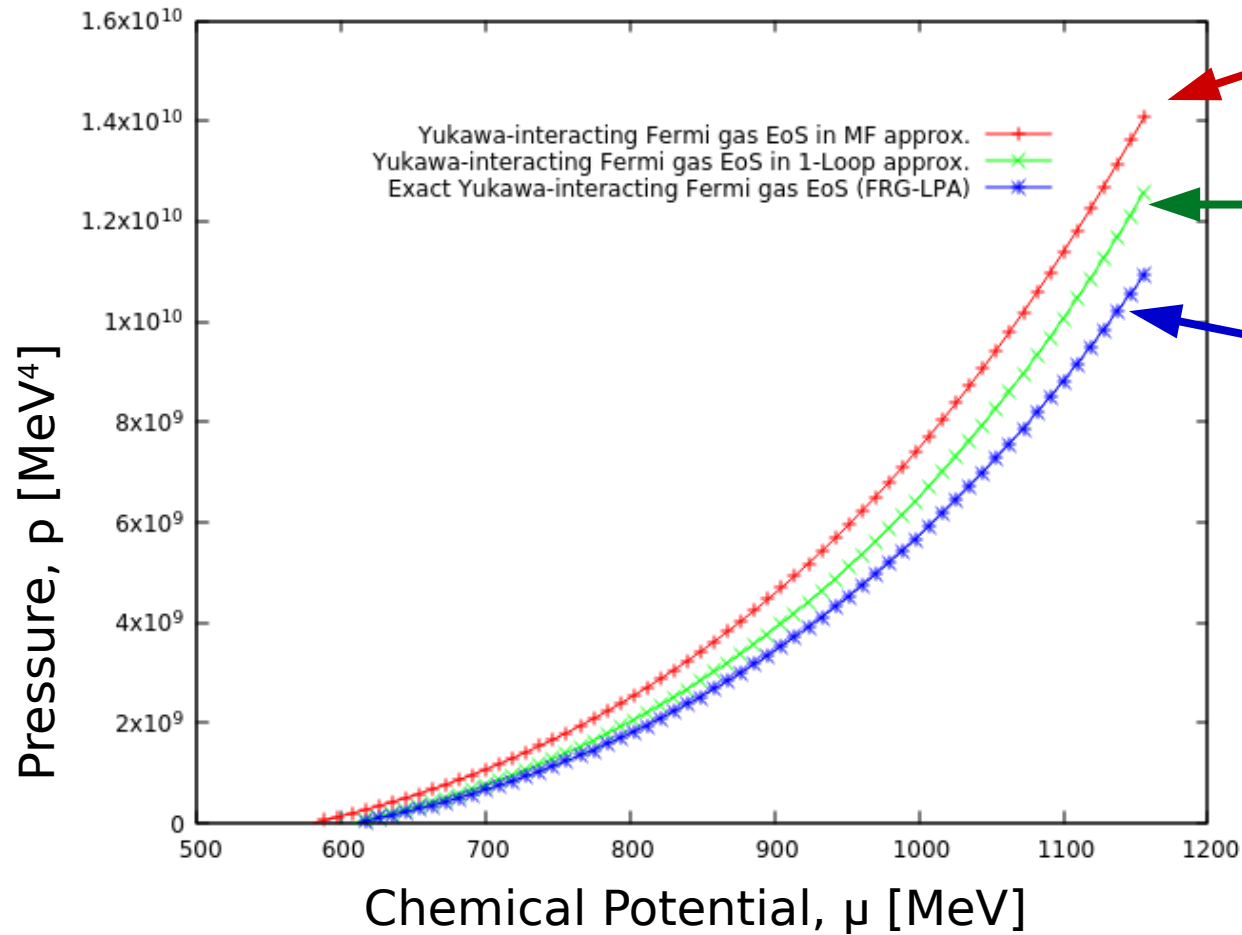
Exact FRG solution counts all quantum fluctuations
1-Loop approximation has only tree diagrams
Mean Filed solution contains averaged effect of interactions

In the phase structure, FRG and 1L are very similar if the LO has the strongest contribution.

Result: Comparison of MF, 1L, & FRG-based EoS



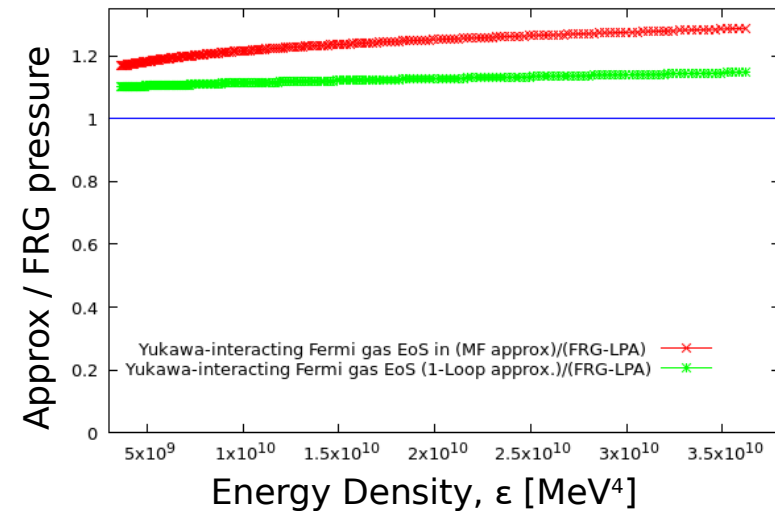
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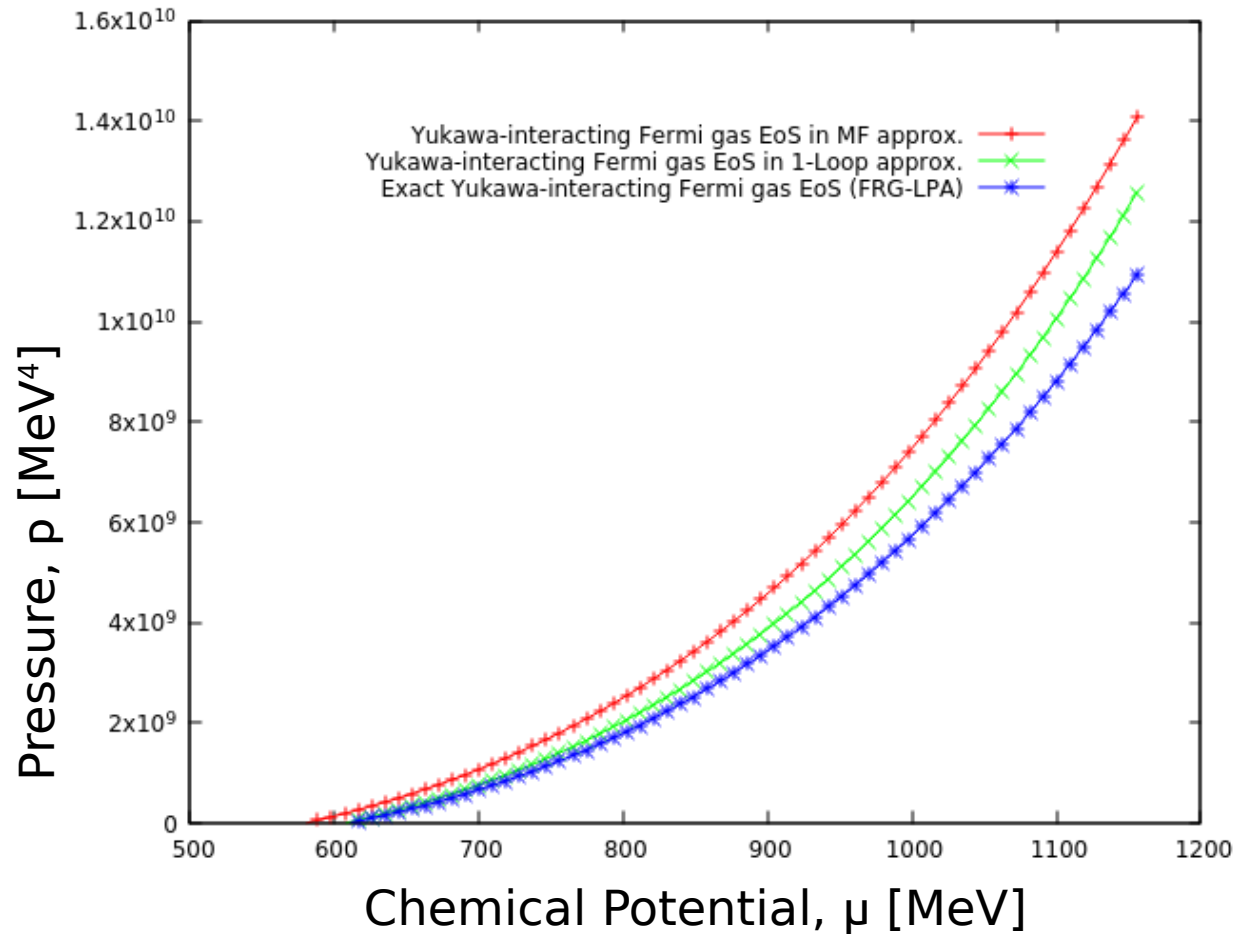
MF is 25% stiffer than the FRG

1L is 10% stiffer than the FRG

Exact FRG solution softest

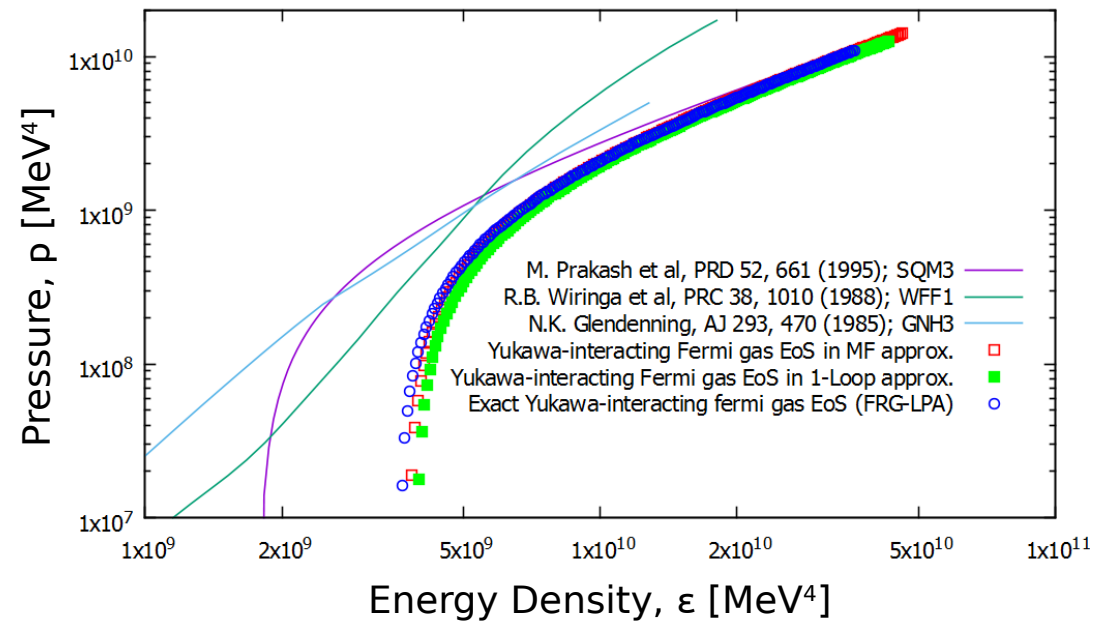


Result: Comparison to other EoS models

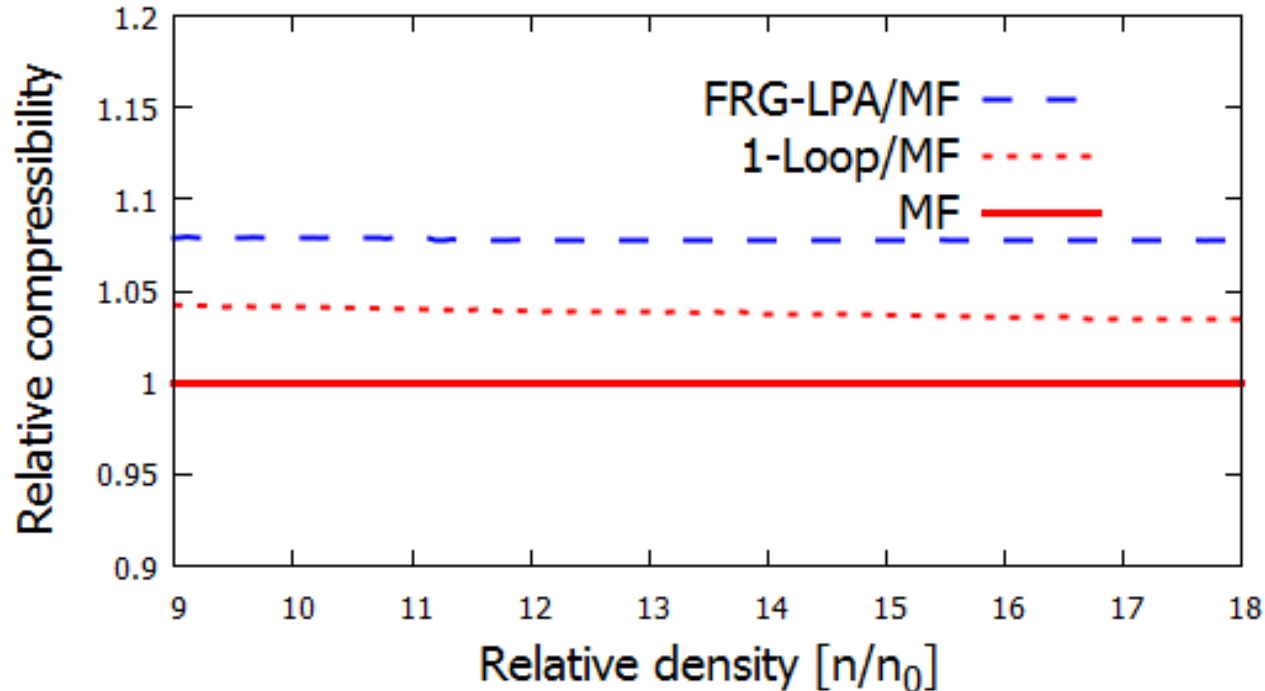


Compare FRG to SQM3, GNH3, WFF1

- Overlap with SQM3 at high ϵ
- Cutoff, ϵ_{cut} is also higher
- Approximations differ slightly



Result: Comparison of compressibility in the models



Compare FRG to 1L and MF

– Compressibility:

$$\frac{1}{\chi} = n \frac{\partial P}{\partial n} = 2n^2 \frac{\partial}{\partial n}(E/A) + n^3 \frac{\partial^2}{\partial n^2}(E/A)$$

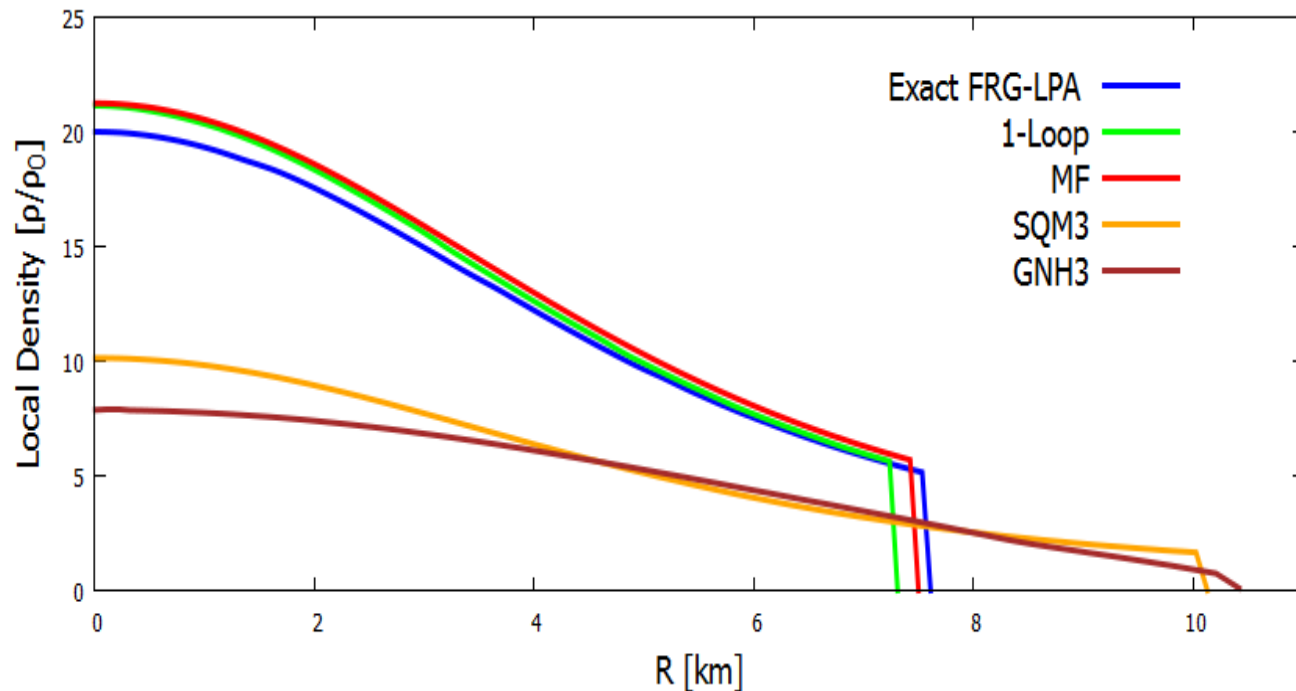
– Compression modulus

$$K = k_F^2 \frac{\partial^2}{\partial k_F^2}(E/A) = \frac{9}{n_0 \chi}$$

– The difference between the models is about ~10%

Result: Test in a Compact Star

- ▶ Compare FRG EoS to SQM3, GNH3 → TOV result: density function



Compare FRG to 1L and MF

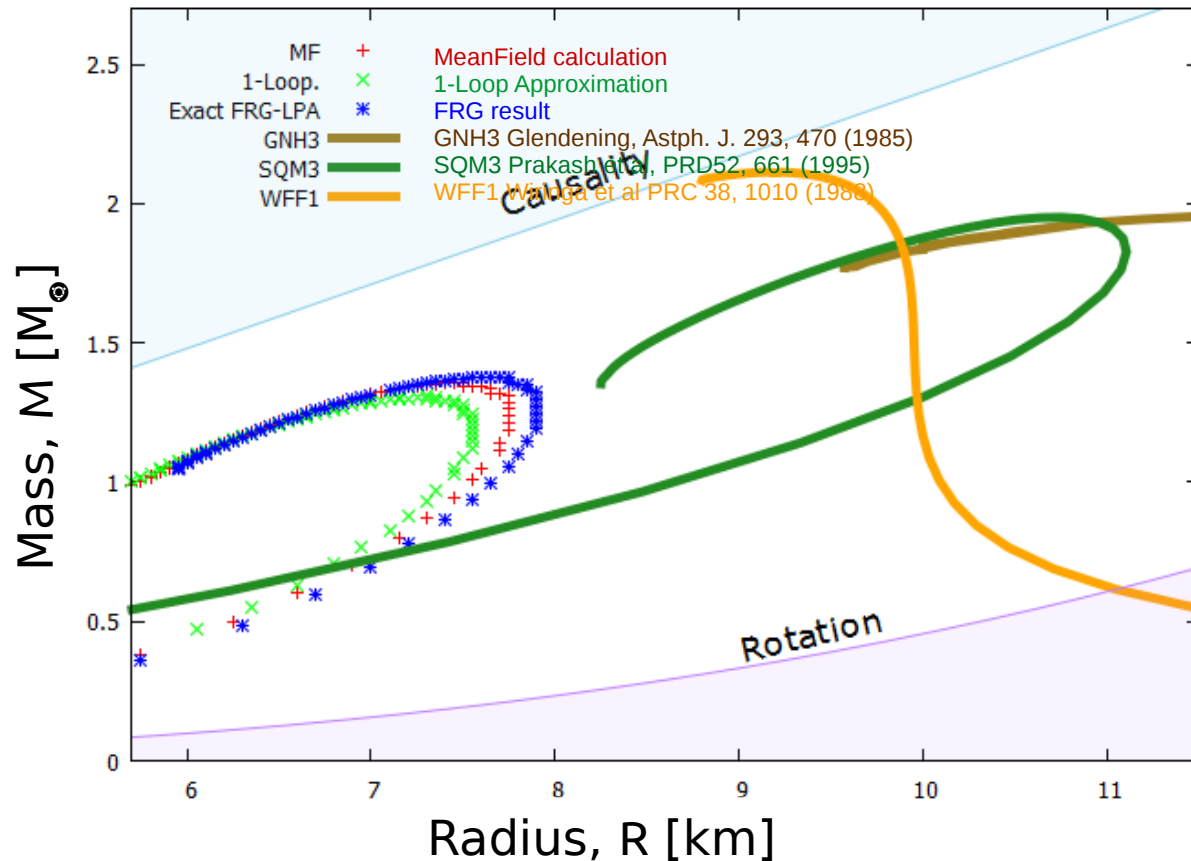
- Soft FRG make biggest star
- High- ϵ part is similar for all
- Difference: $\sim 5\%$ ($.1 M_{\odot}$ and $.5$ km)

FRG to SQM3, GNH3

- FRG: small stars $1.4 M_{\odot}$ and 8 km
- Other models: larger radii and less central density

Result: Test in a Compact Star

- ▶ Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram



Compare FRG to 1L and MF

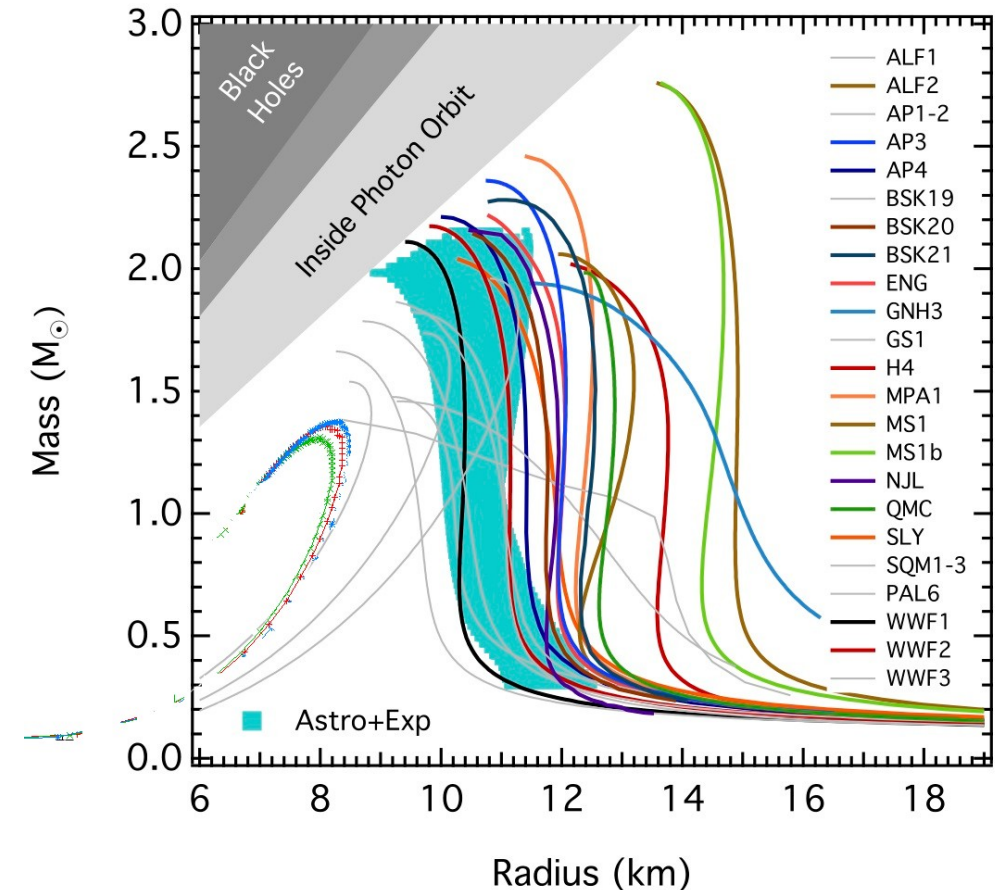
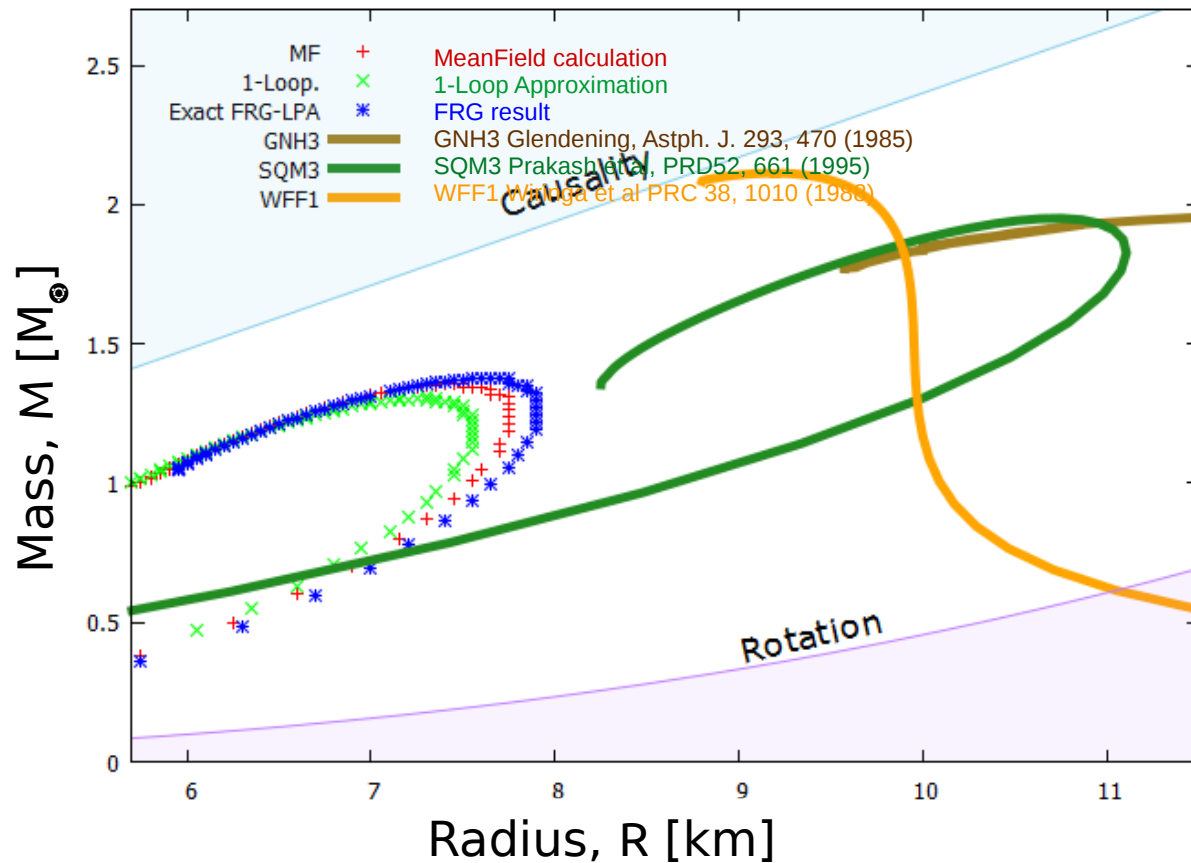
- Soft FRG make biggest star
- High- ϵ part is similar for all
- Difference: $\sim 5\%$ ($.1 M_{\odot}$ and $.5$ km)

FRG to SQM3, GNH3, WFF1

- Small stars $1.4 M_{\odot}$ and 8 km
- Overlap with SQM3 at high ϵ
- Interaction (ω) will increase

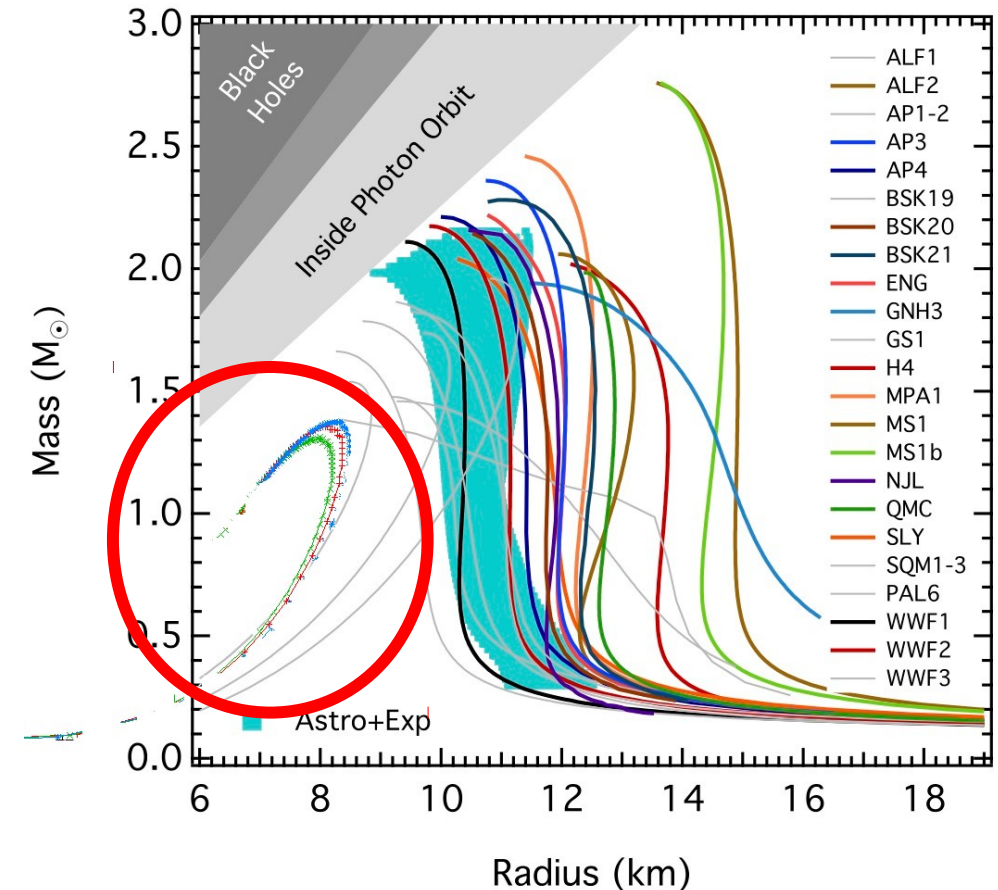
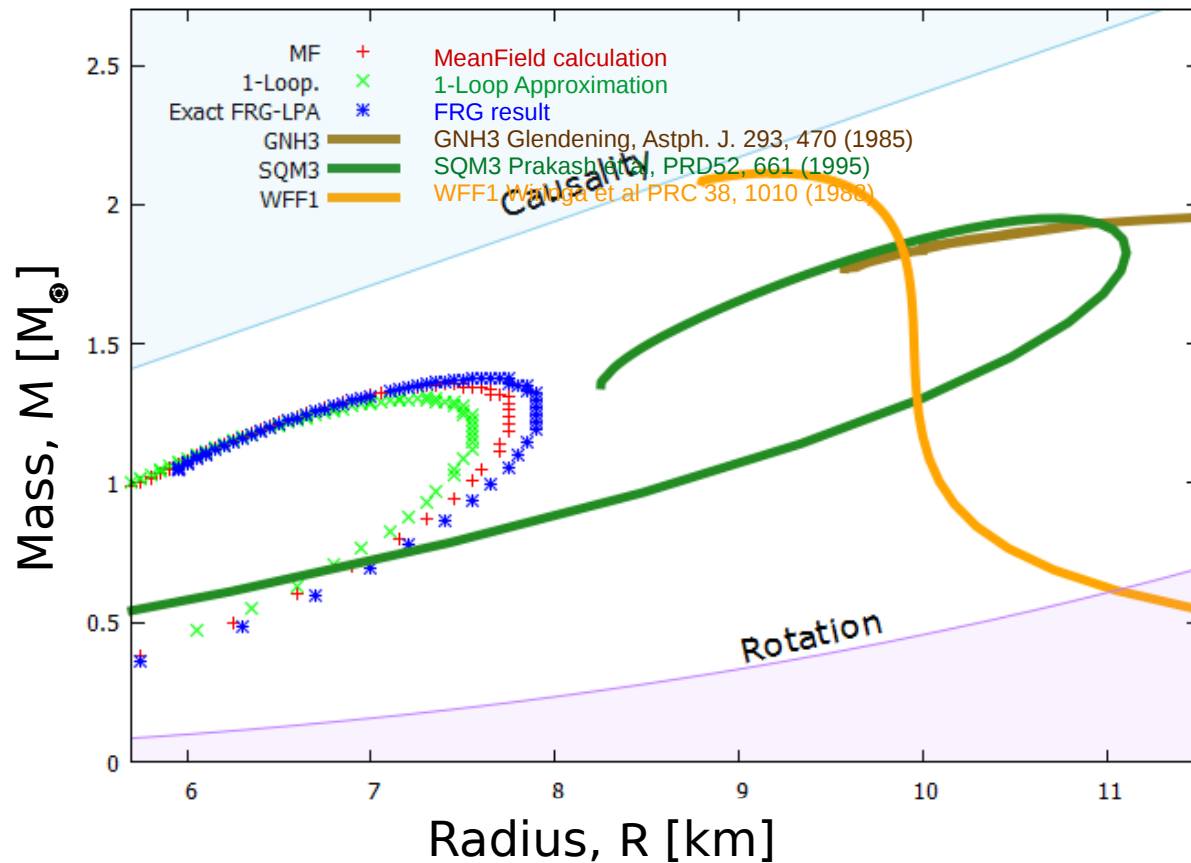
Result: Test in a Compact Star

- ▶ Compare FRG EoS to SQM3, GNH3, WFF1 → TOV result on M(R) diagram



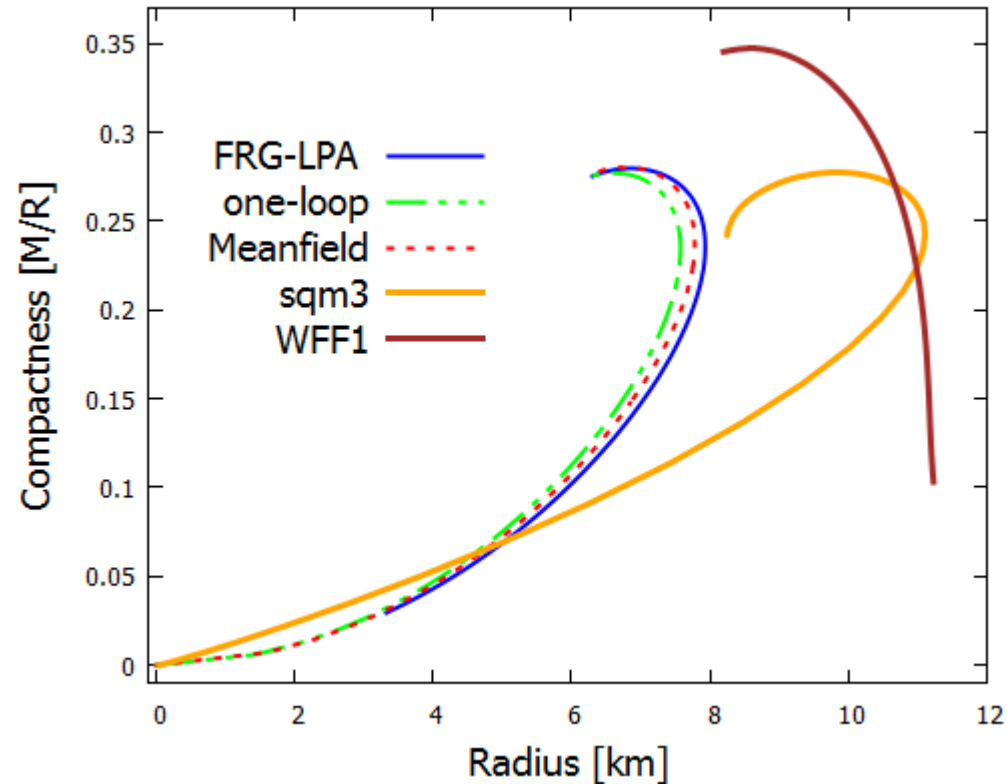
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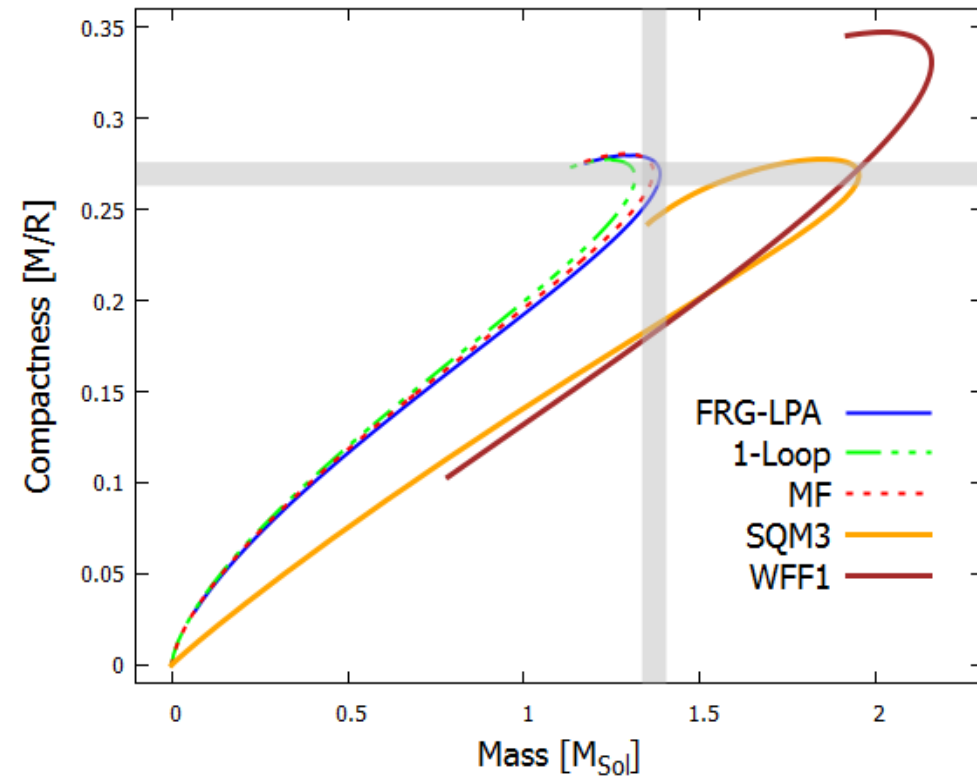
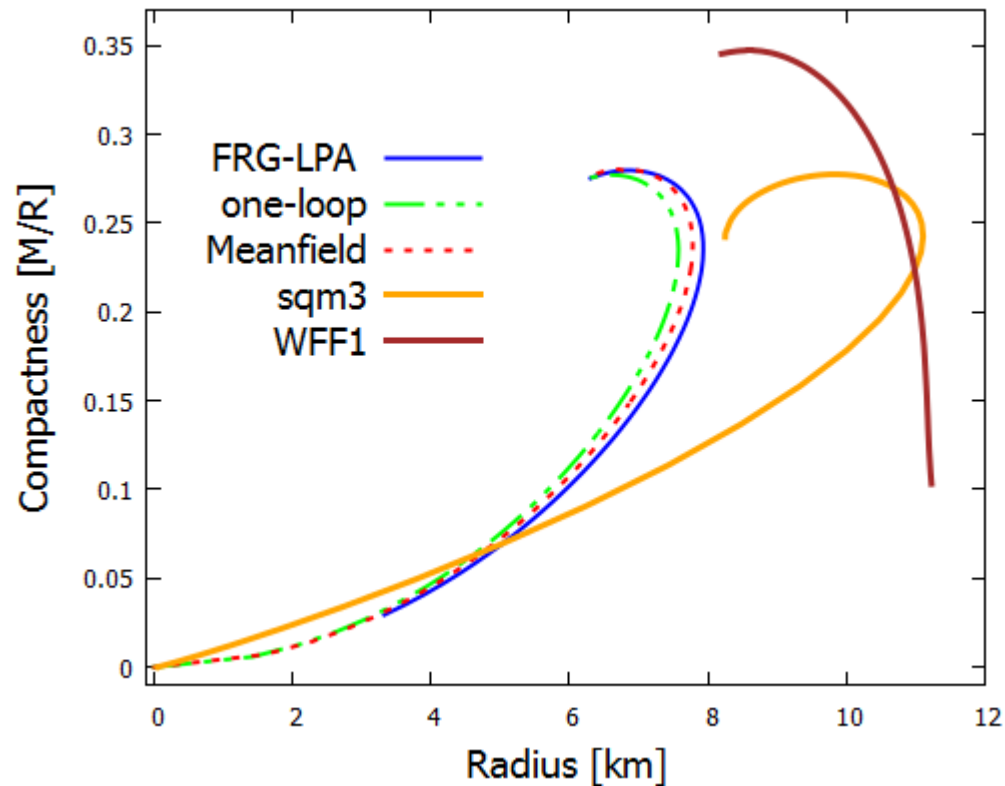
Test: Can we test this by observations?

- ▶ Compare Compactness by FRG, MF, 1L, SQM3, and WFF1 EoS



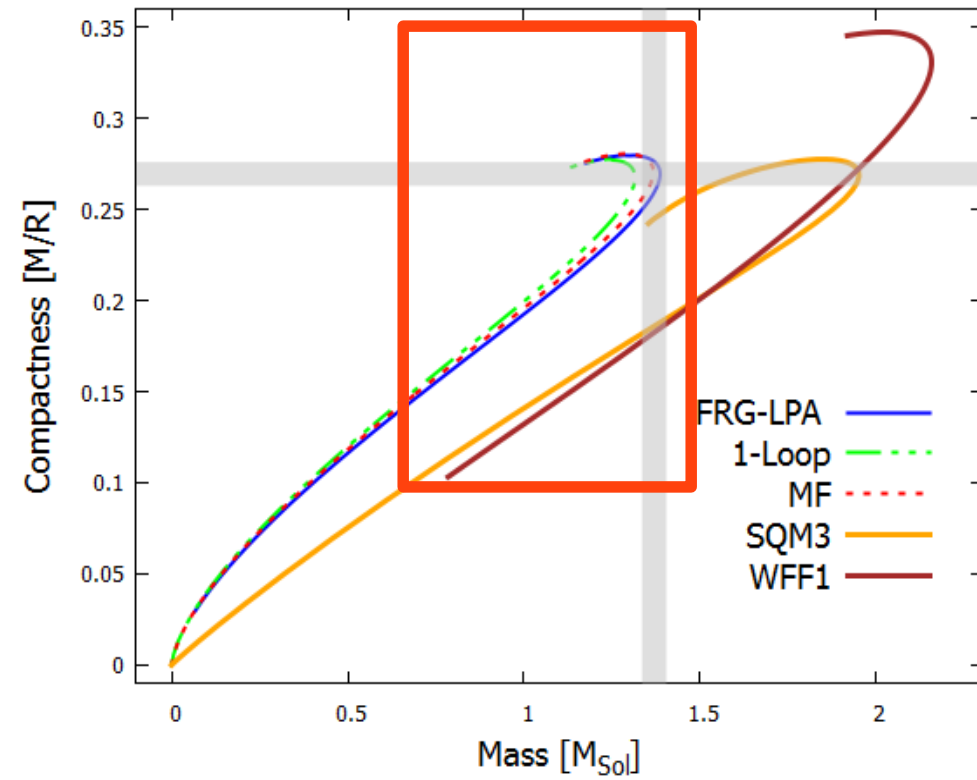
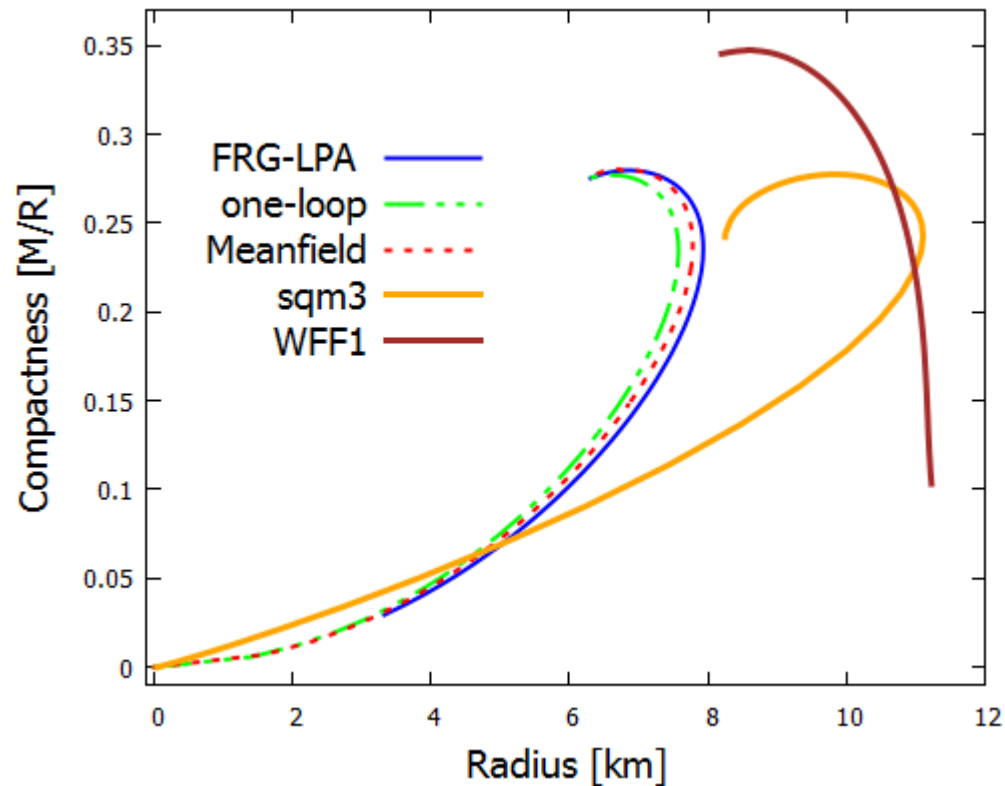
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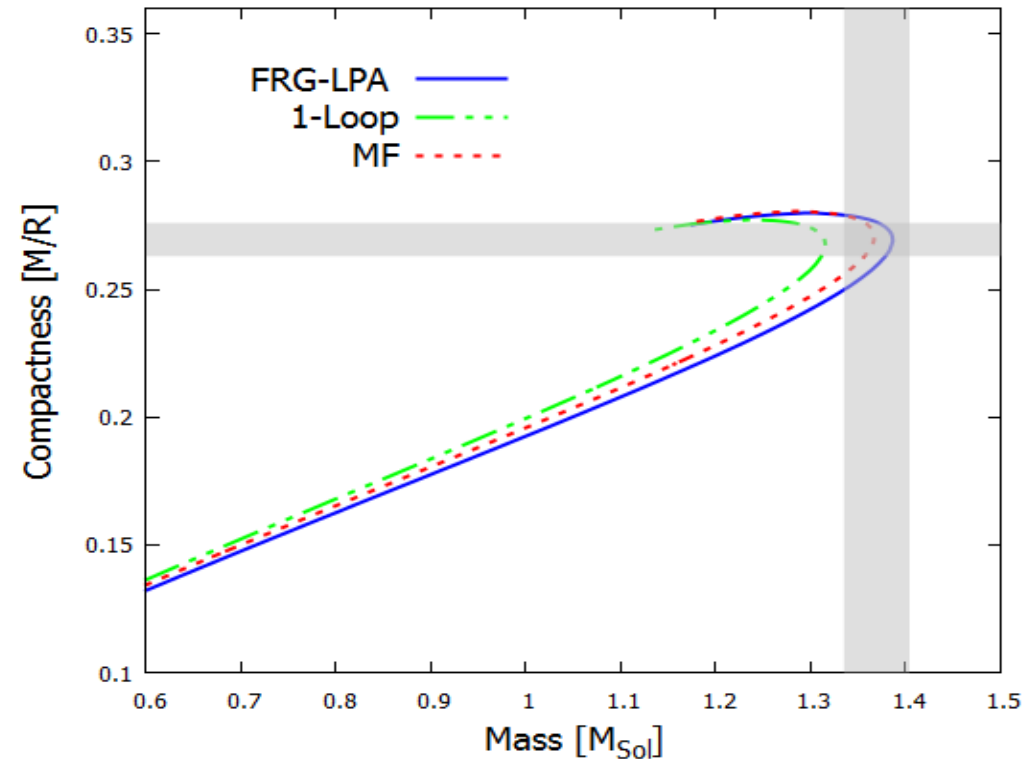
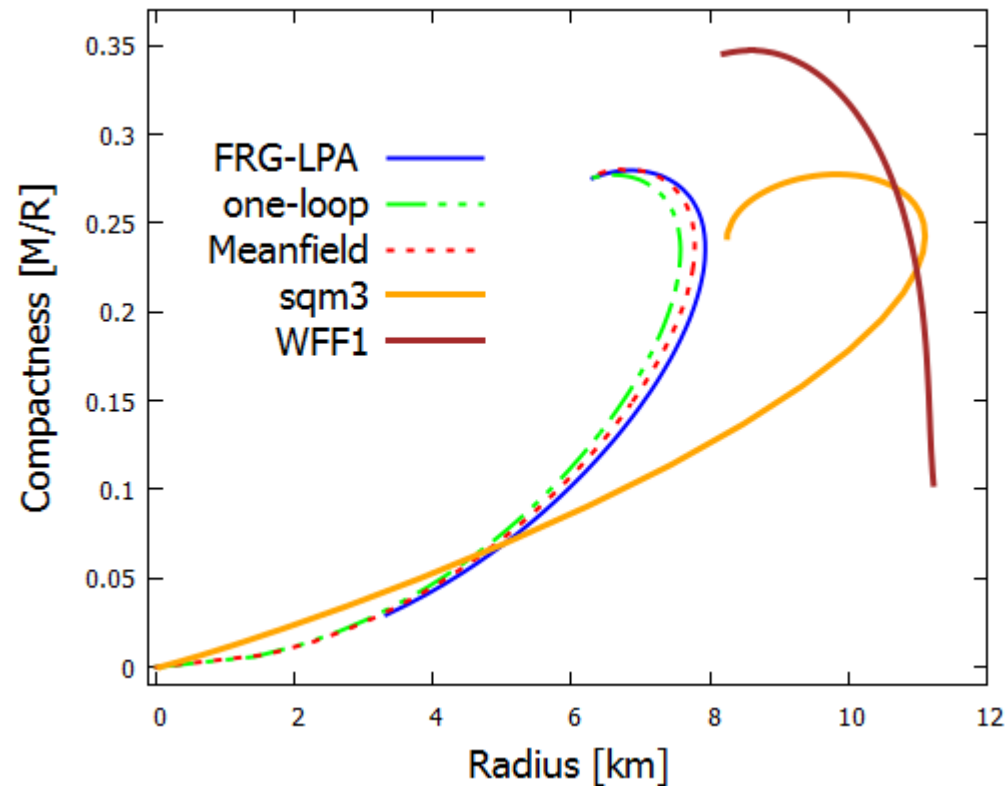
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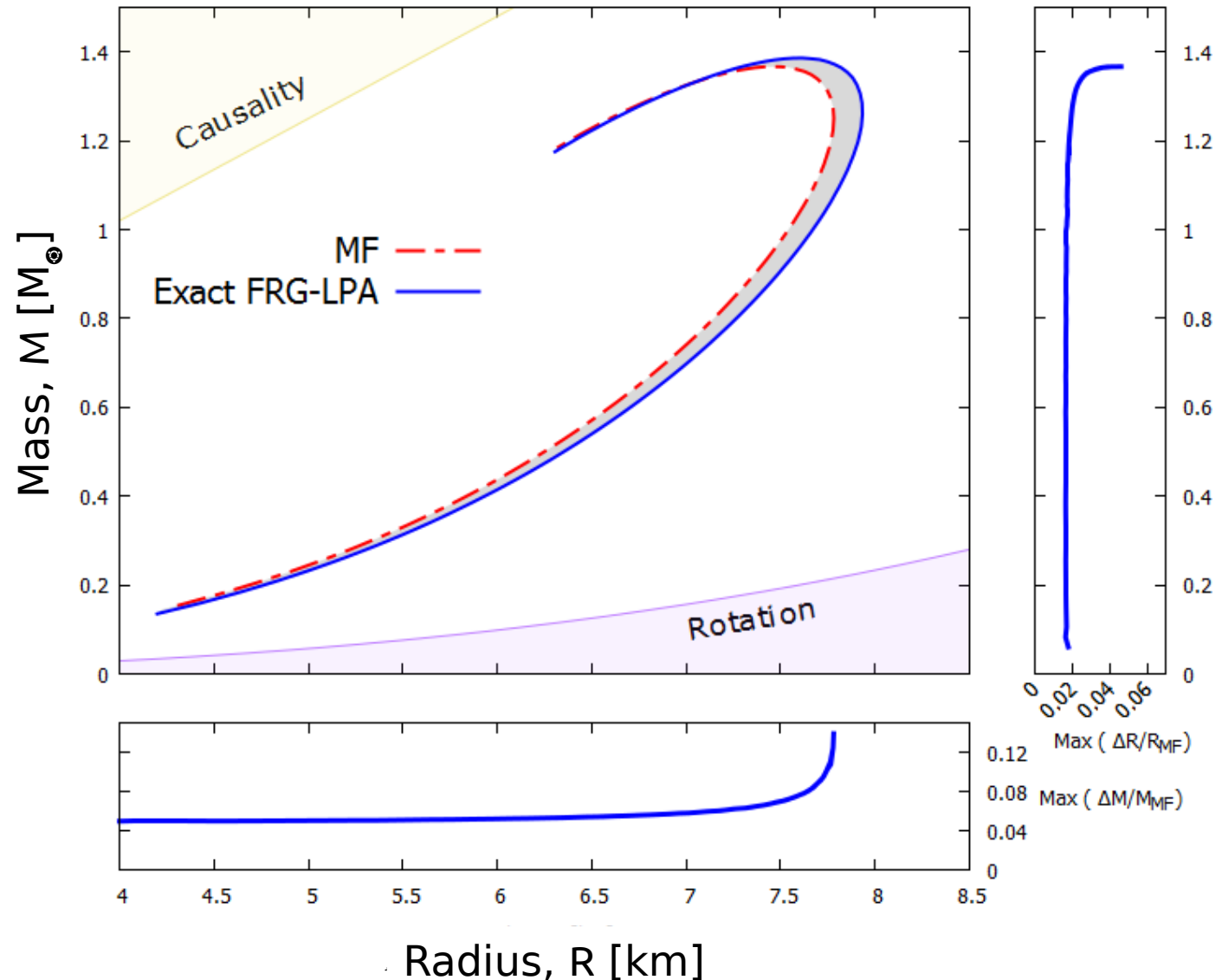
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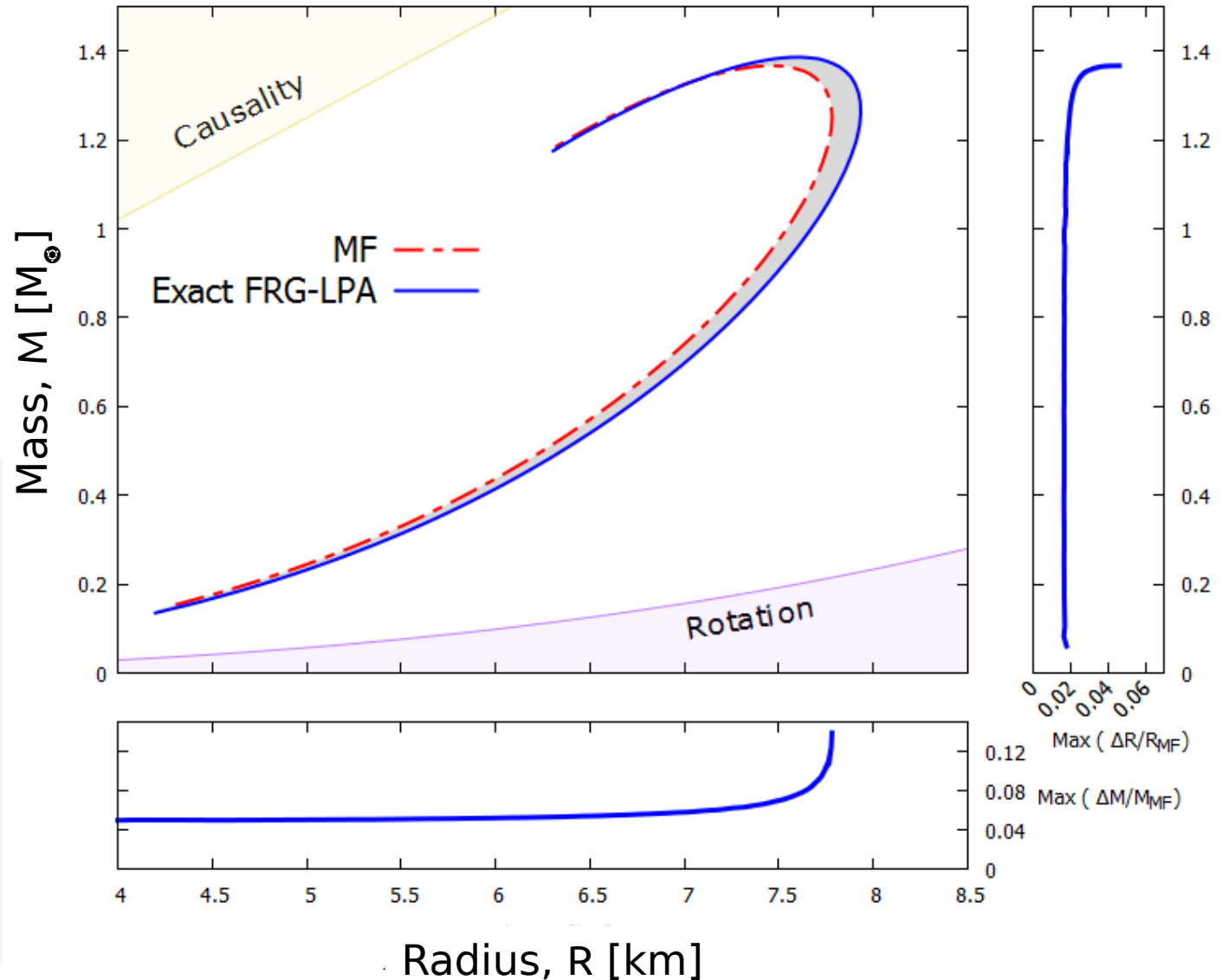
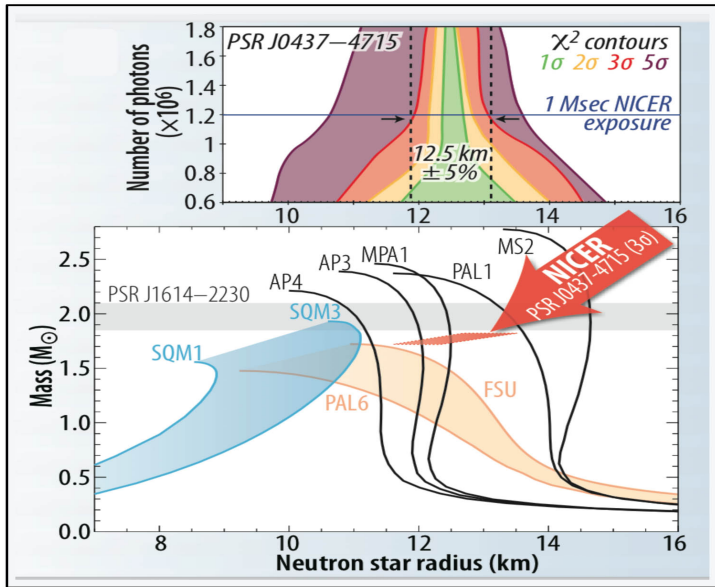
Test: Can we test this by observations?

- ▶ Compare different EoS results on M(R) diagram: MF & FRG
- ▶ Maximal relative differences are also plotted



Test: Can we test this by observations?

- ▶ Compare different EoS results on $M(R)$ diagram: MF & FRG
- ▶ Maximal relative differences are also plotted



Summary

- FRG method were used to obtain the effective potential for
 - One-component Fermi gas with a simple Yukawa-like coupling
 - Concave part of the potential converges slowly to a line → Maxwell construction
 - Convex part of the potential → Coarse Grained action
 - Chiral phase transition is reproduced → Order depends on the applied approximation
- EoS can be compared to other ones, close to the SQM3 (Prakash, 1995)
 - Softness depends on the approximation (FRG → 1L → MF)
 - MF differs 25%, 1L differs 10% from the exact FRG solution, slight evolution at high ϵ
 - Simple model → Relative small compact stars $M < 1.4 M$, and $R < 8$ km
 - Size (both mass and radius) sensitive to quantum fluctuations (5% effect)
- Based on FRG method, now we can have a technique to make:
 - An effective model for the hardly accessible part of the phase diagram ($T=0$, finite μ , high ρ)

Some related events

- New perspectives on Neutron Star Interiors
 - Date: 9-13 October, 2017
 - ECT*, Trento, Italy
 - Web: <http://www.ectstar.eu/node/2230>
- 17th Zimányi Winter School 2017
 - Date: 4-8 December 2017
 - Wigner Research Centre for Physics & THOR
Budapest, Hungary
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ECT* **ECT***

**EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS**
TRENTO, ITALY
Institutional Member of the European Expert Committee NUPECC

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Castello di Trento ("Trin"), watercolor 19.8 x 27.7, painted by A. Dürer on his way back from Venice (1495). British Museum.

New perspectives on Neutron Star Interiors
Trento, October 09 – 13, 2017

Main Topics

- Investigation on the inner structure on neutron stars, existence of quark, hybrid and strange stars,
- Tests of extreme dense QCD phase diagram at finite temperature,
- Crust-core models and pulsar models, strong magnetic field in neutron stars, cooling of neutron stars,
- New observables from X-ray and gamma satellites (NICER, LOFT, ATHENA),
- Gravitational wave signals from isolated neutron stars or merging neutron star binaries,
- Future experimental facilities for neutron star observables.

Keynote participants

Gordon Baym (University of Illinois, Urbana), David Blaschke (University of Wrocław, Poland), Paweł Haensel (Nicolaus Copernicus Astronomical Center, Poland), Andrew Cummings (McGill University, Montreal), Chris Pankow (Northwestern University), Adriana Raduta (IFN-IBU, Bucharest), Michi Banböck (Max Planck Institute Garching, Germany), Marco Limongi (INAF, Rome), Pierre Pizzone (University of Milan), Armen Sedrakian (Frankfurt Institute for Advanced Studies), Fridolin Weber (San Diego State University), Pablo Cerda-Damm (University of Valencia), Szabolcs Bendei (University of Wuppertal, Germany), Michał Bejger (Nicolaus Copernicus Astronomical Center, Poland), Brynjar Haskell (Nicolaus Copernicus Astronomical Center, Warsaw), Máté Csulák (Eötvös University, Hungary), Giampietro Cagnoli (VIRGO, Laboratoire des Matériaux Avancés, France).

Organizers

Gergely Gábor Barnaföldi (Wigner Research Centre for Physics Budapest, Chairman), Gordon Baym (University of Illinois, Urbana), Laura Tolós (Institute of Space Sciences, Universidad de Valencia, Spain).

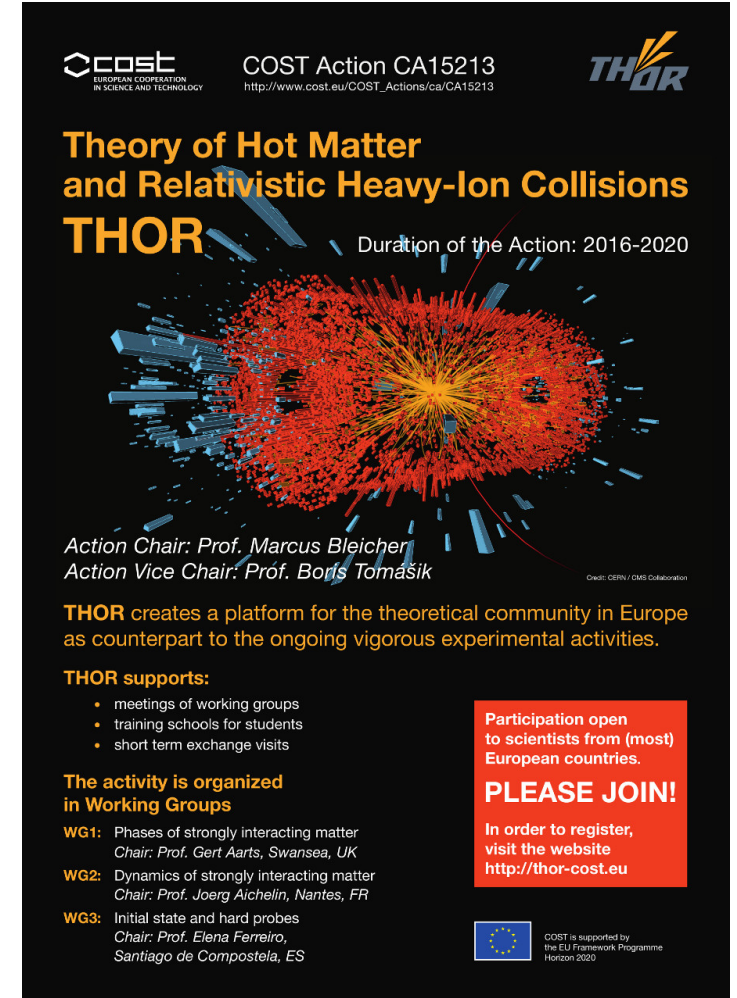
Director of the ECT*: Professor Jochen Wambach (ECT*)

The ECT* is sponsored by the "Fondazione Bruno Kessler" in collaboration with the "Assessorato alla Cultura" (Provincia Autonoma di Trento), funding agencies of EU Member and Associated States and has the support of the Department of Physics of the University of Trento.

For local organization please contact: Christian Fossi - ECT* Local Organizer - Villa Tambosi - Strada delle Tabarelle 286 - 38123 Villazzano (T) Tel: (+39-0461) 314731 Fax: (+39-0461) 314750, E-mail: fossi@ectstar.eu visit <http://www.ectstar.eu>

Advertisement:

- **THOR EU COST Action CA15213**
 - Theory of Hot Matter and Relativistic Heavy Ion Collisions
<http://thor-cost.eu>
- **NewCompStar EU COST Action MP1304**
 - Theory of Compact Stars (ending 2017)
<http://compstar.uni-frankfurt.de>
- **PHAROS EU COST Action CA16214**
 - The multi-messenger physics and astrophysics of neutron stars



The poster for THOR COST Action CA15213 features a central visualization of a heavy-ion collision, showing a dense, glowing red and orange core surrounded by a spray of blue and white particles. The background is black. The text is in white and yellow. The logos for COST and THOR are in the top corners. The text includes the action title, duration, chairs, and a call to action.

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COST Action CA15213
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THOR

Theory of Hot Matter and Relativistic Heavy-Ion Collisions
THOR Duration of the Action: 2016-2020

Action Chair: Prof. Marcus Bleicher
Action Vice Chair: Prof. Boris Tomášik

THOR creates a platform for the theoretical community in Europe as counterpart to the ongoing vigorous experimental activities.

THOR supports:

- meetings of working groups
- training schools for students
- short term exchange visits


The activity is organized in Working Groups

WG1: Phases of strongly interacting matter
Chair: Prof. Gert Aarts, Swansea, UK

WG2: Dynamics of strongly interacting matter
Chair: Prof. Joerg Aichelin, Nantes, FR

WG3: Initial state and hard probes
Chair: Prof. Elena Ferreiro, Santiago de Compostela, ES

Participation open to scientists from (most) European countries.
PLEASE JOIN!
In order to register, visit the website <http://thor-cost.eu>

 COST is supported by the EU Framework Programme Horizon 2020

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Result: Comparison to other EoS models

- ▶ Compare FRG EoS to SQM3, GNH3, WFF1

