Null congruences crossing generic shells

Description of a new method

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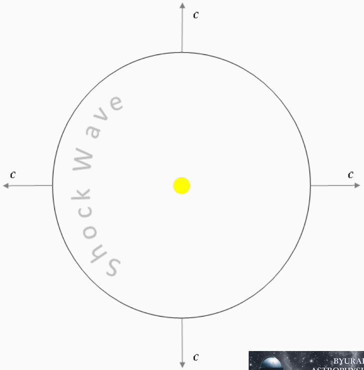
Shock wave theory

There are many models based on General Relativity describing astrophysical shock waves. The model we discuss assumes that the shock wave:

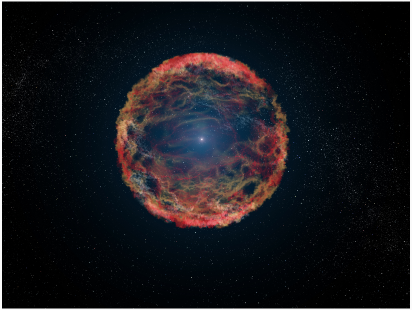
- travels with the speed of light (is lightlike),
- contains both material sources and gravitational waves (is generic),
- has infinitesimal thickness (is singular).



Shock wave theory application

Many cataclysmic astrophysical events produce shock waves. Such models have been applied to:

- Supernova,
- Collision of neutron stars,
- Exploding white holes.



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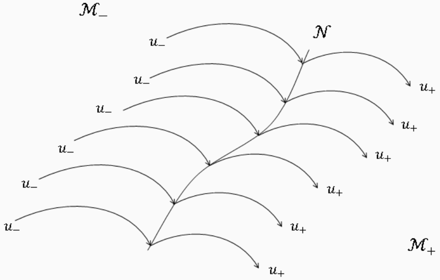
Construction of singular hypersurfaces

Singular hypersurfaces divide the space-time into two domains, each with its own metric, but such that the metric induced on $\mathcal{N}$ is unique.



The derivative of the metric has a jump across $\mathcal{N}$, so $\mathcal{N}$ appears as a Dirac δ-function term in the Riemann curvature tensor of the space-time.

$$R_{k\lambda\mu\nu} = \overbrace{\tilde{R}_{k\lambda\mu\nu}}^{domains} + \overbrace{\hat{R}_{k\lambda\mu\nu}}^{hypersurface} \delta(\Phi)$$

Construction of singular hypersurfaces

It is possible to introduce a coordinate system continuous across the shell which covers the entire space-time.

- This can be done, for example, by expanding the coordinate system in the past to the future domain.
- If both domains are flat spaces, we introduce a soldering function F which defines the matching conditions of two coordinate systems on the shell $x_+|_\mathcal{N} = F(x_-|_\mathcal{N})$, such that the line element on the hypersurface stays unique $ds_-^2|_\mathcal{N} = ds_+^2|_\mathcal{N}$.

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Construction of singular hypersurfaces

- Using this general coordinate system, we can define a general metric, covering the two domains and the shell.
- The jump in the derivative of this metric will depend on the definition of the matching function F.
- As a result, the constituents of the impulsive signal on the shell (the matter source and the gravitational waves) will also depend on the definition of F.

Particles crossing the hypersurface

Barrabes and Hogan used this construction to derive the components of the geodesic deviation vector to the future of the shockwave and in the leading order approximation, for a general space-time and soldering by solving the geodesic deviation equation

$$\frac{D^2 X^\mu}{d\tau^2} = R^\mu{}_{\nu\rho\sigma} T^\nu T^\rho X^\sigma .$$

- Their derivations are quite technical and are not straightforward.
- They only work in the first approximation and only for timelike particles.
- No general technique has been suggested for calculating shear, rotation and expansion of the congruence in the future.

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The suggested approach

We use the same setup as Barrabes and Hogan did.

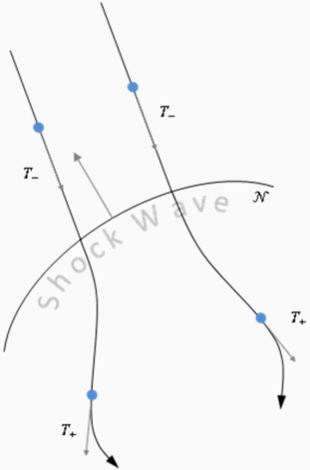
- We take both past and future domains to be flat spaces, as in this case we can define infinite varieties of soldering conditions and also the behavior of the congruence to the future is much easier to describe and interpret.

- We choose the congruence to be parallel before crossing the hypersurface, so that the effect of different shells is directly reflected in the future behavior of the congruence.

The suggested approach

- We assume the shell is given by $u = 0$ where u is also an affine parameter of the particle geodesic.
- We make a coordinate transformation on T on the shell from the past coordinates to the future coordinates.

$$T_+^\alpha\big|_{\mathcal{N}} = \frac{\partial x_+^\alpha}{\partial x_-^\beta} T_-^\beta\bigg|_{\mathcal{N}}$$



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The suggested approach

- This transformation is the mathematical equivalence of the action of the particle crossing the shell.
- To validate this approach, we have derived the jump of the derivative of T across $\mathcal{N}$ which exactly corresponded with the results of Barrabes and Hogan.
- $T_+^\alpha\big|_\mathcal{N}$ will surely be a function of the point X_0 where the particle has crossed the shell, which itself depends on the way the two domains are connected across the shell (let's say $x_- = F(x_+)$).

Shear, expansion and rotation

- The geodesic lines in the future will be straight lines in the future coordinates and one can represent them as follows

$$X = X_0 + u \; T_+^{\alpha}\big|_{\mathcal{N}}$$

- We were able to find a surface S orthogonal to the congruence, i.e. any geodesic in the congruence is orthogonal to a point on the surface:

$$T^{\alpha} = \eta^{\alpha\beta}\partial_{\beta}S$$

- This is an important result, as it follows that the lightlike congruence does not suffer rotation after it crosses a generic shell in Minkowski space.

Shear, expansion and rotation

- In general, shear, expansion and rotation of a congruence are defined through the gradient of the geodesic vector

$$B_{\alpha\beta} = \partial_\beta T_\alpha .$$

- The evaluation of $B_{\alpha\beta}$ for general junction conditions is too complicated and in the presence of caustics might even result into ambiguities.

- Evaluation of $B_{\alpha\beta}$ has been carried out for the special case of infinitesimal BMS supertranslation type solderings.

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Shear, expansion and rotation

- The expansion scalar θ and the projection of shear tensor on transverse direction $\sigma_{AB} = \begin{pmatrix} \sigma_+ & \sigma_\times \\ \sigma_\times & -\sigma_+ \end{pmatrix}$ have been derived and proven to be different from 0.
- Direct calculation show, as expected, that the rotation tensor is 0.

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Eikonal approximation

- The absence of rotation and the presence of a hypersurface orthogonal to the congruence gives the possibility of applying the Eikonal approximation.

- Eikonal approximation allows one to formally represent the congruence as a wave, solution to the d'Alembertian equation $\Box\psi = 0$

- We introduce $\psi(x) = \mathcal{A}(x)e^{i\omega\Theta(x)}$ ansatz, which will result into the following restrictions:
    1. $\partial^\alpha\Theta = T^\alpha$ is a null geodesic
    2. $\frac{\partial \ln \mathcal{A}}{\partial u} = -\frac{1}{2}\theta$, where θ is the congruence expansion scalar.

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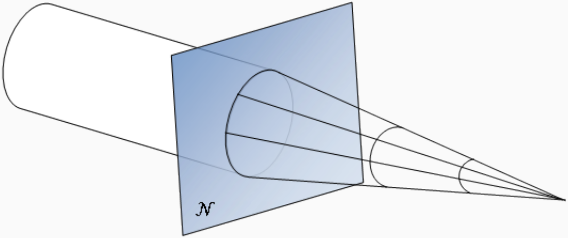
Eikonal approximation

- For BMS supertranslation type soldering we already have the expansion parameter θ and the surface Θ to which the congruence geodesics are orthogonal to, so it is straightforward to construct the wave ψ equivalent to the congruence.
- It is important to check how the wave ψ behaves at the points where caustics form, as a proof of validity of Eikonal approximation.

Caustics

- Points where caustics form mathematically correspond to $\theta \to -\infty$
- We have studied these points and derived focusing conditions.
- The Eikonal approximation breaks down and the amplitude and phase parameters turn to infinities, thus confirming the validity of this approximation.



Conclusion

- New approach has been suggested for studying singular generic shells and geodesics crossing them.
- The jump in the derivative of the geodesic vector has been derived and shown to correspond with the results of Barrabes and Hogan.
- The form of the hypersurface has been derived to which the geodesic lines are orthogonal to on the future.

Conclusion

- Expansion scalar and shear tensor has been derived for infinitesimal BMS supertranslation type solderings and rotation tensor has been proven to be zero for general junction conditions.
- Eikonal approximation has been applied to describe the congruence as a wave.
- Focusing and caustic formation conditions have been studied and shown to correspond with the corresponding results of Eikonal approximation.

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Thank you!