Transport coefficients of two-flavor quark matter from the Kubo formalism

Arus Harutyunyan

based on: PhysRevD.95.114021, PhysRevD.96.034006

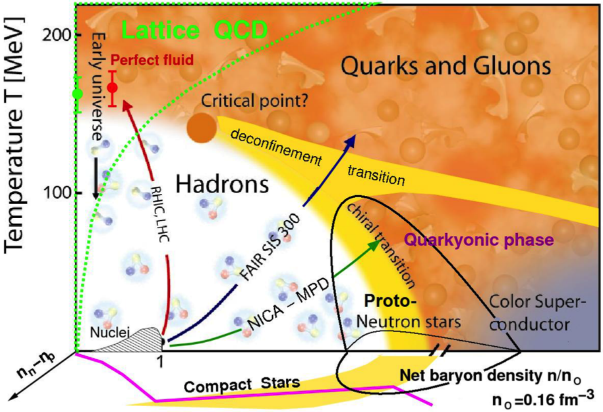
in collaboration with A. Sedrakian, D. Rischke

Institute for Theoretical Physics, Frankfurt am Main, Germany

Yerevan, 18 September, 2017

Outline

QCD phase diagram



1. High temperatures or high densities - perturbative QCD
2. High temperatures and low densities - lattice QCD
3. Intermediate temperatures/densities - effective models: NJL model, MIT bag model

Transport coefficients of hot and dense QCD matter

- Transport coefficients of hot QCD are key inputs in the hydrodynamic description of heavy-ion collisions at RHIC and LHC.
- Transport coefficients of dense QCD are important in the hydrodynamic modelling of various astrophysical phenomena in compact stars.
- The quark-gluon plasma created in heavy-ion collisions is an almost perfect fluid with a very small value of the shear viscosity. The ratio of the shear viscosity η to the entropy density s is close to the lower (KSS) bound conjectured from the gauge/gravity duality [1]

$$\frac{\eta}{s} \geq \frac{1}{4\pi}.$$

- The bulk viscosity of quark matter is negligible compared to the shear viscosity in the perturbative regime, but may become large close to the critical temperature of the chiral phase transition.

[1] P. K. Kovtun, D. T. Son and A. O. Starinets, *Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics*, *Physical Review Letters* **94** (2005) 111601.

Transport coefficients of hot QCD have been computed from several methods:

1. Perturbative QCD methods
   - P. Arnold, D. Moore and L. G. Yaffe, *Transport coefficients in high temperature gauge theories (I): leading-log results*, *Journal of High Energy Physics* **11** (2000) 001.
   - P. Arnold, C. Doğan and D. Moore, *Bulk viscosity of high-temperature QCD*, *Phys. Rev. D* **74** (2006) 085021.
2. Lattice methods + QCD sum rules
   - H. B. Meyer, *Calculation of the shear viscosity in SU(3) gluodynamics*, *Physical Review Letters* **100** (2008) 162001.
   - F. Karsch, D. Kharzeev and K. Tuchin, *Universal properties of bulk viscosity near the QCD phase transition*, *Physics Letters B* **663** (2008) 217-221.
3. Kubo formalism in one-loop approximation
   - R. Lang, N. Kaiser and W. Weise, *Shear viscosities from Kubo formalism in a large-N_c Nambu-Jona-Lasinio model*, *European Physical Journal A* **51** (2015) 127.
   - S. Ghosh, T. C. Peixoto, V. Roy, F. E. Serna and G. Krein, *Shear and bulk viscosities of quark matter from quark-meson fluctuations in the Nambu-Jona-Lasinio model*, *Phys. Rev. C* **93** (2016) 045205.

Linear response theory

Consider a system slightly deviated from thermal equilibrium as a result of thermodynamic forces X_i, which induce dissipative fluxes J_i as a response to these forces.

Linear response theory assumes that these fluxes depend linearly on the forces:

$$J_i = \sum_k \chi_{ik} X^k.$$

Transport coefficients χ_{ik} describe how the system reacts to thermodynamic perturbations. They measure the entropy generation rate in irreversible processes:

$$\frac{dS}{dt} = \sum_i J_i X^i = \sum_{i,k} \chi_{ik} X^i X^k, \qquad \chi_{ik} = \chi_{ki}.$$

In the Green-Kubo formalism χ_{ik} are related to certain two-point retarded equilibrium correlation functions:

$$\chi_{ik} = \lim_{\omega \to 0} \frac{1}{\omega} \int_0^\infty dt e^{i\omega t} \int d\boldsymbol{r} \langle \left[\hat{J}_i(\boldsymbol{r}, t), \hat{J}_k(0) \right] \rangle_0.$$

Green-Kubo formulas are applicable in priciple for *strongly correlated systems*.

Relativistic dissipative hydrodynamics

Consider a relativistic fluid characterized by the conservation laws

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\mu N^\mu = 0. \tag{1}$$

- One can decompose $T^{\mu\nu}$ and N^μ as ($\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$)

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu,$$
$$N^\mu = n u^\mu + j^\mu.$$

- For a perfect fluid $\Pi = \pi^{\mu\nu} = q^\mu = j^\mu = 0$, and the system (1) is closed by an *equation of state* $p = p(\epsilon, n)$.
- For a dissipative fluid the shear stress tensor $\pi_{\mu\nu}$, the bulk viscous pressure Π and the heat current h_μ are given by the relativistic Navier-Stokes relations

$$\pi_{ij} = -\eta\left(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u^k\right), \qquad \Pi = -\zeta\partial_i u^i,$$
$$h_i \equiv q_i - \frac{\epsilon+p}{n} j_i = \kappa\left(\partial_i T - \frac{T}{\epsilon+p}\partial_i p\right).$$

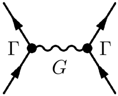
- The coefficients η, ζ and κ are the shear and the bulk viscosities and the thermal conductivity, respectively.

Correlation functions in the Nambu–Jona-Lasinio (NJL) model

Consider two-flavor quark matter described by the NJL Lagrangian

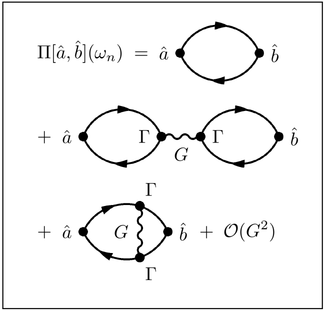
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_0)\psi + \frac{G}{2}\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right]$$

The two bare vertices of the theory are $\Gamma_s = 1$ (scalar) and $\Gamma_{ps} = i\boldsymbol{\tau}\gamma_5$ (pseudoscalar-isovector).



Vertex correction are suppressed by a factor of $1/N_c$, where N_c is the number of colors.

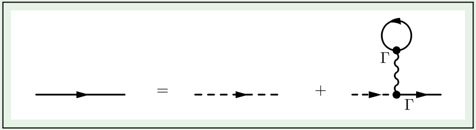
Loop diagrams should be evaluated with *full quark propagators (skeleton expansion).*



- The conductivities and the shear viscosity are given by single-loop diagrams.
- The bulk viscosity includes an infinite geometrical series of multi-loop diagrams.

Quark & meson masses

Dynamically generated quark mass in Hartree approximation is given by the gap equation

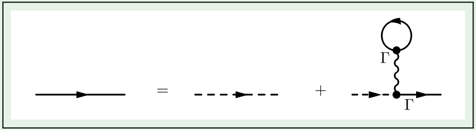


Analytically the gap equation is written as

$$m(T,\mu) = m_0 - G\langle\bar{\psi}\psi\rangle, \qquad \langle\bar{\psi}\psi\rangle = \frac{mN_cN_f}{\pi^2}\int_0^\infty \frac{p^2dp}{E_p}[n^+(E_p)+n^-(E_p)-1].$$

Quark & meson masses

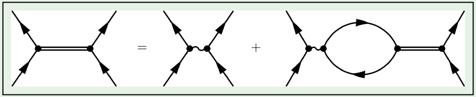
Dynamically generated quark mass in Hartree approximation is given by the gap equation



Analytically the gap equation is written as

$$m(T,\mu) = m_0 - G\langle\bar{\psi}\psi\rangle, \qquad \langle\bar{\psi}\psi\rangle = \frac{mN_cN_f}{\pi^2}\int_0^\infty \frac{p^2dp}{E_p}[n^+(E_p)+n^-(E_p)-1].$$

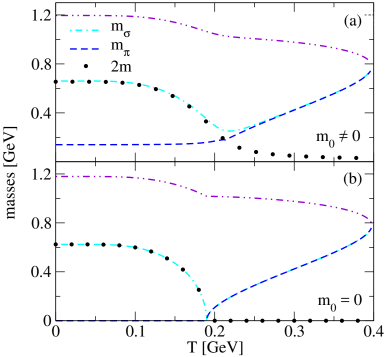
The meson propagators are obtained from the Bethe-Salpeter equation



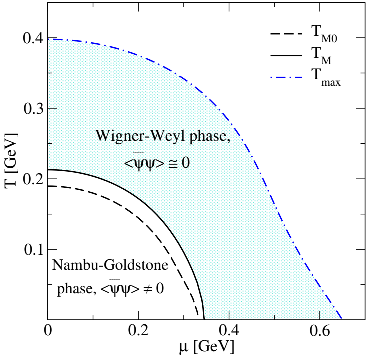
Meson masses are defined as the poles of the propagators in the real space-time

$$D_M(\boldsymbol{p},\omega_n) = G + G\Pi_M(\boldsymbol{p},\omega_n)D_M(\boldsymbol{p},\omega_n) = \frac{G}{1-G\Pi_M(\boldsymbol{p},\omega_n)}, \qquad M=\{\sigma,\pi\}.$$

Phase diagram



The quark and meson masses at $\mu = 0$. If $m_0 \neq 0$, the gap equation always has a non-trivial solution $m > m_0$. If $m_0 = 0$, the chiral symmetry is restored at $T > T_c$.



The Mott temperature T_M defined by the condition $m_\pi(T_M) = 2m(T_M)$ and the temperature $T_{\max}$ as functions of the chemical potential.

Our computations are relevant to the shaded area, where mesonic modes are found.

The full quark propagator

The full quark retarded/advanced propagator and the spectral function are given by

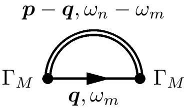
$$G^{R/A}(p_0,\boldsymbol{p}) = \frac{1}{\not p - m - \Sigma^{R/A}(p_0,\boldsymbol{p})}, \qquad A(p_0,\boldsymbol{p}) = -\frac{1}{2\pi i}\left[G^R(p_0,\boldsymbol{p}) - G^A(p_0,\boldsymbol{p})\right].$$

Quark self-energy and spectral function have three Lorentz components

$$A(p_0,\boldsymbol{p}) = -\frac{1}{\pi}(mA_s + p_0\gamma_0 A_0 - \boldsymbol{p\gamma}A_v)$$

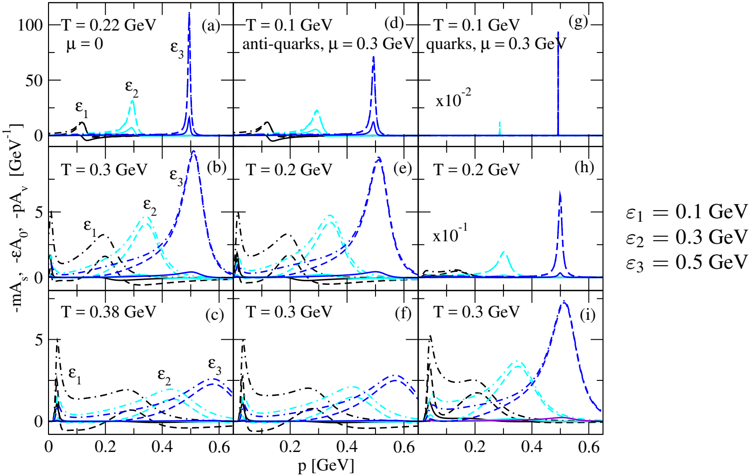
The dominant process in the quark self-energy is the meson decay into quark-antiquark pair and its inverse process above the Mott temperature T_{M}:

$$\Sigma^M(\boldsymbol{p},\omega_n) =$$



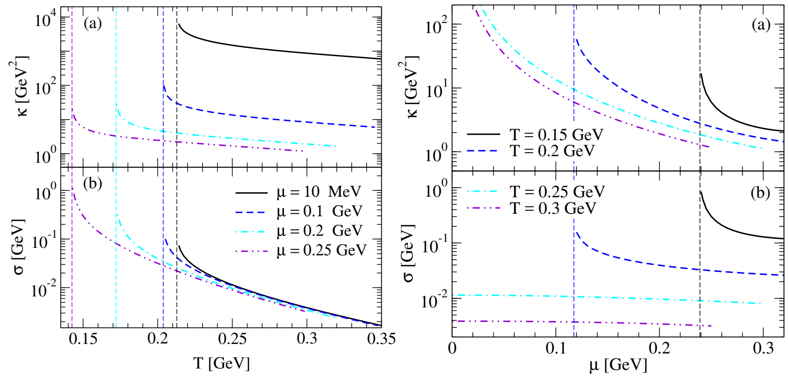
$$\Gamma_\pi = i\gamma_5\tau_i, \quad \Gamma_\sigma = 1.$$

Lorentz components of the spectral function



- The quasiparticle peaks in the spectral functions appear at $p \simeq \varepsilon$.
- The spectral functions are broadened with increasing temperature.

Electrical and thermal conductivities



- The transport coefficients rapidly decrease with the increase of temperature as a result of the broadening of the spectral functions with the temperature.
- At high temperatures we find the scalings $\kappa \propto T^{-3}$, $\sigma \propto T^{-6}$, $\eta \propto T^{-6}$.
- The thermal conductivity diverges in the $\mu \to 0$ limit as $\kappa \propto \mu^{-2}$.

Effective coupling for the bulk viscosity

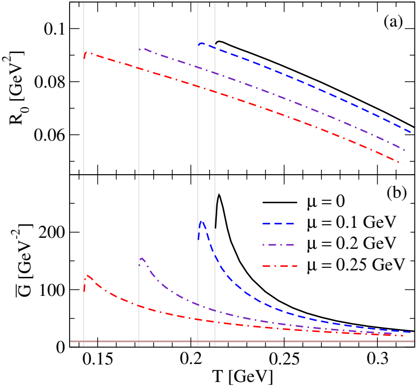
The final result for the bulk viscosity can be written as

$$\zeta = \zeta_0 + \zeta_1 + \zeta_2,$$

where ζ_0 is the one-loop contribution, and the multi-loop contributions are expressed in terms of the effective (resummed) coupling $\bar{G}$

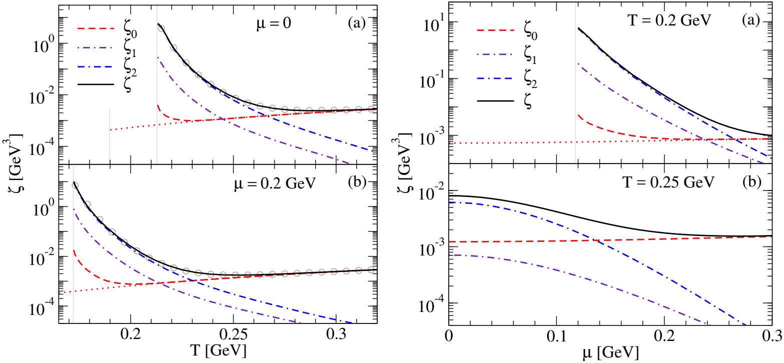
$$\zeta_1 \propto \bar{G}, \quad \zeta_2 \propto \bar{G}^2, \quad \bar{G} = \frac{G}{1 - R_0 G}.$$

In the chiral limit $\zeta_1 = \zeta_2 = 0$.



- For temperatures close to the critical line $R_0 G \simeq 1$ and $\bar{G} \gg G \simeq 10$ GeV^{-2}.
- At high temperatures and chemical potentials $\bar{G}$ tends to its "bare" value $\Rightarrow$ multi-loop contributions should be important close to the Mott transition line.

Results for the bulk viscosity

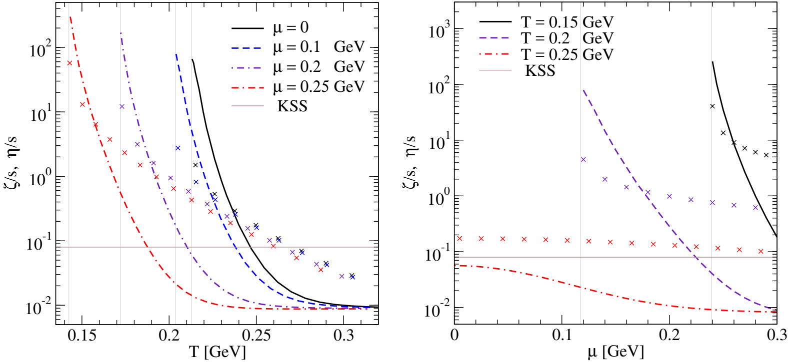


- At low temperature regime close to the Mott line $\zeta \simeq \zeta_2 \gg \zeta_1 \gg \zeta_0$.
- In this regime the bulk viscosity drops exponentially.
- At high temperatures and densities $\zeta \simeq \zeta_0 \propto T^3$.
- In the chiral limit $\zeta_1 = \zeta_2 = 0$, and $\zeta \propto T^3$ in the whole temperature-density range.

Transport coefficients of two-flavor quark matter from the Kubo formalism

A Harutyunyan

Comparison to the shear viscosity



- The ratio η/s decreases in the vicinity of the critical line T_c slower than ζ/s.
- The bulk viscosity dominates the shear viscosity close to T_c by factors $5 \div 20$.
- At high temperatures the shear viscosity undershoots the KSS bound $1/4\pi$.
- At high temperatures shear viscosity becomes the dominant source of dissipation.

Fits to our results

We perform fits to our results for hydrodynamic applications.

- The Mott transition temperature is fitted by the formula

$$T_{\rm M}^{\rm fit}(\mu) = T_0 \begin{cases} 1-\sqrt{\gamma y}e^{-\pi/(\gamma y)} & 0 \le y \le 0.5, \\ \sqrt{1.55(1-y)+0.04(1-y)^2} & 0.5 < y \le 1, \end{cases}$$

  where $y = \mu/\mu_0$, $\mu_0 = 345$ MeV, $T_0 = 213$ MeV and $\gamma = 2.7$.

- The conductivities and the shear viscosity can be fitted by a generic formula

$$\chi_{\rm fit} = C\left(\frac{T}{T_{\rm M}}\right)^{-\alpha} \exp\left(a_1 y^2 + a_2 y^4 + a_3 y^6\right)\chi_{\rm div}, \quad \kappa_{\rm div} = \left(\frac{T}{T_{\rm M}}\right)^2 + y^{-2}.$$

- The bulk viscosity can be fitted by the formula

$$\zeta_{\rm fit}(T,\mu) = a\exp\left(\frac{c}{T/T_M - b}\right) + dT^3,$$

  where a, b, c, d depend on the chemical potential, and $b \lesssim 1$.

Summary

- We computed the transport coefficients of quark matter from the Kubo formalism by applying the $1/N_c$ power counting scheme.
- In the regime of interest the dispersive effects arise from the quark-meson scattering processes above the chiral phase transition temperature T_c.
- The conductivities and the shear viscosity decrease with the temperature above T_c.
- The thermal conductivity diverges quadratically in the limit $\mu \to 0$.
- At high temperatures the shear viscosity undershoots the KSS bound.
- The bulk viscosity dominates the shear viscosity close to the critical line.
- We provide simple fit formulas to our results for hydrodynamic applications.

Summary

- We computed the transport coefficients of quark matter from the Kubo formalism by applying the $1/N_c$ power counting scheme.
- In the regime of interest the dispersive effects arise from the quark-meson scattering processes above the chiral phase transition temperature T_c.
- The conductivities and the shear viscosity decrease with the temperature above T_c.
- The thermal conductivity diverges quadratically in the limit $\mu \to 0$.
- At high temperatures the shear viscosity undershoots the KSS bound.
- The bulk viscosity dominates the shear viscosity close to the critical line.
- We provide simple fit formulas to our results for hydrodynamic applications.

Future outlook

- Our studies can be improved by including vector interactions and/or Polyakov loop contributions in the NJL-model Lagrangian.
- Another possibility is to extend the current studies to the the three-flavor case.
- These studies can be extended also for the low-temperature and high- density quark matter in the presence of leptons, which appear in the quark cores of compact stars.

ՇՆՈՐՀԱԿԱԼՈՒԹՅՈՒՆ

THANK YOU!