Realistic compactification models in Einstein-Gauss-Bonnet gravity

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Based on:

S.P., *Cosmological dynamics of spatially flat Einstein-Gauss-Bonnet models in various dimensions. Vacuum case*, Phys. Rev. D **94**, 024046 (2016) [arXiv:1605.01456]

S.P., *Cosmological dynamics of spatially flat Einstein-Gauss-Bonnet models in various dimensions: low-dimensional Λ-term case*, Phys. Rev. D **94**, 084019 (2016) [arXiv:1607.07347]

S.P., *Cosmological dynamics of spatially flat Einstein-Gauss-Bonnet models in various dimensions: high-dimensional Λ-term case*, Eur. Phys. J. C **77**, 503 (2017) [arXiv:1705.02578]

S.P. and A. Toporensky, *Effects of spatial curvature and anisotropy on the asymptotic regimes in Einstein-Gauss-Bonnet gravity* [arXiv:1709.04258]

Plan of talk:

– Motivation and historical outline;

– General form of EoMs in spatially-flat (Bianchi-I-type) cosmologies;

– For [3+D] spatial splitting:

  – General vacuum case;

  – General Λ-term case;

– Effect of spatial curvature;

– Effect of anisotropy;

– Two-steps scheme (anisotropy + curvature);

– Conclusions

Historical outline

1914 – Nordström's 5D vector theory which unify Nordström's scalar gravity with electromagnetism

Nordström, G., „Über die Möglichkeit, das Elektromagnetische Feld und das Gravitationsfeld zu vereiningen“, Physikalische Zeitschrift **15**, 504 (1914)

1915 – General Relativity, Nordström's gravity proven to be wrong (1919, Solar eclipse)

1919—1921 – Kaluza Hypothesis: 5D Einstein equations → 4D Einstein field equations + Maxwell Equations; cylindrical condition

Kaluza, T., „Zum Unitätsproblem in der Physik“, Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.), 966 (1921)

1926 – Oskar Klein – quantum interpretation: 5^{th} dimension is closed and periodic; elec charge in 5^{th} dim → standing waves → quantization of elec in Borh's model

Klein, O., „Quantentheorie und fünfdimensionale Relativitätstheorie“, Zeitschrift für Physik A **37,** 895 (1926)

Klein, O., „The Atomicity of Electricity as a Quantum Theory Law“, Nature **118**, 516 (1926)

Higher-curvature corrections to the Lagrangian

J. Scherk and J.H. Schwarz, Nucl. Phys. **B81**, 118 (1974) R^2 $R_{\mu\nu}R^{\mu\nu}$

P. Candelas, G.T. Horowitz, A. Strominger and E. Witten, Nucl. Phys. **B258**, 45 (1985)

$R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho}$

B. Zwiebach, Phys. Lett. **156B**, 315 (1985)
B. Zumino, Phys. Rep. **137**, 109 (1986)

$$L_{GB} = R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Euler's topological invariant in (3+1); in higher dimensions gives nontrivial contribution to the equations of motion

Nonlinear!

Nonstandard singularities

The situation with emerges in nonlinear theories:

linear, say, GR $\dot{H} = P_1(H, ...)$

nonlinear theories $\dot{H} = \dfrac{P_2(H, ...)}{P_3(H, ...)}$

H is regular but P3 = 0, dH diverges = (nonstandard) singularity

Spatially-flat (Bianchi-I-type) metric – most generic without imposing any spatial splitting

$$ds^2 = \mathrm{diag}(-1,\, a_1^2(t),\, a_2^2(t),\, \ldots\, a_D^2(t))$$

n=2 for Gauss-Bonnet

$$\mathcal{L} = \sqrt{-g}\,\sum_{n=0}^{t} \alpha_n\, \mathcal{R}^n, \qquad \mathcal{R}^n = \frac{1}{2^n}\delta^{\mu_1\nu_1\ldots\mu_n\nu_n}_{\alpha_1\beta_1\ldots\alpha_n\beta_n}\prod_{r=1}^{n} R^{\alpha_r\beta_r}{}_{\mu_r\nu_r} \qquad \delta^{\mu_1\nu_1\ldots\mu_n\nu_n}_{\alpha_1\beta_1\cdots\alpha_n\beta_n} = \frac{1}{n!}\delta^{\mu_1}_{[\alpha_1}\delta^{\nu_1}_{\beta_1}\cdots\delta^{\mu_n}_{\alpha_n}\delta^{\nu_n}_{\beta_n]}.$$

$$\sum_{n=1}^{d}\zeta_n\left\{\sum_{k\neq m}(\ddot{a}_k+\dot{a}_k^2)\sum_{\{j_1<\cdots<j_{2n-2}\}\neq k,m}\prod_{r=1}^{2n-2}\dot{a}_{j_r} + (2n-1)\sum_{\{j_1<\cdots<j_{2n}\}\neq m}\prod_{r=1}^{2n}\dot{a}_{j_r}\right\} = \kappa^2 T_m^m$$

$$\sum_{n=1}^{d}(2n-1)\zeta_n\sum_{j_1<\cdots<j_{2n}}\prod_{r=1}^{2n}\dot{a}_{j_r} = \kappa^2 T_0^0$$

General vacuum case without any metric ansatz

$H_i = \dot{a}_i(t)/a_i(t)$ spatially-flat – Hubble parameters instead of scale factors

$$2\left[\sum_{j\neq i}(\dot{H}_j + H_j^2) + \sum_{\substack{\{k>l\}\\ \neq i}} H_k H_l\right] + 8\alpha\left[\sum_{j\neq i}(\dot{H}_j + H_j^2)\sum_{\substack{\{k>l\}\\ \neq\{i,j\}}} H_k H_l + 3\sum_{\substack{\{k>l>\\ m>n\}\neq i}} H_k H_l H_m H_n\right] = 0$$

$$2\sum_{i>j} H_i H_j + 24\alpha \sum_{i>j>k>l} H_i H_j H_k H_l = 0$$

$$H_1 = H_2 = H_3 = H \text{ and } H_4 = \ldots = H_{D+3} = h$$

$$2\left[2\dot{H}+3H^2+D\dot{h}+\frac{D(D+1)}{2}h^2+2DHh\right]+8\alpha\left[2\dot{H}\left(DHh+\frac{D(D-1)}{2}h^2\right)+\right.$$

$$+D\dot{h}\left(H^2+2(D-1)Hh+\frac{(D-1)(D-2)}{2}h^2\right)+2DH^3h+\frac{D(5D-3)}{2}H^2h^2+$$

$$\left.+D^2(D-1)Hh^3+\frac{(D+1)D(D-1)(D-2)}{8}h^4\right]=0;$$

$$2\left[3\dot{H}+6H^2+(D-1)\dot{h}+\frac{D(D-1)}{2}h^2+3(D-1)Hh\right]+8\alpha\left[3\dot{H}\left(H^2+2(D-1)Hh+\right.\right.$$

$$\left.+\frac{(D-1)(D-2)}{2}h^2\right)+(D-1)\dot{h}\left(3H^2+3(D-2)Hh+\frac{(D-2)(D-3)}{2}h^2\right)+3H^4+$$

$$+9(D-1)H^3h+3(D-1)(2D-3)H^2h^2+\frac{3(D-1)^2(D-2)}{2}Hh^3+$$

$$\left.+\frac{D(D-1)(D-2)(D-3)}{8}h^4\right]=0;$$

$$2\left[3H^2+3DHh+\frac{D(D-1)}{2}h^2\right]+24\alpha\left[DH^3h+\frac{3D(D-1)}{2}H^2h^2+\frac{D(D-1)(D-2)}{2}Hh^3+\right.$$

$$\left.+\frac{D(D-1)(D-2)(D-3)}{24}h^4\right]=0.$$

D=1 vacuum case

$$4\dot{H} + 6H^2 + 2\dot{h} + 2h^2 + 4Hh + 8\alpha(2(\dot{H} + H^2)Hh + (\dot{h} + h^2)H^2) = 0,$$

$$6\dot{H} + 12H^2 + 24\alpha(\dot{H} + H^2)H^2 = 0,$$

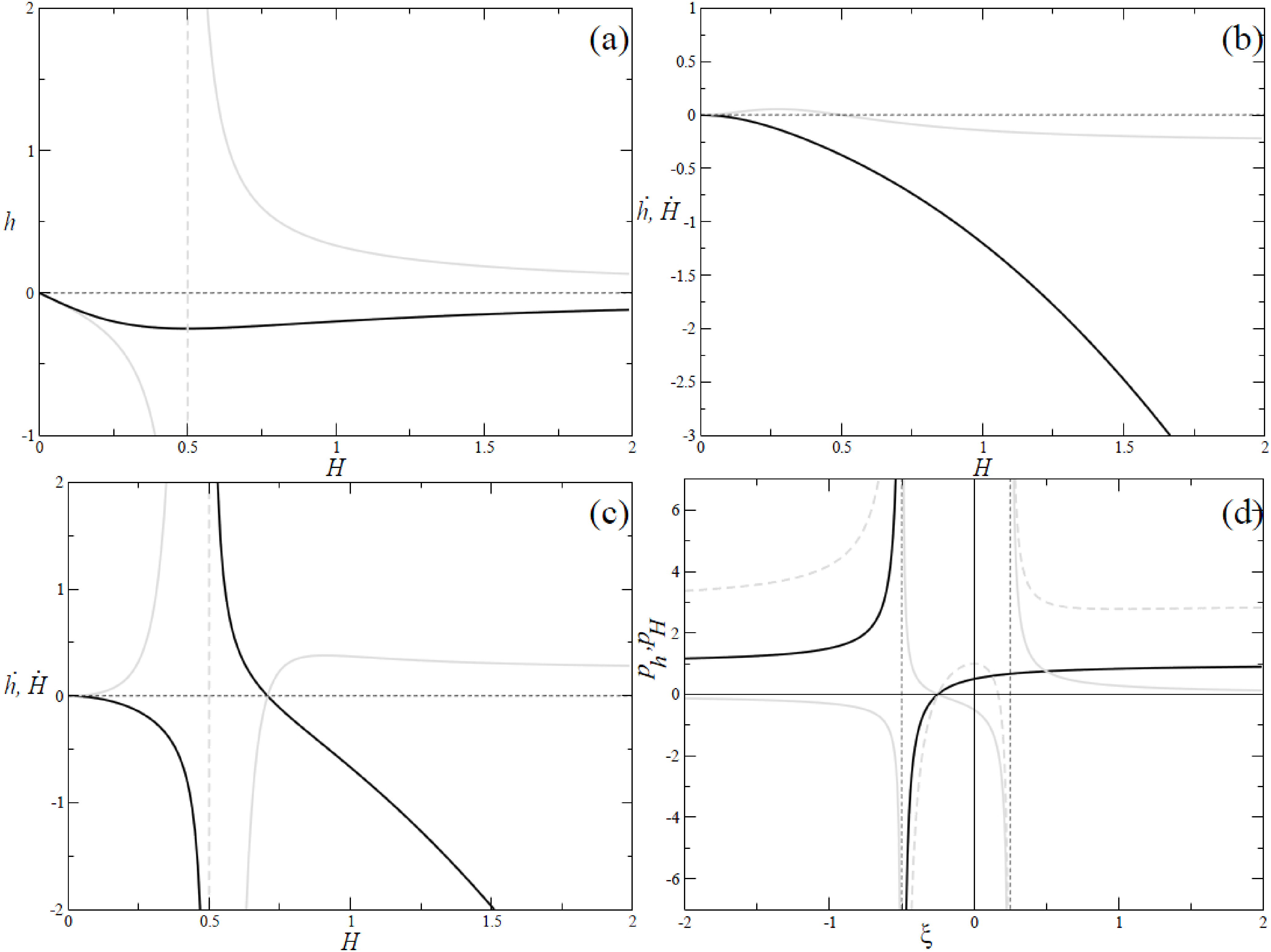
$$h = -\frac{H}{1 + 4\alpha H^2}$$

$$6H^2 + 6Hh + 24\alpha H^3 h = 0.$$

$$\dot{H} = -\frac{2H^2(1 + 2\alpha H^2)}{1 + 4\alpha H^2}$$

$$\dot{h} = -\frac{2H^2(8\alpha^2 H^4 + 2\alpha H^2 - 1)}{(1 + 4\alpha H^2)(16\alpha^2 H^4 + 8\alpha H^2 + 1)}$$

$$p_H = -\dot{H}/H^2 \qquad p_h = -\dot{h}/h^2 \qquad \xi = \alpha H^2$$



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(d)
p_h, p_H
ξ

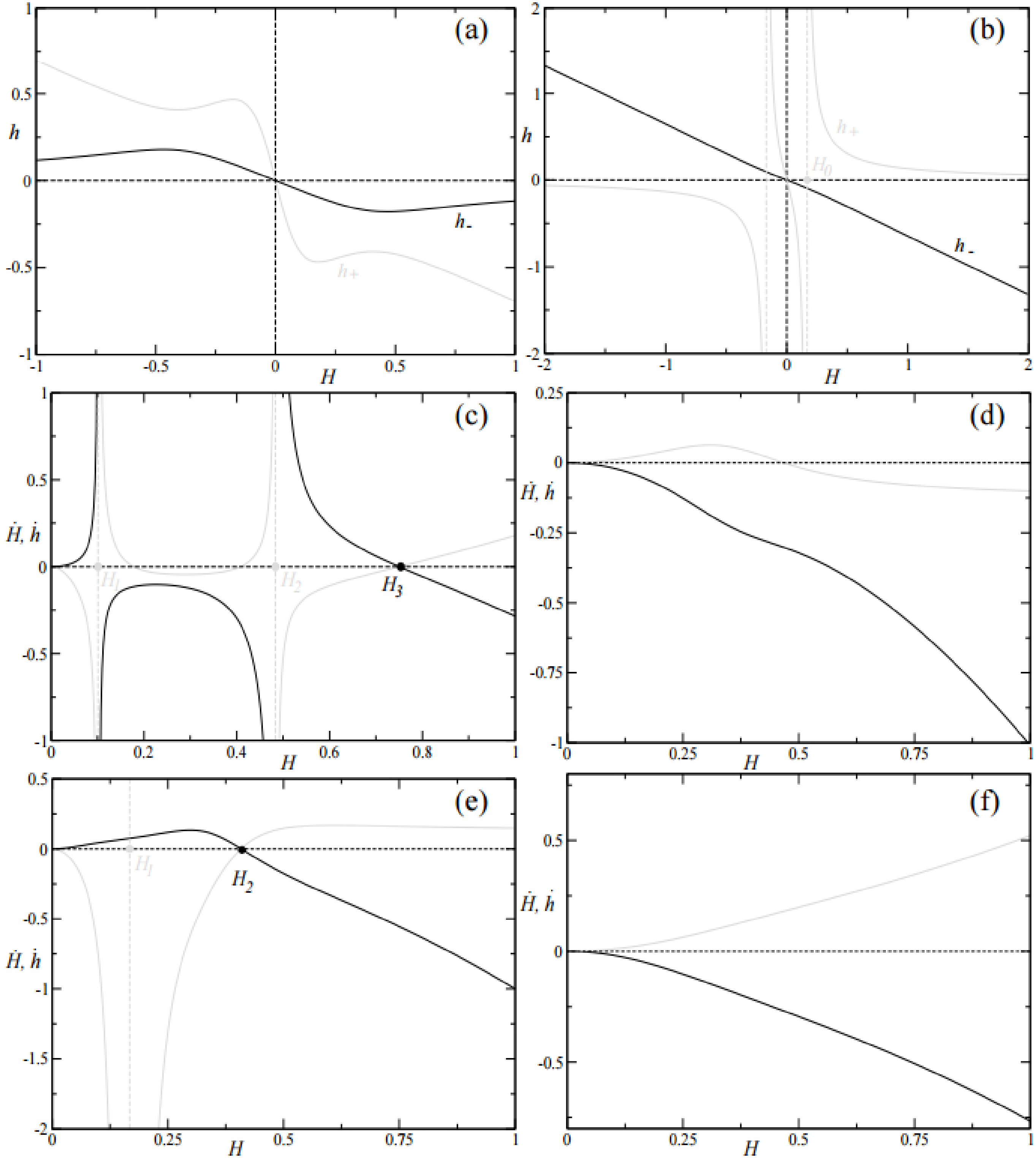
Summary of vacuum D=1 regimes

α	Additional conditions	Regimes
$\alpha > 0$	no	$K_3 \to K_1$
$\alpha < 0$	$H^2 < -\frac{1}{4\alpha}$	$nS \to K_1$
	$-\frac{1}{2\alpha} > H^2 > -\frac{1}{4\alpha}$	$nS \to E_{iso}$
	$H^2 > -\frac{1}{2\alpha}$	$K_3 \to E_{iso}$

D=2 – the same procedure, but two branches:

$$6H^2 + 12Hh + 2h^2 + 24\alpha(2H^3h + 3H^2h^2) = 0$$

$$h_\pm = -\frac{H\left(3 + 12\alpha H^2 \pm \sqrt{6 - 36\alpha H^2 + 144\alpha^2 H^4}\right)}{1 + 36\alpha H^2}$$



(a)
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Summary of vacuum D=2 regimes

<table>
<tr><th>α</th><th>Branch</th><th>Additional conditions</th><th>Regimes</th></tr>
<tr><td rowspan="5">$\alpha > 0$</td><td rowspan="4">h_+</td><td>$H < H_1 = \sqrt{\frac{\xi_1}{\alpha}}$ from (23)</td><td>$K_1 \to nS$</td></tr>
<tr><td>$\sqrt{\frac{\xi_2}{\alpha}} = H_2 > H > H_1 = \sqrt{\frac{\xi_1}{\alpha}}$ from (23)</td><td>$nS \to nS$</td></tr>
<tr><td>$\sqrt{\frac{\xi_0}{\alpha}} = H_3 > H > H_2 = \sqrt{\frac{\xi_2}{\alpha}}$ from (22) and (23)</td><td>$nS \to E_{3+2}$</td></tr>
<tr><td>$H > H_3 = \sqrt{\frac{\xi_0}{\alpha}}$ from (22)</td><td>$K_3 \to E_{3+2}$</td></tr>
<tr><td rowspan="2">h_-</td><td rowspan="2">no</td><td rowspan="2">$K_3 \to K_1$</td></tr>
<tr><td rowspan="4">$\alpha < 0$</td></tr>
<tr><td rowspan="3">h_+</td><td>$H < H_1 = \frac{1}{6\sqrt{-\alpha}}$</td><td>$K_1 \to nS$</td></tr>
<tr><td>$\frac{1}{\sqrt{-6\alpha}} = H_2 > H > H_1 = \frac{1}{6\sqrt{-\alpha}}$</td><td>$nS \to E_{iso}$</td></tr>
<tr><td>$H > H_2 = \frac{1}{\sqrt{-6\alpha}}$</td><td>$K_3 \to E_{iso}$</td></tr>
</table>

Summary of D=3 regimes

Branch	α	Additional conditions	Regimes
h_1	$\alpha > 0$	no	$K_3 \to K_3$
	$\alpha < 0$		$K_3 \to E_{iso}$ (both branches)
h_2	$\alpha > 0$	$H < \sqrt{\dfrac{\xi_3}{\alpha}}$ from (32)	$nS \to K_1$
		$\sqrt{\dfrac{\xi_4}{\alpha}} > H > \sqrt{\dfrac{\xi_3}{\alpha}}$ from (32)	$nS \to E_{3+3}$
		$H > \sqrt{\dfrac{\xi_4}{\alpha}}$ from (32)	$K_3 \to E_{3+3}$
	$\alpha < 0$	no	$K_3 \to K_1$
h_3	$\alpha > 0$	$H < \sqrt{\dfrac{\xi_1}{\alpha}}$ from (32)	$K_1 \to nS$
		$\sqrt{\dfrac{\xi_2}{\alpha}} > H > \sqrt{\dfrac{\xi_1}{\alpha}}$ from (32)	$E_{3+3} \to nS$
		$H > \sqrt{\dfrac{\xi_2}{\alpha}}$ from (32)	$E_{3+3} \to K_3$
	$\alpha < 0$	no	$K_1 \to K_3$

Summary of general D≥4 regimes

Branch	α	Regimes
H_1	$\alpha > 0$	$K_1 \to K_3$
	$\alpha < 0$	$K_3 \to K_1$
H_2	$\alpha > 0$	$nS \to K_1$
		$nS \to nS$
		$E_{3+D} \to nS$
		$E_{3+D} \to K_3$
	$\alpha < 0$	$K_3 \to E_{iso}$ (both regimes)
H_3	$\alpha > 0$	$K_3 \to E_{3+D}$ (both regimes)
	$\alpha < 0$	$K_1 \to K_3$

GR Kasner:

D=1 $p_H = 0.5$

D=2 $p_H = \dfrac{1}{2\sqrt{6}-3} \approx 0.5266$

D=3 $p_H = \dfrac{2}{3\sqrt{5}-3} \approx 0.5294$

general D≥4 $$p_H = \frac{1}{3} - \frac{D+\sqrt{3D^2+6D}}{3(D+3)} \text{ with } \lim_{D\to\infty} p_H = \frac{1}{\sqrt{3}} \approx 0.577.$$

Summary of vacuum regimes:

- high-energy to low-energy Kasner transitions: at $\alpha>0$ for $D\leq 2$ and $\alpha<0$ for $D\geq 2$
- high-energy Kasner to anisotropic exponential transitions: $\alpha>0$ and $D\geq 2$
- the value for p_H could be too high to fit Friedmann cosmology

S.P., Phys. Rev. D **94**, 024046 (2016)

Λ-term cases – the same procedure but results are a bit different:

- no low-energy Kasner regime (replaced with nS, ``dual" regimes or high-energy Kasner)
- more complicated structure of the regimes

Summary of D=1 Λ-term regimes

α	Λ	Additional conditions		Regimes
$\alpha > 0$	$\Lambda > 0$	$H < H_-$ from (14)		$\tilde{K}_1 \to E_{iso}$
		$H > H_-$ from (14)		$K_3 \to E_{iso}$
	$\Lambda < 0$	no		$K_3 \to \tilde{K}_1^S$
$\alpha > 0$	$\Lambda > 0$	$\alpha\Lambda < -3/2$	$H < \frac{1}{2\sqrt{-\alpha}}$	$\tilde{K}_1 \to nS$
			$H > \frac{1}{2\sqrt{-\alpha}}$	$K_3 \to nS$
		$\alpha\Lambda = -3/2$	$H < \frac{1}{2\sqrt{-\alpha}}$	$\tilde{K}_1 \to E_{iso}$
			$H > \frac{1}{2\sqrt{-\alpha}}$	$K_3 \to E_{iso}$
		$\alpha\Lambda > -3/2$	$H < H_-$ from (14)	$\tilde{K}_1 \to E_{iso}^{(1)}$
			$\frac{1}{2\sqrt{-\alpha}} > H > H_-$ from (14)	$nS \to E_{iso}^{(1)}$
			$H_+ > H > \frac{1}{2\sqrt{-\alpha}}$ from (14)	$nS \to E_{iso}^{(2)}$
			$H > H_+$ from (14)	$K_3 \to E_{iso}^{(2)}$
	$\Lambda < 0$	$H < \frac{1}{2\sqrt{-\alpha}}$		$nS \to \tilde{K}_1^S$
		$H_+ > H > \frac{1}{2\sqrt{-\alpha}}$ from (14)		$nS \to E_{iso}$
		$H > H_+$ from (14)		$K_3 \to E_{iso}$

D=2 – even more solutions (3 tables), among them high-energy Kasner to anisotropic exponent for $\alpha>0$, $\alpha\Lambda\leq 1/2$ (including $\Lambda<0$) as well as $\alpha<0$, $\Lambda>0$, $\alpha\Lambda\leq -3/2$.

Additionally for $\alpha<0$, $\Lambda>0$, $\alpha\Lambda=-3/2$ there exist regime with $h \to 0$ – extra dimensions ``stabilize'' (their``size'' in terms of the scale factor reach constant value).

D=3 – similar to D=2, but the regimes are ``doubled''.

Finally, D≥4, $\alpha<0$, $\Lambda>0$, $\alpha\Lambda\leq -3/2$ and $\alpha>0$, $\alpha\Lambda \leqslant \zeta_6$

$$\zeta_6 = \frac{1}{4}\frac{3D^2 - 7D + 6}{D(D-1)}$$

To conclude:

- the only viable regime is the transition from high-energy Kasner to the exponential regime
- D≥2
- just the requirement of the existence of viable cosmologies → constraints on (α, Λ)

<u>S.P.</u>, Phys. Rev. D **94**, 084019 (2016)
<u>S.P.</u>, Eur. Phys. J. C **77**, 503 (2017)

Limits on αΛ from AdS/CFT, causality violation, BHs in GB gravity, shear viscosity-to-entropy ratio etc

$$-\frac{(D+2)(D+3)(D^2+5D+12)}{8(D^2+3D+6)^2} \equiv \eta_2 \leqslant \alpha\Lambda \leqslant \eta_1 \equiv \frac{(D+2)(D+3)(3D+11)}{8D(D+5)^2}$$

Only in AdS (so that $\Lambda<0$)!

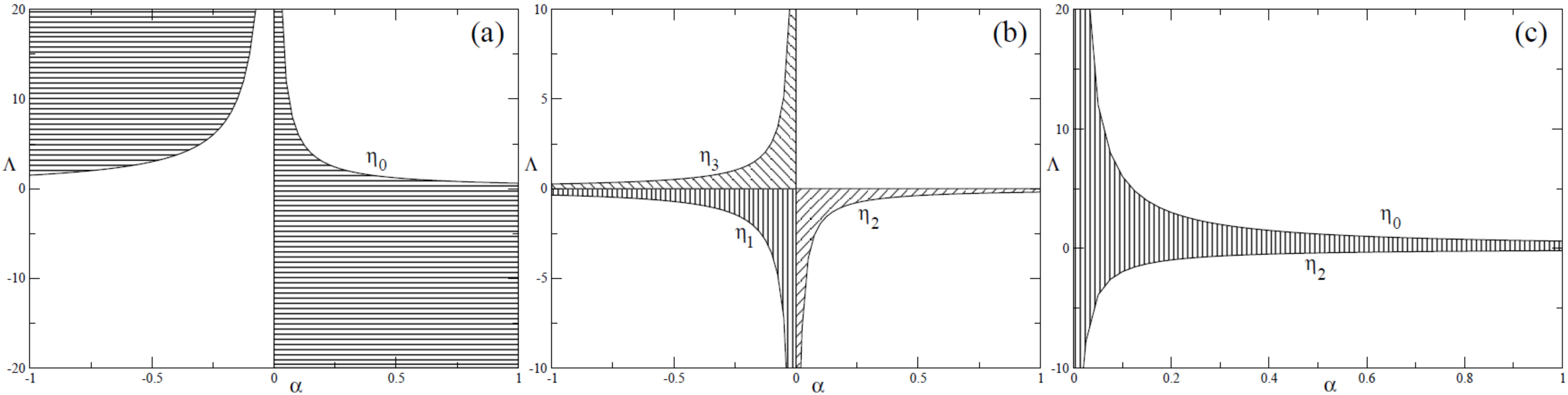
In dS ($\Lambda>0$), the limits are less numerous (BHs, causality etc)

$$\alpha\Lambda \geqslant \eta_3 \equiv -\frac{D^2+7D+4}{8(D-1)(D+2)}$$

Our limits:

$\alpha<0$, $\Lambda>0$, $\alpha\Lambda \leq -3/2$ and $\alpha > 0,\ \alpha\Lambda \leqslant \eta_0 \equiv \zeta_6$

$$\zeta_6 = \frac{1}{4}\frac{3D^2-7D+6}{D(D-1)}$$



$$\alpha > 0, \quad D \geqslant 2, \quad \frac{3D^2 - 7D + 6}{4D(D-1)} \equiv \eta_0 \geqslant \alpha\Lambda \geqslant \eta_2 \equiv -\frac{(D+2)(D+3)(D^2+5D+12)}{8(D^2+3D+6)^2}$$

More complex models:

- spatial curvature
- anisotropy within subspaces

Influence of curvature

$$M_4 \times M_D \qquad ds^2 = -dt^2 + a(t)^2 d\Sigma^2_{(3)} + b(t)^2 d\Sigma^2_{(\mathbf{D})}$$

$$\mathcal{E}_i = 0$$

$$\Leftrightarrow 0 = \alpha + \beta\left(B_{(2)} + \frac{4A_{(1)}}{D(D-1)} + \frac{2B_{(1)}}{D-1} + \frac{2A_{(2)}}{D(D-1)} + \frac{4C}{(D-1)}\right) + \gamma\left(B^2_{(2)} + \frac{16A_{(1)}C}{(D-1)(D-2)(D-3)} + \frac{8B_{(2)}C}{D-3} + \frac{8A_{(1)}B_{(2)}}{(D-2)(D-3)} + \frac{8A_{(2)}B_{(1)}}{(D-1)(D-2)(D-3)} + \frac{16B_{(1)}C}{(D-2)(D-3)} + \frac{4B_{(1)}B_{(2)}}{(D-3)} + \frac{4A_{(2)}B_{(2)}}{(D-2)(D-3)} + \frac{8C^2}{(D-2)(D-3)}\right),$$

$$\mathcal{E}_a = 0 \Leftrightarrow 0$$

$$= \frac{D}{(D-4)}\alpha + \frac{(D-2)}{(D-4)}\beta\left(B_{(2)} + \frac{6A_{(1)}}{(D-1)(D-2)} + \frac{2B_{(1)}}{D-2} + \frac{6A_{(2)}}{(D-1)(D-2)} + \frac{6C}{(D-2)}\right) + \gamma\left(B^2_{(2)} + \frac{48A_{(1)}C}{(D-2)(D-3)(D-4)} + \frac{12B_{(2)}C}{D-4} + \frac{24C^2}{(D-3)(D-4)} + \frac{12A_{(1)}B_{(2)}}{(D-3)(D-4)} + \frac{24A_{(2)}B_{(1)}}{(D-2)(D-3)(D-4)} + \frac{24B_{(1)}C}{(D-3)(D-4)} + \frac{4B_{(1)}B_{(2)}}{(D-4)} + \frac{12A_{(2)}B_{(2)}}{(D-3)(D-4)} + \frac{24A_{(2)}C}{(D-2)(D-3)(D-4)} + \frac{24A_{(1)}A_{(2)}}{(D-1)(D-2)(D-3)(D-4)}\right).$$

$$\mathcal{E}_0 = 0 \Leftrightarrow 0$$

$$= \alpha + \beta\left(B_{(2)} + \frac{6}{D-1}C + \frac{6}{D(D-1)}A_{(2)}\right) + \gamma\left(B^2_{(2)} + \frac{12A_{(2)}B_{(2)}}{(D-2)(D-3)} + \frac{24C^2}{(D-2)(D-3)} + \frac{12B_{(2)}C}{(D-3)} + \frac{24A_{(2)}C}{(D-1)(D-2)(D-3)}\right),$$

$$A_{(1)} = \frac{\ddot{a}}{a}, \qquad C = \frac{\dot{a}\,\dot{b}}{ab}, \qquad B_{(1)} = \frac{\ddot{b}}{b},$$

$$A_{(2)} = \frac{[\gamma_{(3)} + (\dot{a})^2]}{a^2}, \qquad B_{(2)} = \frac{[\gamma_{(\mathbf{D})} + (\dot{b})^2]}{b^2}.$$

Vacuum $K_3 \to K_1$

$(\gamma_{(3)}, \gamma_{(\mathbf{D})})$	Regime
$\gamma_{(\mathbf{D})} = 0$	$K_3 \to K_1$
$\gamma_{(\mathbf{D})} > 0$	$K_3 \to nS$
$\gamma_{(\mathbf{D})} < 0$	$K_3 \to K_D$

Vacuum $K_3 \to E_{3+D}$

$(\gamma_{(3)}, \gamma_{(\mathbf{D})})$	Regime
$\gamma_{(\mathbf{D})} = 0$	$K_3 \to E_{3+D}$
$\gamma_{(\mathbf{D})} \neq 0$	$K_3 \to nS$

Λ-term $K_3 \to E_{3+D}$

$(\gamma_{(3)}, \gamma_{(\mathbf{D})})$	Regime
$\gamma_{(\mathbf{D})} = 0$	$K_3 \to E_{3+D}$
$\gamma_{(\mathbf{D})} > 0$	$K_3 \to nS$
$\gamma_{(\mathbf{D})} < 0,\ D = 2$	$K_3 \to nS$ or $K_3 \to E_{iso}$
$\gamma_{(\mathbf{D})} < 0,\ D \geqslant 3$	$K_3 \to nS$ or $K_3 \to E_3$

K_D: $H \to 0$ $(a(t) \to \mathrm{const})$, $b(t) \propto t$ $(p_h = 1)$ ``stabilization" of three dimensions, only D≥3

E_3: $H \to \mathrm{const}$, $b(t) \to \mathrm{const}$ ``stabilization" of extra dimensions, only D≥3

Only negative curvature of extra dimensions brings new realistic regime(s) and only in D≥3

K_D could be viable in D=3, but p_h is too large...

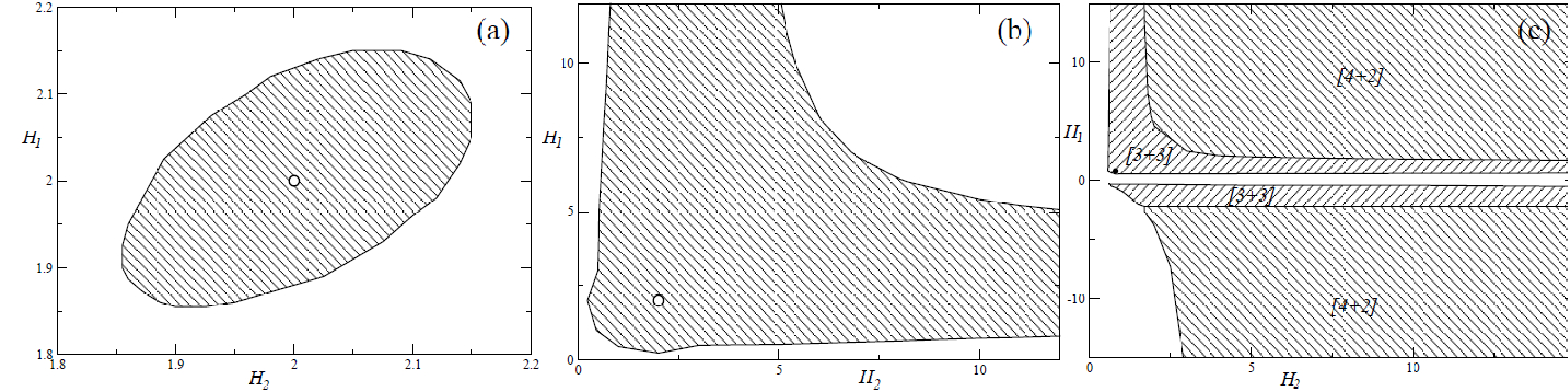
Influence of anisotropy

$$2\left[\sum_{j\neq i}(\dot{H}_j + H_j^2) + \sum_{\substack{\{k>l\}\\ \neq i}} H_k H_l\right] + 8\alpha\left[\sum_{j\neq i}(\dot{H}_j + H_j^2)\sum_{\substack{\{k>l\}\\ \neq\{i,j\}}} H_k H_l + 3\sum_{\substack{\{k>l>\\ m>n\}\neq i}} H_k H_l H_m H_n\right] = 0$$

$$2\sum_{i>j} H_i H_j + 24\alpha \sum_{i>j>k>l} H_i H_j H_k H_l = 0$$

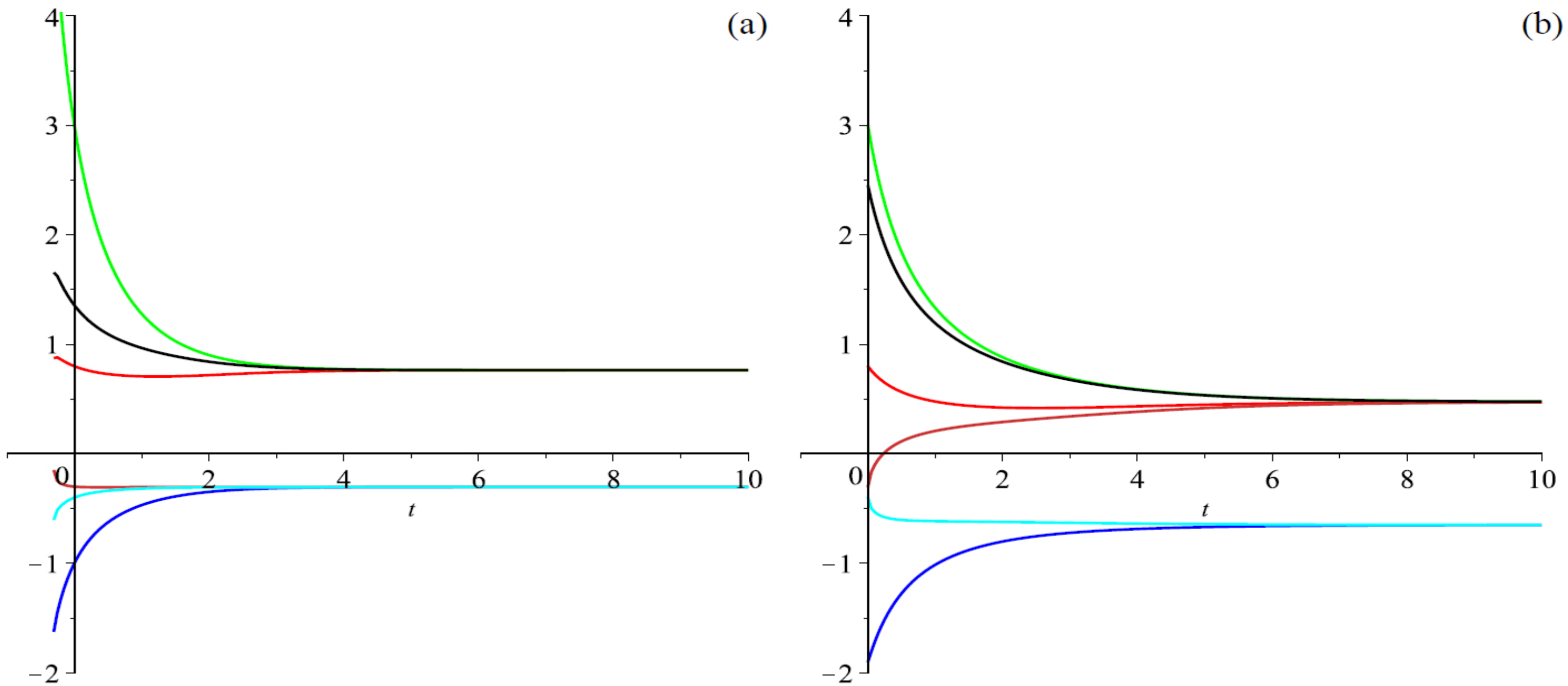
$H_i = \dot{a}_i(t)/a_i(t)$ without [3+D] spatial splitting

- start in the vicinity of the exact solution;
- set and fix all but three initial Hubble parameters;
- vary two of the remaining Hubble parameters;
- calculate the remaining from the constraint equation.



$K_3 \to K_1$ $\qquad$ $K_3 \to E_{3+2}\ (D=2)$ $\qquad$ $K_3 \to E_{3+D}\ (D \geqslant 3)$

Kasner asymptote – ``metastable''
Exponential – stable for a wide range of the initial conditions
But in D≥3 there exist more then one stable anisotropic exponential solution



Two-steps scheme

Anisotropy – exponential solution with [3+D] spatial splitting from quite wide initial conditions;

Negative cirvature of the extra dimensions – stabilization of the extra dimensions, if started from anisotropic exponential solution.

Combine them!
Start from i.c. with small curvature → exponential solution → curvature of the extra dimensions begins to play role → stabilization of the extra dimensions

… works only in $D \geq 3$ (from curvature), where the anisotropy could lead to another (non-viable) exponential solution...

Results

- analytically found power-law and exponential exact solutions are presented in the general scheme without any prior metric ansatz (including constant-volume exponential solutions);

- requirement of existence of viable (realistic) regimes (standard cosmological singularity as a past attractor and power-law/exponential expansion of 3D + contraction of extra dimensions as a future attractor) allows us to put constraints on (α, Λ);

- comparisong of our constraints with those from other considerations allows to put more tighter constraints; in particular, only $\alpha>0$ is allowed; $D\geq 2$

- negative curvature of the extra dimensions brings new potentially viable regimes;

- initially anisotropic cosmologies tend to exponential solutions;

- demonstrated existence of the transition from the initially anisotropic curved space to the exponential solution with expanding three and constant-sized extra dimensions with no fine-tuning; $D\geq 3$