Phase diagram of hadron matter in effective theories of QCD

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The modern sketch of HIC

Experiment

The picture of the heavy ion collision's evolution

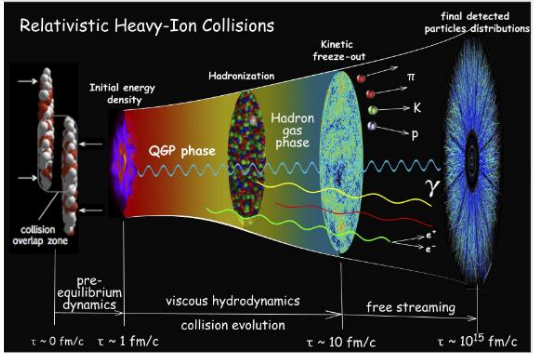
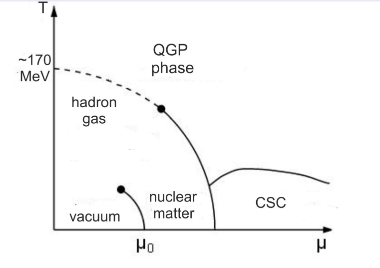


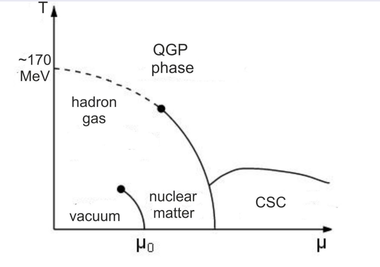
Figure 1: arXiv (nucl-th): 1304.3634

The QCD phase diagram



- chiral symmetry restoration (constituent quarks $\rightarrow$ current quarks);
- deconfinement;

The QCD phase diagram



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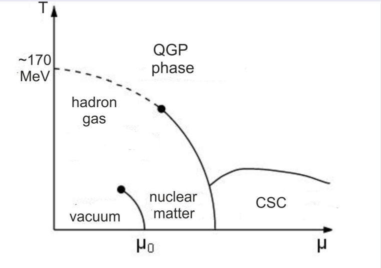
Do they coincide?

The QCD phase diagram



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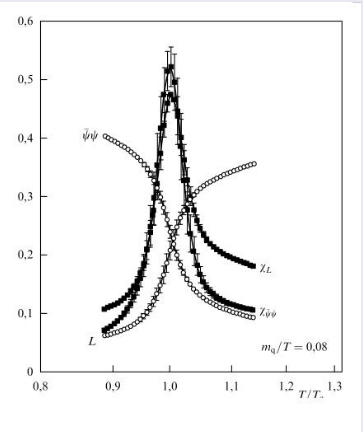
Lattice QCD

Hands S. Contemp. Phys. 42, 209 [2001], $T_c = 0.17$ GeV (SU(2))



The Nambu-Jona-Lasinio model

The Lagrangian

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\mathrm{q}}\left(\mathrm{i}\not{\partial} - \hat{\mathrm{m}}_0 - \gamma_0\mu\right)\mathrm{q} + \mathrm{G_s}\left[\left(\bar{\mathrm{q}}\mathrm{q}\right)^2 + \left(\bar{\mathrm{q}}\mathrm{i}\gamma_5\vec{\tau}\mathrm{q}\right)^2\right] \ ,$$

$\mathrm{G_s}$ the effective coupling strength,
$\bar{\mathrm{q}}$ и q - quark fields
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M. K. Volkov, Ann. Phys. 157,282 (1989); Sov. J. Part and Nuclei 17, 433 (1986) S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).

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- describe chiral phase transitions.
- describe light quarks and mesons properties,

The mean - field approximation

We can introduce the partition function

$$\mathcal{Z}[\bar{\mathrm{q}},\mathrm{q}] = \int \mathcal{D}\bar{\mathrm{q}}\mathcal{D}\mathrm{q}\exp\left\{\int_0^\beta \mathrm{d}\tau \int_{\mathrm{v}} \mathrm{d}^3\mathrm{x}\,[\mathcal{L}_{\mathrm{NJL}}]\right\}. \quad (1)$$

Then, using the mean-field approximation procedure, we get

$$\mathcal{Z}_{\mathrm{MF}}[\bar{\mathrm{q}},\mathrm{q}] = \exp\left\{-\int_0^\beta \mathrm{d}\tau \int_{\mathrm{v}} \mathrm{d}^3\mathrm{x}\frac{\sigma'^2_{\mathrm{MF}}}{4\mathrm{G}} + \mathrm{Tr}\ \ln\mathrm{S}^{-1}_{\mathrm{MF}}[\mathrm{m}]\right\}. \quad (2)$$

And then

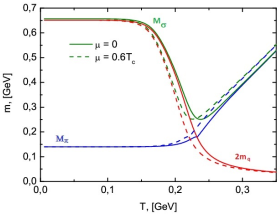
$$\Omega_{\mathrm{NJL}}(\mathrm{T},\mu) = -\frac{\mathrm{T}}{\mathrm{V}}\ln\mathrm{Z}_{\mathrm{MF}}(\overline{\mathrm{q}},\mathrm{q}). \quad (3)$$

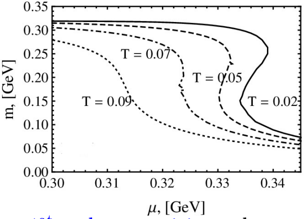
The grand potential

$$\Omega_{\mathrm{NJL}} \;=\; \mathrm{G_s}\langle\bar{\mathrm{q}}\mathrm{q}\rangle^2 - 2\mathrm{N_c}\mathrm{N_f}\int\frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\mathrm{E_p} - 2\mathrm{N_c}\mathrm{N_f}\mathrm{T}\int\frac{\mathrm{d}^3\mathrm{p}}{(2\pi)^3}\left[\ln\mathrm{N}^+(\mathrm{E_p}) + \ln\mathrm{N}^-(\mathrm{E_p})\right] \quad (4)$$

where $\mathrm{N}^+(\mathrm{E_p}) = 1 + \mathrm{e}^{-\beta(\mathrm{E_p}-\mu)}$, $\mathrm{N}^-(\mathrm{E_p}) = 1 + \mathrm{e}^{-\beta(\mathrm{E_p}+\mu)}$
$\mathrm{E_p} = \sqrt{\mathrm{p}^2+\mathrm{m}^2}$ - quark energy and $\beta = 1/\mathrm{T}$.

Symmetries restoration and breaking



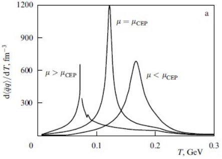


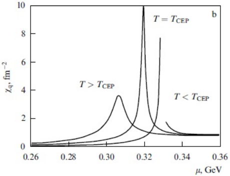
Crossover transition

$$\frac{\partial < \bar{\mathrm{q}}\mathrm{q} >}{\partial \mathrm{T}}\Big|_{\mu=\mathrm{const}}$$

1^{st} order transition: the quark susceptibility

$$\frac{\chi_{\mathrm{q}}(\mathrm{T},\mu)}{\mathrm{T}^2} = \frac{\partial^2(\mathrm{p}/\mathrm{T}^4)}{\partial(\mu/\mathrm{T})^2} = \frac{\partial}{\partial(\mu/\mathrm{T})}(\rho/\mathrm{T}^3).$$





The model parameters

The model has free parameters:

- m_0 - current quark mass,
- Λ - three-momentum cut-off,
- G_s - the effective coupling strength

To fix the parameters we use the experimental data:

- The pion decay constant $f_\pi = 0.092$ GeV,
- The pion mass $M_\pi = 0.139$ GeV
- The quark condensate $< \bar{q}q >= (-0.25 \text{ GeV})^3$

	m_0 [MeV]	Λ [GeV]	G_s [GeV $]^{-2}$	f_π [GeV]	m_π [GeV]	m [GeV]
Set A	5.5	0.639	5.227	0.092	0.139	0.319
Set B	5.6	0.646	5.56	0.099	0.141	0.394

Table 1: The NJL parameters.

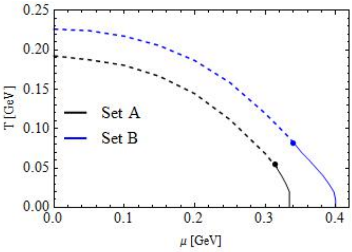


Figure 2: The NJL phase diagram.

Set A: $T_c = 0.186$ GeV ,
T_{CEP} (0.05, 0.3165)
Set B: $T_c = 0.2265$ GeV,
T_{CEP} (0.08, 0.3425)
A. V. Friesen, Yu. L. Kalinovsky, Phys. Part. Nucl. Lett. 6, 737 (2015)

The critical temperatures:

- T_{Mott} $(M_\pi = 2m_q)$
- T_c - crossover line $\max \frac{\partial < q\bar{q} >}{\partial T}$
- 1^{st}-order transition - $\max \frac{\partial^2 \Omega}{\partial \mu^2}|_{T=const}$
- T_{CEP}

The NJL model:

- can reproduce chiral phase transition;
- shows crossover phase transition at low density and high temperature;
- shows 1^{st} order transition at low temperature and high density;
- is local theory and cannot describe confinement/deconfinement properties.

The Polyakov-loop extended Nambu-Jona-Lasinio model

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0 - \gamma_0\mu\right)q + G_s\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - \mathcal{U}\left(\Phi, \bar{\Phi}; T\right)$$

C. Ratti, M. Thaler, W. Weise, PRD 73, 014019 (2006)

$q = (q_u, q_d)$ quark fields,
$\hat{m}_0 = \mathrm{diag}\,(m_u^0, m_d^0)$-current quark masses, $m_u^0 = m_d^0 = m_0$
$D^\mu = \partial^\mu - iA^\mu$ - covariant derivative,
$A^\mu(x) = g\mathcal{A}_a^\mu \frac{\lambda_a}{2}$, $\mathcal{A}_a^\mu$ the gauge field SU(3),
$A^\mu = \delta_0^\mu A^0 = -i\delta_4^\mu A_4$,
λ_a - Gell-Mann matrices,
G_s - scalar coupling strength.

The Polyakov field Φ is determined as: $\Phi[A] = \dfrac{1}{N_c}\mathrm{Tr}_c L(\vec{x})$,

$$L(\vec{x}) = \mathcal{P}\exp\left[i\int_0^\beta d\tau A_4(\vec{x},\tau)\right],$$

$$\langle l(\overrightarrow{x})\rangle = e^{-\beta\Delta F_Q(\overrightarrow{x})}.$$

The effective potential

Polynomial fit:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{b_2\left(T\right)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3+\bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2,$$

$$b_2\left(T\right) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2 + a_3\left(\frac{T_0}{T}\right)^3 .$$

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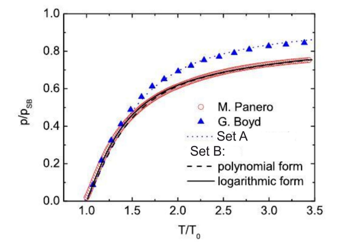
Logarithmic fit:

$$\frac{\mathcal{U}\left(\Phi,\bar{\Phi};T\right)}{T^4} = -\frac{1}{2}a\left(T\right)\bar{\Phi}\Phi + b\left(T\right)\ln\left[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\right],$$

$$a\left(T\right) = \tilde{a}_0 + \tilde{a}_1\left(\frac{T_0}{T}\right) + \tilde{a}_2\left(\frac{T_0}{T}\right)^2, b\left(T\right) = \tilde{b}_3\left(\frac{T_0}{T}\right)^3.$$

The effective potential parametrization



M. Panero, PRL 103, 232001 (2009)
G. Boyd et. al, NPB 469, 419 (1996)

- $\Phi \to 1$, $p/T^4 \to 1.75$, where $T \to \infty$
- $\Rightarrow \tilde{a}_0 = 3.51$ for logarithmic fit $1.75 = a_0/2 + b_3/3 - b_4/4$ for polinomial fit
- $\frac{\partial \mathcal{U}(\Phi,\bar{\Phi},T)}{\partial \Phi}|\mu = 0$ ($\Phi = \bar{\Phi}$ at $\mu = 0$) $\Rightarrow$ the mean square method $\Rightarrow a_i$, b_i

A. V. Friesen et al., IJMP A27, 1250013 (2012)

Parameters

	$\tilde{a}_0$	$\tilde{a}_1$	$\tilde{a}_2$	$\tilde{b}_3$	a_0	a_1	a_2	a_3	b_3	b_4
Set A	3.51	-2.47	15.2	-1.75	6.75	-1.95	2.625	-7.44	0.75	7.5
Set B	3.51	-5.121	20.99	-2.09	6.47	-4.62	7.95	-9.09	1.03	7.32

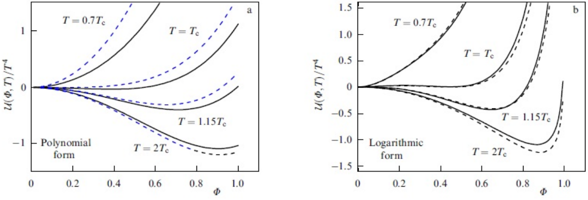


Figure 3: Effective potential as function Φ for different temperatures.

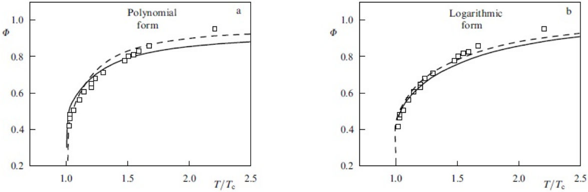


Figure 4: Polyakov loop field Φ for (a) polynomial and (b) logarithmic effective potentials. The solid (dashed) curve corresponds to the sets B (A) parameters. (Lattice data from [Karsch F, Laermann E, Peikert A Phys. Lett. B 478 447 (2000)].)

The mean-field approximation

- The PNJL grand potential ($N_f = 2$):

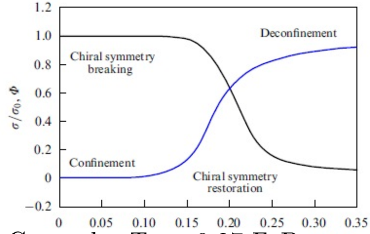
$$\Omega(\Phi, \bar{\Phi}, m, T, \mu) = \mathcal{U}\left(\Phi, \bar{\Phi}; T\right) + G\langle\bar{q}q\rangle^2 -$$
$$-2N_cN_f\int\frac{d^3p}{(2\pi)^3}E_p - 2N_fT\int\frac{d^3p}{(2\pi)^3}\left[\ln N_\Phi^+(E_p) + \ln N_\Phi^-(E_p)\right],$$

where $N_\Phi^\pm(E_p) = \left[1 + 3\left(\Phi + \bar{\Phi}e^{-\beta E_p^\pm}\right)e^{-\beta E_p^\pm} + e^{-3\beta E_p^\pm}\right]$
and $E_p = \sqrt{p^2 + m^2}$ - quark energy; $E_p^\pm = E_p \mp \mu$.

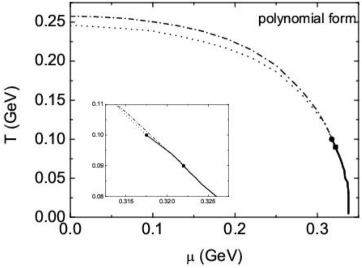
- the equations of motion

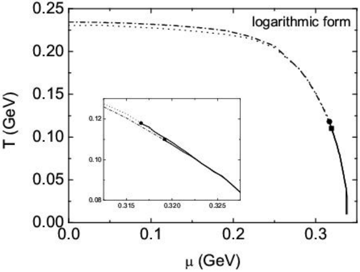
$$\frac{\partial\Omega_{MF}}{\partial\sigma_{MF}} = 0, \quad \frac{\partial\Omega_{MF}}{\partial\Phi} = 0, \frac{\partial\Omega_{MF}}{\partial\bar{\Phi}} = 0.$$

Phase diagram of PNJL model



Parameters: $m_0, \Lambda, G_s,\ a_i,\ b_i,\ T_0 = 0.27$ ГэВ





PNJL with vector interaction

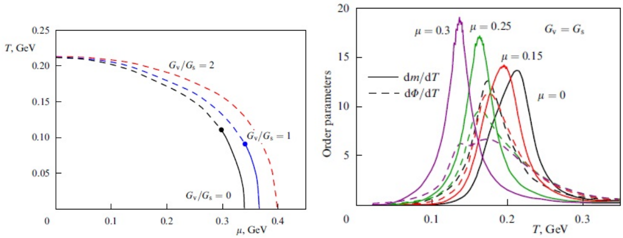
Introduction of vector interaction into model

$$\mathcal{L}_{\mathrm{PNJL}} = \bar{q}\left(i\gamma_\mu D^\mu - \hat{m}_0 - \gamma_0\mu\right)q + G_s\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] - G_v\left(\bar{q}\gamma_\nu q\right)^2 - \mathcal{U}\left(\Phi,\bar{\Phi};T\right)$$

leads to re-normalization of chemical potential:

$$\tilde{\mu} = \mu - 4G_vN_cN_f\int_\Lambda \frac{d^3p}{(2\pi)^3}\frac{m}{E_p}\left[f_\Phi^+ + f_\Phi^-\right].$$

$T_0 = 0.19$ GeV



A. V. Friesen et al. IJMP A 30, 1550089 (2015); Phys. Usp. 59 (4) 367 (2016)

Extended PNJL

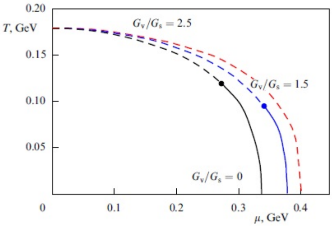
It is possible to introduce a phenomenological dependence of $G_s(\Phi)$ and $G_v(\Phi)$:

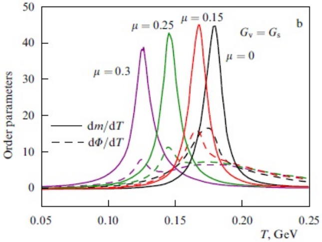
$$\tilde{G}_s(\Phi) = G_s[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)],$$
$$\tilde{G}_v(\Phi) = G_v[1 - \alpha_1 \Phi\bar{\Phi} - \alpha_2(\Phi^3 + \bar{\Phi}^3)],$$

with $\alpha_1 = \alpha_2 = 0.2$.

Y. Sakai et al PRD 82, 076003 (2010)

P. de Forcrand, O. Philipsen NPB 642, 290(2002)



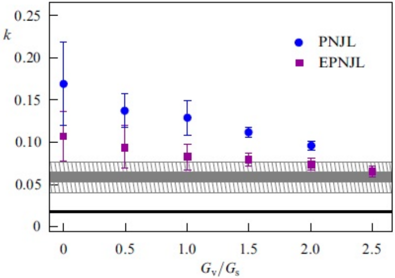


A. V. Friesen et al. IJMP A30 1550089 (2015)

Crossover curvature

It was suggested that critical curves for all physical quantities (chiral condensate, quark susceptibility, strange quark susceptibility, Polyakov loop) must meet at one point, which is the CEP (Kaczmarek O. et al. PRD 83, 014504 (2011)).

$$\frac{T_c(\mu)}{T_c(0)} = 1 - k\left(\frac{\mu}{T_c(\mu)}\right)^2 .$$



Enrödi G. JHEP (4) 1, 2011; Cea P. PRD89, 074512 (2014)

Conclusions and outlook

- NJL-like models let to describe the structure of phase diagram, light mesons and quarks properties, scattering and decay processes;
- describe the confinement properties & describe the chiral symmetry; check how additional interactions (vector interaction and extended couplings) effect on phase diagram;
- how correlations btw quarks and mesons can effect on thermodynamic properties of system in critical region? (taking into account the part of the functional integral responsible for correlations, which usually is ignored in mean field approximation)
  D. Blaschke: Phys.Part.Nucl.Lett. 10 (2013) 660-668; arXiv: 1612.09556; 1511.00338;

(thanks to the grant RSF-17-12-01427)

Thank you for attention