Higgs Cosmology

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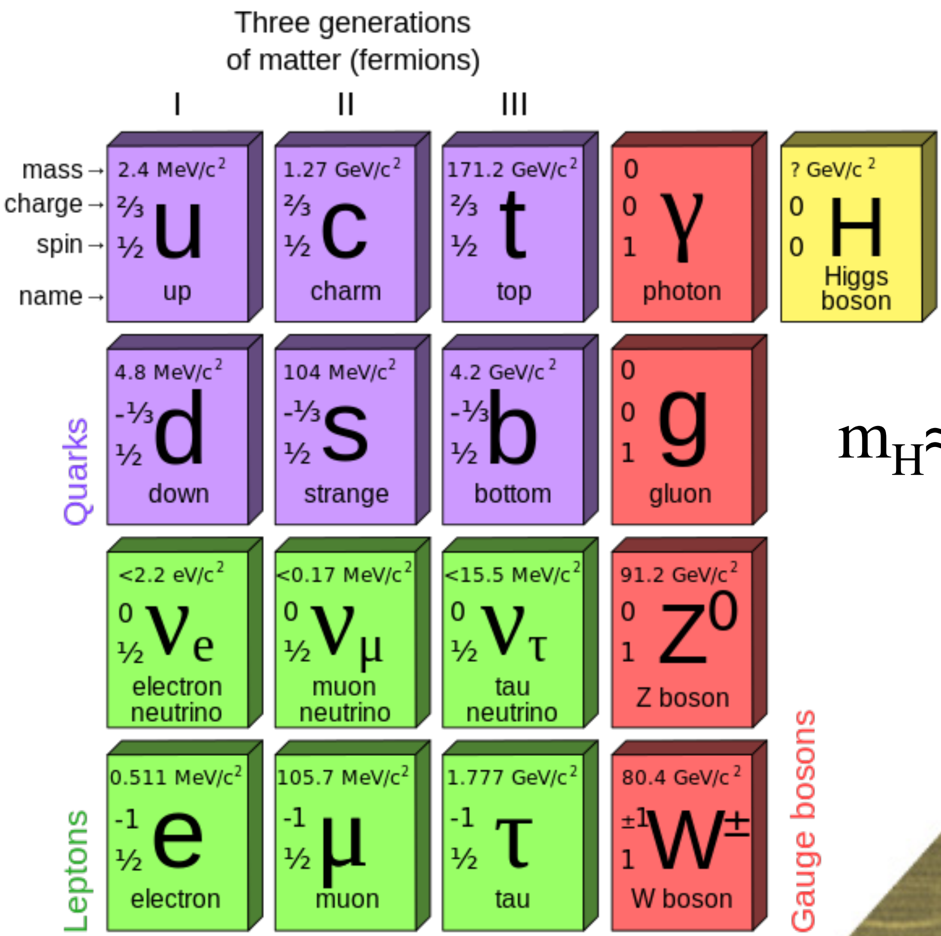
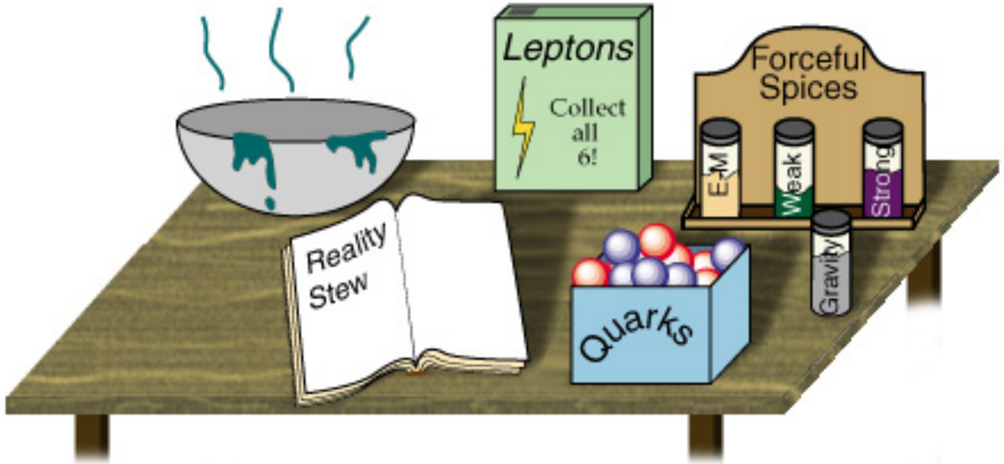
Based on arXiv:1701.02140 with I. Kuntz and I.G. Moss

standard model

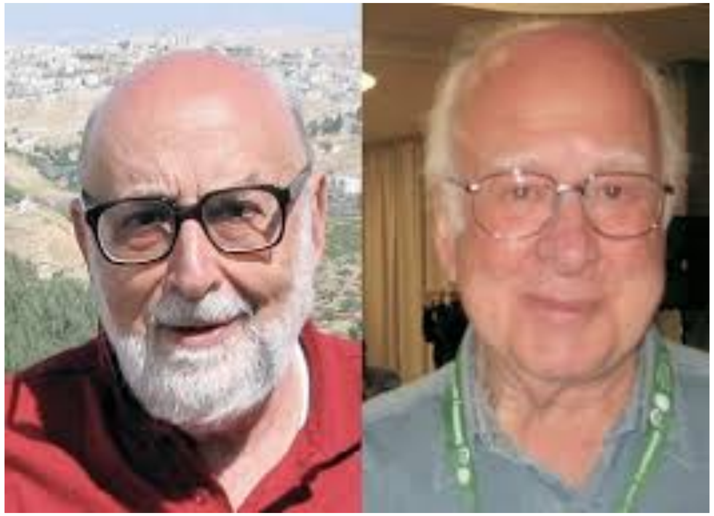


$m_H \approx 125$ GeV



50 years between prediction and discovery

Nobel prize 2013 for François Englert and Peter Higgs



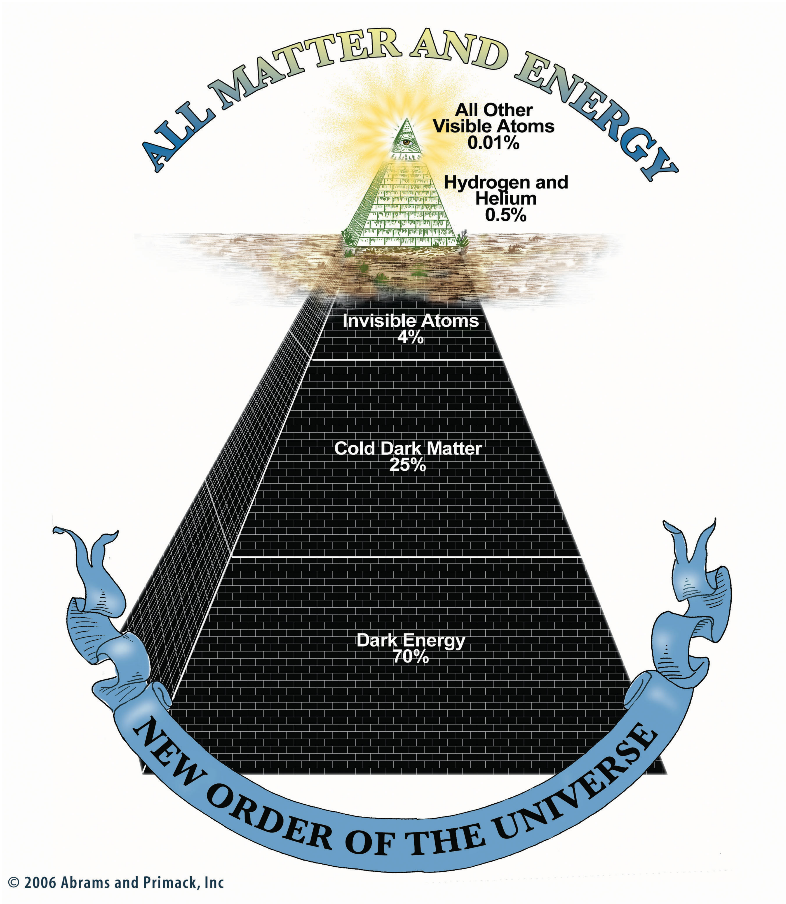
Many people have contributed

Kibble, Guralnik, Hagen, Englert, and Brout

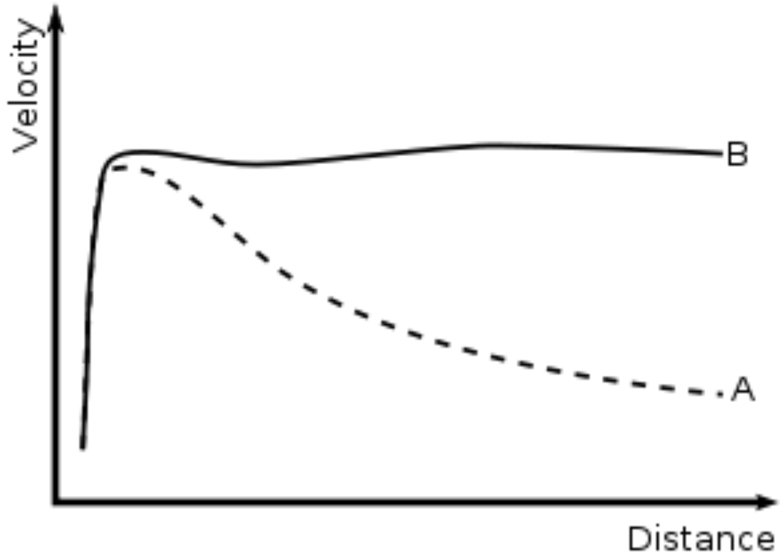
Anderson

So the standard model works (maybe too well), what do we next?

We know that the standard model is not complete.



Dark matter is not explained by the standard model



Galaxy rotation curves are an indication that some matter is dark

Still given the lack of new physics at the LHC we must try to understand as much as possible with the SM. Also new physics may not couple to the SM directly maybe only gravitationally.

Gravitational extension of the SM

- From what we know at this point, all physics beyond the SM could be of gravitational origin:
  - Dark energy could be a not-so-lousy constant
  - Dark matter could be primordial black holes, or some new form of gravitational degrees of freedom not connected to weak scale physics.
- A key problem is that the electroweak potential could be unstable, depending on the top quark mass.
- New physics required by astrophysical/cosmological observations may not affect the Higgs potential.
- If our universe started with a large Higgs field value, how come we ended up in the false electroweak vacuum?
- This is generic problem for inflationary cosmology coupled to the standard model.
- Recent progress in effective field theory applied to quantum gravity enables one to build models of inflation coupled to the SM taking into account quantum gravity effects.

Effective Quantum Gravity

- We have an EFT valid up to a scale $M_\star \sim \sqrt{\frac{120\pi}{N G_N}}$

- The leading order terms are

$$S = \int d^4x \sqrt{-g}\left(\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R - \Lambda_C^4 + c_1 R^2 + c_2 C^2 + c_3 E + c_4 \Box R + \right.$$
$$\left. -L_{SM} - L_{DM} + O(M_\star^{-2})\right)$$ + non local terms (see next slide)

$$E = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$C^2 = E + 2R_{\mu\nu}R^{\mu\nu} - 2/3R^2$$

NB the Wilson coefficients of these operators must be measured in experiments.

Predictions of EQG

- The Wilson coefficients of these operators are universal predictions of quantum gravity:

$$S_{QL} = \int d^4x\sqrt{g}\left(\alpha R \log\left(\frac{\Box}{\mu_\alpha^2}\right) R + \beta R_{\mu\nu} \log\left(\frac{\Box}{\mu_\beta^2}\right) R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta} \log\left(\frac{\Box}{\mu_\gamma^2}\right) R^{\mu\nu\alpha\beta}\right)$$

	α	β	γ
Scalar	$5(6\xi - 1)^2$	-2	2
Fermion	-5	8	7
Vector	-50	176	-26
Graviton	430	-1444	424

All numbers should be divided by $11520\pi^2$.

(Donoghue et al, Codello et al.)

Universal features of quantum gravity

Using EFT techniques, we have identified universal (model independent) features of quantum gravity:

- The scale of quantum gravity is dynamical,

$$\sqrt{\frac{120\pi}{G_N N}}$$

  it depends on the number of fields in the theory.

- Strong interactions kick in at this energy scale.

- Space-time becomes non-local at this energy scale.

Summary of EQG and bounds on its parameters

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S=\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2+\xi H^\dagger H\right)\mathcal{R}-\Lambda_C^4+c_1\mathcal{R}^2+c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{L}_{SM}+\mathcal{O}(M_\star^{-2})\right]$$

- Planck scale $(M^2+\xi v^2)=M_P^2 \qquad M_P=2.4335\times 10^{18}\ \mathrm{GeV}$
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1 = 9.7\times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6\times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

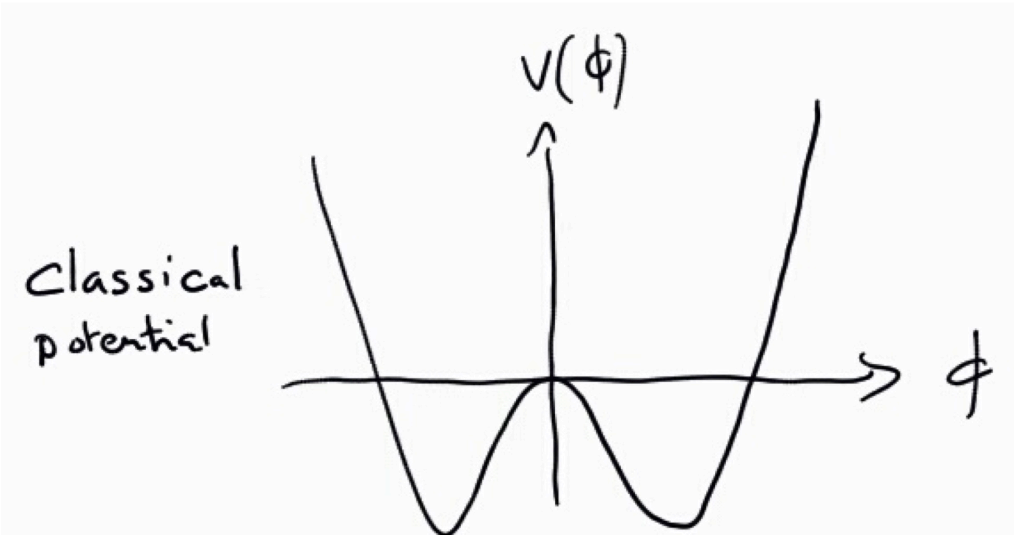
Higgs in cosmology

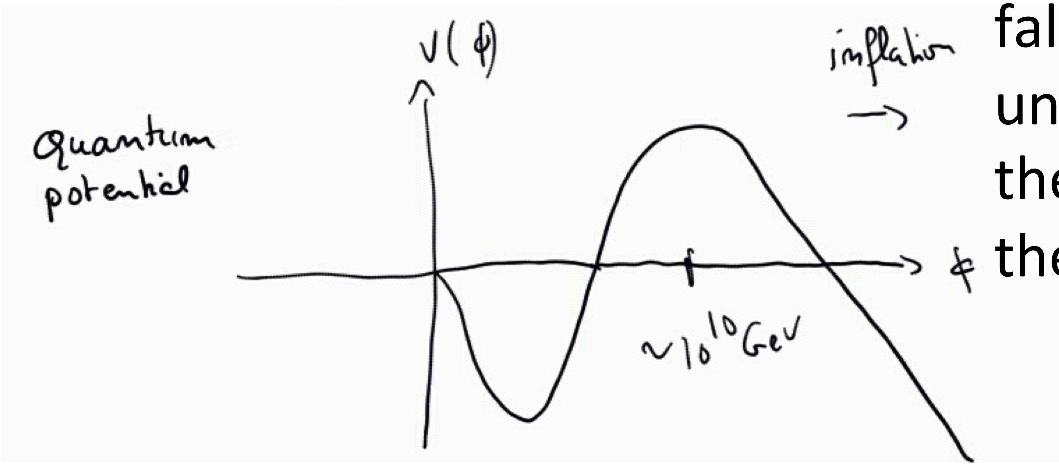
- In an expanding universe with Hubble scale H, the evolution of the Higgs boson h is given by

$$\ddot{h} + 3H\dot{h} + \frac{\partial V(h)}{\partial h} = 0$$

  where V (h) is the potential of the scalar field.
- Even if one imposes as an initial condition at the start of our universe that the Higgs field starts at the origin, it will most likely be excited to higher field values during inflation.
- Indeed, because the mass of the Higgs boson is very small compared to the scale of inflation, it is essentially massless.
- Quantum fluctuations of the Higgs field will drive it away from the minimum of the potential.
- Its quantum fluctuations are of order the Hubble scale H.

Higgs potential instability





For H > Λ, it is likely that
the Higgs will
overshoot the barrier
between the
false vacuum in which our
universe lives and
the lower state true vacuum of
the theory.

Non-minimal coupling of scalars to curvature

- In inflationary cosmology one often deals with actions of the type

$$S_{scalar} = \int d^4x \sqrt{-g} \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\xi\phi^2 R \right)$$

- which lead to

$$(\Box + m^2 - \xi R)\phi = 0,$$

- It is often argued that the term ξ R is a curvature dependent mass term for the scalar field ϕ.

- One needs to be more careful!

- The problem is that the non-minimal coupling induces a mixing between the kinetic term of the scalar field and of the metric field.

Higgs in cosmology

- Starting with the standard model coupled to GR, we have

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{2} M^2 + \xi \mathcal{H}^\dagger \mathcal{H} \right) R - (D^\mu \mathcal{H})^\dagger (D_\mu \mathcal{H}) - \mathcal{L}_{SM} \right]$$

- After electroweak symmetry breaking, the scalar boson gains a non-zero vacuum expectation value, v = 246 GeV, M and ξ are then fixed by the relation

$$(M^2 + \xi v^2) = M_P^2 \,.$$

- Key message: $\mathcal{H}$ is not the Higgs field, as the non-minimal coupling induces a mixing between the kinetic terms of the Higgs and the metric.

Higgs mixing

- Let's transform the action to the Einstein frame

$$S = \int d^4x \sqrt{-\tilde{g}} \left[\frac{1}{2} M_P^2 \tilde{\mathcal{R}} - \frac{3\xi^2}{M_P^2 \Omega^4} \partial^\mu (\mathcal{H}^\dagger \mathcal{H}) \partial_\mu (\mathcal{H}^\dagger \mathcal{H}) - \frac{1}{\Omega^2} (D^\mu \mathcal{H})^\dagger (D_\mu \mathcal{H}) - \frac{\mathcal{L}_{SM}}{\Omega^4} \right]$$

- with

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = (M^2 + 2\xi \mathcal{H}^\dagger \mathcal{H}) / M_P^2$$

- Let's now use the unitary gauge $\mathcal{H} = \frac{1}{\sqrt{2}}(0, \phi + v)^\top$

- The physical Higgs boson is given by

$$\frac{d\chi}{d\phi} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2 v^2}{M_P^2 \Omega^4}}\,.$$

- Expanding $1/\Omega$, we see at leading order the field redefinition simply has the effect of a wave function renormalization

$$\phi = \chi/\sqrt{1+\beta}$$

- where

$$\beta = 6\xi^2 v^2/M_P^2$$

- Thus the canonically normalized scalar field, i.e., the true Higgs boson, does not have any special coupling to gravity.

- It couples like any other field to gravity in accordance with the equivalence principle.

Same effect in the Jordan frame

- After fully expanding the Higgs boson around its vacuum expectation value and also the metric around a fixed background,

$$g_{\mu\nu} = \bar{\gamma}_{\mu\nu} + h_{\mu\nu}$$

- we find

$$\begin{aligned}\mathcal{L}^{(2)} &= -\frac{M^2+\xi v^2}{8}(h^{\mu\nu}\Box h_{\mu\nu} + 2\partial_\nu h^{\mu\nu}\partial_\rho h^{\mu\rho} - 2\partial_\nu h^{\mu\nu}\partial_\mu h^\rho_\rho - h^\mu_\mu \Box h^\nu_\nu \\ &\quad +\frac{1}{2}(\partial_\mu\phi)^2 + \xi v(\Box h^\mu_\mu - \partial_\mu\partial_\nu h^{\mu\nu})\phi\end{aligned}$$

- and we see the same mixing appearing! We need to diagonalize the fields

$$\begin{aligned}\phi &= \chi/\sqrt{1+\beta} \\ h_{\mu\nu} &= \frac{1}{M_P}\tilde{h}_{\mu\nu} - \frac{2\xi v}{M_P^2\sqrt{1+\beta}}\bar{\gamma}_{\mu\nu}\chi.\end{aligned}$$

Physics of the non-minimal coupling

- These results demonstrate that the non-minimal coupling does not introduce stronger gravitational interactions for the Higgs boson once its field has been correctly canonically normalized.
- The underlying reason is that there is no violation of the equivalence principle.
- For very large ξ, the Higgs boson decouples from all particles of the standard model because of the wavefunction renormalization.
- The fact that the Higgs boson behaves like the Higgs boson of the standard model at the LHC leads to the following bound on ξ:

$$\xi < 2.6 \times 10^{15}$$

xc & Atkins, Phys.Rev.Lett. 110 (2013) no.5, 051301

- While the fact that we may be living in a metastable vacuum is problematic for the Higgs boson in an inflationary context, the non-minimal coupling of the Higgs boson to curvature does not create a new problem.

- On the contrary, we shall now show that this non-minimal coupling could solve the stability issue.

- The non-minimal coupling will induce a coupling between the Higgs boson and the inflaton whether we had one in the original theory or not.

- Even if no direct coupling between the Higgs boson is assumed in the Jordan frame, it will be induced in the Einstein frame:

$$V_I(\sigma) \to \frac{V_I(\sigma\Omega)}{\Omega^4} = \frac{V_I(\sigma\Omega)}{\left(1 + \frac{2\xi v\phi(\chi)+\xi\phi(\chi)^2}{M_P^2}\right)^2}$$

Small Higgs field values

- Let us first consider the range $\phi \ll v$
- We find immediately: $$\frac{V_I(\sigma\Omega)}{\Omega^4} \approx V_I(\sigma)\left(1 - 4\xi v\phi/M_p^2\right)$$
- A coupling between the inflaton and the Higgs field is induced by the transformation to the Einstein frame.
- Note that there is a priori no reason to exclude a coupling of the type $V_I H^\dagger H$ in the Jordan frame where the theory is defined.
- There could be cancelations between this coupling and that generated by the map to the Einstein frame.
- The magnitude of the coupling between the Higgs boson and the inflaton appearing in the mapped inflationary potential thus cannot be regarded as a prediction of the model.

Large Higgs field values: early universe

- Let us now consider the range:

$$v \ll \phi \ll M_P|\xi|^{-1/2}$$

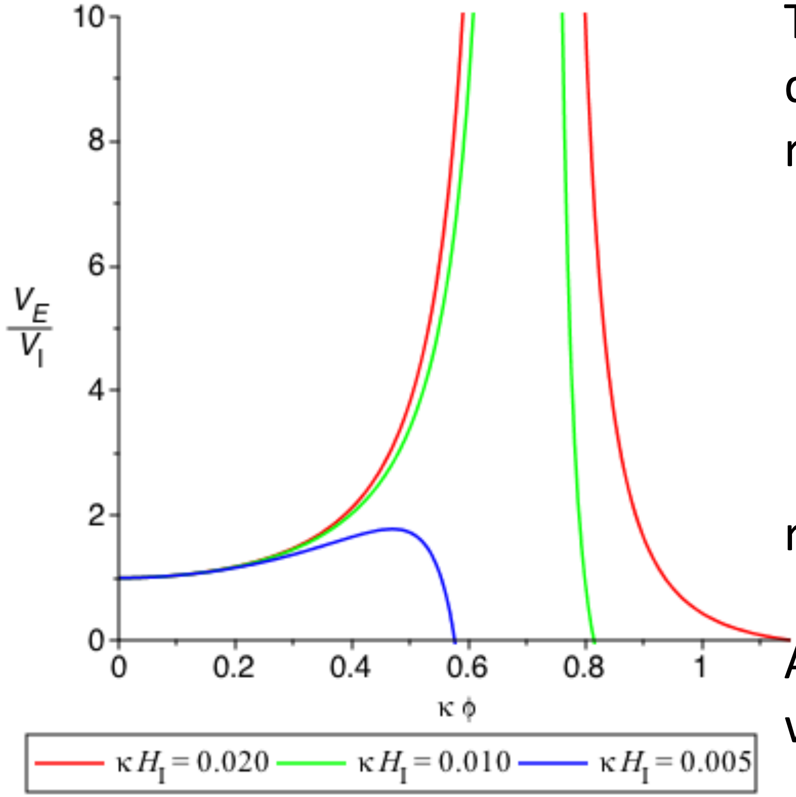
- As explained previously, even if one is willing to fine-tune the initial condition for the value of the Higgs field, it will experience quantum fluctuations of the order of the Hubble scale.

- Unless the Hubble scale is much smaller than the energy scale at which the electroweak vacuum becomes unstable, the Higgs field is likely to swing into the lower true vacuum of the theory.

- Even if we start the universe with a large Higgs field value, the induced coupling to the inflaton can drive the Higgs boson to the false vacuum.

Driving the Higgs to the false vacuum

- To drive the Higgs boson to the false vacuum during inflation, the interaction of the Higgs with the inflaton must act like a mass term for the Higgs during inflation (sign of the non-minimal coupling matters!) and dominate over the inflaton potential.



The Einstein frame Higgs potential $V_E(\phi)$ for different values of the false-vacuum inflation rate H_I for $\xi = -2$.

The potential vanishes at $\phi = \phi_m$, and there is an asymptote at $\phi = \phi_c$.

Consistency of the model (no ghosts) requires $\phi < \phi_c$.

An initial condition $V_E \sim M_P^4$ can be achieved with the initial ϕ close to ϕ_c.

- The early evolution of the Higgs field is described by the equation

$$\ddot{\chi} + 3H\dot{\chi} + \frac{dV_E}{d\chi} = 0$$

- For the inflaton, one has

$$\ddot{\sigma} + \left(\frac{1}{g_\sigma}\frac{dg_\sigma}{d\chi}\right)\dot{\chi}\dot{\sigma} + 3H\dot{\sigma} + \frac{1}{g_\sigma}\frac{dV_E}{d\sigma} = 0.$$

$$g_\phi = \frac{1 + \xi\kappa^2\phi^2 + 6\xi^2\kappa^2\phi^2}{1 + \xi\kappa^2\phi^2}, \qquad g_\sigma = \frac{1}{1 + \xi\kappa^2\phi^2}$$

- where the expansion rate is given by

$$3H^2 = \kappa^2\left(\frac{1}{2}g_\sigma\dot{\sigma}^2 + \frac{1}{2}\dot{\chi}^2 + V_E\right)$$

- For $\chi > M_P$, we have

$$V_E \approx (V_I + V_\phi)e^{\sqrt{8/3}\kappa(\chi-\chi_0)}, \qquad g_\sigma \approx e^{\sqrt{8/3}\kappa(\chi-\chi_0)}.$$

- There is thus rapid evolution of χ and slow evolution of σ (assuming slow-roll conditions on V_I).

A specific example

- For large Higgs field values we have

$$\frac{d\chi}{d\phi} = \sqrt{\frac{1}{\Omega^2} + \frac{6\xi^2\phi^2}{M_P^2\Omega^4}}$$

- To remain within the range of calculability, we request that $\phi \sim H < 0.1M_P$ as for larger values than this we would have to consider a series of higher dimensional operators of the type

$$(H^\dagger H)^n / M_P^{(2n-4)}$$

which would spoil the predictability of any model.

- For $(6\xi^2+\xi) < 100$, one finds $\chi \approx \phi$

$$\frac{V_{inf}(\sigma\Omega)}{\Omega^4} \approx V_{inf}(\sigma\Omega)\left(1 - 2\xi\frac{\chi^2}{M_P^2}\right)$$

- For an inflationary potential of the type $m^2_\sigma \sigma^2 + \lambda_\sigma \sigma^4$ we obtain

$$\frac{V_{inf}(\sigma\Omega)}{\Omega^4} = m_\sigma^2 \sigma^2 \left(1 - \xi \frac{\chi^2}{M_P^2}\right) + \lambda_\sigma \sigma^4$$

- We thus find a coupling between the inflaton and the Higgs given by

$$\lambda = -\frac{m_\sigma^2}{M_P^2}\xi.$$

- Lebedev and Westphal (Phys. Lett. B 719, 415 (2013)) that the Higgs boson is driven to the unstable vacuum during inflation if the coupling λ of the dimension four operator $\sigma^2\chi^2$ is bounded by

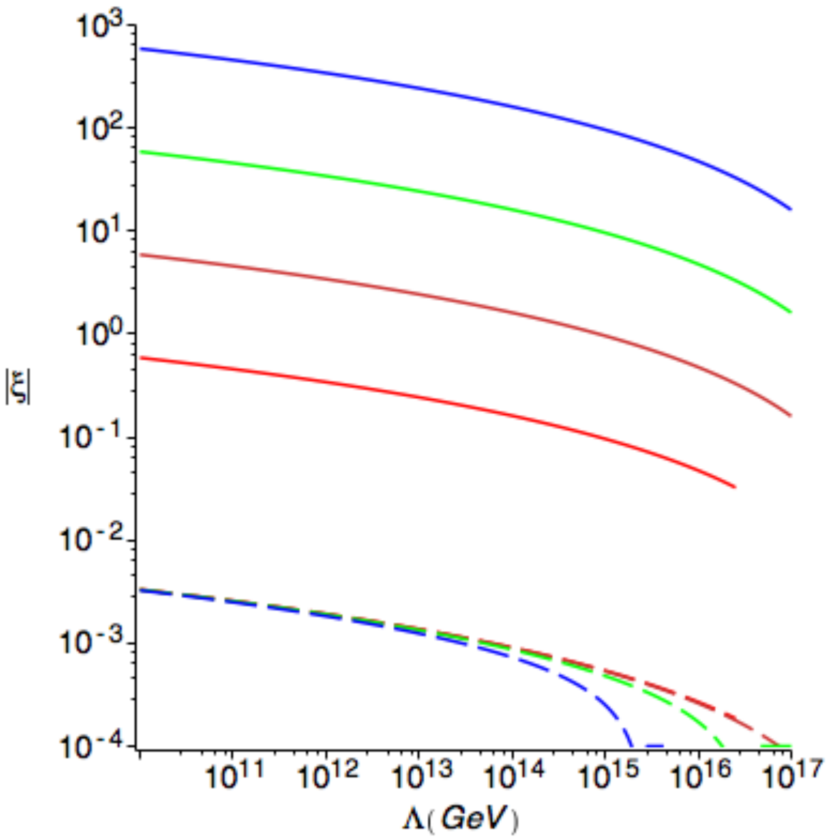
$$\lambda \geq 5 \times 10^{-11}$$

- Here it translates into a bound on the non-minimal coupling

$$\xi \leq -0.5$$

Generic case

For generic values of ξ, we cannot expand the potential as we did previously. However, we can see the same effect from requiring that the coupling of the Higgs to the inflaton dominates over the inflaton potential which fixes the initial conditions for which the Higgs is driven to the false vacuum.



- The lower bound on $-\xi$, for consistent chaotic initial conditions on the Higgs field which will lead the Higgs into the false vacuum.
- The horizontal axis is the Higgs stability scale.
- The different curves from bottom to top are for the false vacuum Hubble parameter 0.1 M_P to $10^{-4}M_P$.
- The dashed lines show the lower bound for quantum stability of the false vacuum.

Conclusion

- The standard model is not compatible with high scale inflation because of the at best metastability of the electroweak vacuum.
- A non-minimal coupling of the Higgs boson to curvature does not generate any new issues in cosmology.
- On the contrary it can generate a coupling between the Higgs boson and the inflaton and drive it to the false vacuum.
- Such investigations are possible thanks to the progress in our understanding of effective field theories applied to quantum gravity which enable one to make serious calculations in early universe cosmology.

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Thanks for your attention!