Alternative mechanism to SUSY
(Conservative extensions of the Poincaré group)

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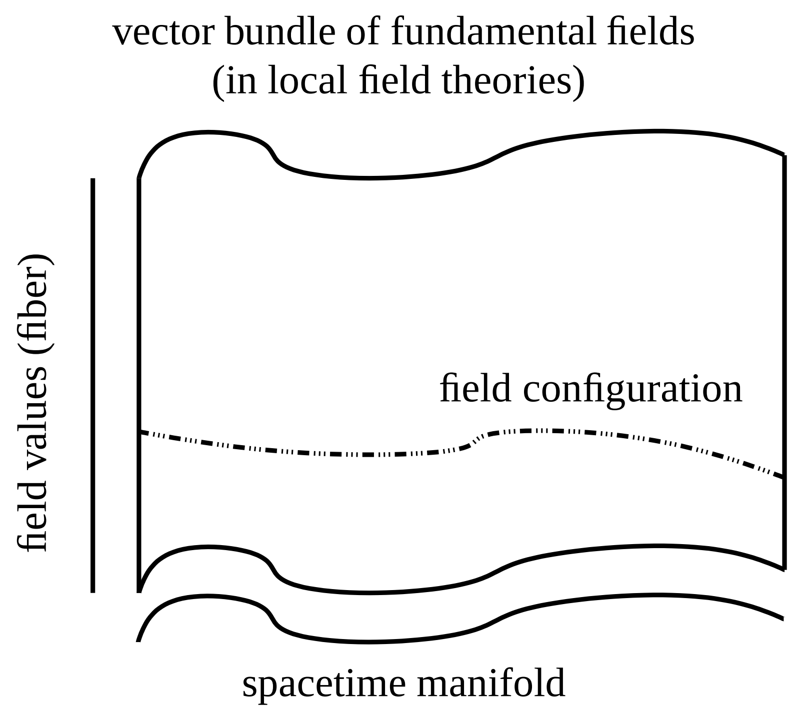
Outline

- Introduction to local field theories and their symmetries
- General structure of Lie groups and the SUSY
- All possible extensions of the spacetime symmetry group
- Non-SUSY gauge and spacetime symmetry unification
- Summary

Introduction to local field theories and their symmetries

- **What is a (classical) local field theory?** We have a vector bundle of fundamental fields over some spacetime manifold:



A local Lagrangian is given as a function of field configurations and their gradients.

The action functional is an intergal of the Lagrangian.

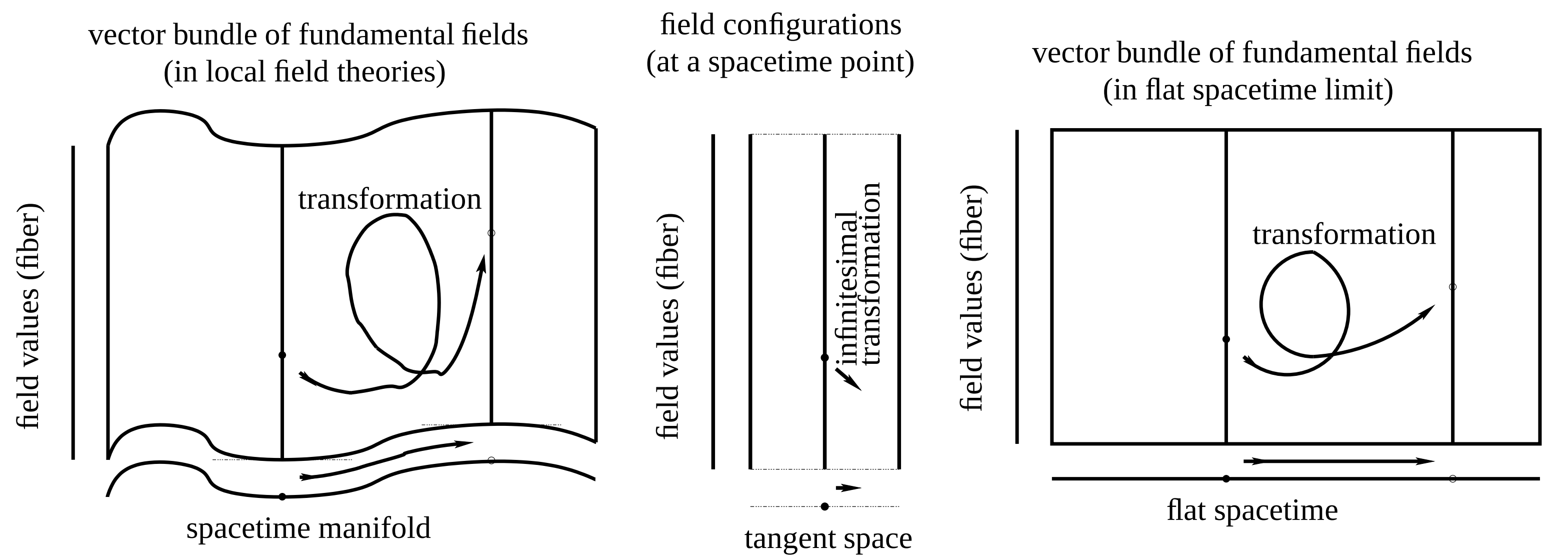
Physical field configurations: where the derivative of the action functional vanishes.

Local symmetries of a theory: deformations to which the action is invariant.

- **More symmetries simplify a theory.** Larger symmetry requirement reduces the number of variants of a field theoretical Lagrangian, and relates its coupling constants.
- **Grand unification (GUT) strategy.** Models with large, direct-indecomposable symmetry group is looked for.
- **Unification no-go theorems.** Spacetime symmetries (Poincaé group) and compact internal symmetries (compact gauge group) cannot be simply unified (McGlinn1964, Coleman-Mandula1967).
- **Supersymmetry (SUSY).** With this, the no-go theorems are circuimventable (Haag-Lopuszanski-Sohnius1975).
  SUSY seemed justified by its convenient properties.
  But a bit unusual, it is “super-Lie algebra”.
- **SUSY is not seen experimentally.** At present status (ICHEP2016 conference).
- **Alternative exists.** We found a group theoretical mechanism to possibly substitute SUSY.

Remark: the no-go theorems are based on global symmetry arguments.
This happens to be enough because:

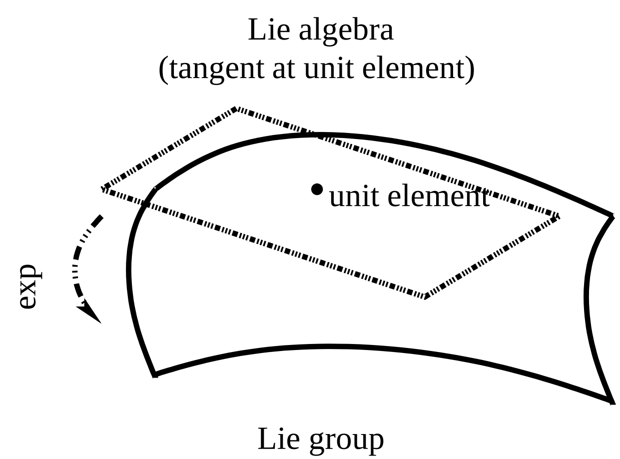
1. the no-go theorems use infinitesimal (symmetry generator level) arguments,
2. the global symmetries infinitesimally act in the same way on the fields at a point of spacetime as local symmetries would do.



For studying symmetries at a point it is enough to study finite dimensional Lie algebras.
Lie algebras are infinitesimal versions of Lie groups, i.e. of parametric groups.

General structure of Lie groups and the SUSY

- **Group.** A collection of transformations, which can be composed, inverted, and there is unit transformation within the collection.
- **Lie group.** A parametric group, parametrized by a finite collection of real parameters. E.g.: rotation group, symmetry group of flat plane, Poincaré group, $\mathrm{SU}(N)$ etc.
- **Lie algebra.** Derivatives (or, equivalently, the tangent) of a Lie group at the unit element.



Thus, a Lie algebra is the infinitesimal version of a Lie group.
Exponential map makes a Lie group element from Lie algebra element.

- **Ado's theorem.** Lie algebra completely characterizes Lie group, modulo global topology.
- **Lie bracket.** Lie algebra has the Lie bracket $[\ ,\ ]$ (commutator).

- **Subgroup.** A sub-group of a group.
- **Normal, or invariant subgroup.** A subgroup N is normal, whenever $g\,N\,g^{-1} \subset N$, for all group elements g.
  E.g. the translations in the symmetry group of flat plane.
- **Extension, or semi-semi-direct product.** Just synonym to above. If N is normal subgroup, and H is a complementing part, then we sometimes write $N.H$ for the entire group.
- **Semi-direct product.** As above, but complementing part H is subgroup. Notation: $N \rtimes H$.
  E.g. the symmetry group of flat plane is semi-direct product of translations and rotations.
- **Direct product.** As above, but H is also normal subgroup. Notation: $N \times H$ or $H \times N$.
  Direct product means that the large group is built of completely independent parts N, H.
  E.g. Standard Model gauge group: $\mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$.
  GUT strategy tries to avoid direct product (requirement of direct-indecomposability).

- **Killing-form**: it is an invariant scalar product $x \cdot y = \mathrm{Tr}\left(\mathrm{ad}_x\, \mathrm{ad}_y\right)$ on any Lie algebra, where $\mathrm{ad}_x(\,) = [x,\,]$. (E.g. it appears in the $F_{ab} \cdot F^{ab}$ Yang-Mills Lagrangian.)

- **Levi decomposition theorem**:

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}\text{, or } \textit{solvable part})}} \rtimes \underbrace{L}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}\text{, or } \textit{semisimple part})}}$$

holds for any Lie group, where $\rtimes$ denotes semi-direct product.
The symmetries of flat plane (translations $\rtimes$ rotations) is a typical example.
Groups like $\mathrm{SU}(N)$, $\mathrm{SL}(2, \mathbb{C})$ only have Levi factor, i.e. they are semisimple.

- **Poincaré group**:

$$\underbrace{\mathcal{P}}_{\text{Poincaré group}} = \underbrace{\mathcal{T}}_{\text{translation group (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

is an educative demonstration of Levi's decomposition theorem.

- **super-Poincaré group (SUSY group)**:

$$\underbrace{\mathcal{P}_s}_{\text{SUSY group}} = \underbrace{\mathcal{S}}_{\text{supertranslation group (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

is a similar example, with a bit larger radical.

Supertranslations: it is a transformation group on the vector bundle of superfields. They act as

$$\begin{pmatrix} \theta^A \\ x^a \end{pmatrix} \mapsto \begin{pmatrix} \theta^A + \epsilon^A \\ x^a + d^a + \sigma^a_{AA'} \mathrm{i}\big(\theta^A \bar{\epsilon}^{A'} - \epsilon^A \bar{\theta}^{A'}\big) \end{pmatrix}$$

on the “supercoordinates” and the affine spacetime coordinates.

One can write:

$$\underbrace{\mathcal{P}_s}_{\text{SUSY group}} = \underbrace{\underbrace{\Big(\underbrace{\mathcal{T}}_{\text{translations}} \cdot \underbrace{\mathcal{Q}}_{\text{supercharges}} \Big)}_{=\mathcal{S}\text{ (group of supertranslations)}} \rtimes \underbrace{\mathcal{L}}_{\text{Lorentz group}}}_{\text{super-Poinceré group (SUSY group)}}$$

where arrows indicate which subgroup acts nontrivially on which normal subgroup.

super-Lie algebra presentation: traditionally they are not presented as Lie algebras, but as "super-Lie algebras".

$$
\begin{array}{lll}
\checkmark & [\, P_a \, , P_b \,] = 0, \\
\checkmark & [\, P_a \, , Q_A \,] = 0, \\
\checkmark & [\, P_a \, , \bar{Q}_{A'} \,] = 0, \\
!!! \rightarrow & \{\, Q_A \, , Q_B \,\} = 0, \\
!!! \rightarrow & \{\, \bar{Q}_{A'} \, , \bar{Q}_{B'} \,\} = 0, \\
!!! \rightarrow & \{\, Q_A \, , \bar{Q}_{A'} \,\} = 2\, \sigma^a_{\;AA'}\, P_a .
\end{array}
$$

What does that mean? It has also ordinary Lie algebra presentation!

[Nucl.Phys.**B76**(1974)477, Phys.Lett.**B51**(1974)239]:
Introduce alternative generators $\delta_{(i)} = \epsilon^A_{(i)}\, Q_A$ instead of Q_A, where $\epsilon^A_{(i)}$ ($i = 1, 2$) is "supercoordinate" (Grassmann valued two-spinor) basis. Will make ordinary Lie algebra.

Via exponentiating this Lie algebra: SUSY Lie group obtained. SUSY is not that exotic! Ordinary Lie group / Lie algebra theory applies!

$\Rightarrow$
With super-Lie algebra, one can exactly generate those ordinary Lie algebras (Lie groups) which have the structure

$$\underbrace{\left(\mathcal{T} \;\cdot\; \mathcal{Q}\right)}_{=\mathcal{S}} \;\rtimes\; \mathcal{L}$$

with

- $\mathcal{L}$ being a subgroup,
- $\mathcal{S}$ being a complementing normal subgroup,
- $\mathcal{T}$ being a normal subgroup within $\mathcal{S}$, and
- $\mathcal{T}$ as well as $\mathcal{Q} \cong \mathcal{S}/\mathcal{T}$ being abelian. $\leftarrow$!!!
  (translations are abelian,
  supercharges without considering contribution of translations are abelian.)

$\uparrow$
This makes it possible to switch the sign of “odd” part ($\mathcal{Q}$) with Grassmannian basis.

All possible extensions of the spacetime symmetry group

Now we understand that SUSY is just a Lie group / Lie algebra extension of Poincaré group. (Infinitesimal symmetries of any relativistic local field theory must be a Poincaré extension.)

Poincaré extensions are classified by O'Raifeartaigh theorem (1965), via Levi decomposition:

- **Either:**

$$\begin{array}{ccccc} E & = & R & \rtimes & L \\ \bigcup & & \bigcup & & \bigcup \\ \mathcal{P} & = & \mathcal{T} & \rtimes & \mathcal{L} \end{array}$$

(A) $E = \mathcal{P} \times \{\text{some further Lie group}\}$
(*trivial extension, case of no-go theorems*)

(B) Not (A), $\mathcal{L} \subset L$, R is solvable extension of $\mathcal{T}$.
(*SUSY, and our new example is such*)

- **Or:**

$$\begin{array}{ccccc} E & = & R & \rtimes & L \\ \bigcup & & & & \bigcup \\ \mathcal{P} & = & & & (\mathcal{T} \rtimes \mathcal{L}) \end{array}$$

(C) L contains entire $\mathcal{P}$, and L is simple Lie group.
(*conform, and* E_8, $\mathrm{SO}(1,13)$ *theories*)

Consequently: if non-trivial extension is looked for, and we do not want to achieve this via symmetry breaking of a giant symmetry group, then we need to extend the radical (case B).

How (extended) SUSY works?

- **Unification via *extended SUSY group***:

$$\underbrace{\underbrace{\Big(\big(\underbrace{\mathcal{T}}_{\text{translations}} \times \underbrace{\mathcal{Z}}_{\text{central charges}}\big) \cdot \underbrace{\mathcal{Q}}_{\text{supercharges}}\Big)}_{\text{group of supertranslations}} \rtimes \Big(\underbrace{\mathcal{G}}_{\text{compact internal symmetries}} \times \underbrace{\mathcal{L}}_{\text{Lorentz group}}\Big)}_{\text{group of global extended supersymmetries}}$$

Arrows: indicate nontrivial subgroup action.

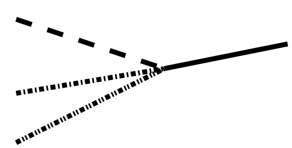
Parts not connected by arrows: are independent.

- **The extended SUSY group is direct-indecomposable.**

$\Rightarrow$ Connects spacetime symmetries with compact internal (gauge) symmetries.

$\Rightarrow$ Connects potentially independent compact internal symmetries with each-other.

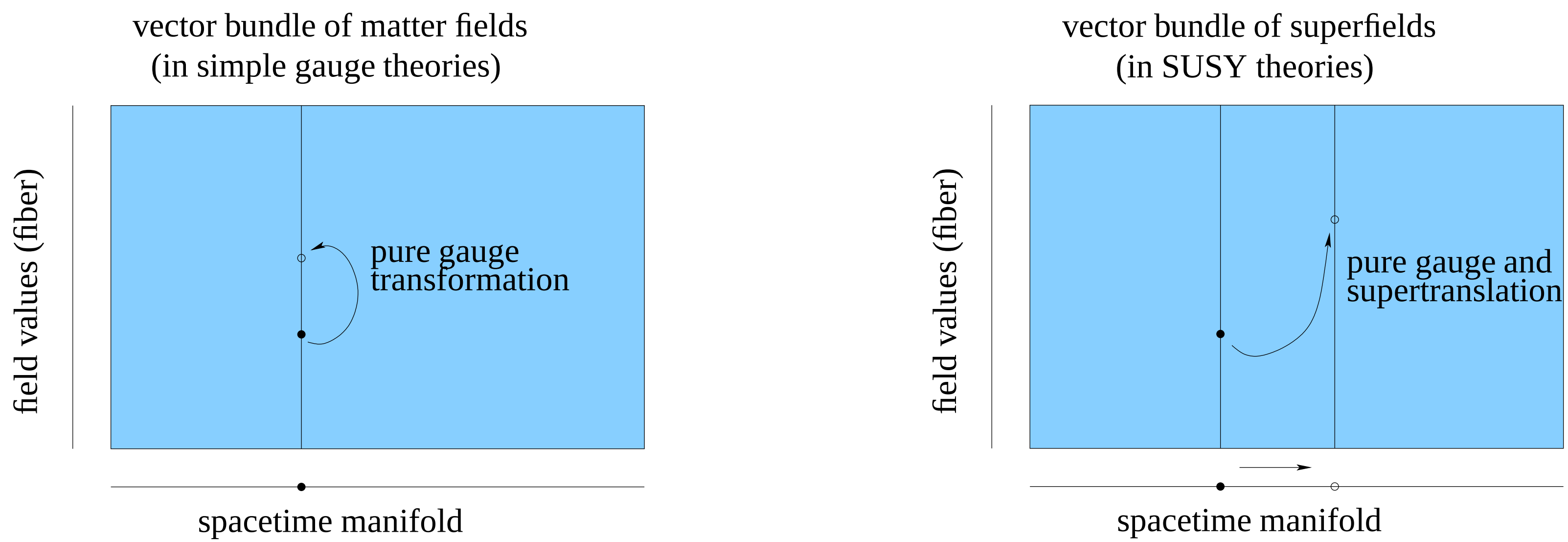
$\Rightarrow$ Running of coupling factors do unify.



Running of gauge couplings

- **Operated by O'Raifeartaigh theorem case B.** Via the extension of the radical.

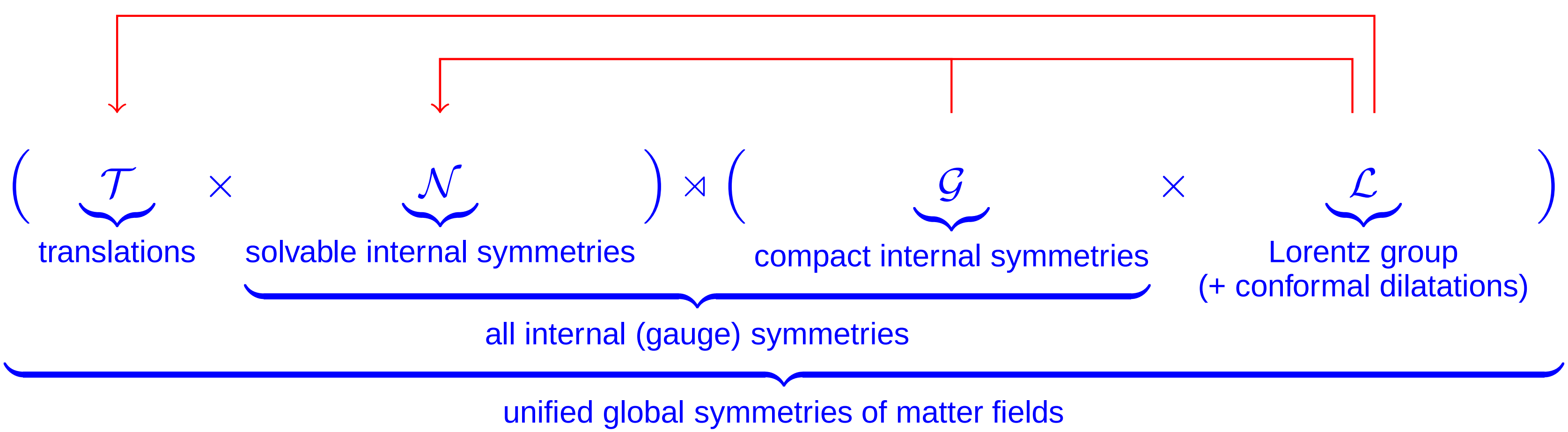
- **Symmetry breaking needed.** Because in the (extended) SUSY the complementing symmetries to spacetime symmetries couple too strongly to spacetime symmetries.



Experimental evidence not seen for this, so symmetry breaking needed for an SM-like limit.

Non-SUSY gauge and spacetime symmetry unification

- ***Conservative* extentsions of the Poincaré group.**
  $\Leftrightarrow$ The complementing symmetries are all *inner*, i.e. do not act on spacetime.
  $\Leftrightarrow$ There exists $\mathcal{P} \xrightarrow{i} E \xrightarrow{o} \mathcal{P}$ homomorphisms, such that $o \circ i =$ identity.
  $\Leftrightarrow$ Extension and its invariant restriction both exists.
  $\Leftrightarrow$ The extended symmetries do not need symmetry breaking for an SM-like limit.
- **The (extended) SUSY group is a non-conservative extension of the Poincaré group.**
- **All possible conservative extensions of the Poincaré group:**



Arrows: indicate nontrivial subgroup action.
O'Raifeartaigh theorem + energy non-negativity $\Rightarrow$ these are the only possible ones.
Similar gauge - spacetime symmetry unification as extended SUSY, via extended radical.

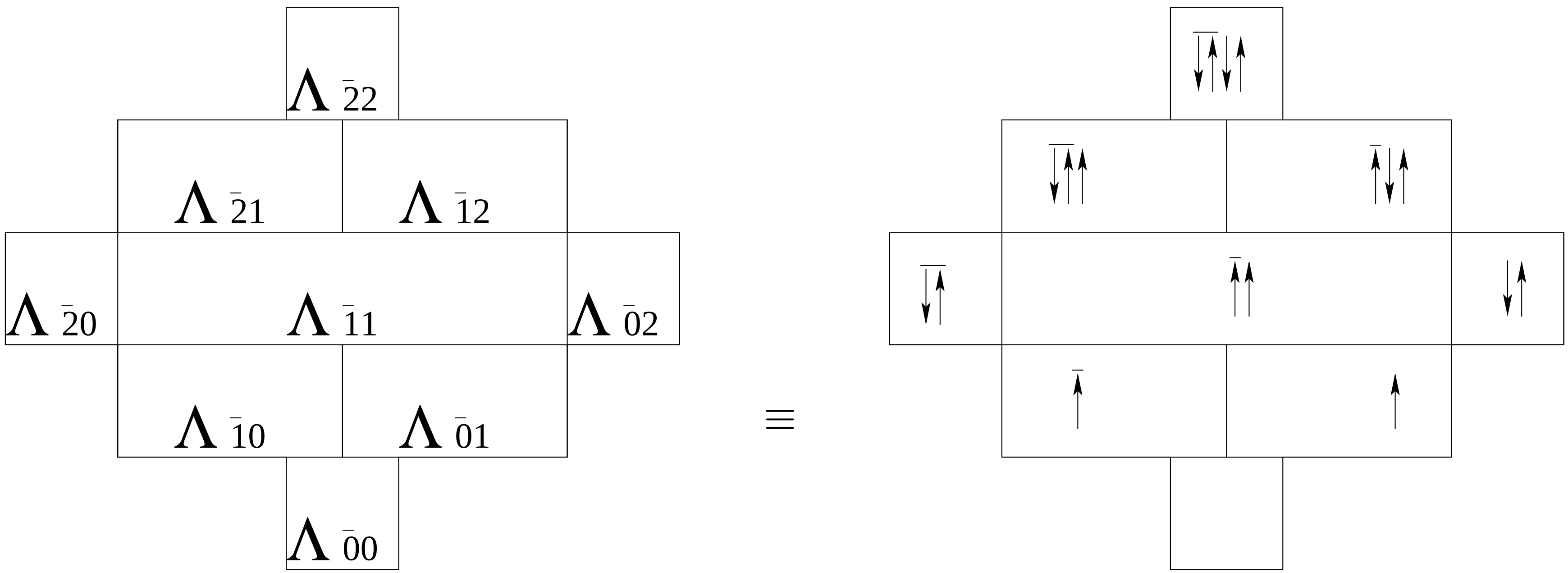
- **A conservative Poincaré group extension can be made local.** The discussed group structure can be equivalently reordered as:

$$\underbrace{\underbrace{\mathcal{T}}_{\text{translations}} \rtimes \Big(\underbrace{\underbrace{\underbrace{\mathcal{N}}_{\text{solvable internal symmetries}} \rtimes \big(\underbrace{\mathcal{G}}_{\text{compact internal symmetries}}}_{\text{all internal (gauge) symmetries}} \times \underbrace{\mathcal{L}}_{\substack{\text{Lorentz group} \\ \text{(+ conformal dilatations)}}} \big)}_{\text{unified local symmetries of matter fields at points of spacetime}} \Big)}_{\text{unified global symmetries of matter fields}}$$

- **The local, i.e. spacetime pointwise acting part is the key ingredient:**

$$\underbrace{\underbrace{\underbrace{\mathcal{N}}_{\text{solvable internal symmetries}} \rtimes \big(\underbrace{\mathcal{G}}_{\text{compact internal symmetries}}}_{\text{all internal (gauge) symmetries}} \times \underbrace{\mathcal{L}}_{\substack{\text{Lorentz group} \\ \text{(+ conformal dilatations)}}} \big)}_{\text{unified local symmetries of matter fields at points of spacetime}}$$

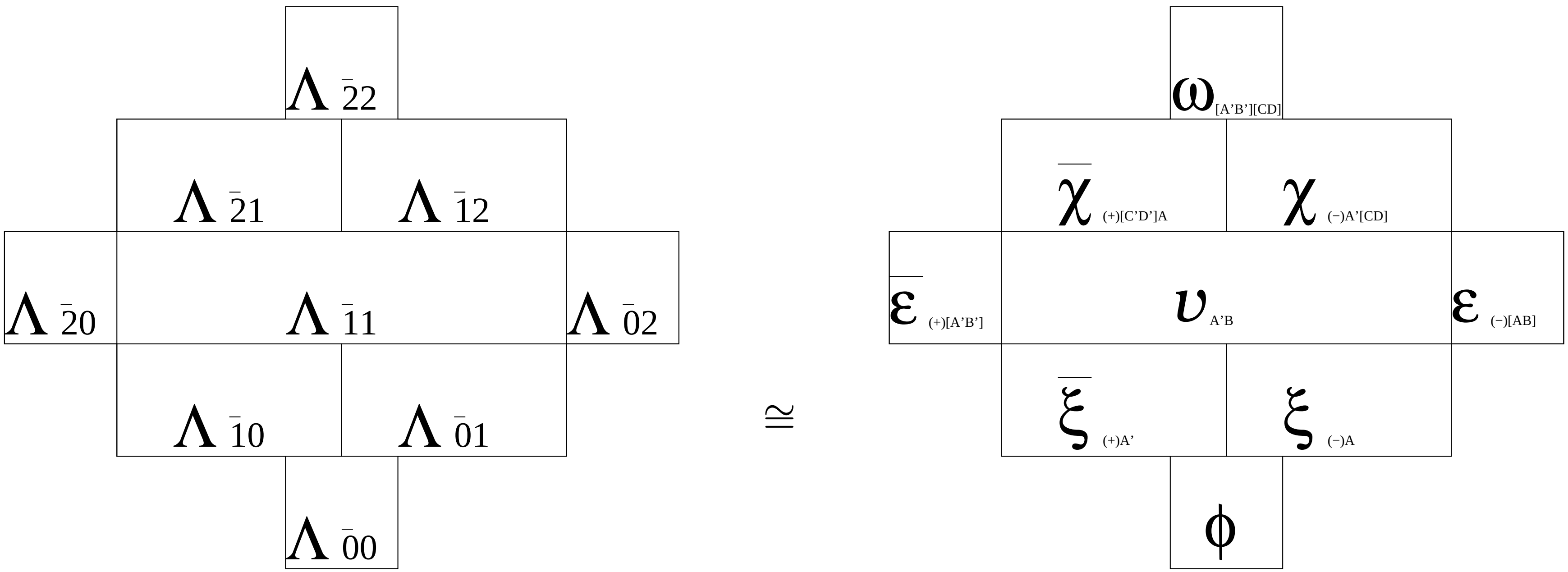
Constructed example for $\mathcal{G} = \mathrm{U}(1)$ in JPhys**A50**(2017)115401 and IJMP**A32**(2016)1645041.
It is a symmetry group of a QFT-related algebra:



$\sim$ algebra of creation operators of fermion and antifermion in the limit if we only had spin.
(Encoding 2 fundamental degrees of freedom, Pauli principle, and charge conjugation.)

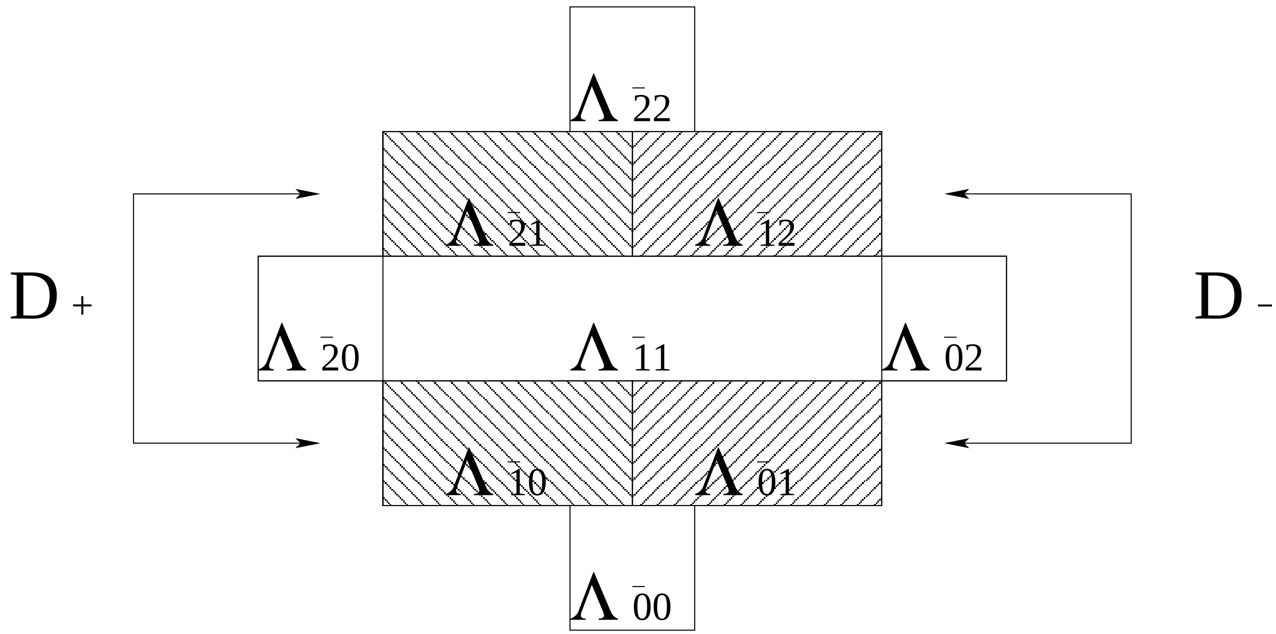
We named it spin algebra.

Spin algebra can also be represented via two-spinor formalism:



$\sim\ \Lambda(\bar{S}^*) \otimes \Lambda(S^*)$, where S^* is lower index two-spinor space (it is a 9-tuple of spinor fields).

Within spin algebra, the Dirac bispinors are contained:



Possible to define Dirac gamma map $\gamma: \; T \to \; \mathrm{Lin}(A)$, such that it obeys Clifford relations

$$\gamma_a \gamma_b + \gamma_b \gamma_a \quad = \quad 2\, I\, g_{ab}$$

when restricted to D_+ and D_-. (But that does not hold on entire spin algebra.)

However, the spin tensor

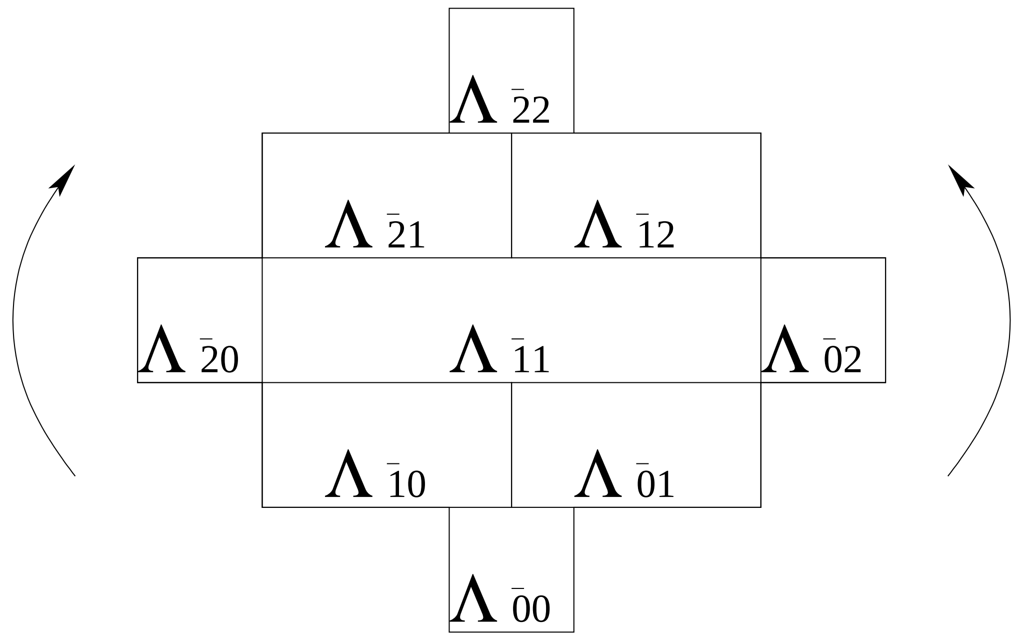
$$\Sigma_{ab} \quad := \quad \frac{\mathrm{i}}{2}\big(\gamma_a \gamma_b - \gamma_b \gamma_a\big)$$

implements the Lorentz group action over the entire spin algebra hierarchy, correctly.

The exotic symmetries ($\mathcal{N}$) act as “dressing” transformations.

$$\underbrace{e_i}_{\text{generators}} \overset{\mathcal{N}}{\longmapsto} \underbrace{e_i + \text{higher polynomials}}_{\text{new generators}}$$

$(i=1,2)$



In QFT analogy: 1-particle spaces not conserved. Are “dressed” by higher particle content.

These symmetries are invisible when truncated to 1-particle theory.

Price to pay for such unification of gauge and spacetime symmetries:

- Full gauge group is $\mathcal{N} \rtimes \mathcal{G}$, i.e. “zero-energy” non-propagating gauge field modes ($\mathcal{N}$).
- In QFT analogy: CM is bypassed by relaxing preservation of 1-particle space.

Mechanism singled out group theoretically by “conservativeness” of the Poincaré extension.

Summary

- **SUSY experimentally not visible at present.** See e.g.: ICHEP2016 conference.
- **Mathematical alternatives to SUSY exist.** These are also O'Raifeartaigh B type, as SUSY.
- **The alternative: "conservative" extensions of the spacetime symmetries.** The complementing symmetries to spacetime symmetries are all inner. Symmetry breaking not needed.
- **Concrente example constructed.** At present, merely with $\mathrm{U}(1)$ as compact gauge group.
- **It connects gauge and spacetime symmetries.** Just like extended SUSY.
- **Harmonizes with present experimental situation.** Extra symmetries are inherently "hidden".

Backup slides

- **General structure of Lie groups / Lie algebras** (modulo topology):

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}\text{, or } \textit{solvable part})}} \rtimes \underbrace{L}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}\text{, or } \textit{semisimple part})}}$$

Some info about properties of radical:

- **solvability of radical R** $\Leftrightarrow$ for the Lie algebra r of R, the sequence $r^0 := r, \quad r^1 := [r^0, r^0], \quad r^2 := [r^1, r^1], \quad \ldots, \quad r^k := [r^{k-1}, r^{k-1}] = \{0\}$ for finite k.
- **special case: R is nilpotent.** There is finite k such that $\mathrm{ad}_{x_1} \ldots \mathrm{ad}_{x_k} = 0$ for all $x_1, \ldots, x_k \in r$.
- **special case: R is abelian.** For all $x \in r$ one has $\mathrm{ad}_x = 0$.

- **General structure of Lie groups / Lie algebras** (modulo topology):

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}\text{, or }\textit{solvable part})}} \rtimes \underbrace{L_1 \times \cdots \times L_n}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}\text{, or }\textit{semisimple part})}}$$

Some info about the Levi factor:

- The Levi factor (semisimple part) is direct product of simple parts: L_i ($i = 1, \ldots, n$) are themselves semisimple and have no normal subgroups within.

- Simple Lie groups / Lie algebras a completely classified (complete list available): $\mathrm{SU}(N)$, $\mathrm{SL}(2, \mathbb{C})$, etc.

- General structure of Lie groups / Lie algebras (modulo topology):

$$\underbrace{E}_{\text{Lie group}} = \underbrace{R}_{\substack{\text{degenerate directions of Killing form} \\ (\textit{radical}\text{, or } \textit{solvable part})}} \rtimes \underbrace{L_1 \times \cdots \times L_n}_{\substack{\text{non-degenerate directions of Killing form} \\ (\textit{Levi factor}\text{, or } \textit{semisimple part})}}$$

where L_i ($i = 1, \ldots, n$) are *simple* (no normal subgroup within).

- Structure of Poincaré group:

$$\underbrace{\mathcal{P}}_{\text{Poincaré group}} = \underbrace{\mathcal{T}}_{\text{translations (radical)}} \rtimes \underbrace{\mathcal{L}}_{\text{homogeneous Lorentz group (Levi factor)}}$$

- General structure of compact Lie groups / compact Lie algebras:

$$\underbrace{G}_{\text{compact Lie group}} = \underbrace{\mathrm{U}(1) \times \cdots \times \mathrm{U}(1)}_{\text{compact abelian part (radical)}} \times \underbrace{G_1 \times \cdots \times G_m}_{\text{compact non-abelian part (Levi factor)}}$$

where G_j ($j = 1, \ldots, m$) are *compact simple* (no normal subgroup within).