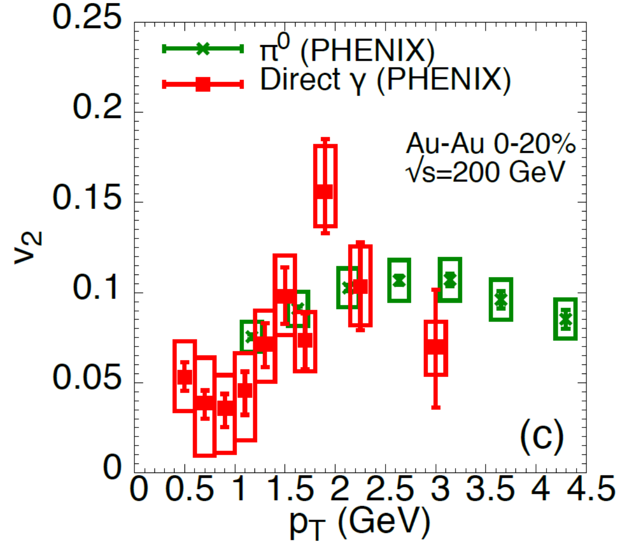
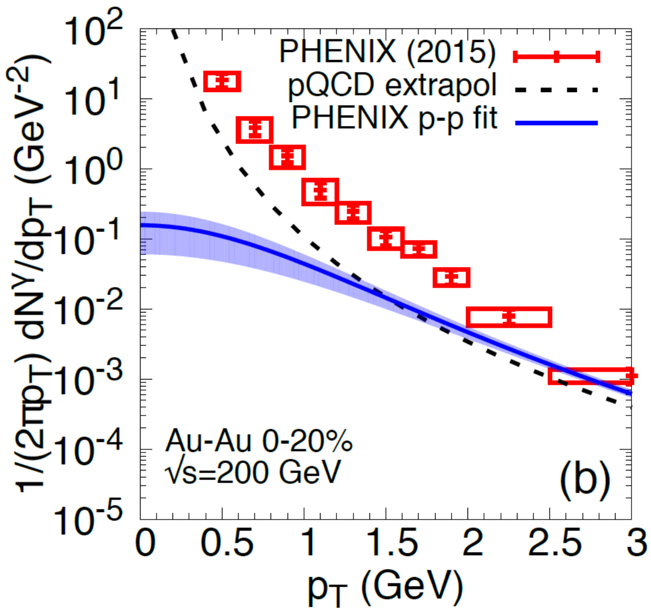
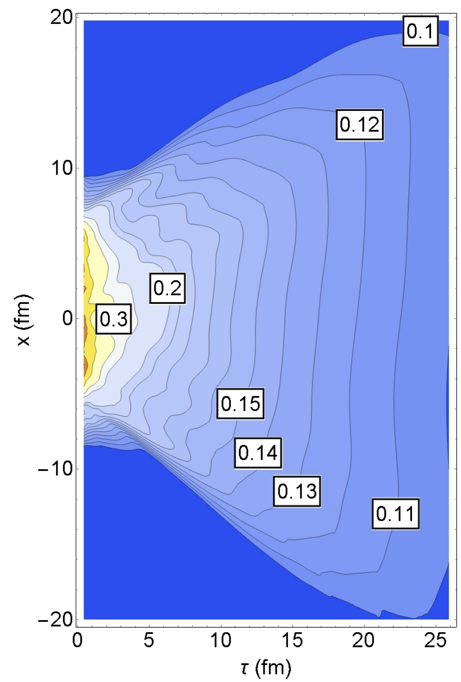
The direct photon puzzle



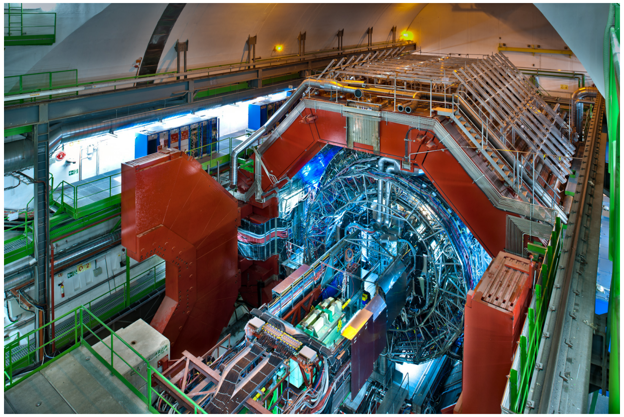




Jean-François Paquet

Stony Brook University

January 16, 2017



ALICE Journal Club

What is the "direct photon puzzle"?

<<< Let me try a simple definition >>>

Background information:

1) "Direct photons"=Photons not produced by a hadronic decays ~~$\pi^0 \to \gamma\gamma$~~ ~~$\eta^0 \to \gamma\gamma$~~ …

2) Thermal photons (radiated by the quark-gluon plasma) are thought to dominate direct photon measurements

Puzzle

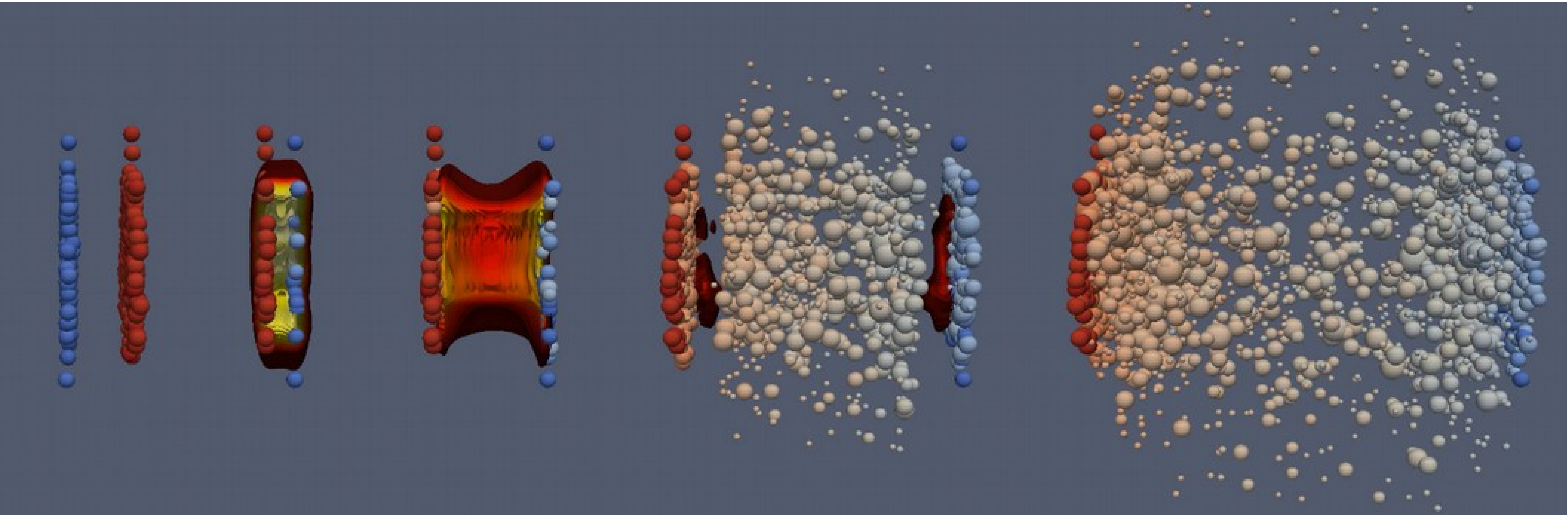
Calculations struggle to explain all direct photon measurements (spectra and v_n) at the same time

Overview

- Why direct photons (and why heavy ion collisions)?
- Sources of photons, with emphasis on prompt photons
- Understanding thermal photons
- Where do we go from here?

Why direct photons (and why heavy ion collisions)?

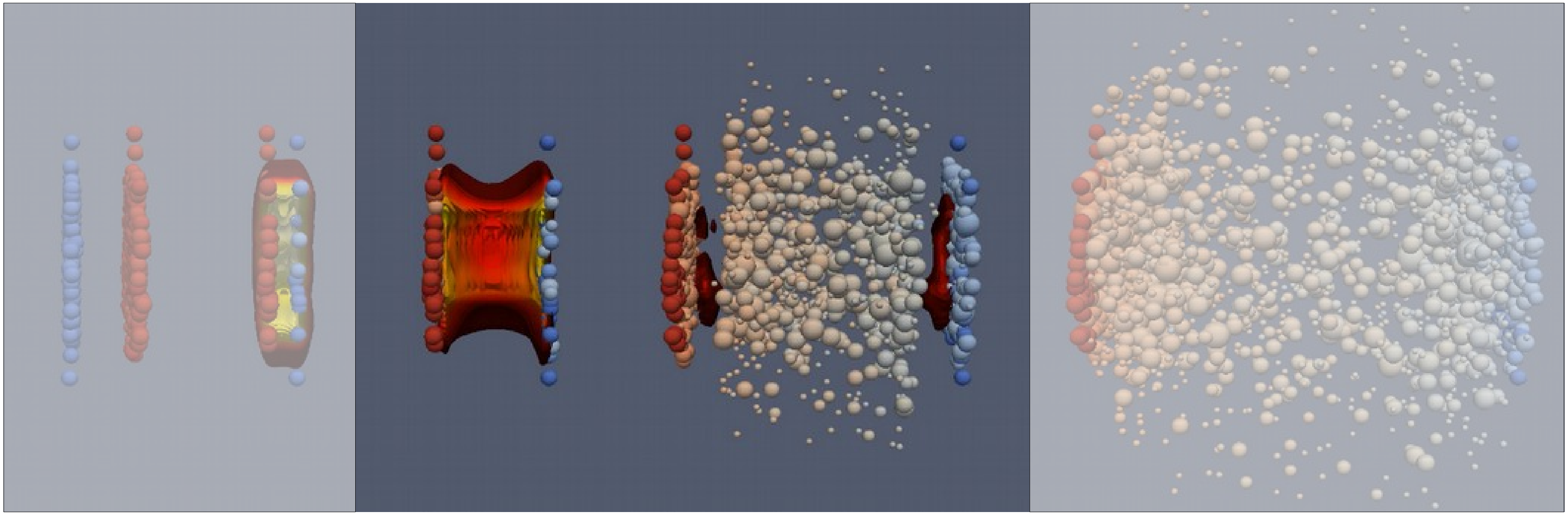
Why study heavy ion collisions?



Time

(Picture credit: J. Bernhard, Duke)

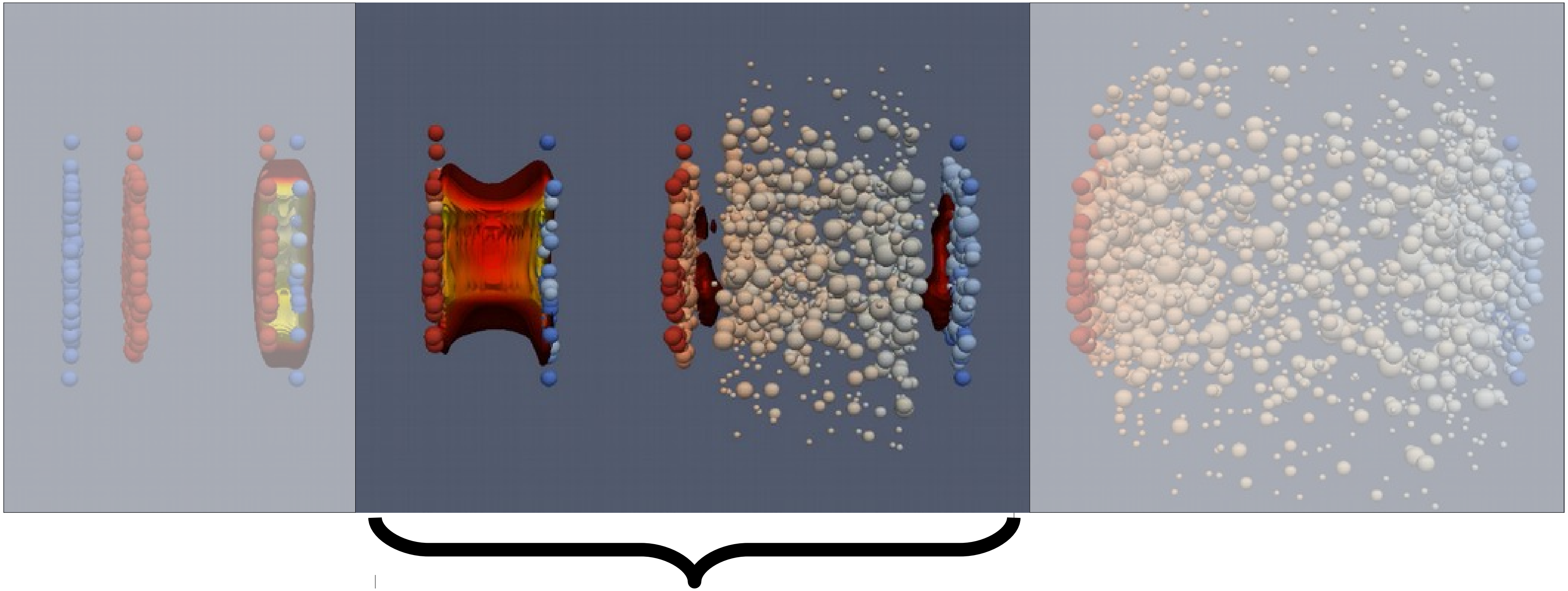
The "quark-gluon plasma"



The "quark-gluon plasma" (QGP)

Study many-body properties of deconfined nuclear matter (quantum chromodynamics):
e.g. <u>shear and bulk viscosities</u>

How to study the plasma?

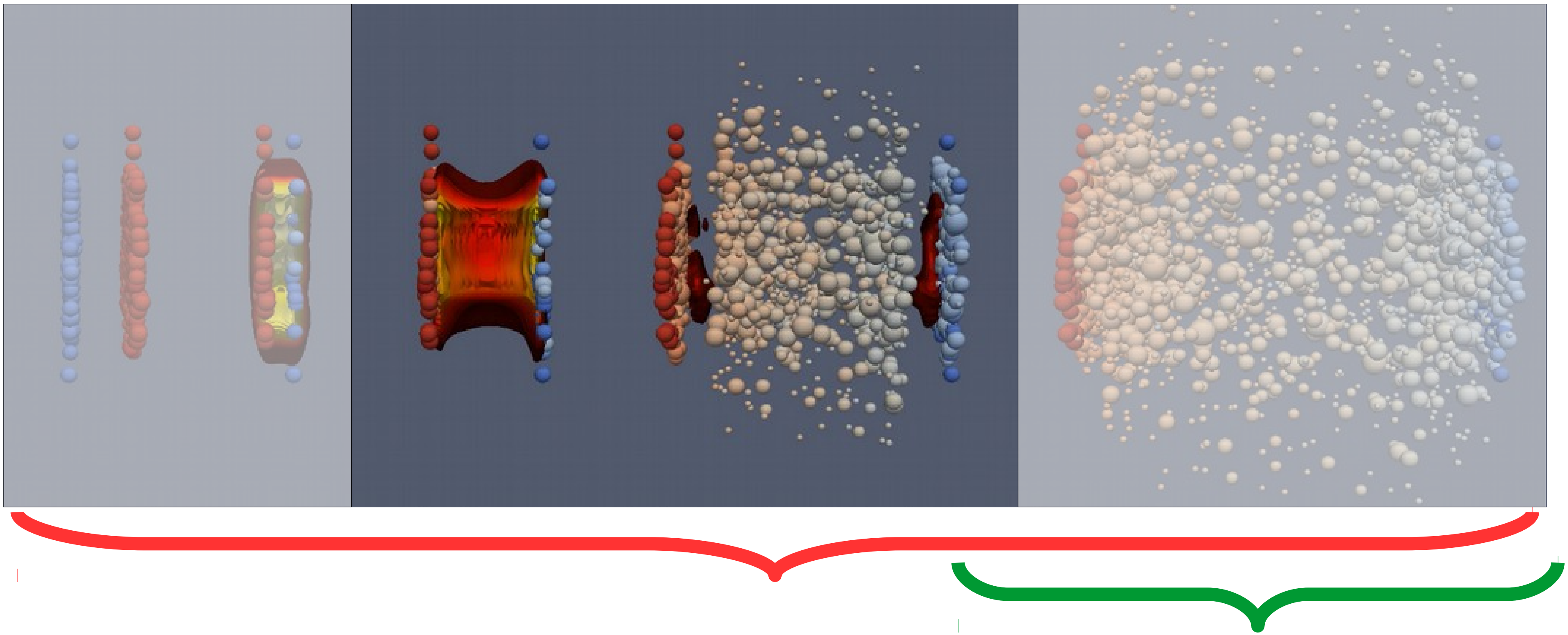


The "quark-gluon plasma" (QGP)

How do we study the properties of the QGP?

Photons/dileptons, soft hadron observables (p_T spectra, v_n), jet energy loss, heavy quarks, ...

How to study the plasma?



<u>How do we study the properties of the QGP?</u>

Photons/dileptons, soft hadron observables (p_T spectra, v_n),

jet energy loss, heavy quarks, ...

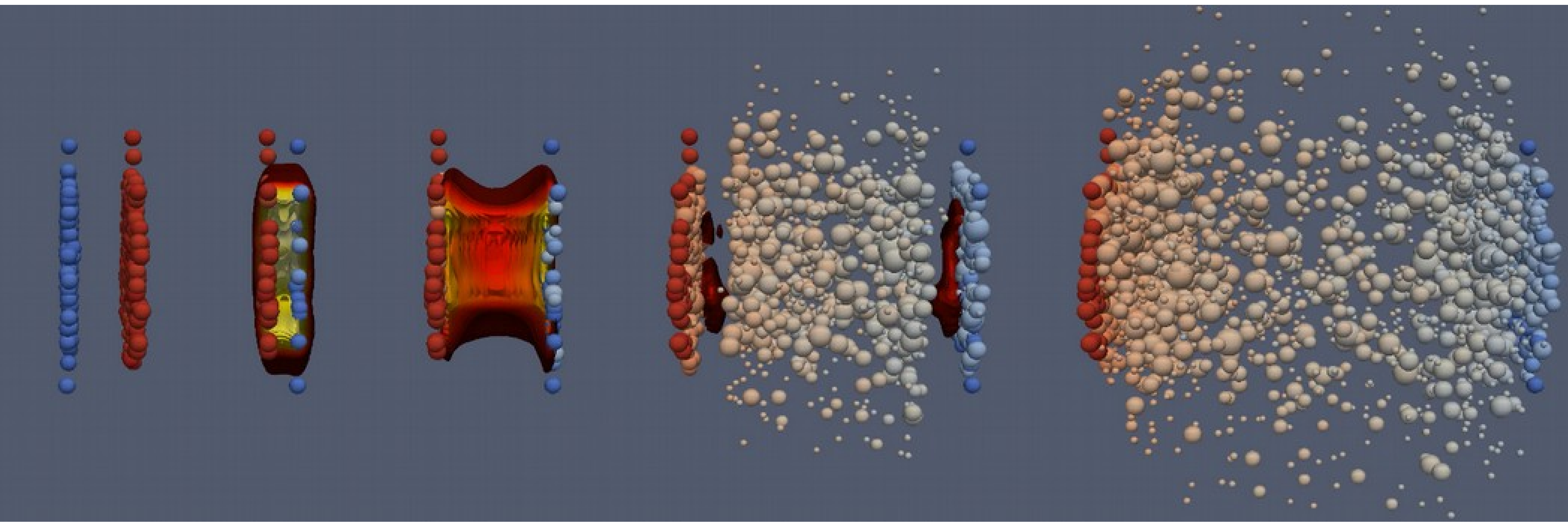
Sources of photons in heavy ion collisions

Sources of photons

Prompt photons

"Thermal" photons

Decay photons (e.g. $\pi^0 \rightarrow \gamma\gamma$)



Pre-equilibrium emission

Late stage emission (e.g. $\pi\rho \rightarrow \pi\gamma$)

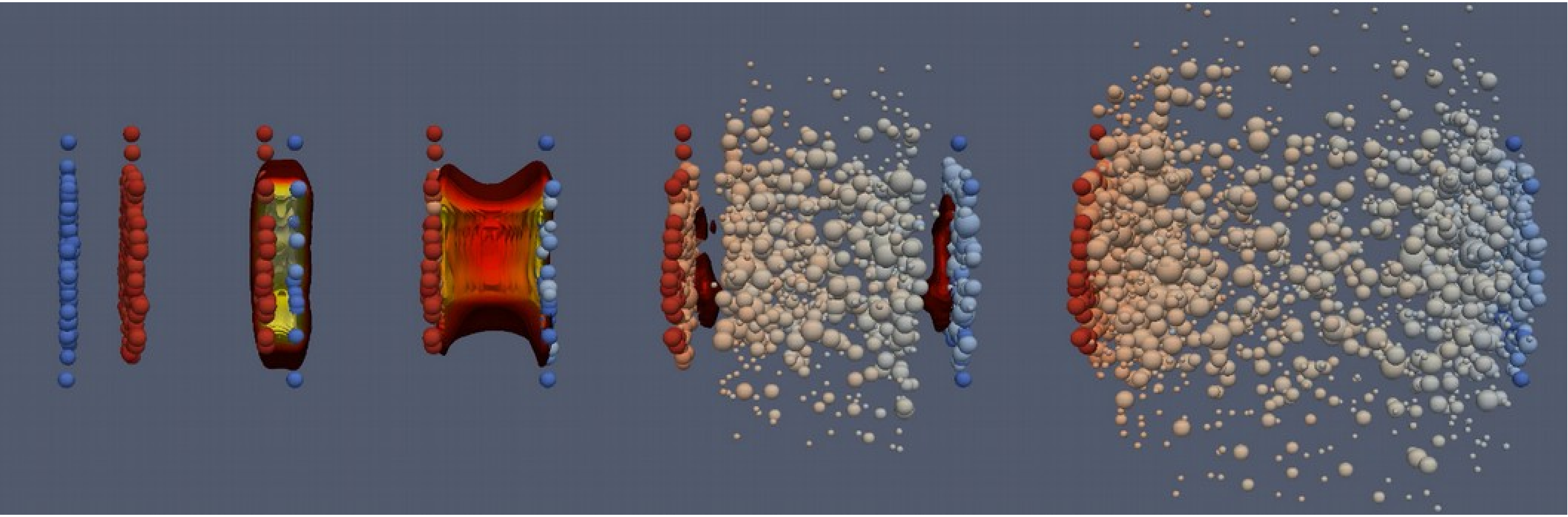
Also: photons from jet-plasma interactions, B-field, ...

Direct photons

Prompt photons

"Thermal" photons

~~Decay photons~~
(e.g. ~~$\pi^0 \rightarrow \gamma\gamma$~~)



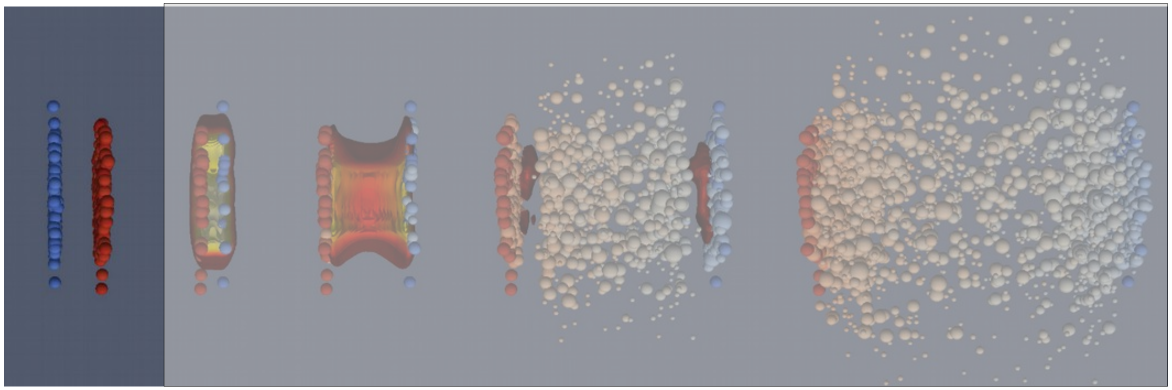
Pre-equilibrium emission

Late stage emission
(e.g. $\pi\rho \rightarrow \pi\gamma$)

Also: photons from jet-plasma interactions, B-field, ...

Prompt photons: in p+p collisions

Think about proton-proton collisions



A parton (quark/gluon) from each proton interact and produce a photon (or produce a parton that then fragments into a photon)

$$E_h \frac{d^3\sigma}{dp_h^3} = \sum_{a,b,c} \int dx_a dx_b \frac{dz_c}{z_c} f_{a/A}(x_a, Q_{\rm fac}) f_{b/B}(x_b, Q_{\rm fac}) \left[E_c \frac{d^3\hat{\sigma}}{dk_c^3} (Q_{\rm ren}) \right] D_{h/c}(z_c, Q_{\rm frag})$$

Parton distribution function

Parton-parton cross-section

Photon fragmentation function

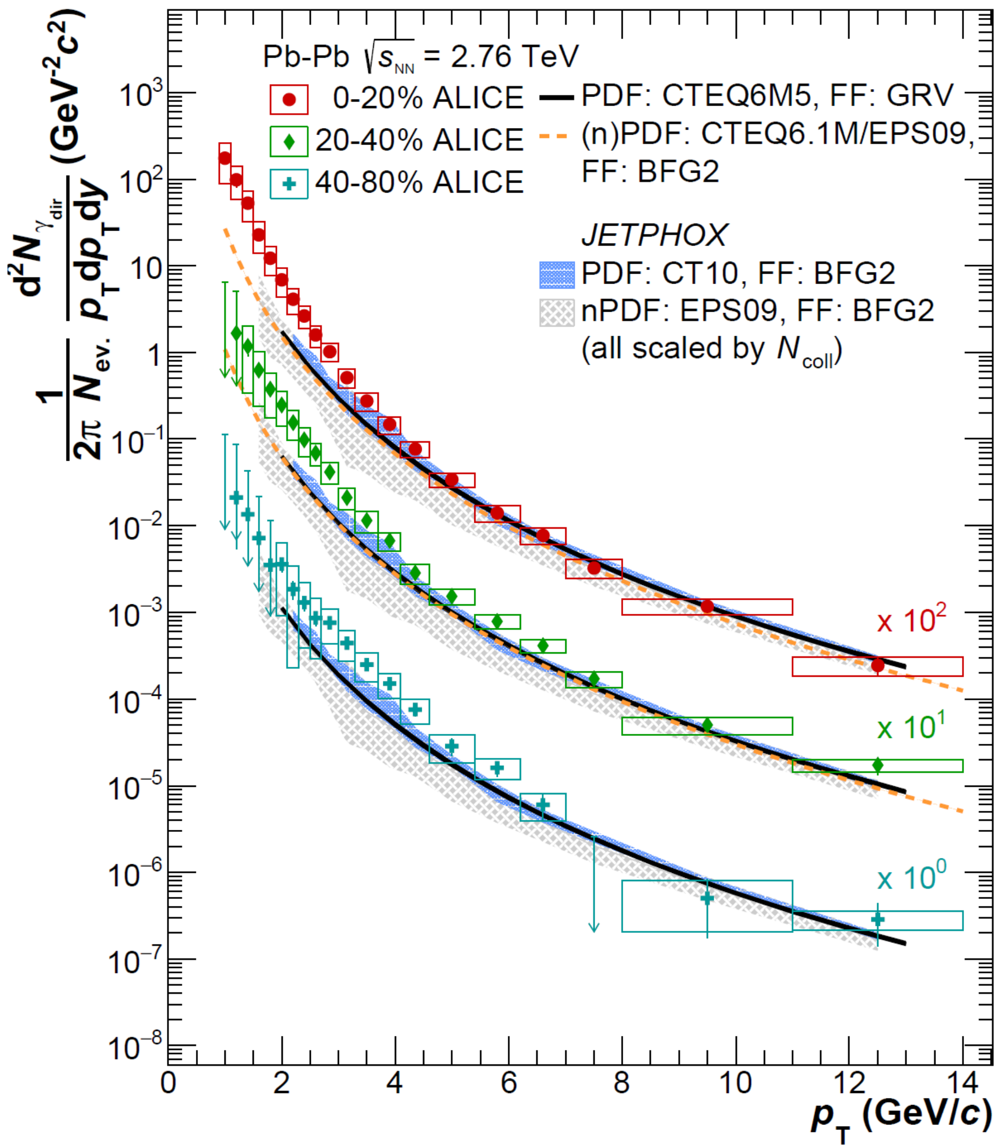
Prompt photons: A+A, high p_T

At high p_T: binary scaling of prompt photons

$$E\frac{d^3N_{AA}}{d\mathbf{p}} \approx \frac{N_{bin}}{\sigma_{pp}^{inel}} \boxed{E\frac{d^3\sigma_{pp}}{d\mathbf{p}}}$$

Can include small corrections from nuclear parton distribution function

But there's more: parton energy loss



Prompt photon channels

Two distinct contributions: "isolated" & fragmentation

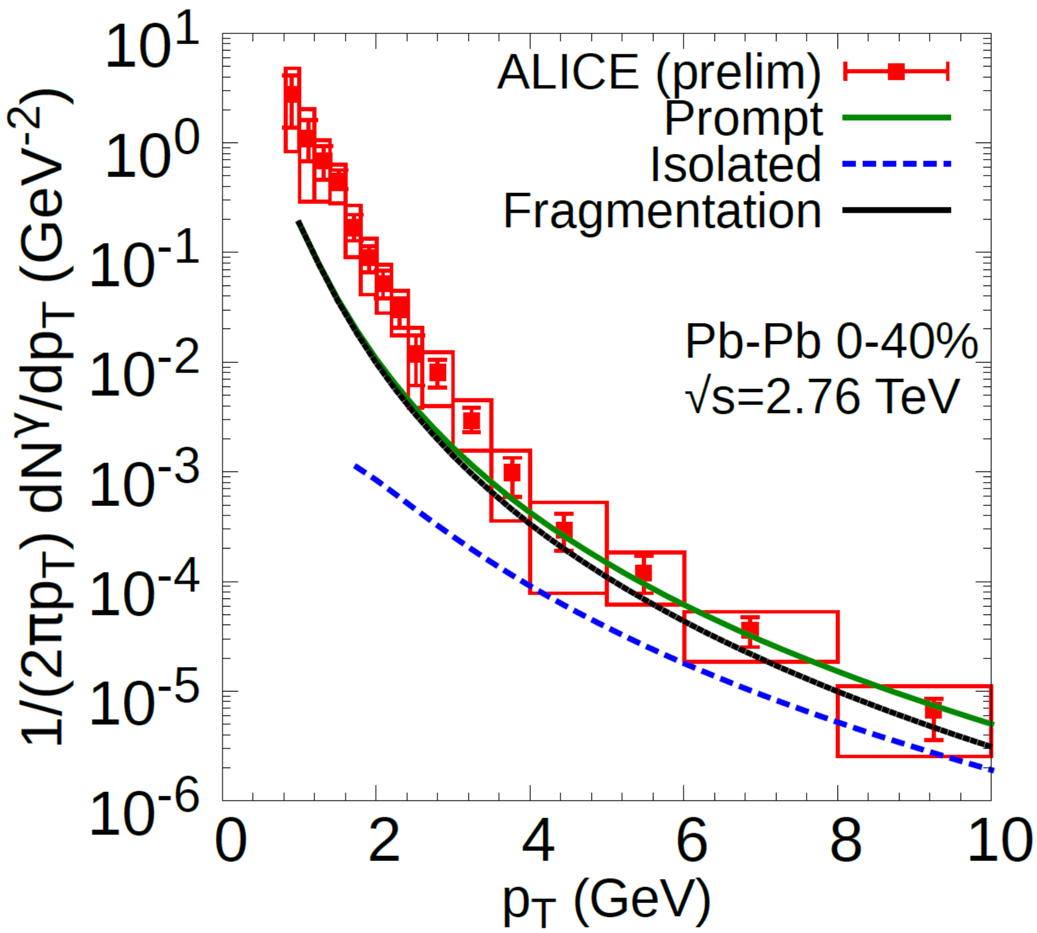
By "isolated", I mean e.g. Compton scattering

$$\text{gluon} + \text{quark} \rightarrow \text{gluon} + \gamma$$

By fragmentation, I mean e.g.

$$\text{gluon} + \text{quark} \rightarrow \text{gluon} + \text{quark}$$

and γ is produced during hadronisation



Prompt photon & energy loss

Two distinct contributions: "isolated" & fragmentation

Not affected by energy loss

By "isolated", I mean e.g. Compton scattering

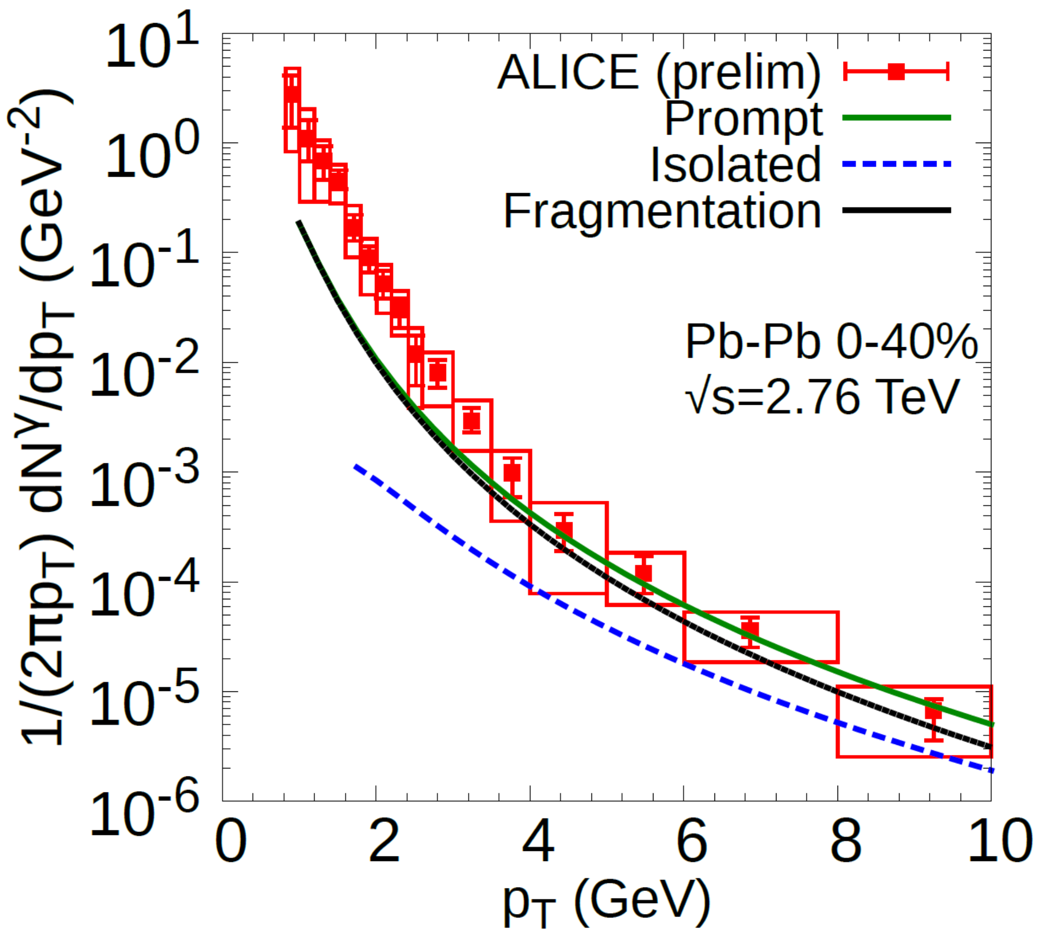
gluon + quark → gluon + γ

By fragmentation, I mean e.g.

gluon + quark → gluon + quark

and γ is produced during hadronisation

Affected by energy loss



(Also, related to jet-plasma photons)

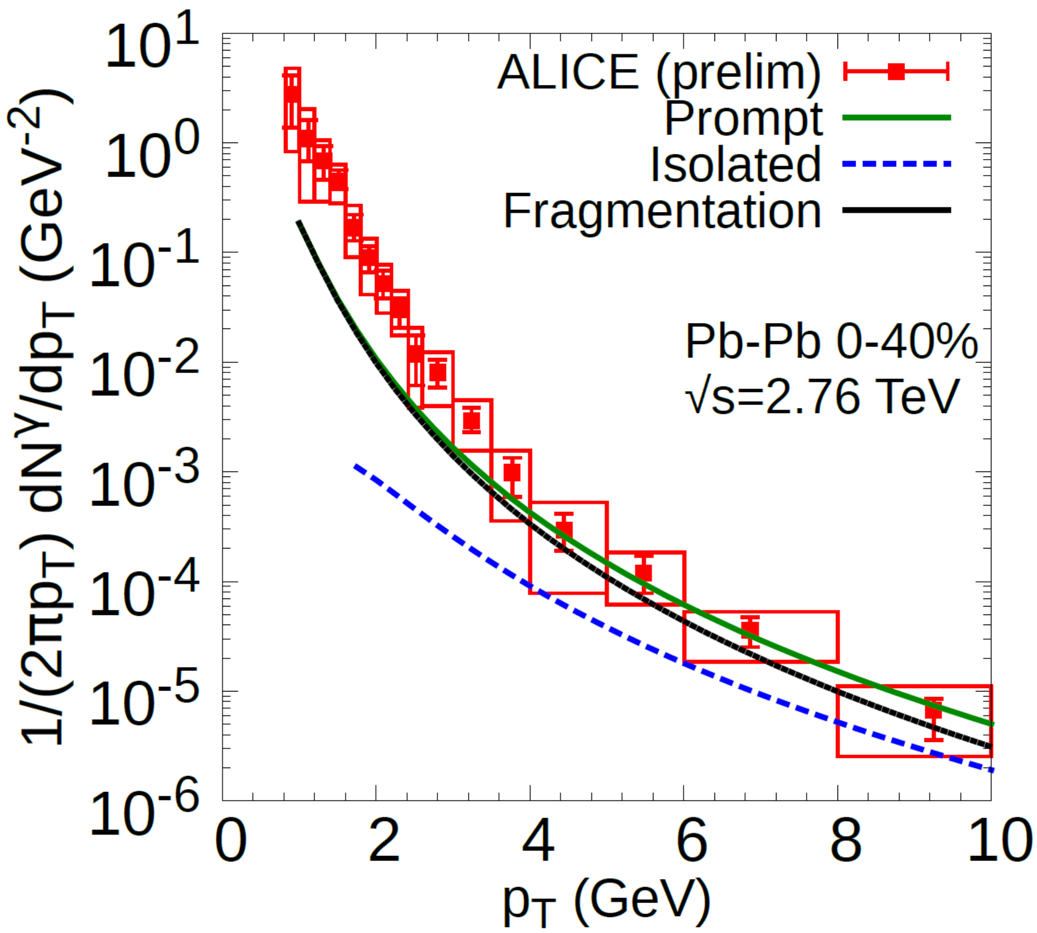
Prompt photon channels

Two distinct contributions: "isolated" & fragmentation

Dominate at high p_T

By "isolated", I mean e.g. Compton scattering

$$\text{gluon} + \text{quark} \rightarrow \text{gluon} + \gamma$$



By fragmentation, I mean e.g.

$$\text{gluon} + \text{quark} \rightarrow \text{gluon} + \text{quark}$$

and γ is produced during hadronisation

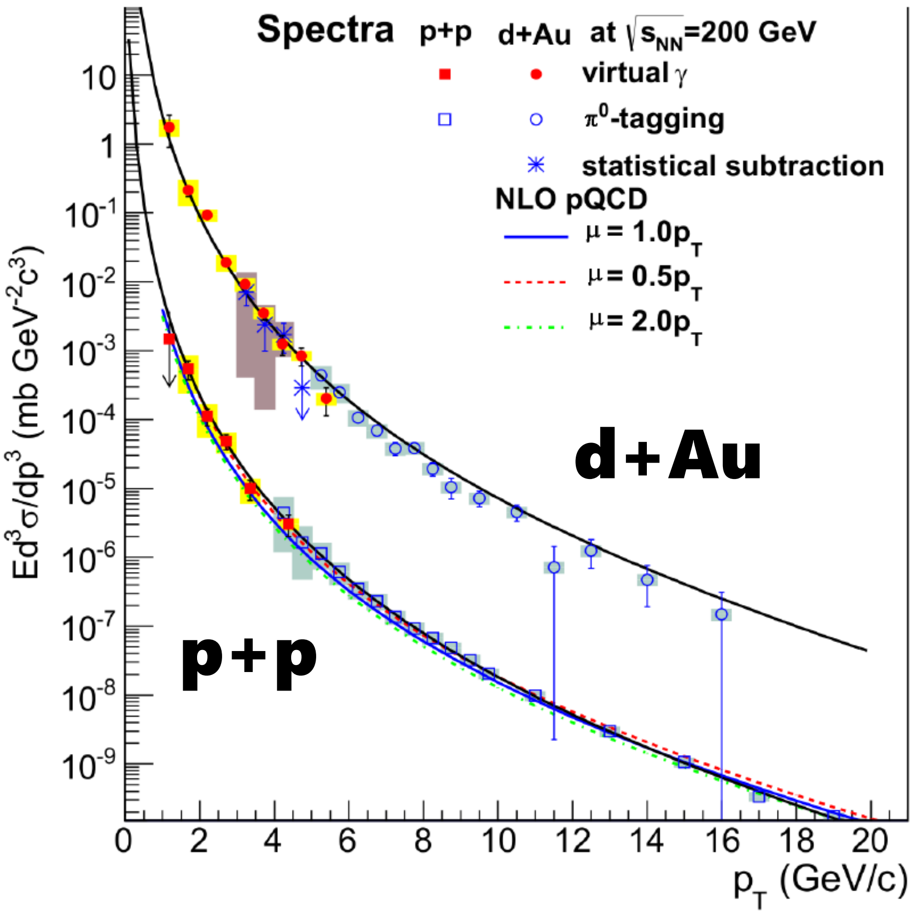
Dominate at low p_T

Prompt photons: why do we care?

Why so much discussion about prompt photons?

Because they are the **reference**

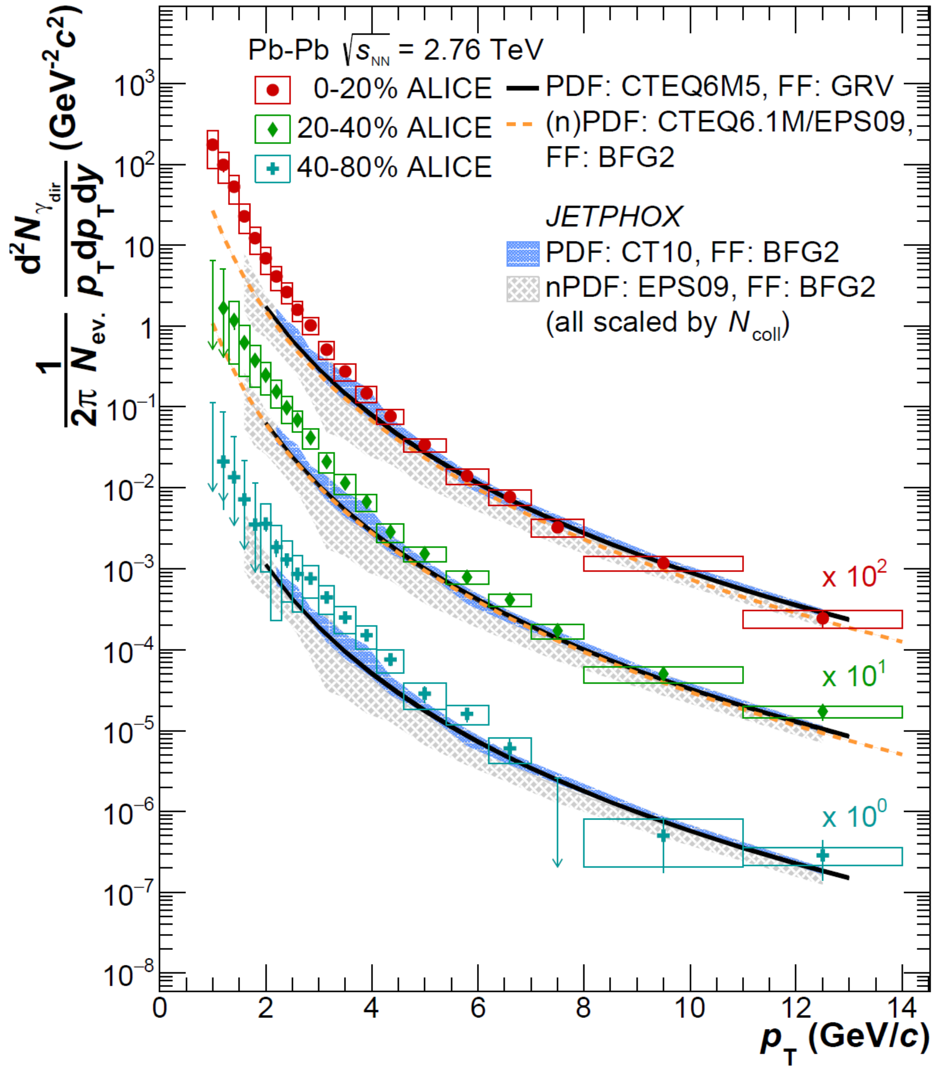
RHIC



In proton-proton and proton-nucleus collisions, prompt photons dominate at all p_T > ~1 GeV

Prompt photons: A+A

But in nucleus-nucleus collisions, do we know the reference?

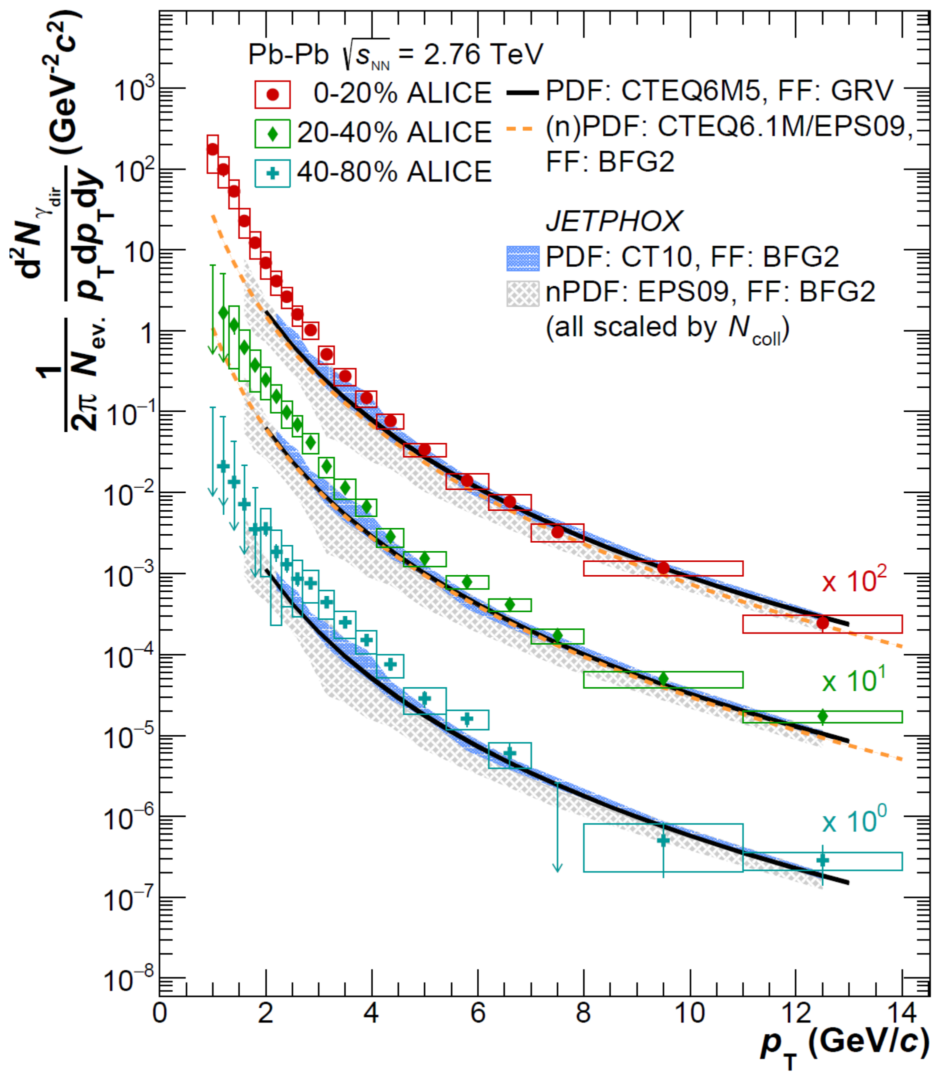


Prompt photons describe well the high p_T data in A+A

Prompt photons underestimate the low p_T data in A+A:
direct photon excess!

but...
jet energy loss and jet-medium photons not accounted for

Prompt photons: bottom line



Prompt photons can be a tricky reference

<u>unless plasma effects are taken into account</u>

(however: model bias)

A photon excess **is** observed, but remember how this excess is defined

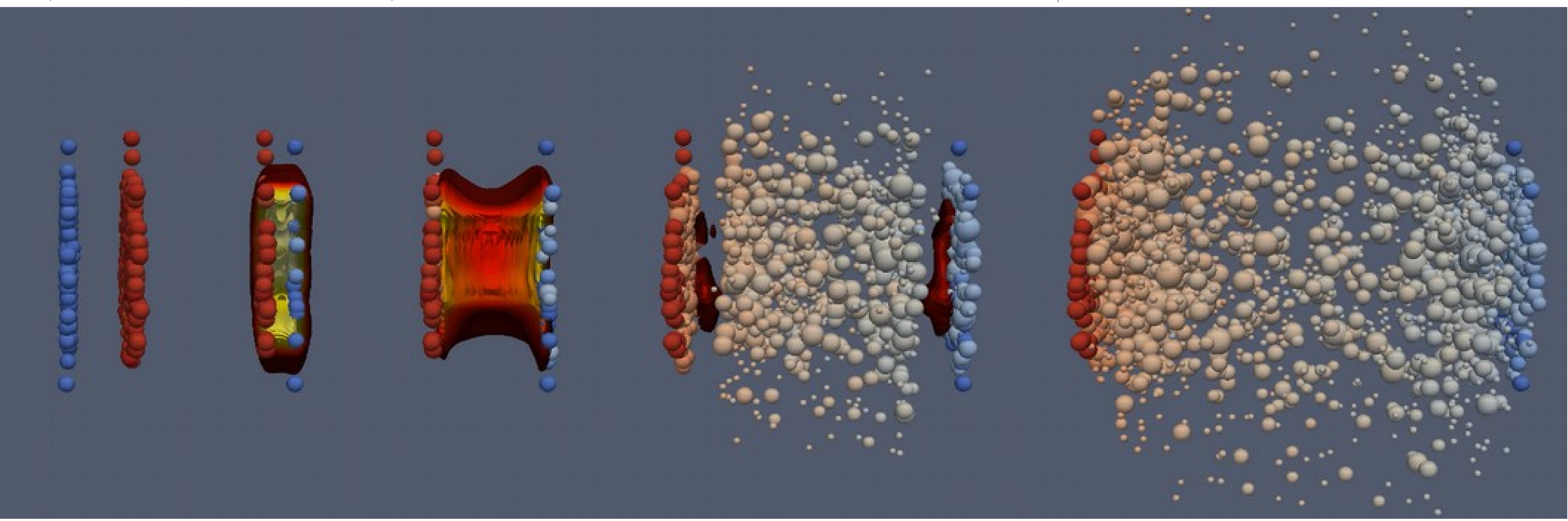
Thermal photons

Reminder: sources of photons

Prompt photons

"Thermal" photons

~~Decay photons~~ (e.g. ~~$\pi^0 \to \gamma\gamma$~~)

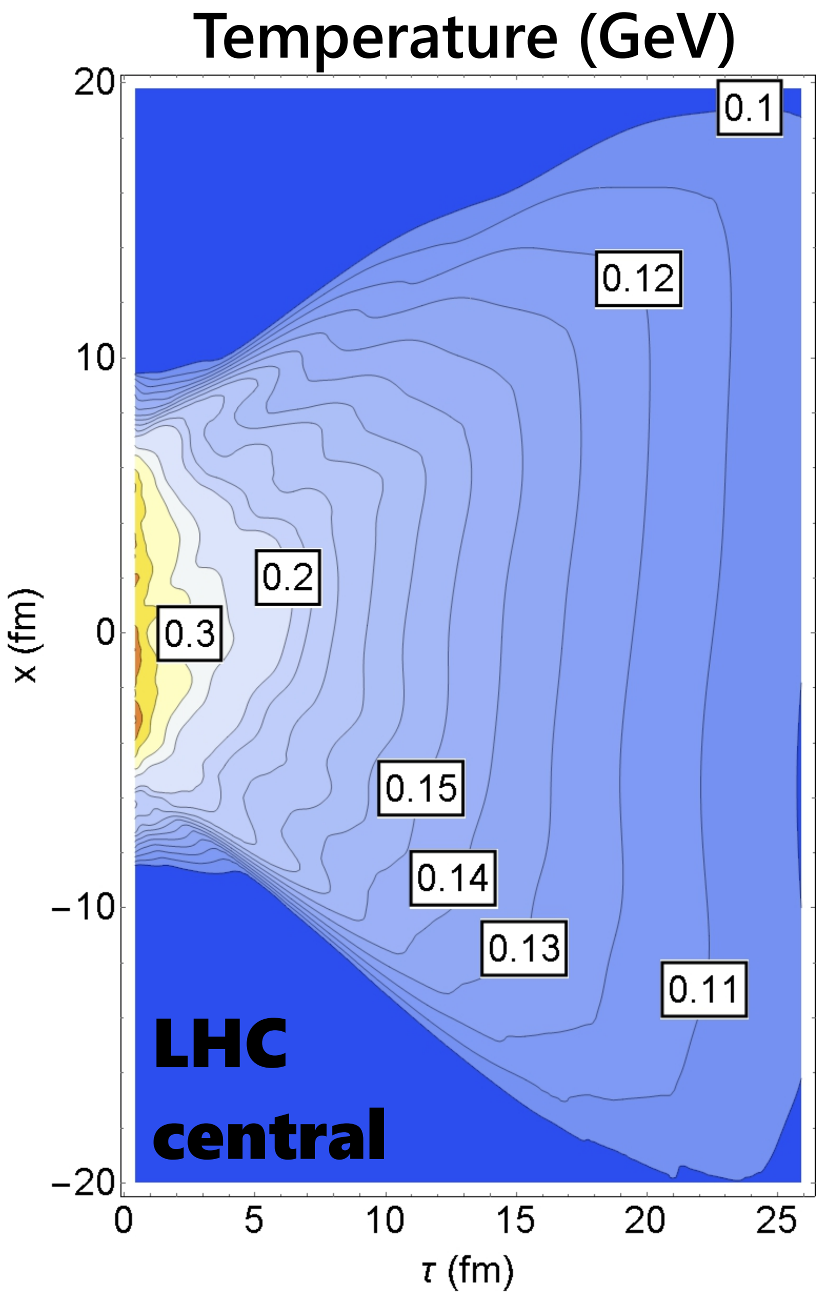


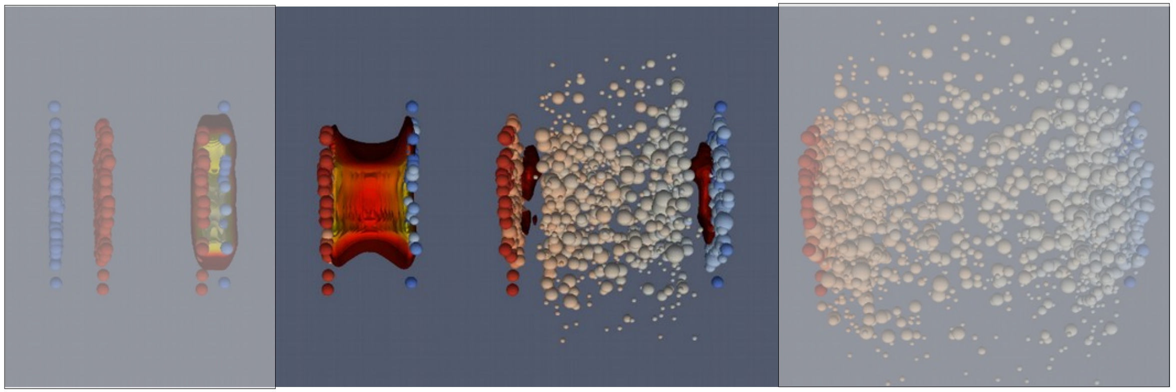
Pre-equilibrium emission

Late stage emission (e.g. $\pi\rho \to \pi\gamma$)

Also: photons from jet-plasma interactions, B-field, ...

Thermal photons & temperature profile



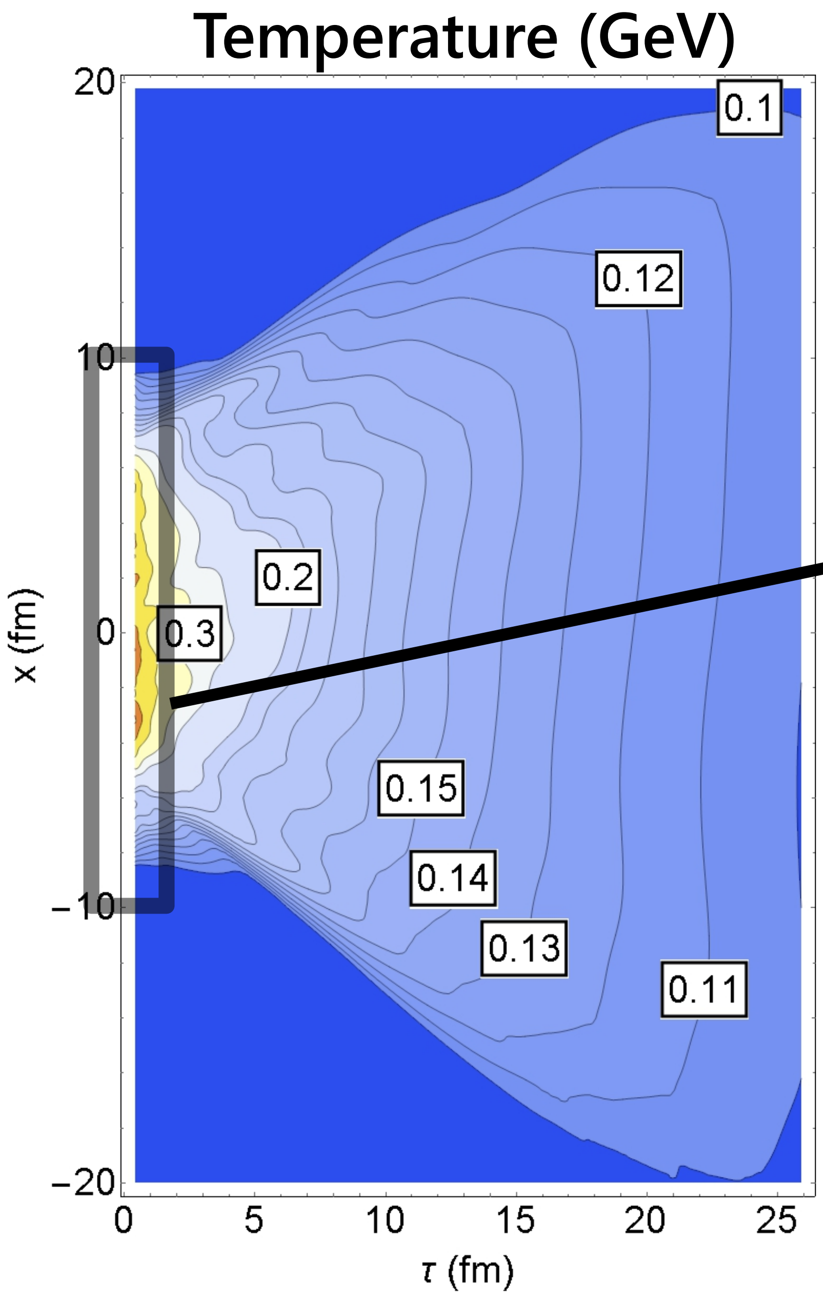


Plasma seems to reach & maintain local equilibrium:

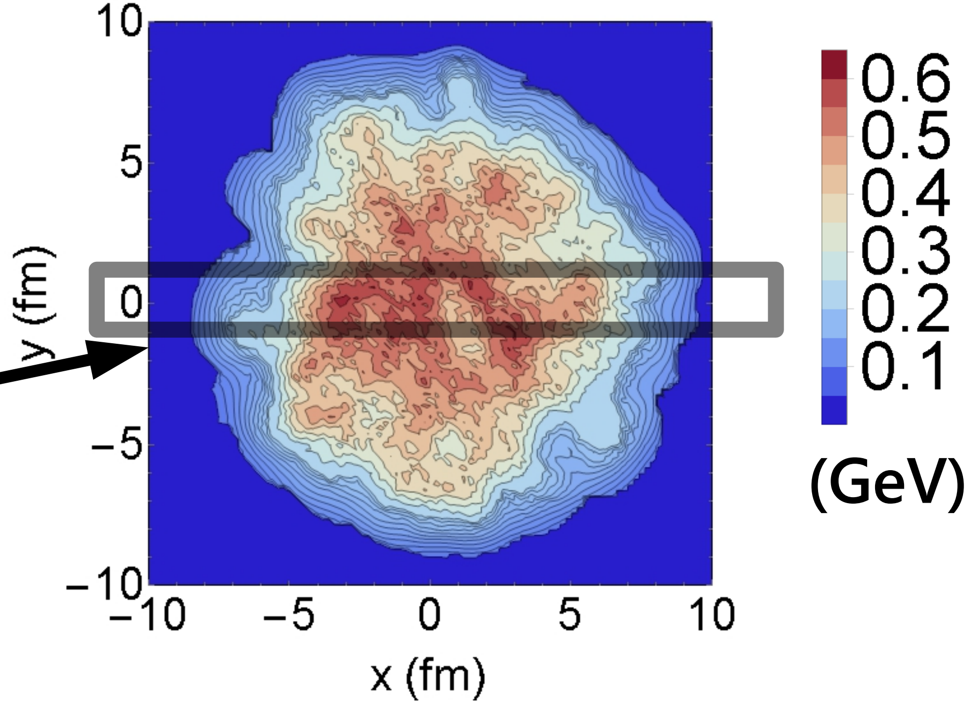
A <u>local</u> temperature can be defined

Evolution of the temperature & flow velocity given by **relativistic hydrodynamics**

Initial temperature



Simple answer:



Real answer:

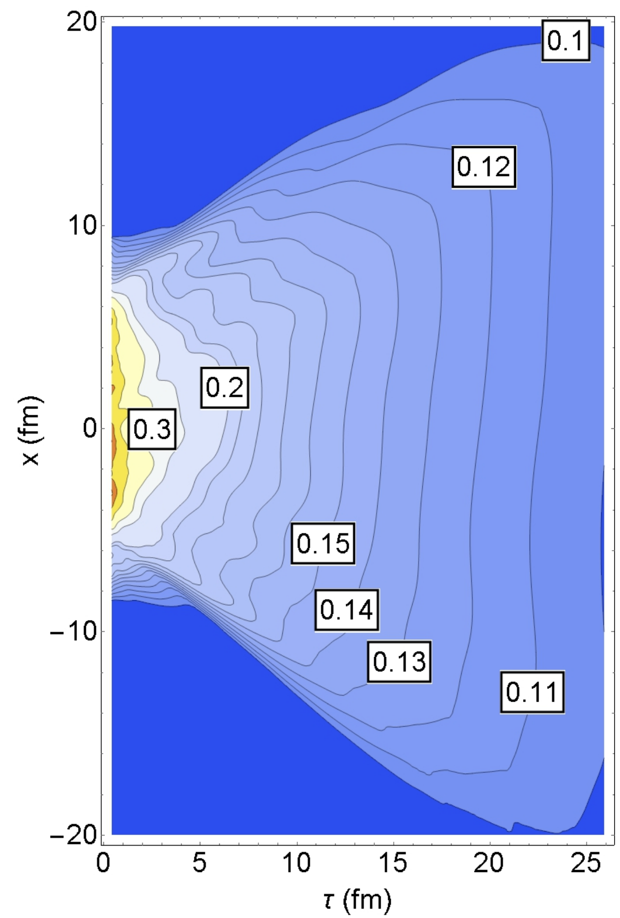
Not very well defined.

Depends on initial time & averaging procedure

Thermal photons: rate & plasma profile

$$E\frac{d^3N}{d\mathbf{k}} = \int d^4X\, E\frac{d^3\Gamma}{d\mathbf{k}}\left(K^\mu, u^\mu(X), T(X), \pi^{\mu\nu}(X), \Pi(X)\right)$$

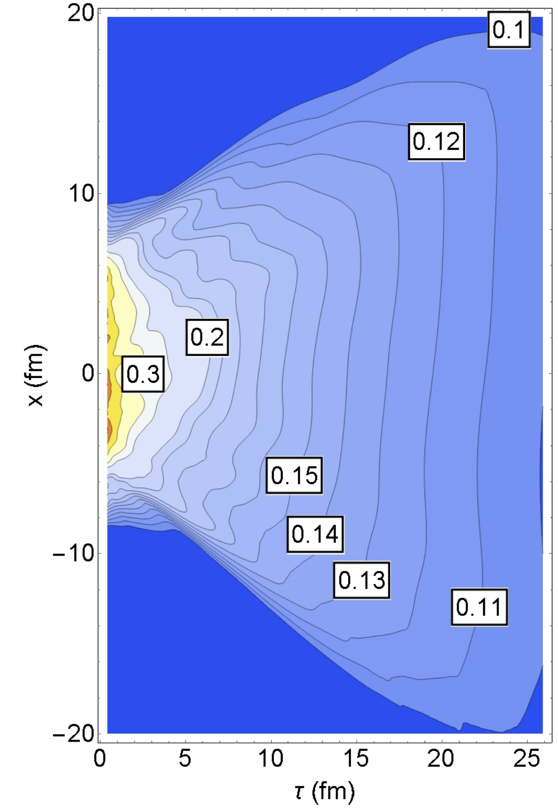
Thermal emission rate:
Given local plasma properties (e.g. temperature, flow velocity), how much photons are radiated?



Spacetime profile of plasma: hydro

Hydrodynamic model of plasma:

- **Initial conditions**
- **Hydrodynamic equations**
- **Production of hadrons from hydro (Cooper-Frye + afterburner)**



$$\partial_\mu T^{\mu\nu}(X) = 0$$

Flow

$$T^{\mu\nu}(X) = \epsilon(X)u^\mu(X)u^\nu(X) - [\mathcal{P}(X) + \Pi(X)]\,\Delta^{\mu\nu}(X) + \pi^{\mu\nu}(X)$$

Energy density (related to temperature through equation of state)

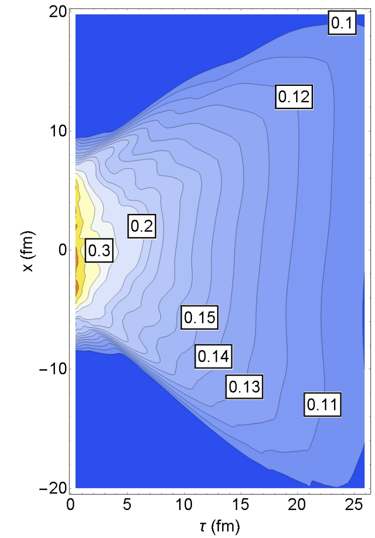
Bulk pressure

Shear stress tensor

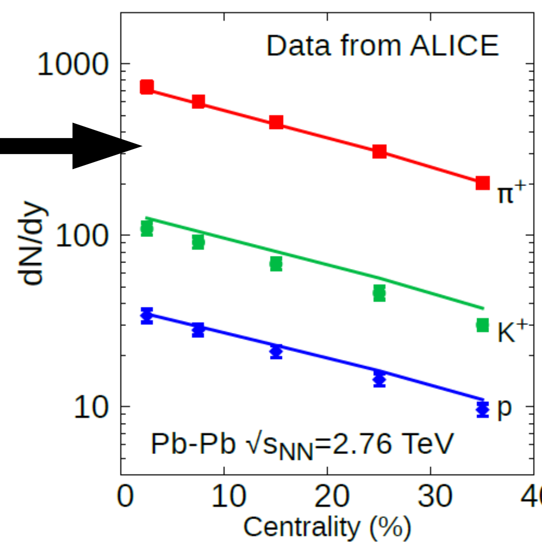
Fix hydro with hadrons

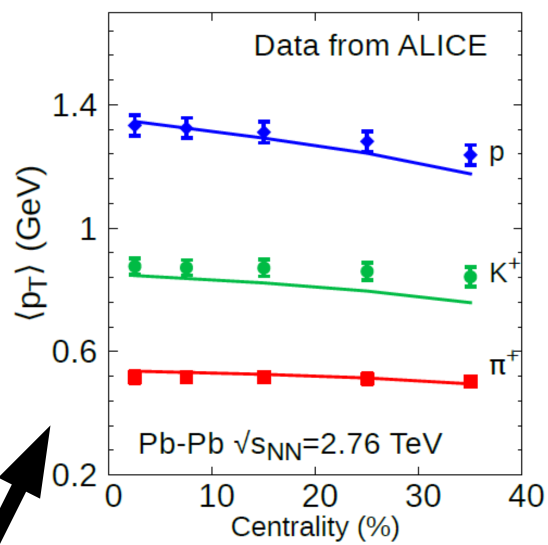
Hydrodynamic model of plasma:

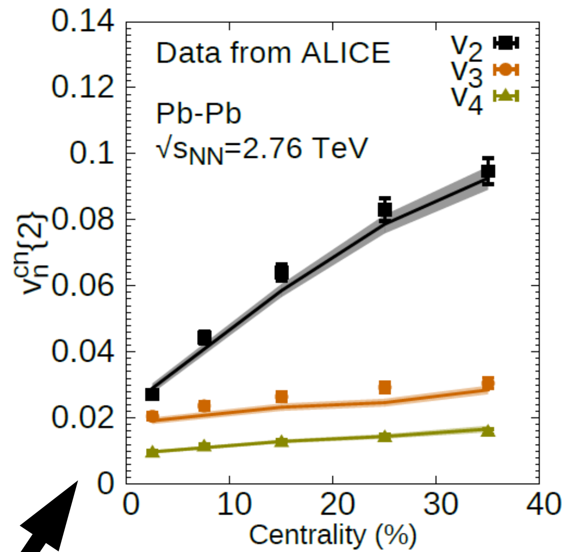
- **Initial conditions**
- **Hydrodynamic equations**
- **Production of hadrons from hydro (Cooper-Frye + afterburner)**



Number of hadrons?





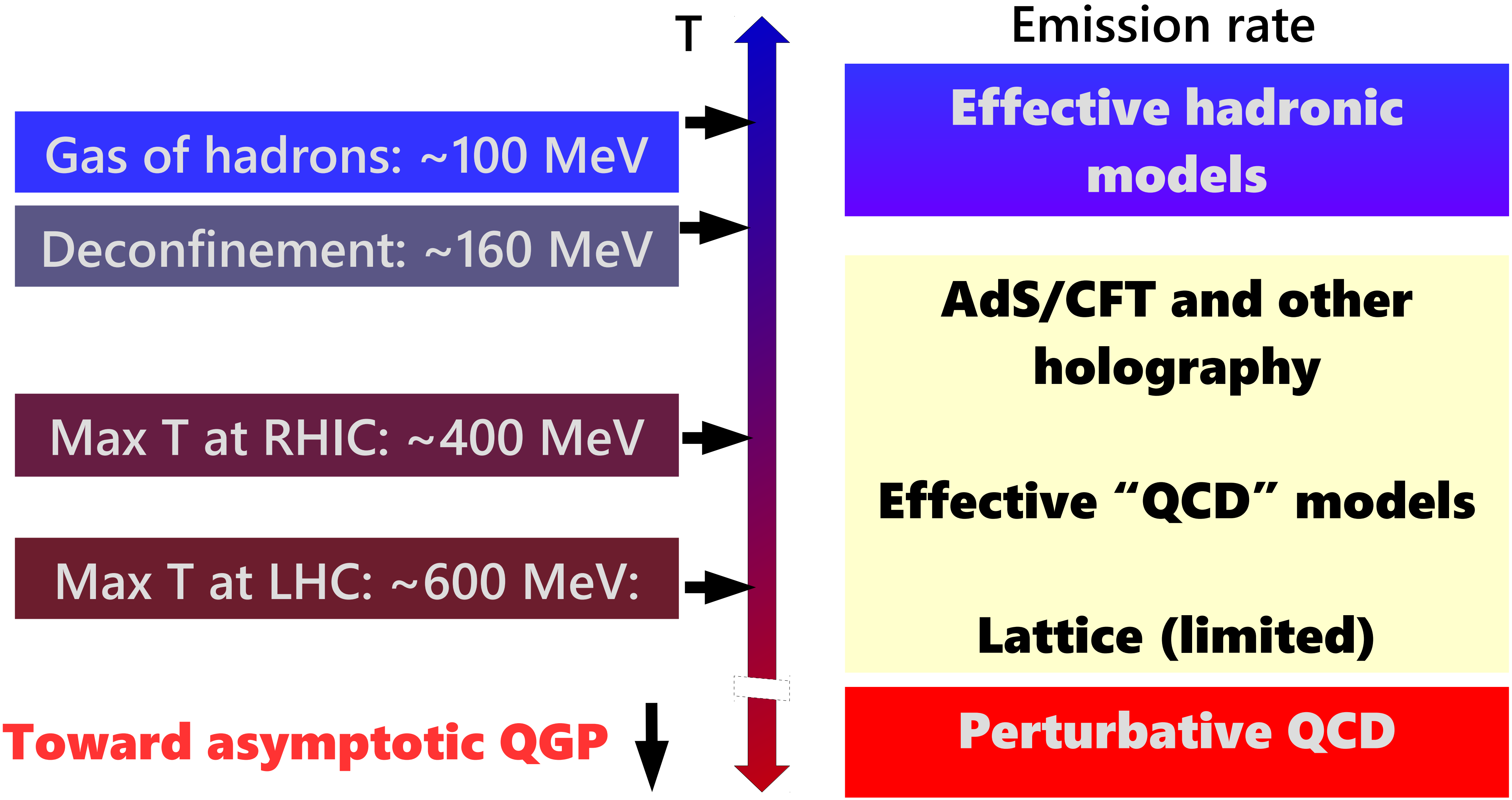


Average energy of hadrons?

Azimuthal distribution?

Photon emission rate

How much does a plasma of nuclear matter at temperature T radiate?



Electromagnetic emission rate

How much does a plasma of nuclear matter at temperature T radiate?

T | Emission rate

Gas of hadrons: ~100 MeV

Deconfinement: ~160 MeV

Max T at RHIC: ~400 MeV

Max T at LHC: ~600 MeV:

Toward asymptotic QGP

Effective hadronic models

Refs: Texas A&M (Rapp et al) and McGill (Gale et al) groups; Stony Brook group (Dusling & Zahed) and others

AdS/CFT and other holography

Effective "QCD" models

Refs: Caron-Huot et al (AdS/CFT); Finazzo and Rougemont (bottom-up holography), ...

Perturbative QCD

Refs: Arnold, Moore, Yaffe (AMY); Ghiglieri, Teaney; Laine, ...

Thermal photons: bottom line

$$E\frac{d^3N}{d\mathbf{k}} = \int d^4X\, E\frac{d^3\Gamma}{d\mathbf{k}}(K^\mu, u^\mu(X), T(X), \pi^{\mu\nu}(X), \Pi(X))$$

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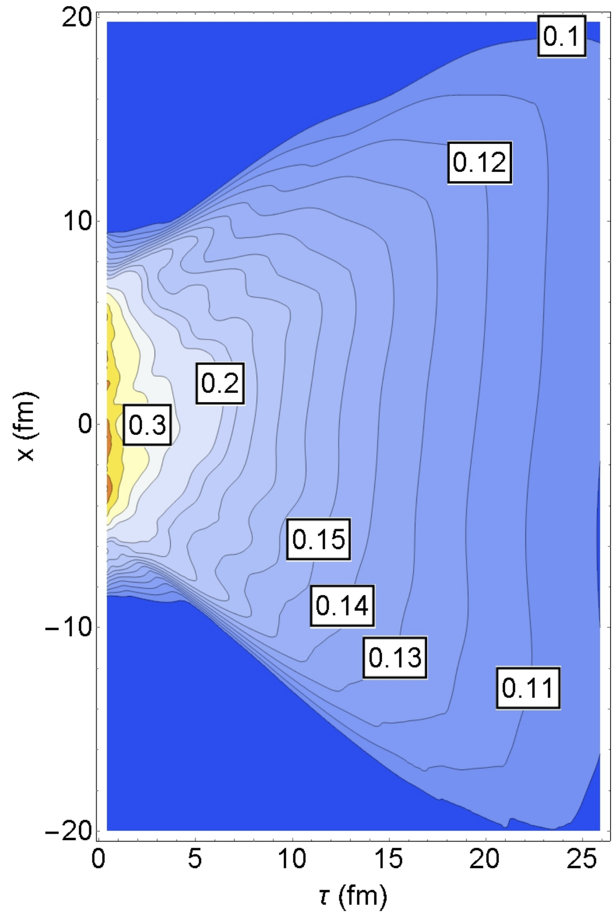
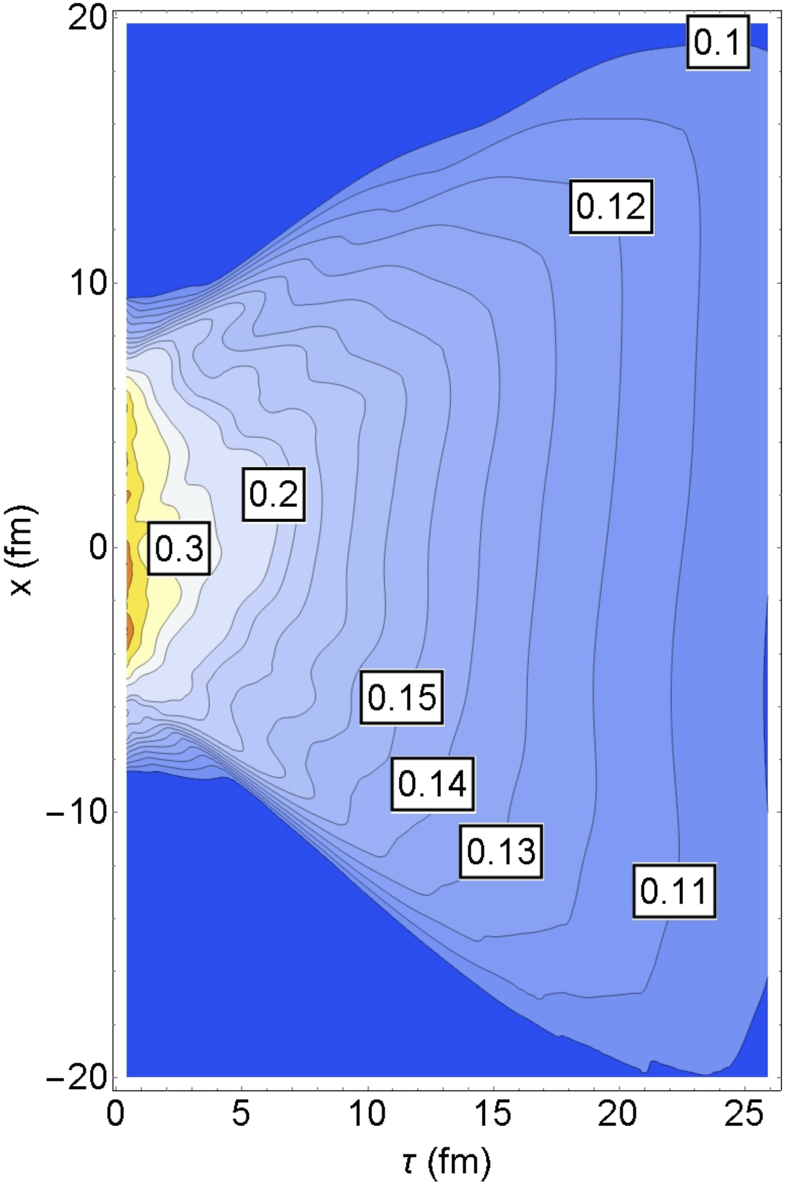
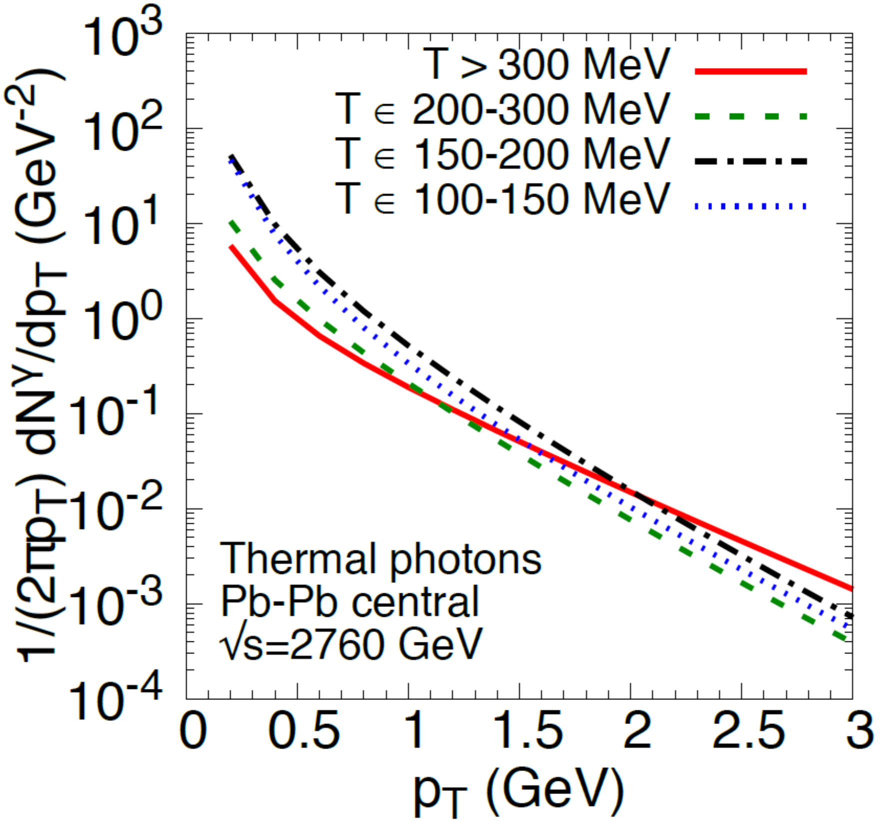


Thermal photons

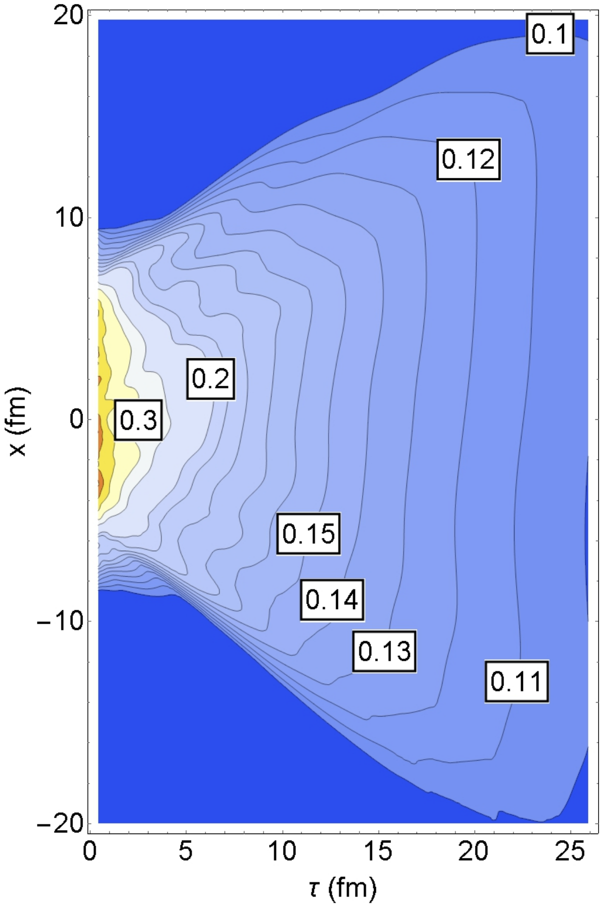


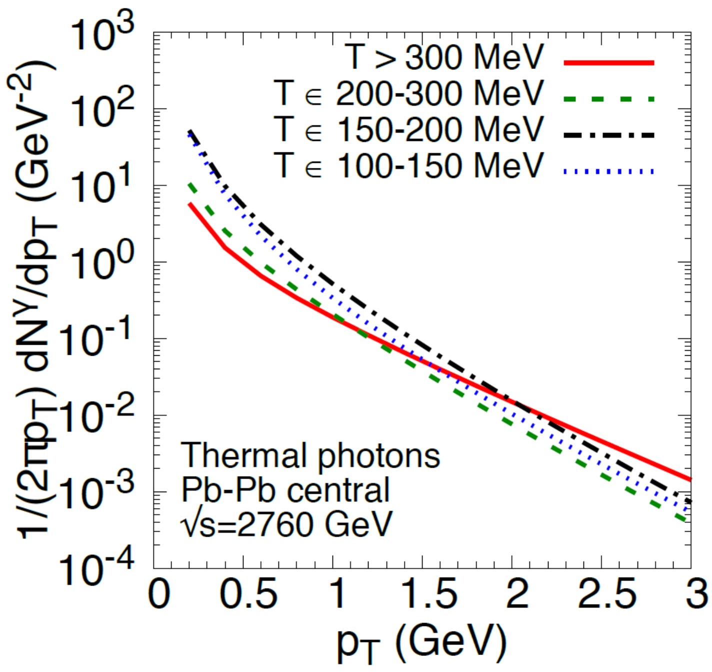
Thermal photons from different temperature ranges:



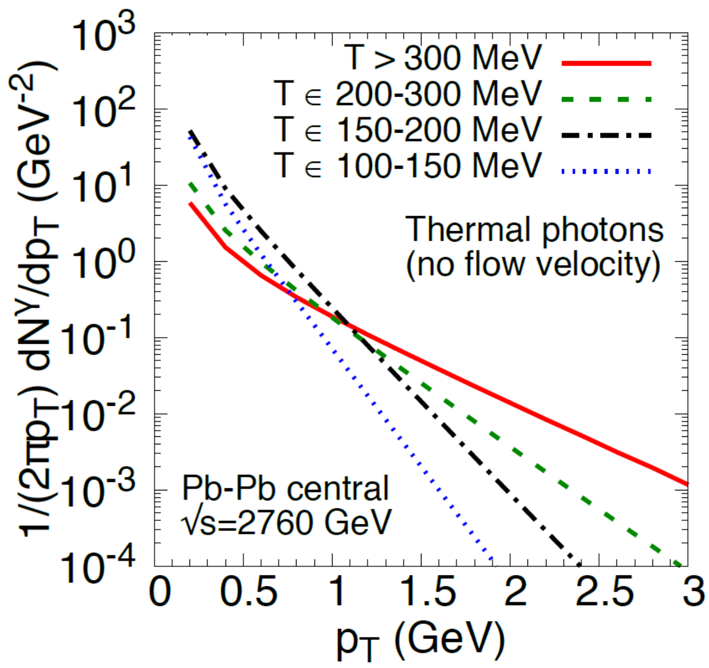
Not just temperature!

Thermal photons vs flow





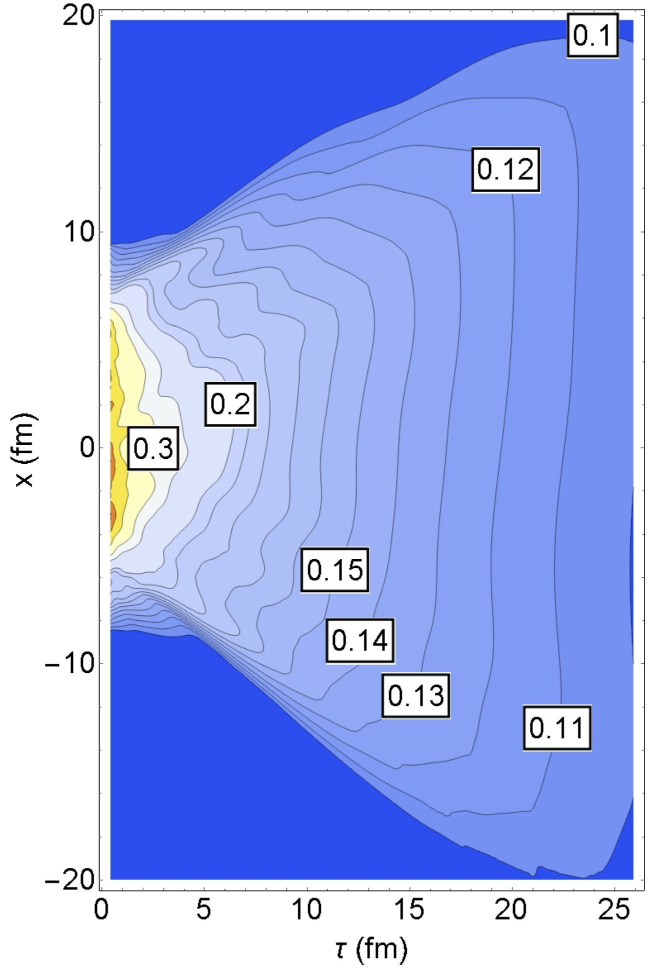
With flow

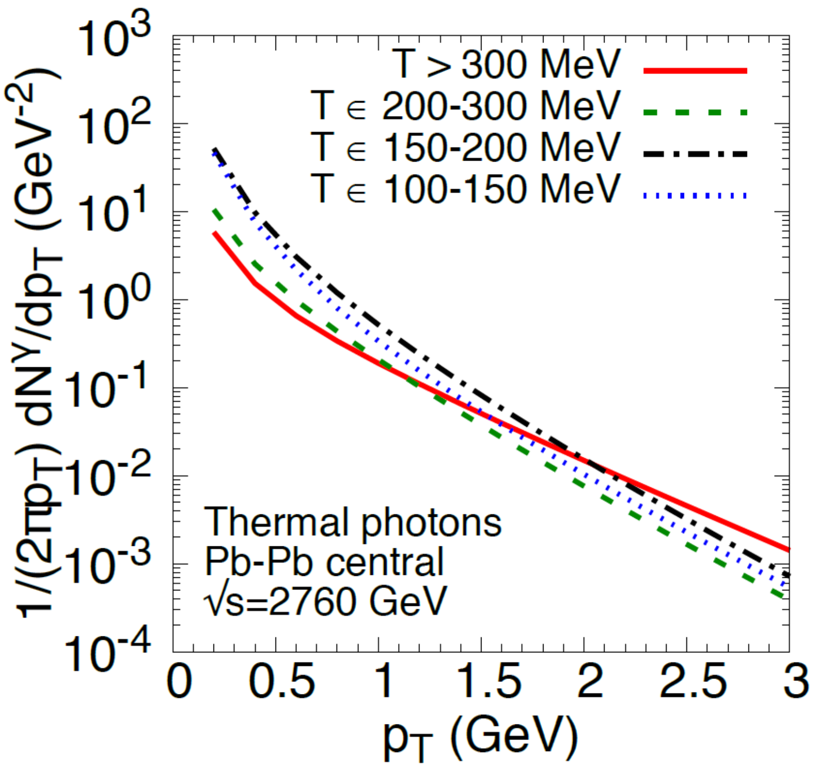


Without flow

Number of photons stays the same, but the flow changes (boosts) the momentum distribution

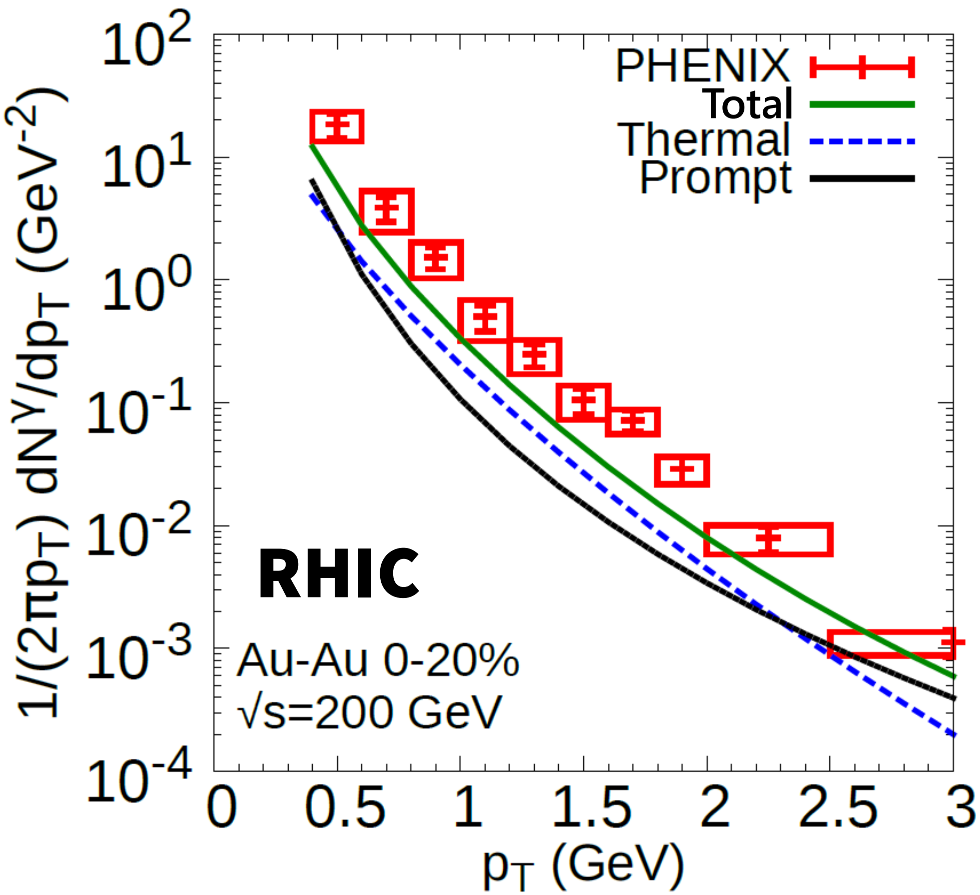
Thermal photons: p_T ranges

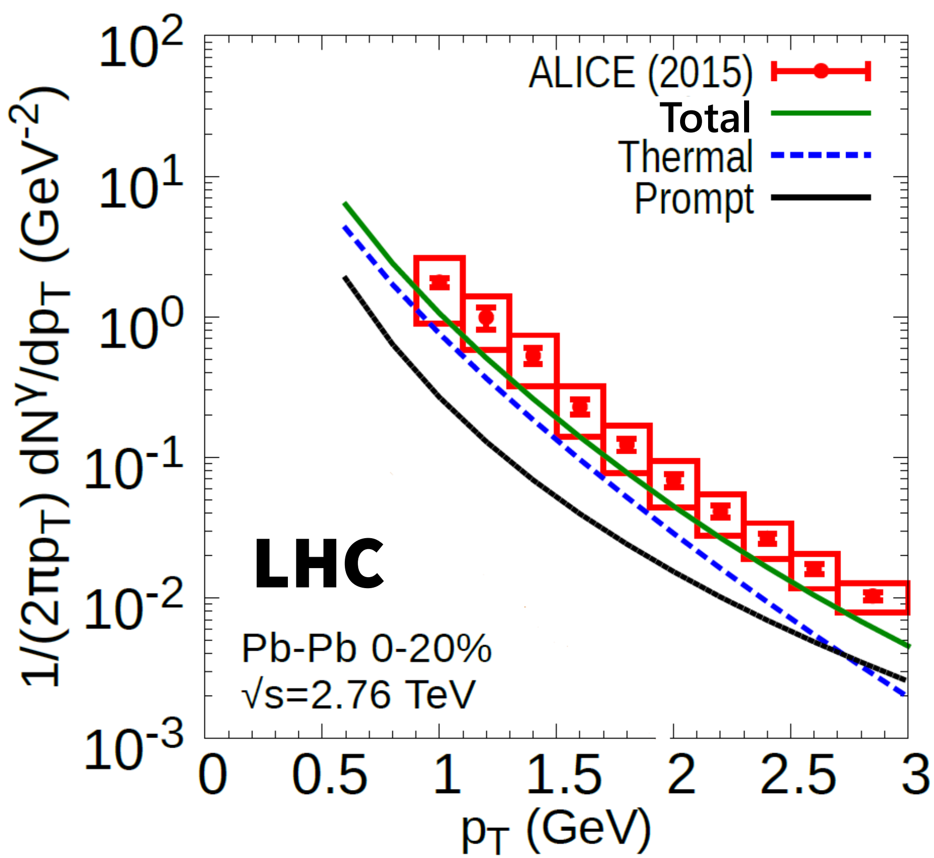




Low p_T photons are from low temperatures (late times)

Direct photon spectra vs data



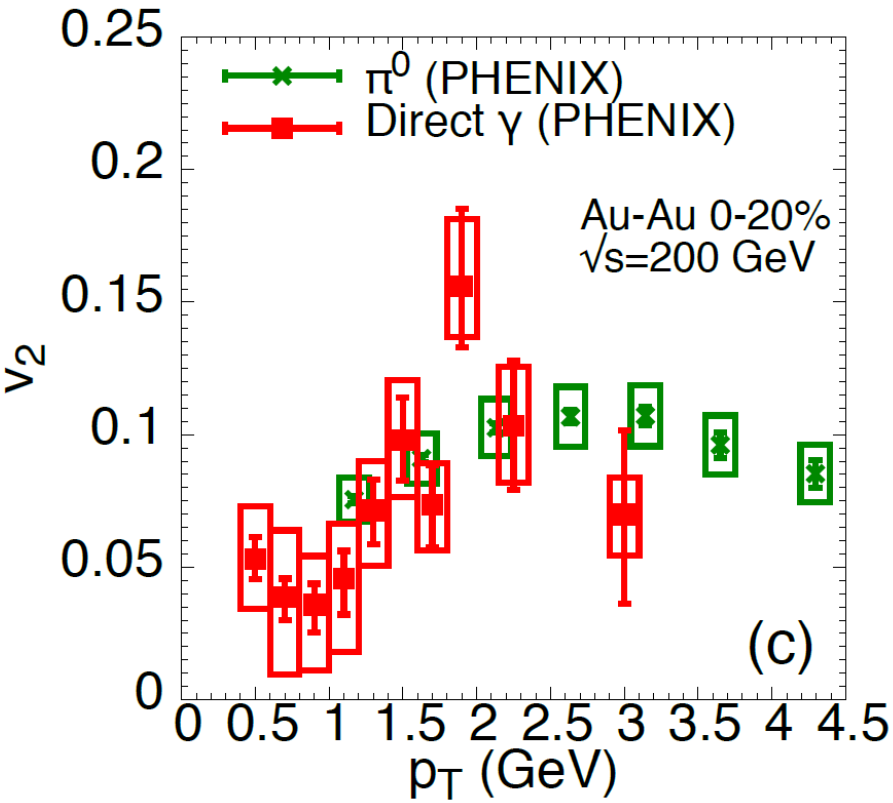


Agreement with data?
(better at LHC than RHIC)

Beware: prompt photons

Momentum anisotropy: v_2

First measurement of direct photon v_2 (PHENIX collaboration, 2011)



Direct photon v_2 as large as pion v_2

Why is this surprising?

Reason 1: Prompt photons

Reason 2: How v_2 develops

Summing different sources of v_2's

$$\frac{1}{2\pi p_T}\frac{dN}{dp_T d\phi} = \left(\frac{1}{2\pi p_T}\frac{dN}{dp_T}\right)\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_n))\right]$$

but what if there are two sources of direct photons?
(say thermal and prompt)

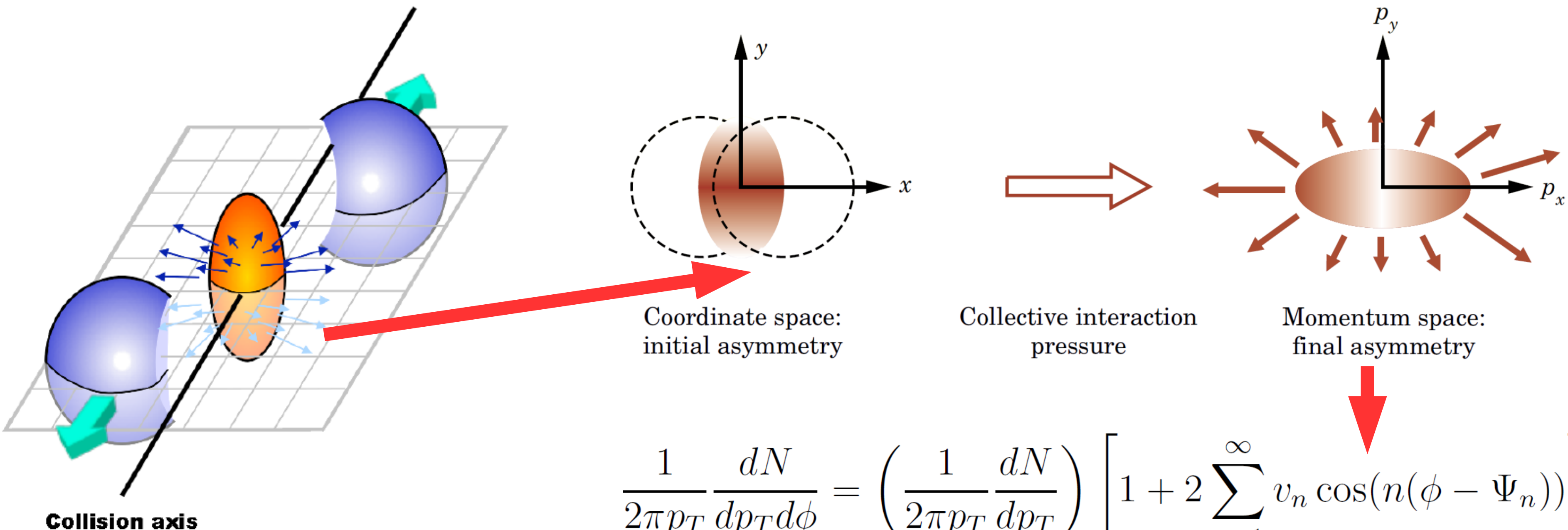
$$v_2^{\text{direct }\gamma}(p_T) = \frac{[\text{spectra of thermal } \gamma \text{ at } p_T]\times v_2^{\text{thermal}}(p_T) + [\text{spectra of prompt } \gamma \text{ at } p_T]\times v_2^{\text{prompt}}(p_T)}{[\text{spectra of thermal } \gamma \text{ at } p_T] + [\text{spectra of prompt } \gamma \text{ at } p_T]}$$

v_2 is a weighted average

Since prompt photons are expected to have a small v_2, they dilute (decrease) the thermal photon v_2

How is v_2 of thermal photons developed?

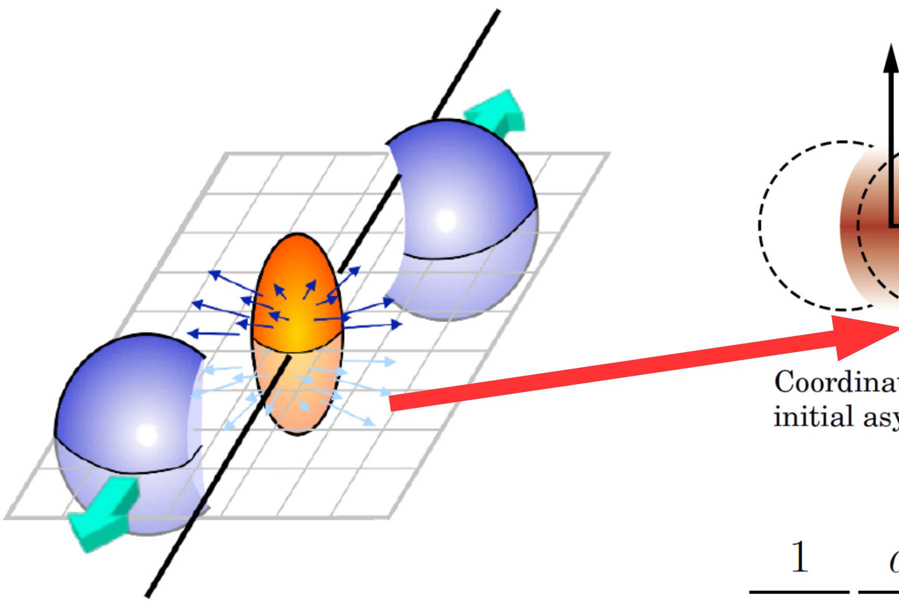
Spatial anisotropy leads to anisotropic expansion (different pressure gradients in x and y)

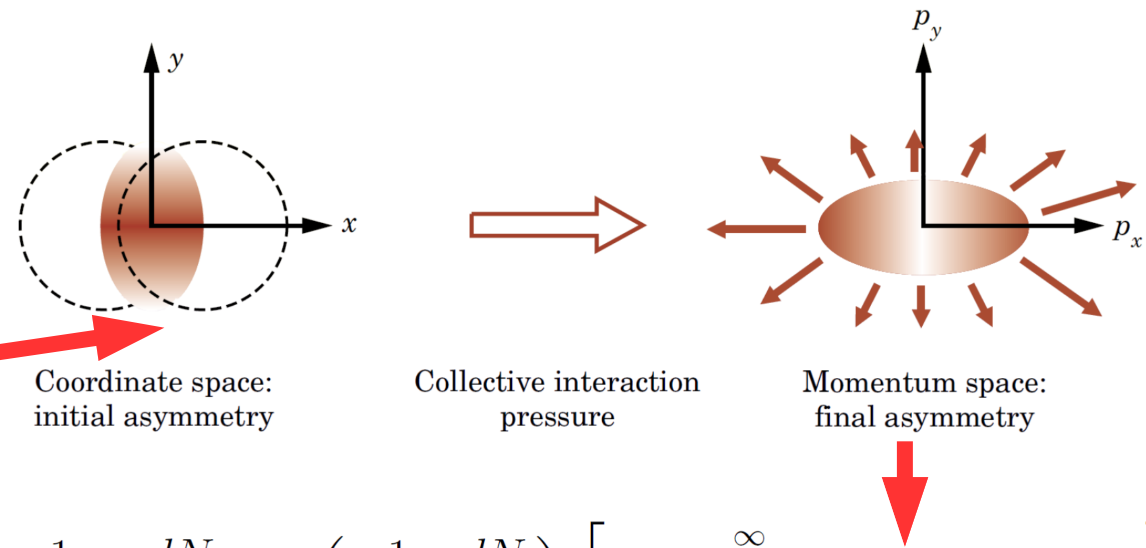


$$\frac{1}{2\pi p_T}\frac{dN}{dp_T d\phi} = \left(\frac{1}{2\pi p_T}\frac{dN}{dp_T}\right)\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_n))\right]$$

Anisotropic flow velocities boosts particles by different amount, which lead to a momentum anisotropy (v_n)

Thermal photon v_2 and time





$$\frac{1}{2\pi p_T}\frac{dN}{dp_T d\phi} = \left(\frac{1}{2\pi p_T}\frac{dN}{dp_T}\right)\left[1 + 2\sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_n))\right]$$

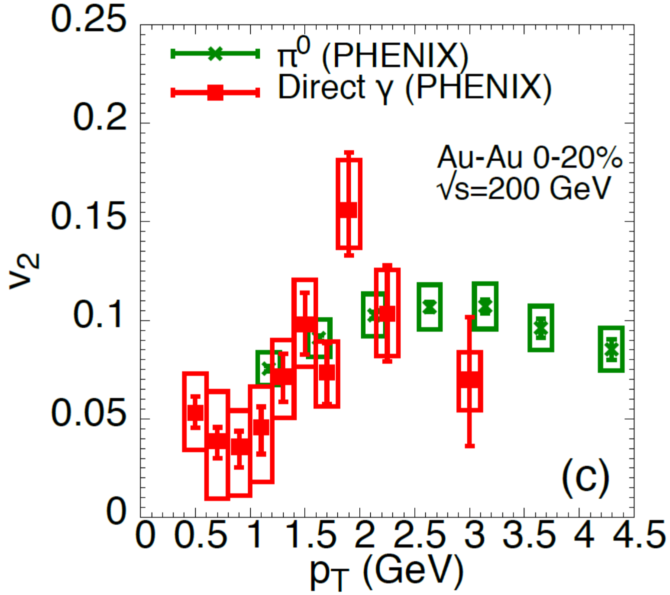
Early times: v_2 much smaller than pion's

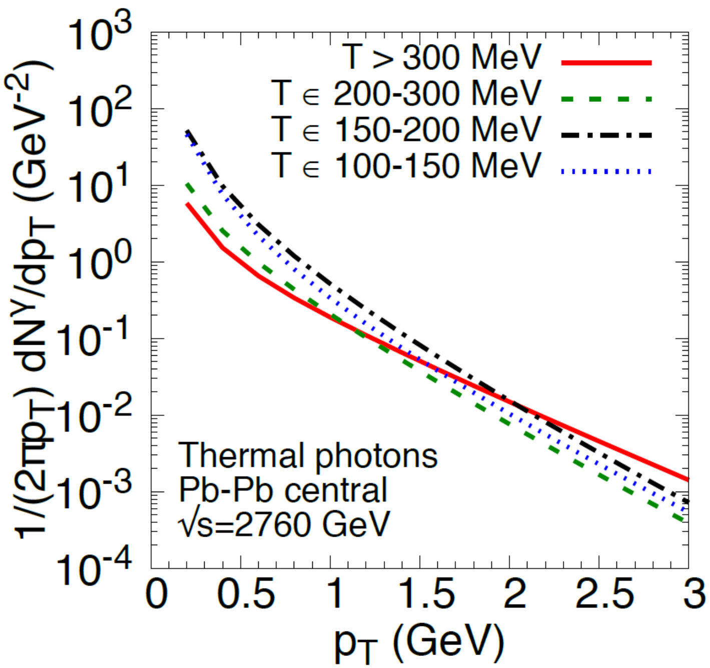
Late times: v_2 similar pion's

Intermediate times: "interpolate" between two limits

Overall result: expect thermal photon v_2 smaller than pion

Direct photon v_2: qualitative expectations



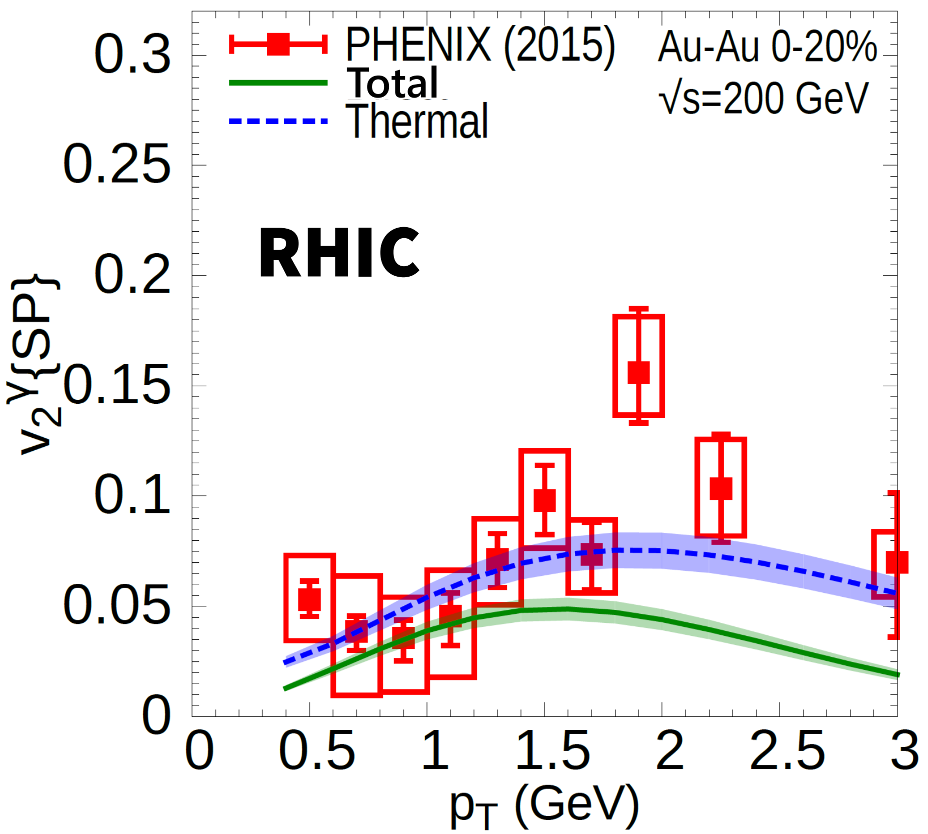


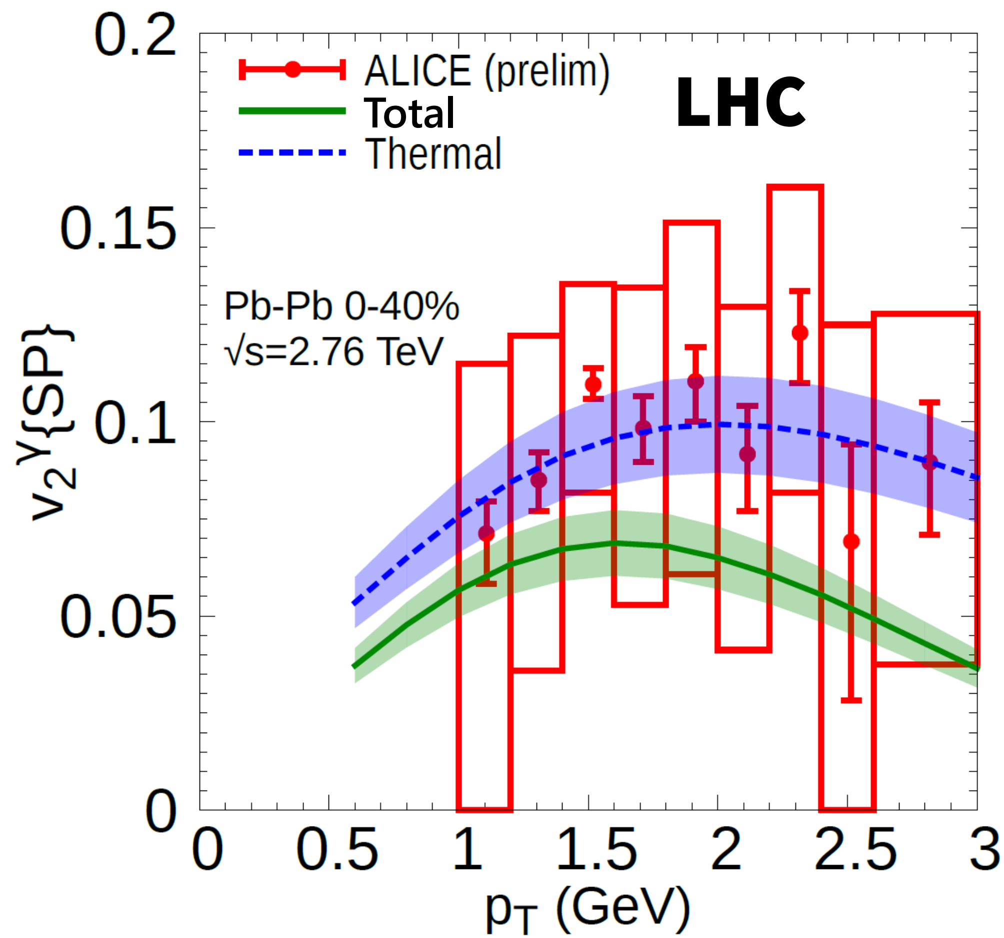
Thermal photon v_2 smaller than the pion one because of early time thermal photon emission w/ low v_2

Prompt photons should reduce the direct photon v_2 even more (assuming significant spectra and small v_2)

Bottom line: expect direct photon v_2 smaller than pion (but depends on p_T!)

Direct photon v_2 vs data





Low p_T dominated by late times thermal photons

High p_T smaller due to early times thermal photons and prompt photons

Progress that improved agreement w/ data

$$E\frac{d^3N}{d\mathbf{k}} = \int d^4X\, E\frac{d^3\Gamma}{d\mathbf{k}}(K^\mu, u^\mu(X), T(X), \pi^{\mu\nu}(X), \Pi(X))$$

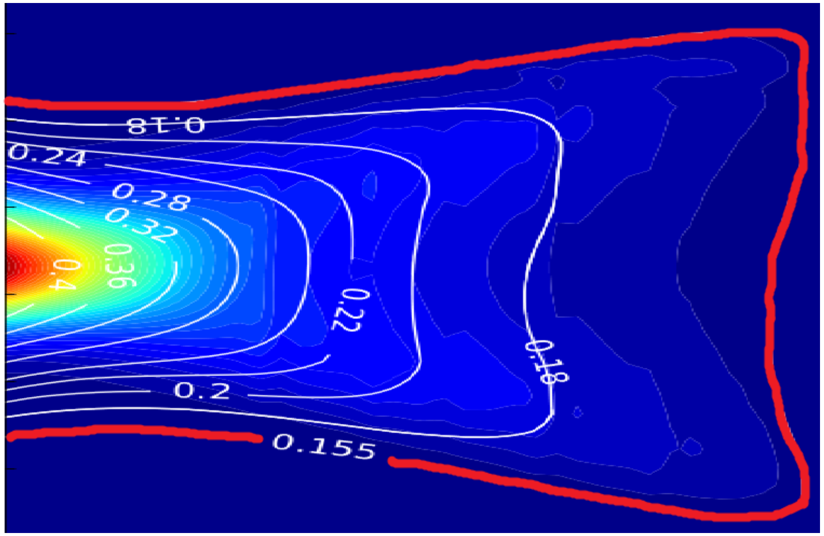
T

Emission rate

Effective hadronic models

More complete low temperature photon emission rate

Perturbative QCD



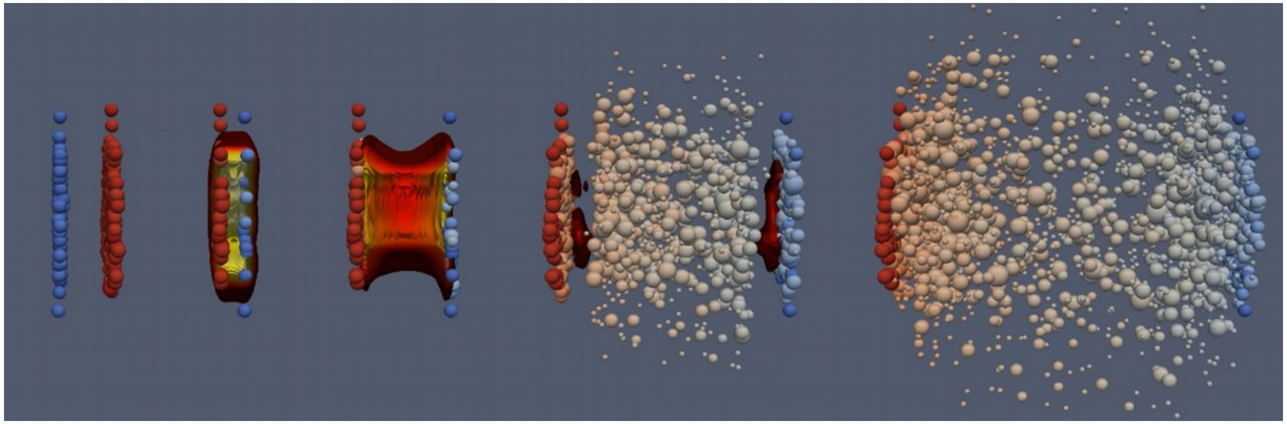
Better initial state (fluctuations, initial flow); more realistic EOS

Where do we go from here?

Still tension with data ("photon puzzle"), but lots of progress in understanding thermal photons over the past years

Future improvement

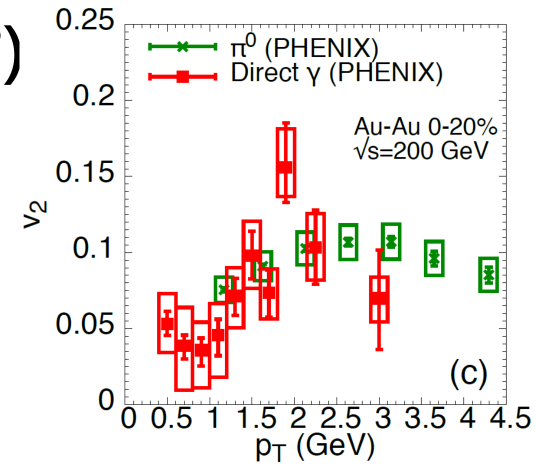
- Pre-equilibrium emission?
- Late stage emission?
- Jet-plasma photons & fragmentation photons energy loss
- Other sources?

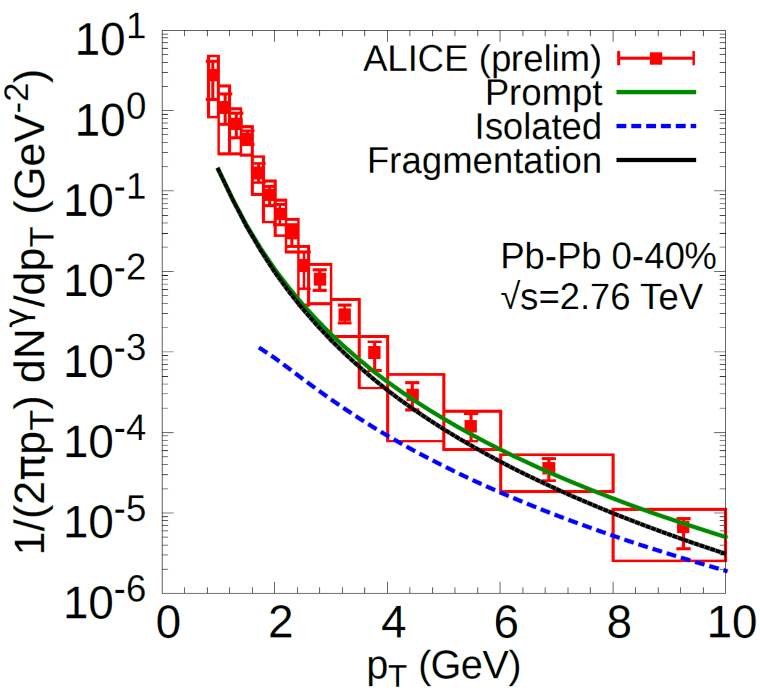


Also, future improvements on thermal photons coming from improved hydrodynamics description and better-constrained thermal photon emission rates?

From experimental side?

- **Trend of photon v_n** at low p_T (does it go to 0?) and high p_T (how fast does it decrease?)
- **Integrated observables**? e.g. multiplicity, integrated v_2?
- Low p_T isolated photons? Or use p_T~4-10 GeV at LHC (2.76 TeV) to better understand fragmentation photons?
- Centrality/center-of-mass-energy/system-size dependence?
- Low p_T photons in p+p?



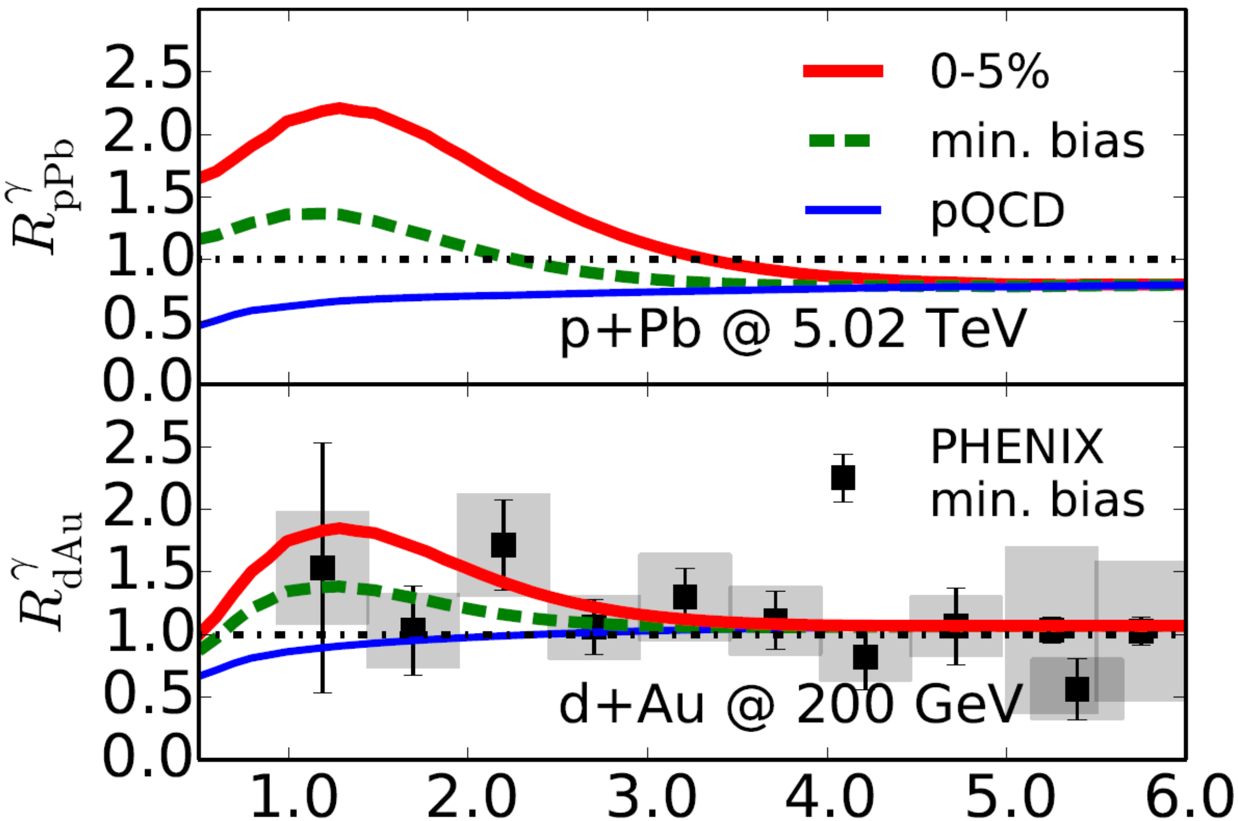


Questions?

Back-up slides

Photon production in small systems

Thermal photons in small systems: R_{pA}



(Ref.: Shen, Paquet, Denicol, Jeon & Gale, arXiv:1609.02590)

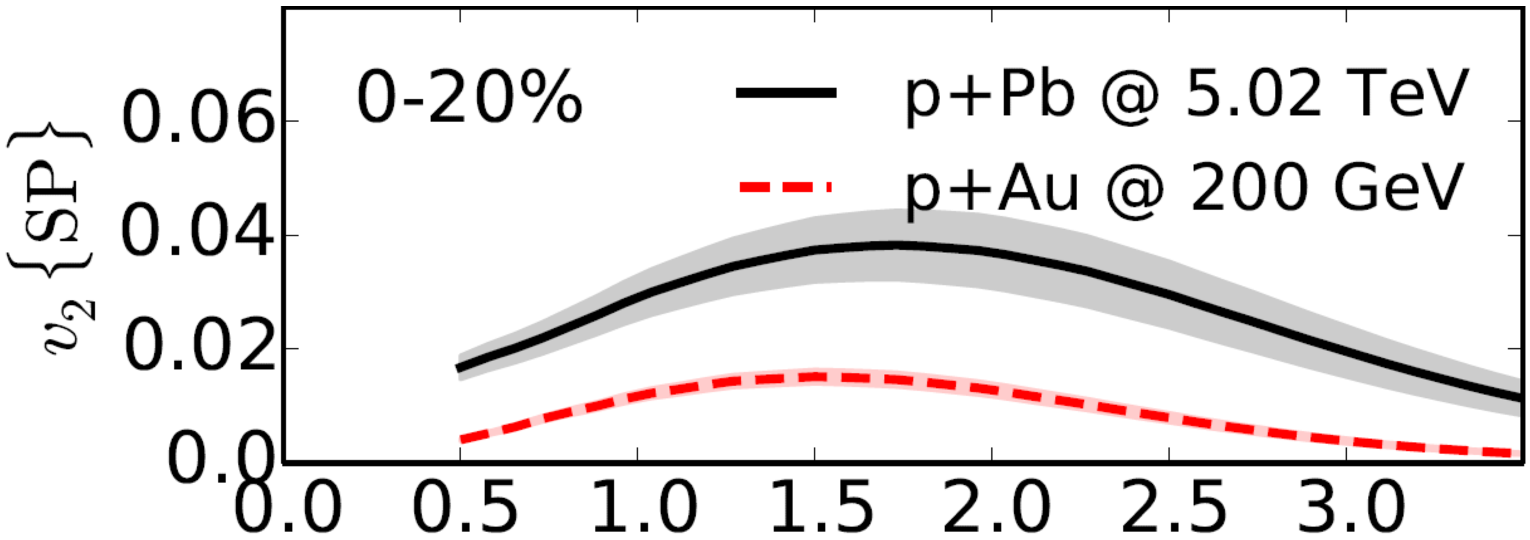
Factor of ~2-3 enhancement at low p_T for central small systems

Enhancement ~1.5 at minimum bias

<u>Theory vs data:</u>
Need good constraints on low p_T photons w/ cold nuclear effects

Thermal photons in small systems: v_2

Predictions for direct photon v_2 at RHIC and LHC



(Ref.: Shen, Paquet, Denicol, Jeon & Gale, arXiv:1609.02590)

v_2~2% at the RHIC ({p,d,He}+Au)

v_2~4% at the LHC

Eventually: Additional tool to study collectivity(?) in small systems

Viscosity & thermal photons

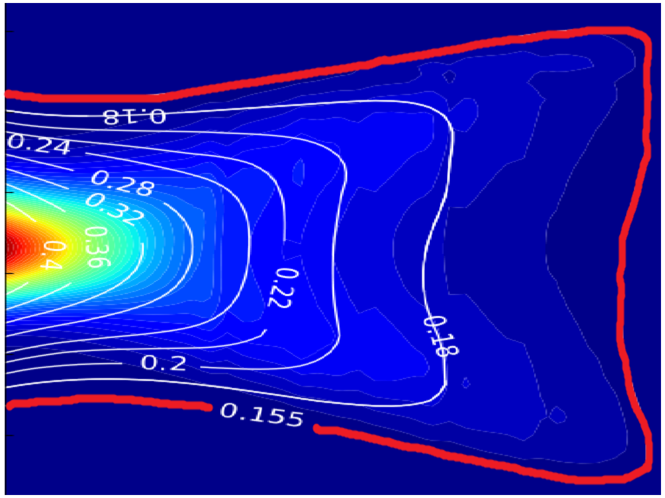
Viscous corrections to rate

$$k\frac{d^3\Gamma_\gamma}{d\boldsymbol{k}} = k\frac{d^3\Gamma_\gamma^{ideal}}{d\boldsymbol{k}} + \pi^{\mu\nu}K_\mu K_\nu d\Gamma_\gamma^{viscous,shear} + \Pi d\Gamma_\gamma^{viscous,bulk}$$

Rate	Ideal	Shear correction	Bulk correction
QGP — $2 \to 2$	[52]	Yes [57]	Forward scattering approximation
QGP — Bremsstrahlung	[52]	No	No
Hadronic — Meson gas (π, K, ρ, K^*, a_1)	[26]	Yes [18, 58]	Yes [this work]
Hadronic — ρ spectral function (incl. baryons)	[26, 27]	No	No
Hadronic — $\pi + \pi$ bremsstrahlung	[27, 59]	No	No
Hadronic — π-ρ-ω system	[28]	No	No

Viscosity and emission rate

Medium evolution: hydrodynamics



<u>Ideal hydro = local thermal equilibrium</u>

f(p)="Fermi-Dirac" (quark, baryons)

or "Bose-Einstein" (gluon, mesons)

<u>Viscous hydro = some deviation from local thermal equilibrium</u>

f(p)="Fermi-Dirac"/"Bose-Einstein" + δf(p)

δf(p) is related to the viscosity (shear and bulk) of the plasma

Deviation from equilibrium = correction to EM probe rate

$$k\frac{d^3\Gamma_{\gamma/\gamma^*}}{d\boldsymbol{k}}=k\frac{d^3\Gamma^{ideal}_{\gamma/\gamma^*}}{d\boldsymbol{k}}+\pi^{\mu\nu}K_\mu K_\nu d\Gamma^{viscous,shear}_{\gamma/\gamma^*}+\Pi\, d\Gamma^{viscous,bulk}_{\gamma/\gamma^*}$$

Refs: Dusling, NPA (2010); Shen, Paquet, Heinz & Gale, PRC (2015)

Effect on photons of viscous rate correction

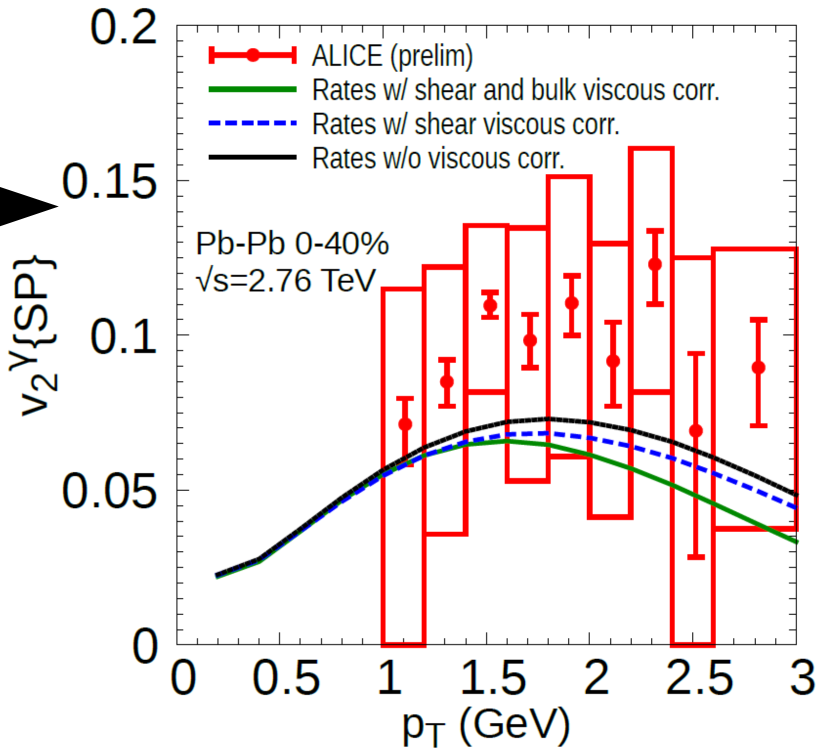
$$k\frac{d^3\Gamma_\gamma}{d\boldsymbol{k}}=k\frac{d^3\Gamma_\gamma^{ideal}}{d\boldsymbol{k}}+\underbrace{\pi^{\mu\nu}K_\mu K_\nu d\Gamma_\gamma^{viscous,shear}+\Pi d\Gamma_\gamma^{viscous,bulk}}$$

Effect on photon spectra and v_2?

Spectra: small effect

v_2: decrease at high p_T →

(consistent with effect on hadrons)



Summary: "thermal" EM probes

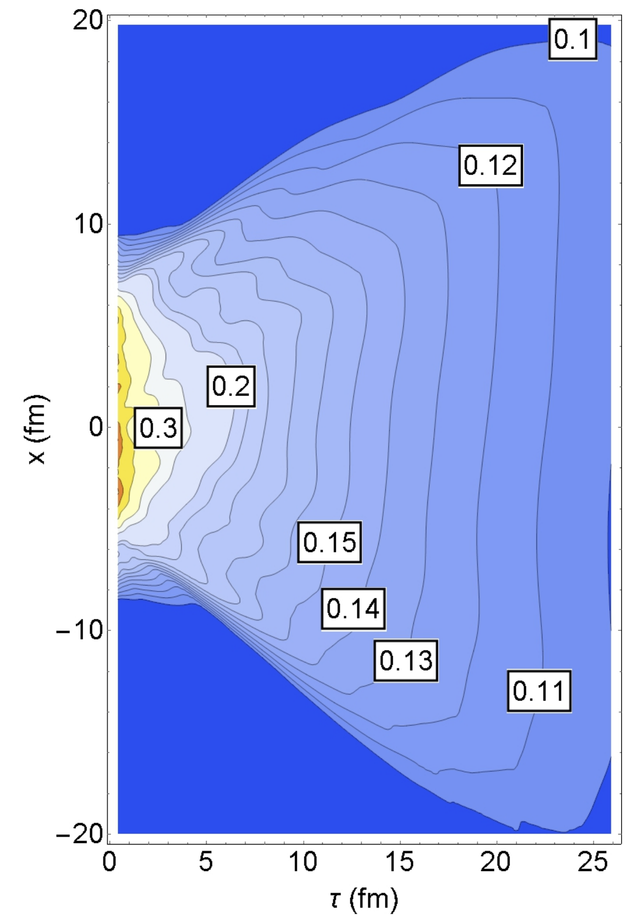
$$E\frac{d^3N}{d\mathbf{k}} = \int d^4X\, E\frac{d^3\Gamma}{d\mathbf{k}}\left(K^\mu, u^\mu(X), T(X), \pi^{\mu\nu}(X), \Pi(X)\right)$$

T **Emission rate**

Effective hadronic models

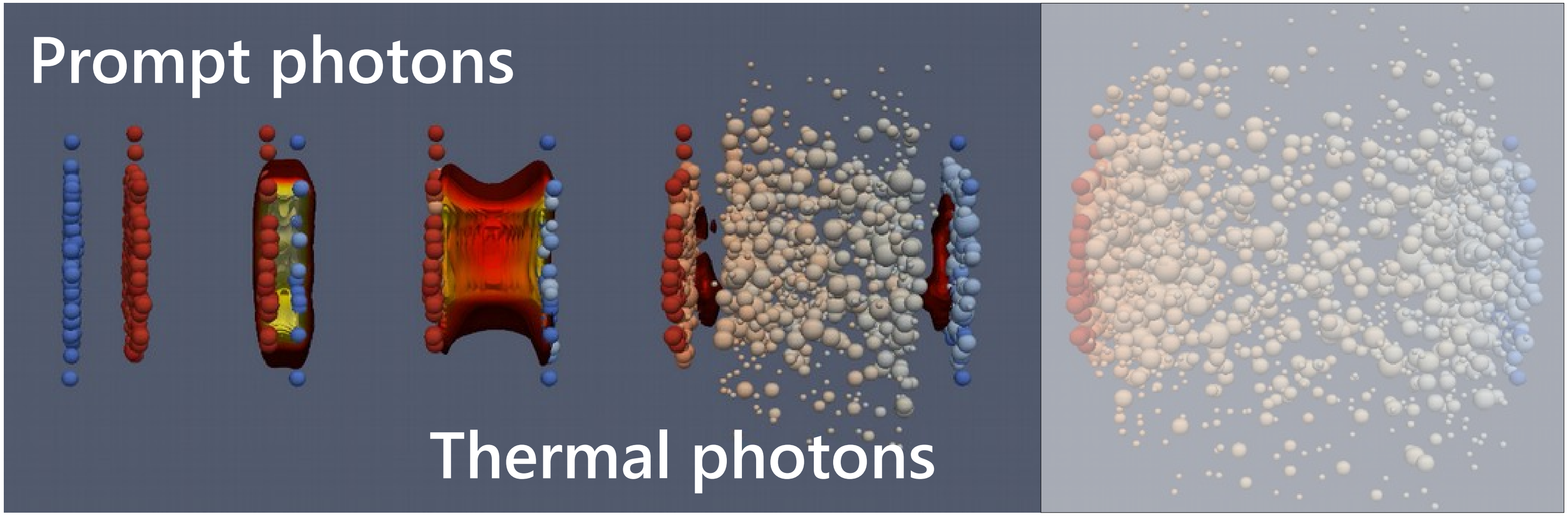
$$k\frac{d^3\Gamma_{\gamma/\gamma^*}}{d\boldsymbol{k}} = k\frac{d^3\Gamma^{ideal}_{\gamma/\gamma^*}}{d\boldsymbol{k}} + \pi^{\mu\nu}K_\mu K_\nu\, d\Gamma^{viscous,shear}_{\gamma/\gamma^*} + \Pi\, d\Gamma^{viscous,bulk}_{\gamma/\gamma^*}$$

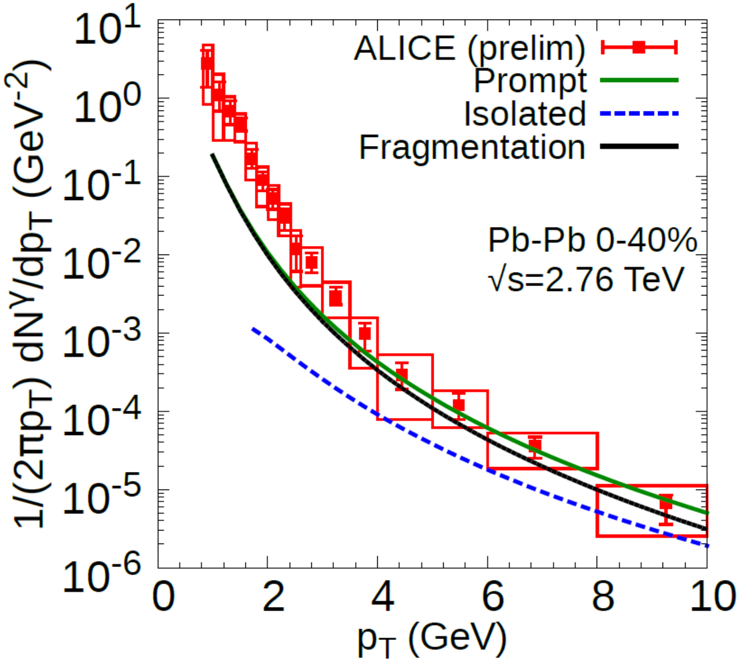
Perturbative QCD

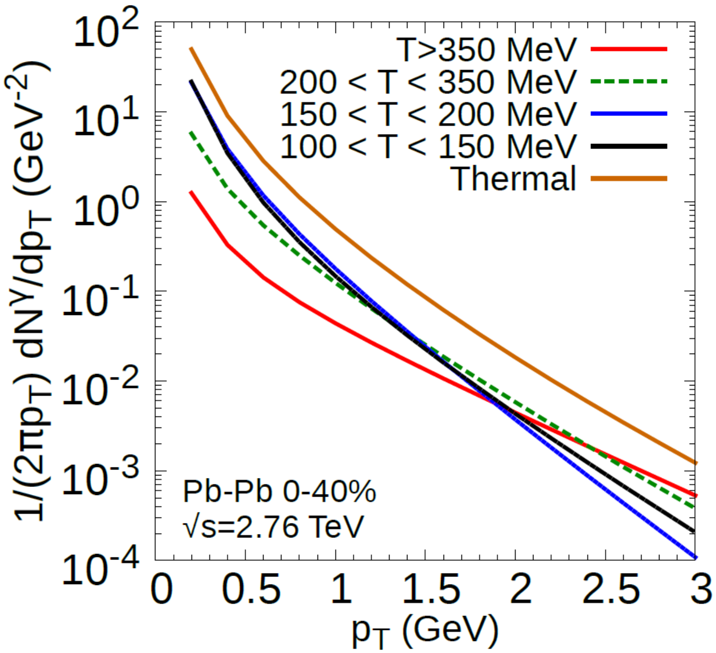


Prompt photons

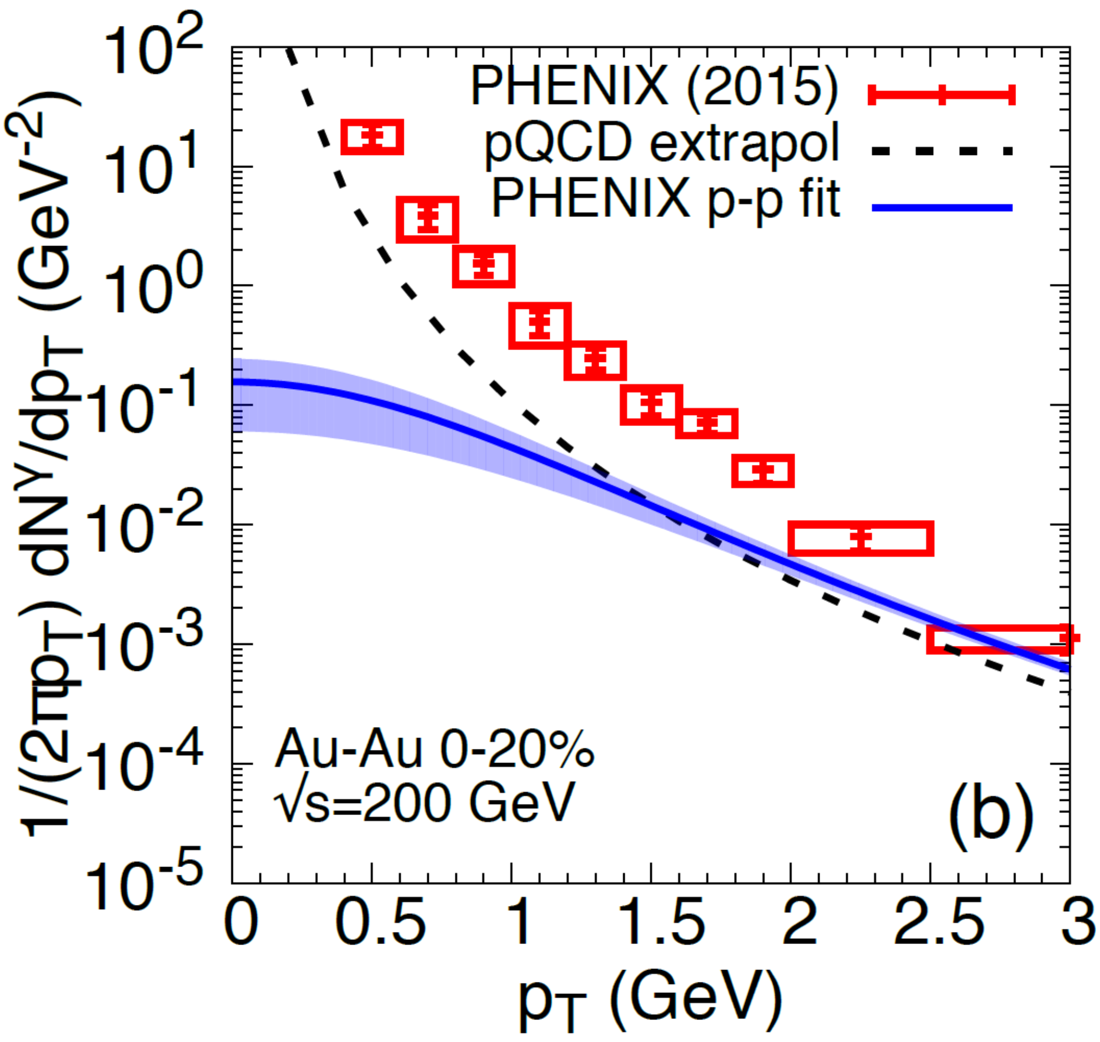
Prompt and thermal photons



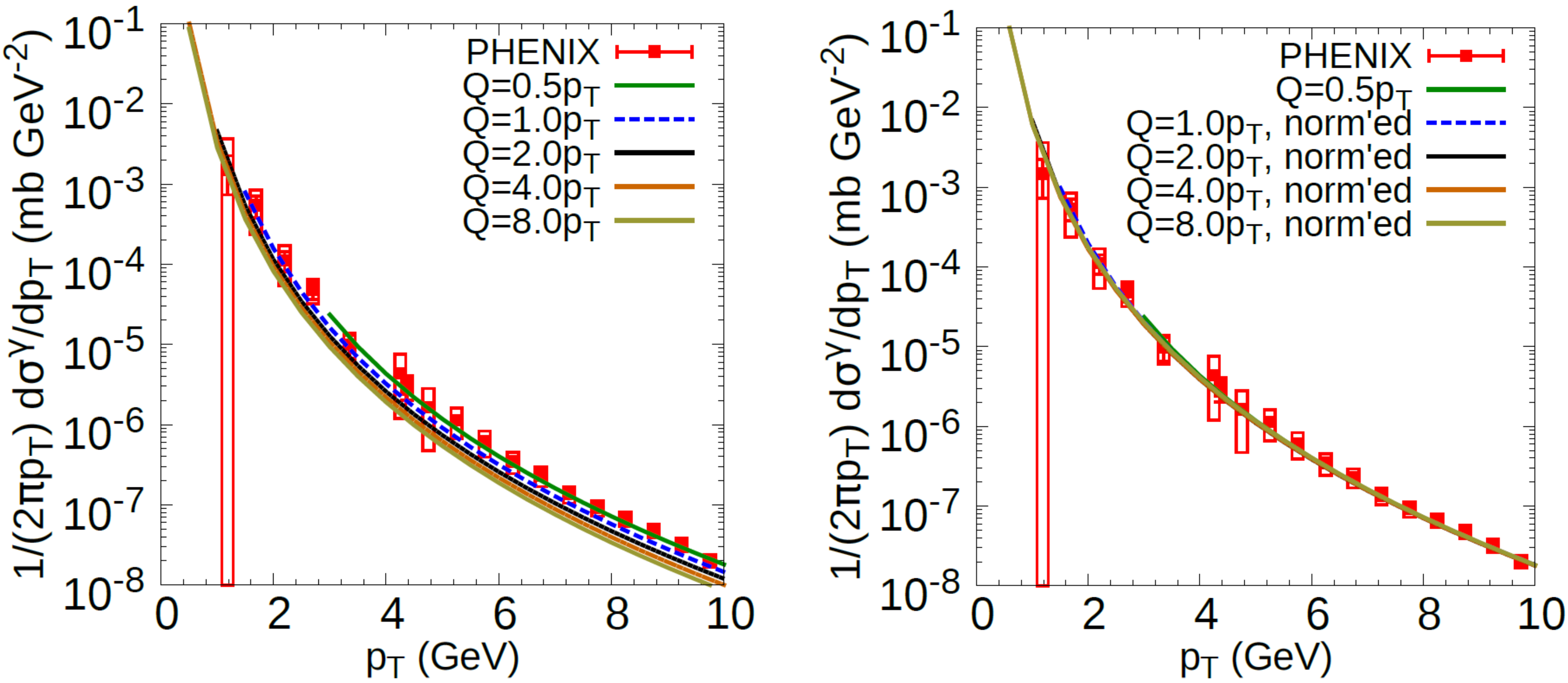




Prompt photon reference?

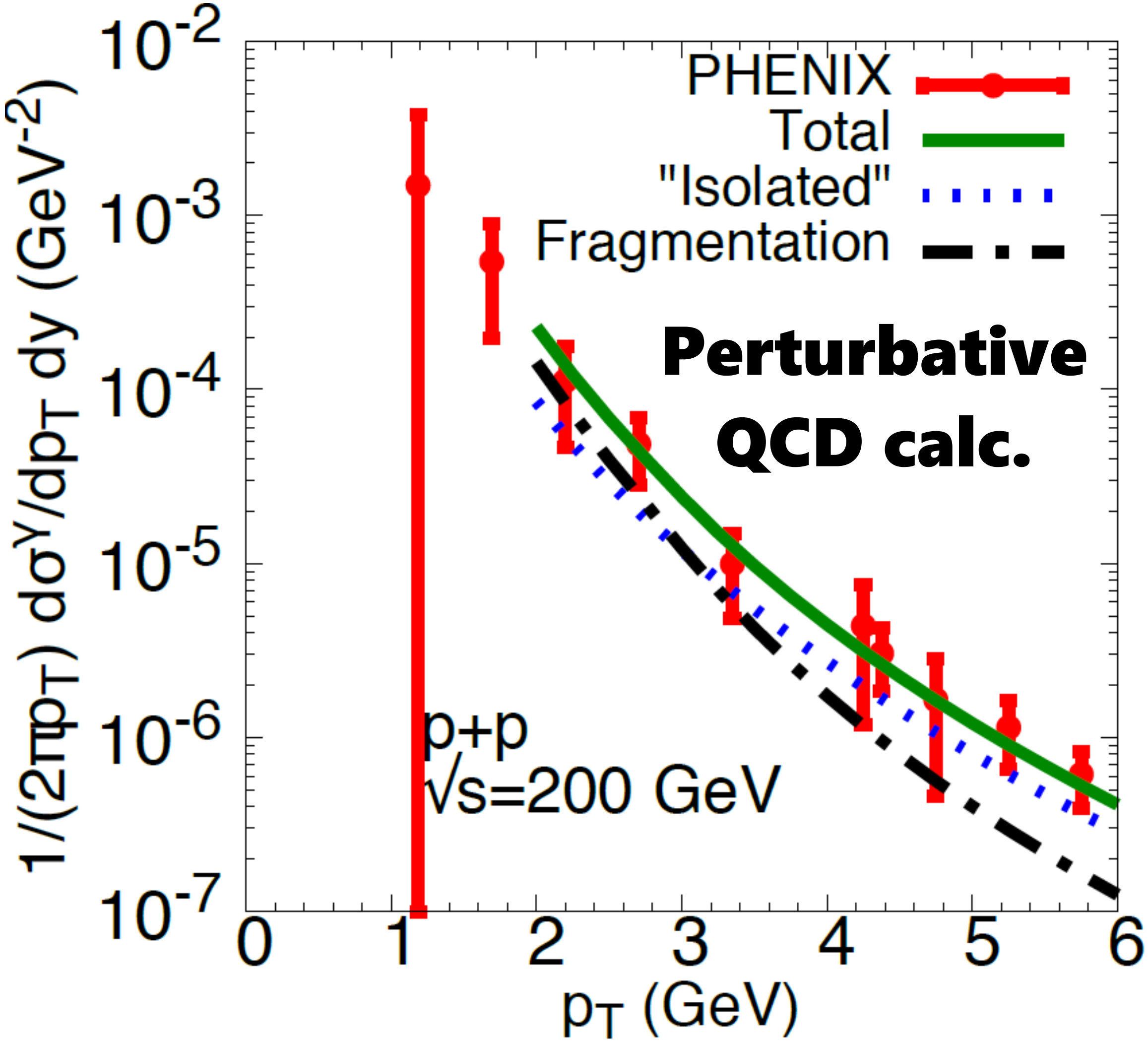


Prompt photons

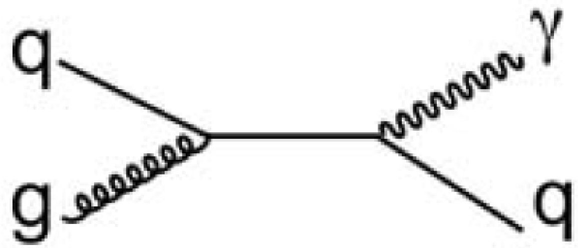


$$E\frac{d^3\sigma_{pp}}{d\mathbf{p}} = \sum_{a,b,c,d} f_{a/p}(x_a, Q_{fact}) \otimes f_{b/p}(x_b, Q_{fact}) \otimes d\hat{\sigma}(Q_{ren}) \otimes D_{\gamma/c}(z_c, Q_{frag})$$

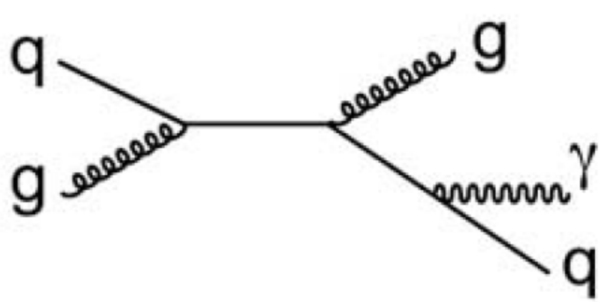
Prompt photons in vacuum



"Isolated" photons



Fragmentation γ



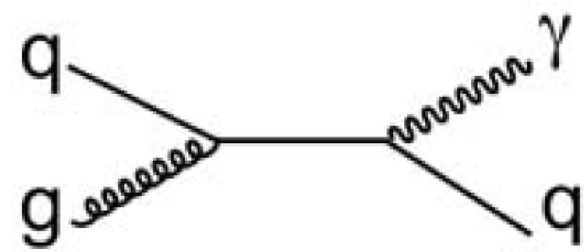
Note: At LHC, fragmentation photons increasingly dominant at low p_T

(Figure credits: Stankus, ARNPS 2005)

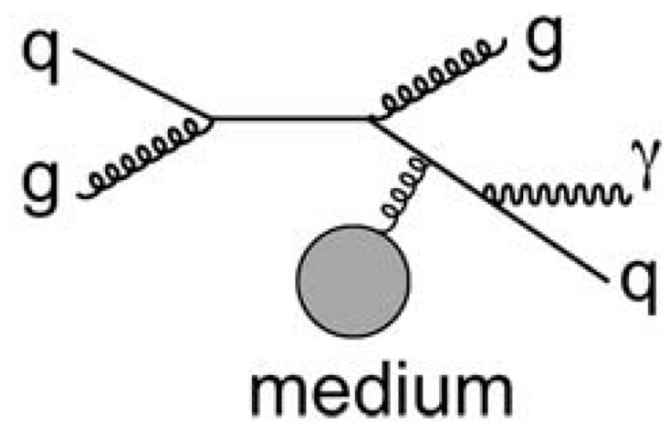
Prompt photons in medium

Parton energy loss =
Final parton less energetic =
Less photons from fragmentation
& positive v_2

"Isolated" photons



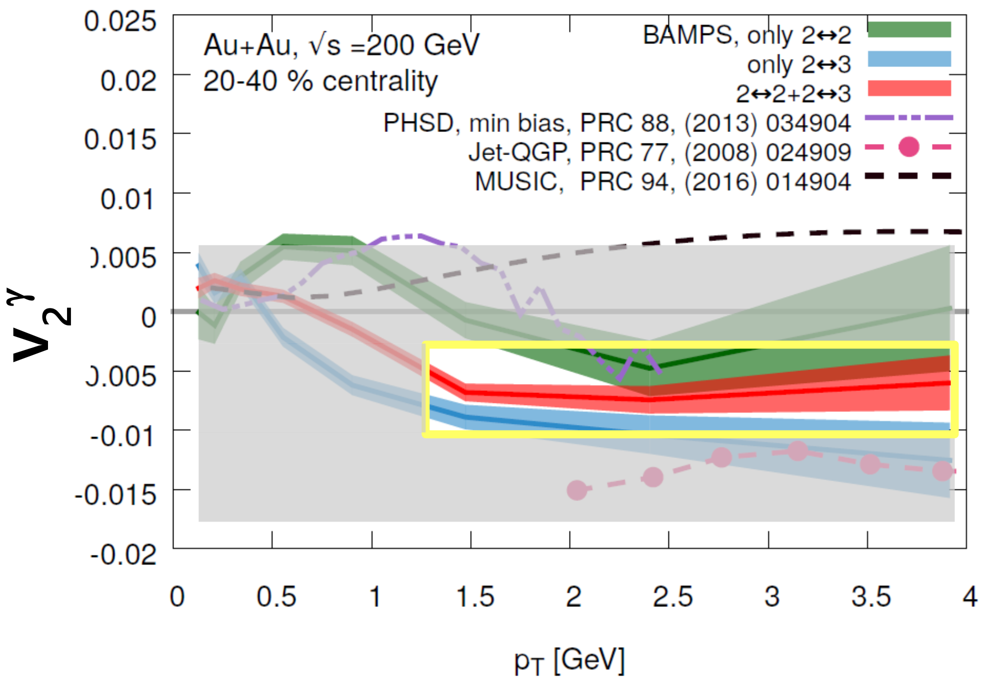
Fragmentation photon energy loss and jet-medium photons



Parton energy loss =
Medium-induced photon emission=
More photons from final showering
& negative v_2

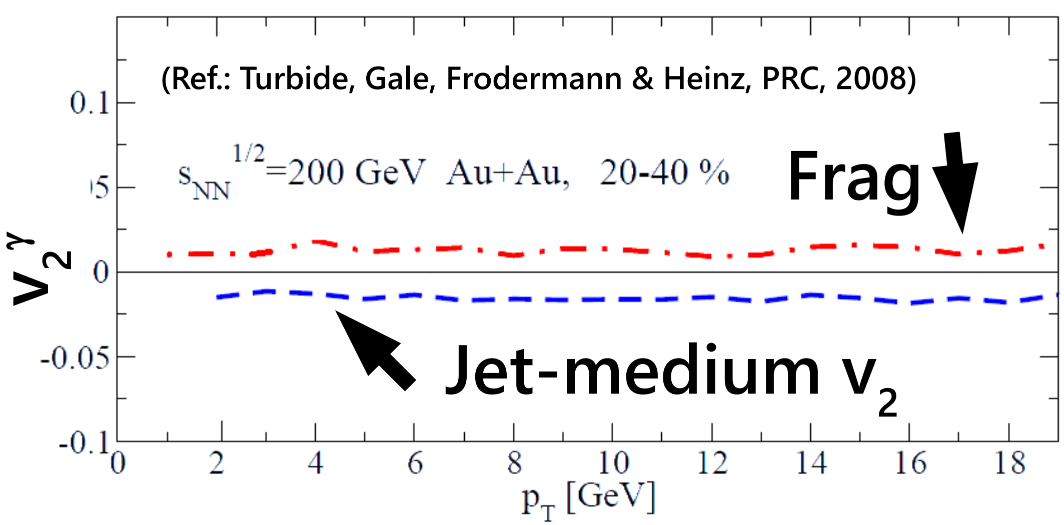
(Figure credits: Stankus, ARNPS 2005)

Jet-medium photon v_2

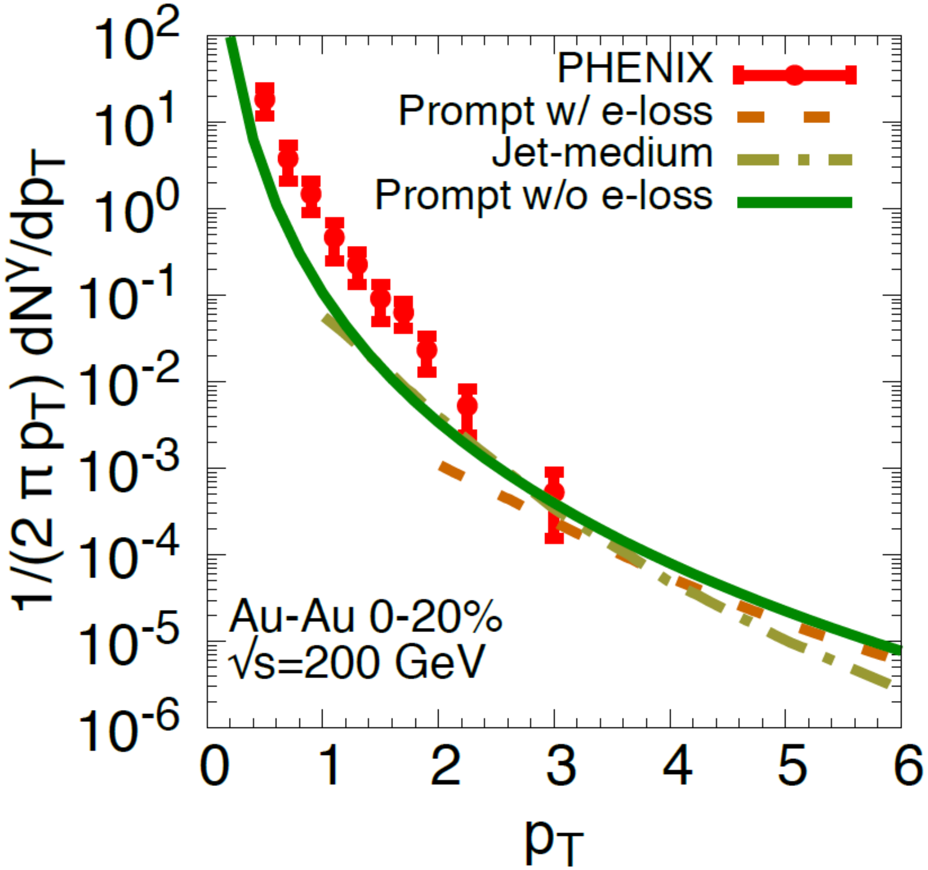


Small negative v_2 of <u>jet-medium photons</u> from BAMPS (v_2~0.5-1% in 20-40% centrality)

Quantitatively similar to Turbide et al (2008)

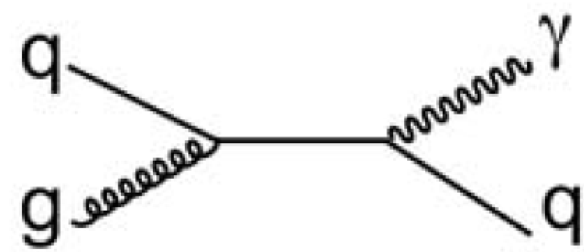


Prompt photons in medium



(Prompt w/ e-loss and jet-medium calculations from Turbide, Gale, Frodermann & Heinz, PRC, 2008)

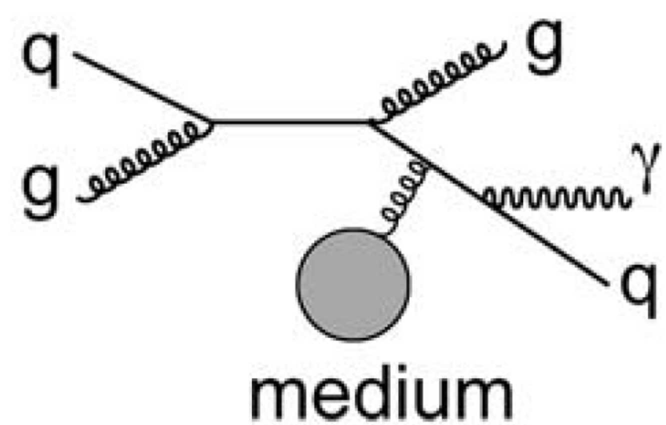
"Isolated" photons



Fragmentation photon energy loss

and

jet-medium photons



Also: can this be investigated with ~low p_T photons with isolated cuts?

Thermal photons

Evaluating the EM emission rate

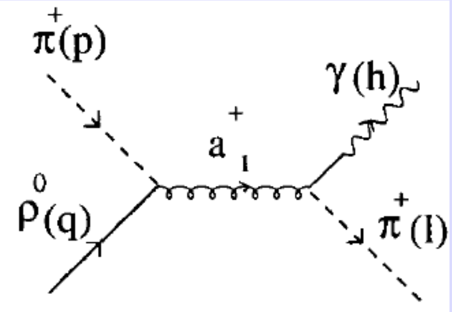
QCD plasma at low temperature: effective hadronic models

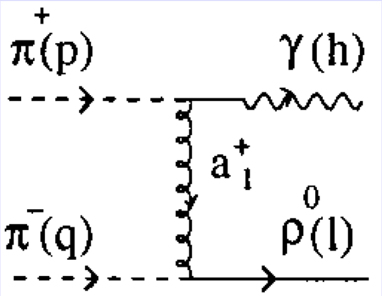
Example: Turbide, Rapp and Gale (2003)

Photon production of a gas of (π, K, ρ, K*, a_1) mesons

$$k\frac{d^3\Gamma_\gamma}{d\mathbf{k}} = \frac{1}{2(2\pi)^3}\int\frac{d^3p_1}{2P_1^0(2\pi)^3}\frac{d^3p_2}{2P_2^0(2\pi)^3}\frac{d^3p_3}{2P_3^0(2\pi)^3}(2\pi)^4\delta^4(P_1+P_2-P_3-K)|\mathcal{M}|^2$$
$$\times f_{B/F}(P_1)f_{B/F}(P_2)(1+\sigma_{B/F}f_{B/F}(P_3))$$

Evaluated from effective Lagrangian fitted to hadronic data





Thermal photon emission rate

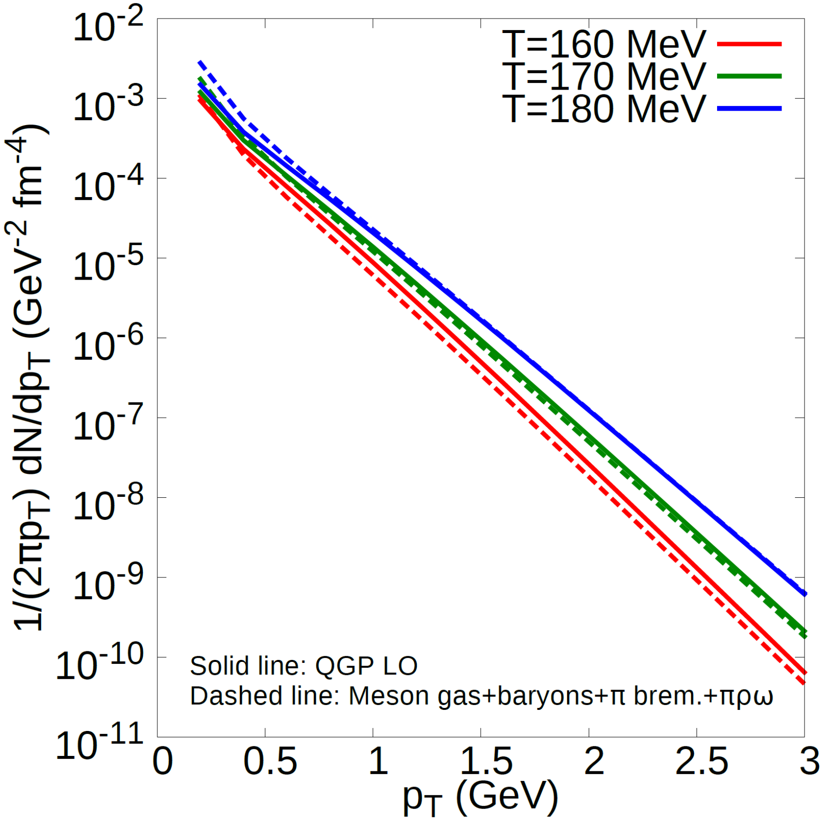
less hot (hadronic D.O.F.) **Temperature** *very hot (QGP)*

Effective Lagrangian

(Switch at 180 MeV)

Perturbative expansion in α_s

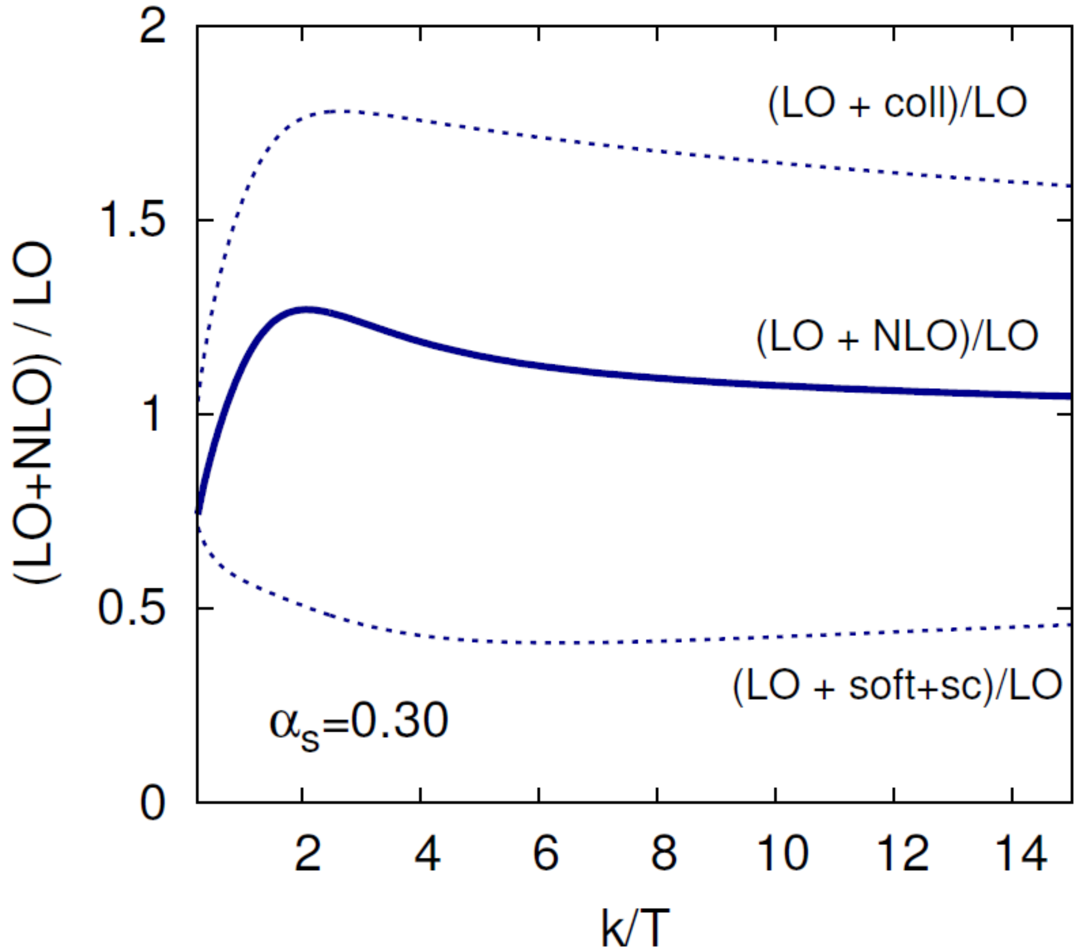
Texas A&M/McGill (Turbide, Rapp, Gale et al)



QGP LO (Arnold, Moore, Yaffe. 2002)

Perturbative rate at LO and NLO

(Ref.: Ghiglieri, Hong, Kurkela, Lu, Moore & Teaney, JHEP, 2013)



NLO (g_s^3) correction to photon rate is <u>small</u>

(Unlike for e.g. heavy quark energy loss and shear viscosity)

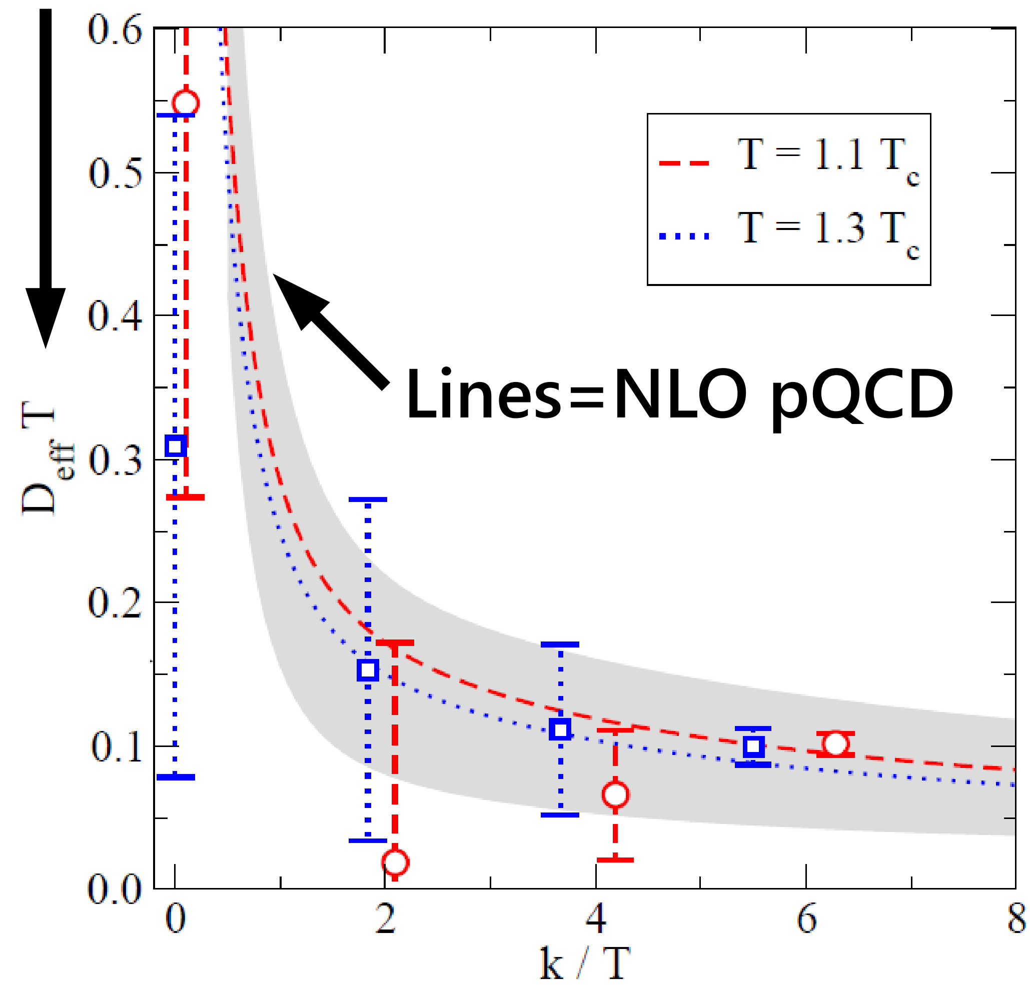
NLO="LO"+"collinear (coll)"+"soft"+"semi-collinear (sc)"

Perturbative rate vs lattice

Perturbative photon rates remarkably consistent with lattice results around $T \sim T_c^+$

(note: quarks are quenched in both the lattice and pQCD calculations)

Proportional to photon rate



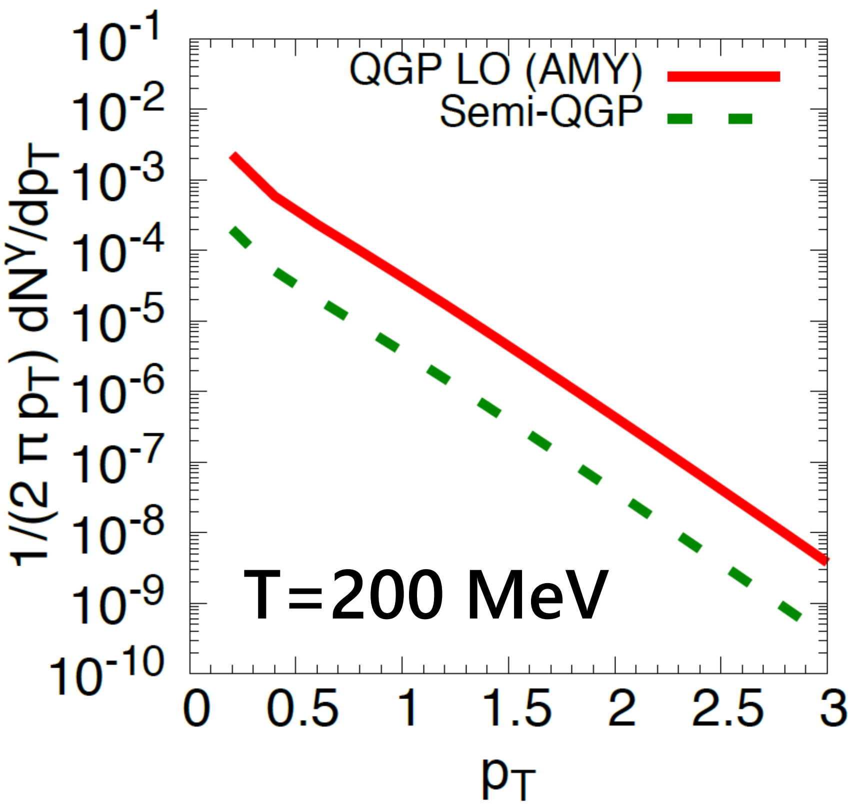
Lines=NLO pQCD

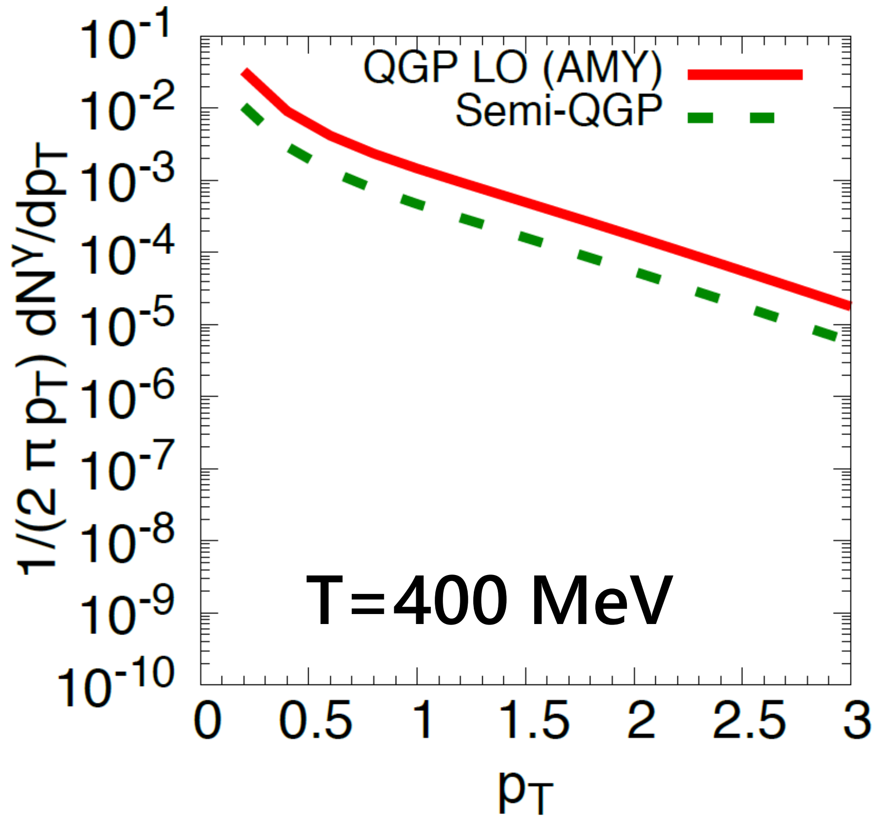
(Ref.: Ghiglieri, Kaczmarek, Laine & Meyer, PRD, 2016)

Other calculations at T~200-400 MeV

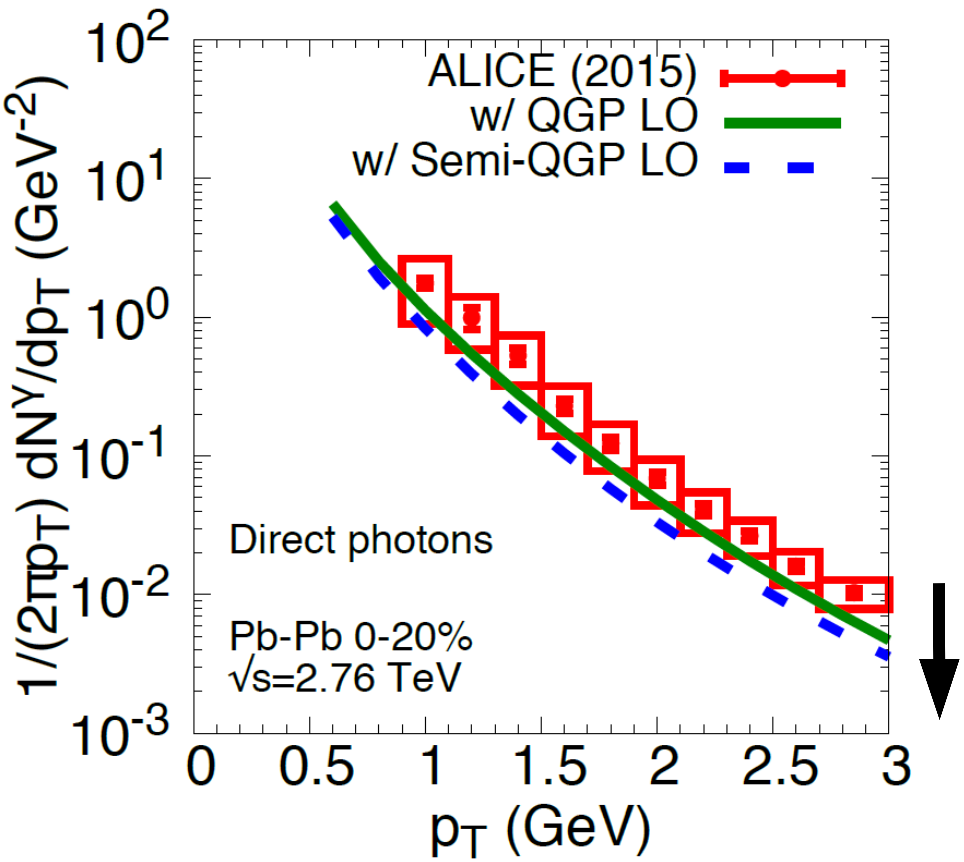
Semi-QGP: Mean field with suppressed Polyakov loop

[Gale et al, PRL, 2015; Hidaka, Lin, Pisarski & Satow, JHEP, 2015]

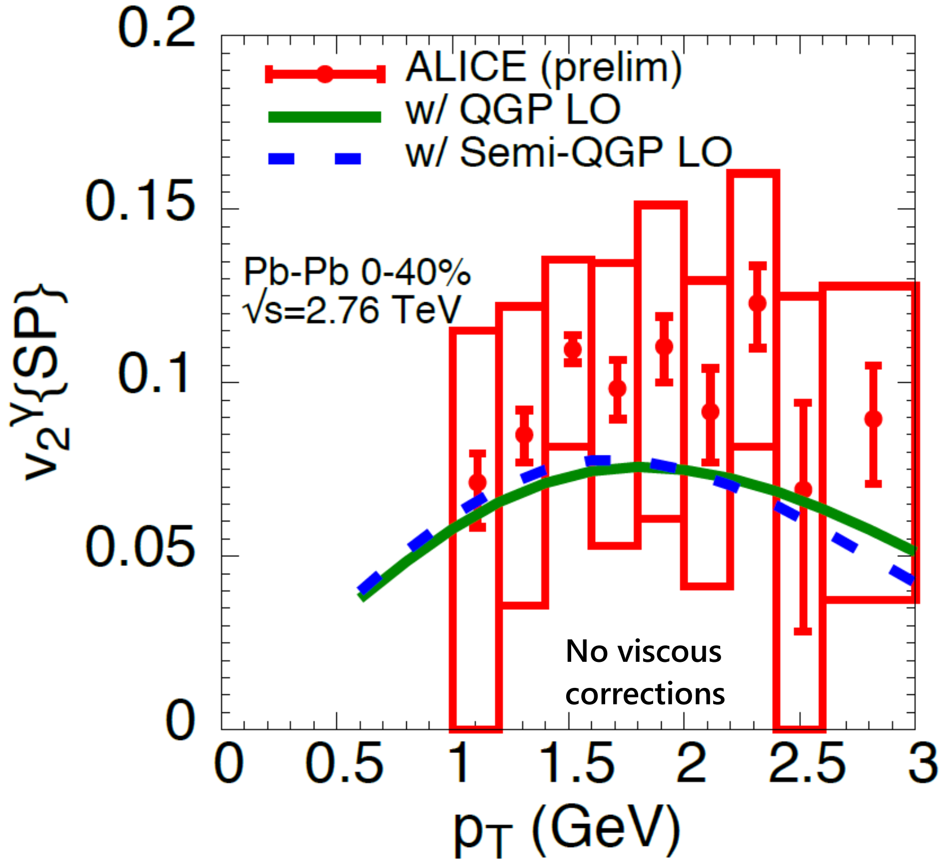




Effect of suppressed QGP rate



Spectra suppressed



Small effect on v_2 (counterbalance of thermal and prompt photons)

Hydrodynamics flow and boost

Thermal rate is (almost always) evaluated in rest frame

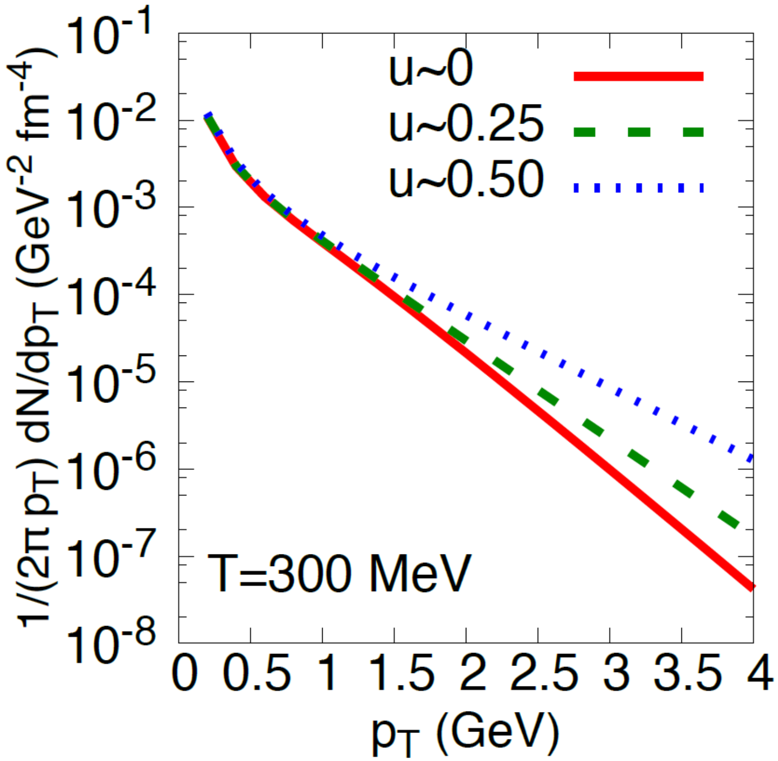
$$k\frac{d^3\Gamma_\gamma}{d\mathbf{k}} = \frac{1}{2(2\pi)^3}\int\frac{d^3p_1}{2P_1^0(2\pi)^3}\frac{d^3p_2}{2P_2^0(2\pi)^3}\frac{d^3p_3}{2P_3^0(2\pi)^3}(2\pi)^4\delta^4(P_1+P_2-P_3-K)|\mathcal{M}|^2 \times f_{B/F}(P_1)f_{B/F}(P_2)(1+\sigma_{B/F}f_{B/F}(P_3))$$

Rate in other frames?

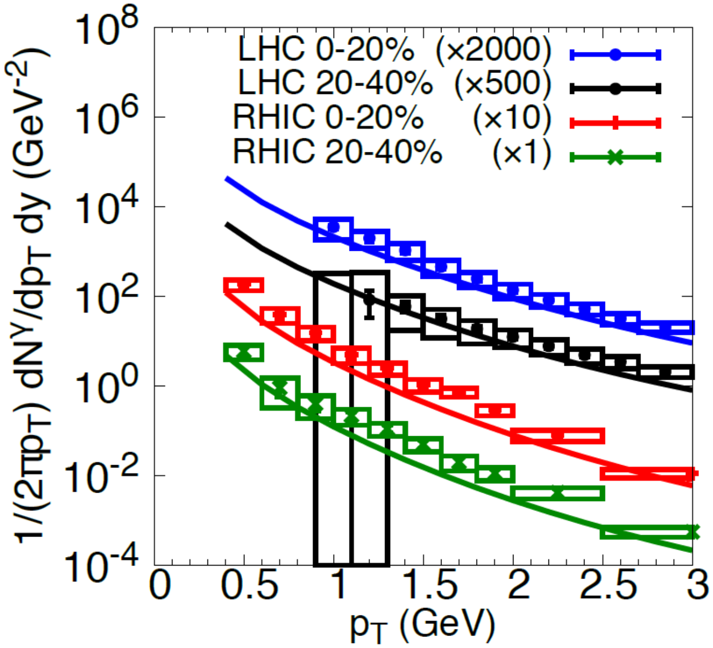
Just boost (Lorentz transform):

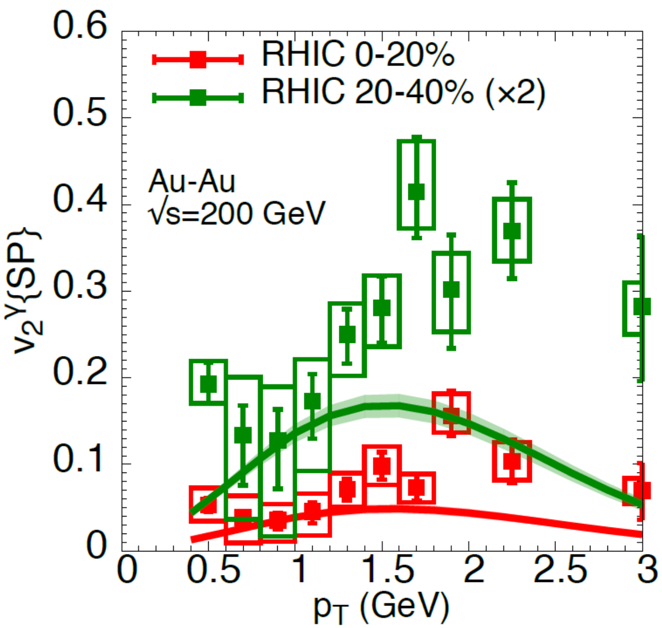
$$k\frac{d^3\Gamma}{d\boldsymbol{k}}(k,\boldsymbol{u})=k\frac{d^3\Gamma}{d\boldsymbol{k}}(k=K\cdot\boldsymbol{u},\boldsymbol{u}=\mathbf{0})$$

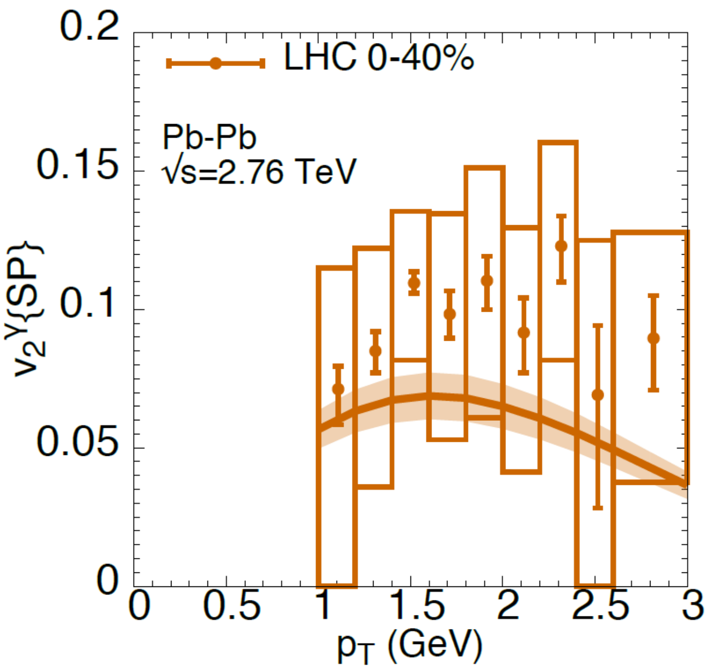
Hydro flow velocity



Centrality dependence







Relativistic viscous hydrodynamics

Bulk pressure

Shear stress tensor

$$\partial_\mu T^{\mu\nu}(X) = 0$$

$$T^{\mu\nu}(X) = \epsilon(X) u^\mu(X) u^\nu(X) - [\mathcal{P}(X) + \Pi(X)]\,\Delta^{\mu\nu}(X) + \pi^{\mu\nu}(X)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{K} + \mathcal{R} + \mathcal{J}$$

$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta}\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \mathcal{K}^{\mu\nu} + \mathcal{R}^{\mu\nu} + \mathcal{J}^{\mu\nu}$$

The bulk viscosity ζ and the shear viscosity η characterizes the quark-gluon plasma's response to deviation from equilibrium