3rd Workshop

20-22 March 2017

Stretched-wire systems for the magnetic measurement of small-aperture magnets

Domenico Caiazza

Outline

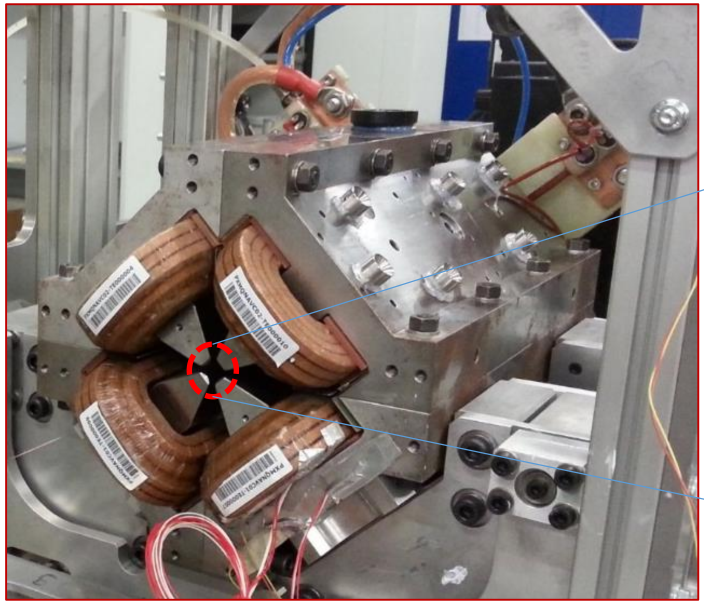
- INTRODUCTION
  - magnet characterization and wire methods
- OBJECTIVES
- PART I - Performance enhancement for magnet alignment
  - Background fields
  - Multipole field error effects
  - Random errors, sensitivity, nonlinearities
- PART II - Field strength, locating the magnet, field harmonics
- SUMMARY

INTRODUCTION

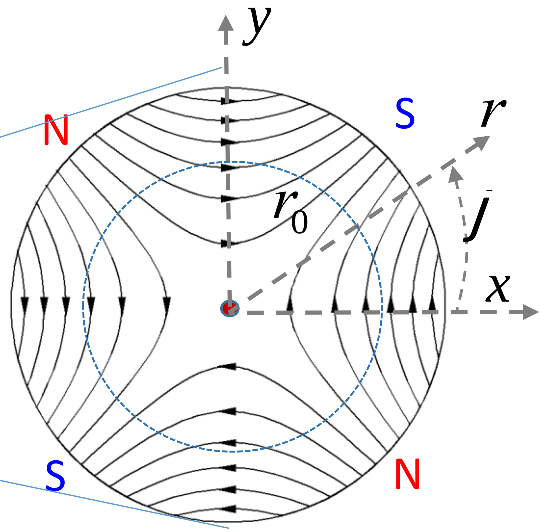
Magnet characterization

2-D formulation

- Integration on the entire magnet length

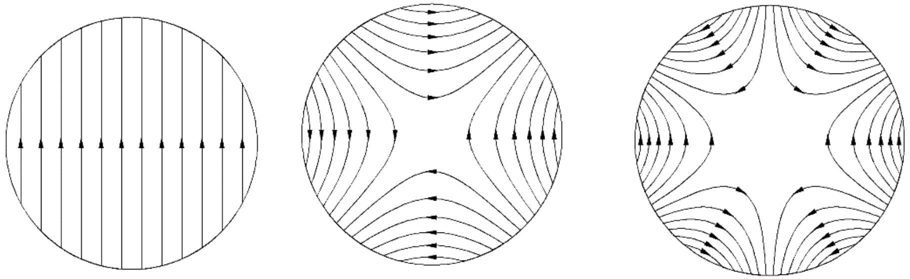


CLIC quadrupole
ϕ10 mm



Multipole field model

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{x + iy}{r_0} \right)^{n-1}$$



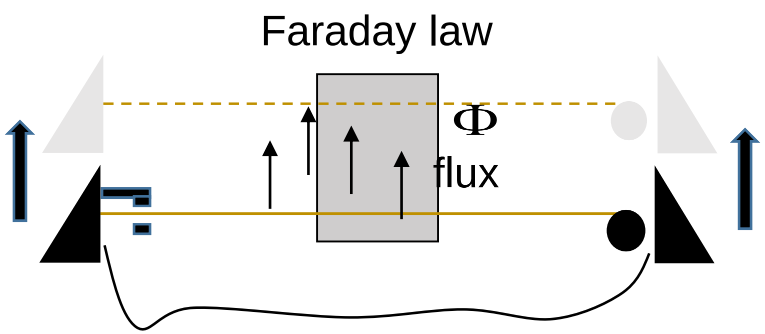
DIPOLE (*n*=1) QUADRUPOLE (*n*=2) SEXTUPOLE (*n*=3)

Measurements

- ✓ Magnetic axis: locus of points with zero field
- ✓ Magnetic field strength and direction (roll angle)
- ✓ Magnetic field quality: harmonic content

Wire methods

Translating wire

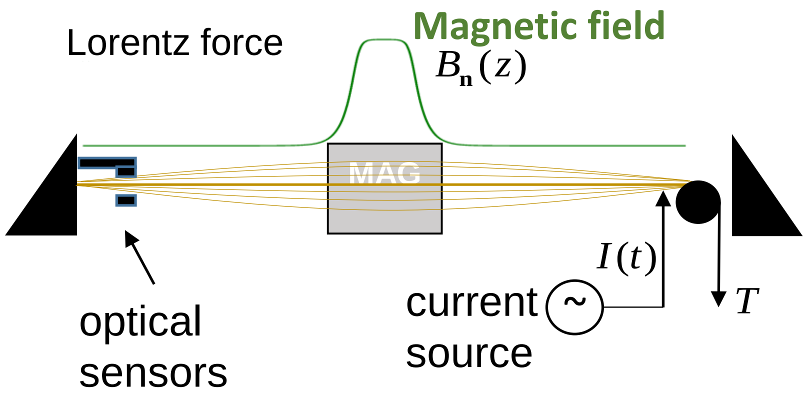


Measurements for LHC magnets

- ✓ integrated field strength (10^{-5} uncertainty)
- ✓ field direction
- ✓ magnetic axis (50-100 μm)

J. Di Marco et al., "Field alignment of quadrupole magnets for the LHC interaction Regions". *IEEE Transactions on Applied Superconductivity*, 2000.

Oscillating/vibrating wire



- ✓ High sensitivity also for low field and small apertures
- ✓ Used in particular for alignment and magnetic field quality (multipoles)

- **A. Temnykh**. "Vibrating wire field-measuring technique". *Nuclear Instruments and Methods in Physics Research,* **1997**.
- **P. Arpaia, M. Buzio, J. G. Perez, C. Petrone, S. Russenschuck, L. Walckiers**. "Measuring field multipoles in accelerator magnets with small-aperture by an oscillating moved on a circular trajectory". *JINST – Journal of Instrumentation,* **2012**.

Objectives

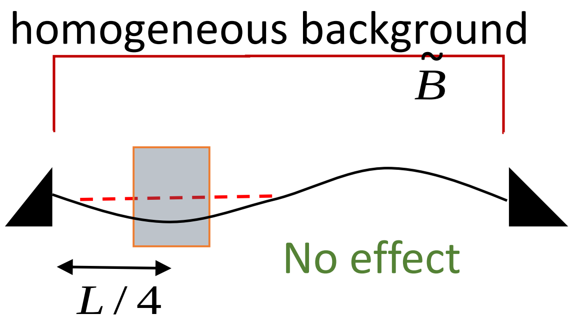
- Starting from state-of-the-art systems
- Design and implementation of methods for the enhancement of the metrological performance
- Development of a new measurement station for the experimental validation of these methods in the frame of PACMAN

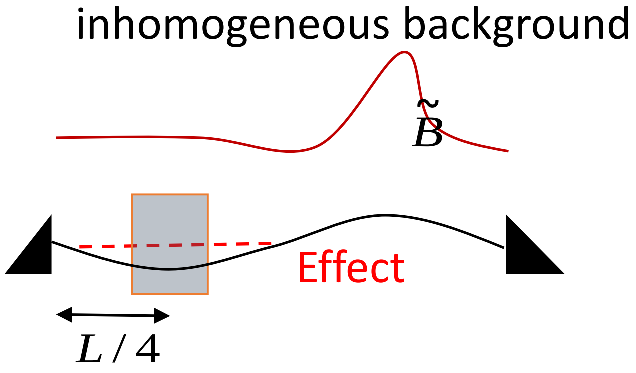
PART I - PERFORMANCE ENHANCEMENT FOR ALIGNMENT

Background fields

Background field influence\ Problem

- Background field: Earth magnetic field, stray field from equipment
- The wire senses magnet field + background
- State of the art solutions
  - Rotating the magnet and averaging measurements
  - Displacing the magnet at L/4

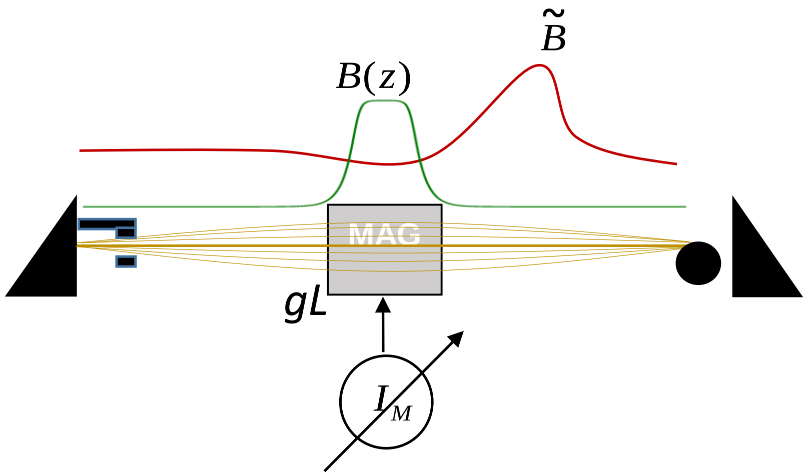




- PACMAN constraints:
  - Magnet rotation not practicable
  - Limited space: CMM table length is 1.2 m
  - No magnet cooling → low power magnet excitation →background effect amplification (several tens of microns)

Background field correction\ Proposal

- A correction method based on varying the magnet excitation
- The measured axis moves as a function of the magnet strength



Model for the translating wire

$$x_{\mathrm{a}} = x_{\mathrm{c}} - \frac{\tilde{\Phi}(x, \bar{y})}{x \bar{g} L}$$

Model for the vibrating wire

$$x_{\mathrm{a}} = x_{\mathrm{c}} - \frac{\int_0^L \tilde{B}_y(z) \sin\left(\frac{m\pi}{L} z\right) \mathrm{d}z}{\int_0^L g(z) \sin\left(\frac{m\pi}{L} z\right) \mathrm{d}z}$$

x_a: apparent axis coordinate
x_c: actual center coordinate
$\bar{g}L$: magnet strength
x_a-x_c: error

Core idea
- ➢ measure the magnetic axis and strength for different magnet excitations
- ➢ Fit the measurements to the model

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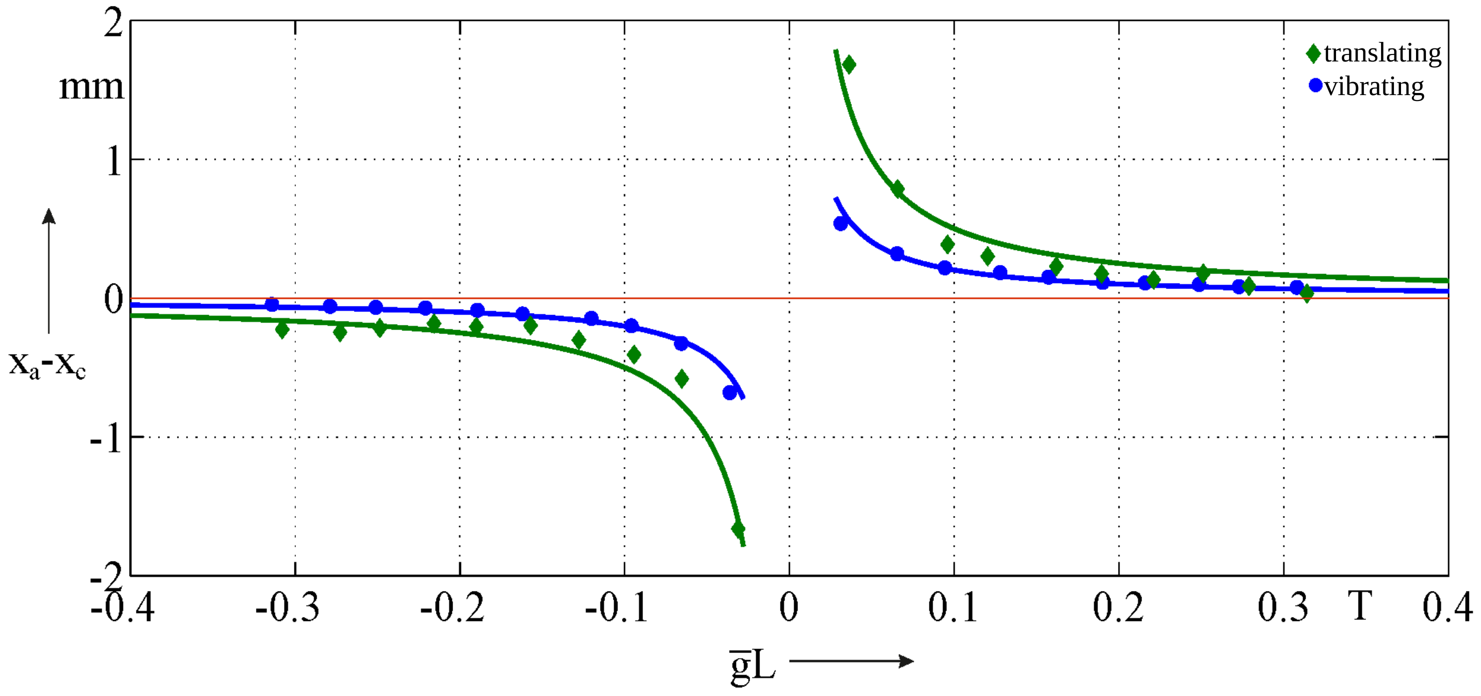
Background field correction\ Results

Experimental validation: locating the magnetic axis of a quadrupole magnet

Apparent – actual magnetic center



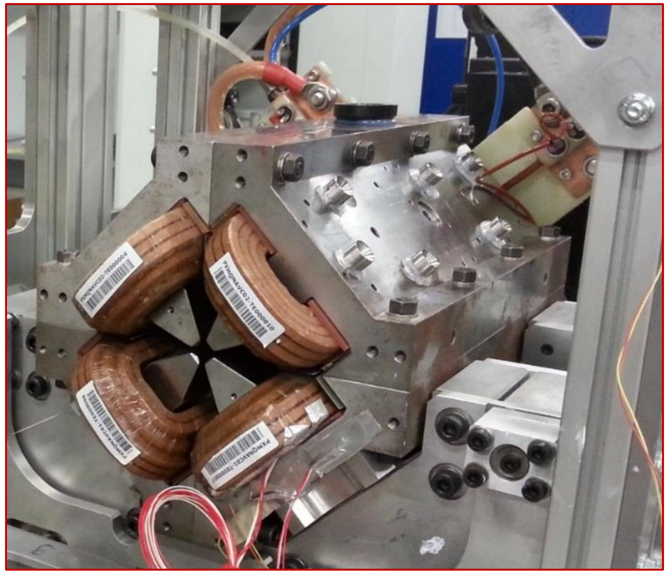
Translating and vibrating wire:

- different sensitivity to background
- Correction amount:
  - translating wire: 227 µm at 0.31 T
  - vibrating wire: 71 µm at 0.31 T
- The model coefficients are suitable for any magnet if the system configuration is not changed

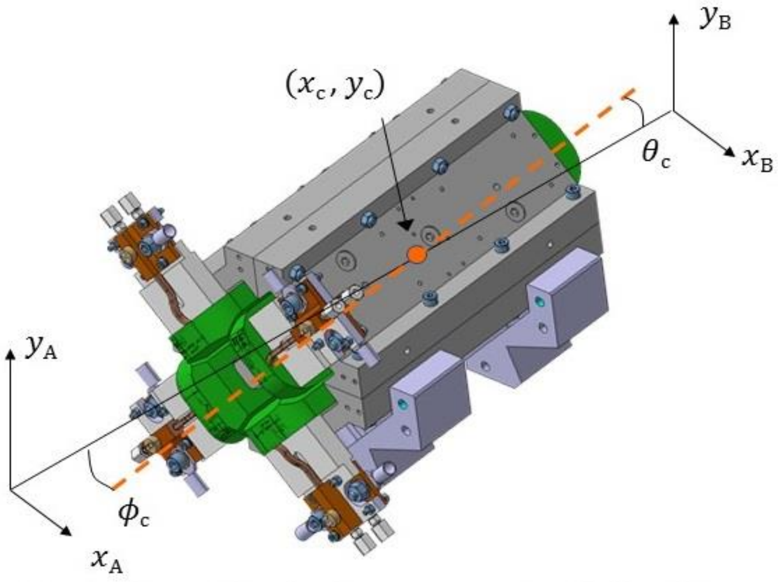
P. Arpaia, D. Caiazza, C. Petrone, S. Russenschuck. "Background and multipole field effects in locating quadrupole magnetic axes". *In review phase.*

Background field correction\ Results

Experimental validation on the CLIC quadrupole



CLIC main beam quadrupole



Measurements taken at different magnet currents and referred to the axis at nominal gradient
(after correction)

Magnet current	Integrated gradient	x_c	y_c	ϕ_c	θ_c
126	70.61	0	0	0	0
65	40.91	3.8	-0.9	0.9	4.6
4	3.64	2.9	3.1	-2.3	-5.1
A	T	µm	µm	µrad	µrad

Repeatability

- Within ±0.2 µm for the centers
- Within ±0.9 µrad for the angles
- Also at 4 A

PART I - PERFORMANCE ENHANCEMENT FOR ALIGNMENT

Multipole field error effects

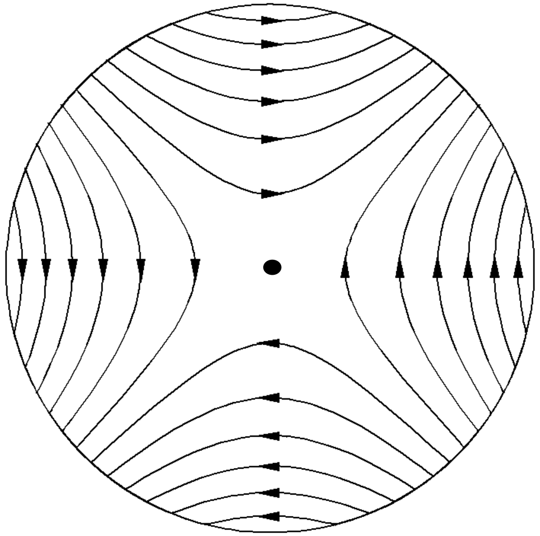
Multipole field error effect/ Problem

Real magnets contain multipole components

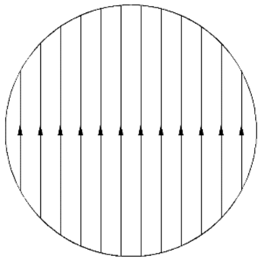
- Related to the symmetry (finite poles)
- Related to construction defects

Multipole field model

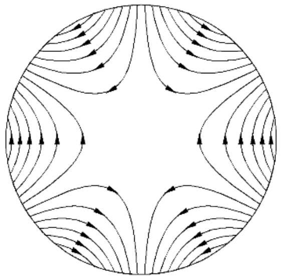
$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{x+iy}{r_0} \right)^{n-1}$$



QUADRUPOLE (n=2)



DIPOLE (n=1)



SEXTUPOLE (n=3)

…

The magnetic axis is found by assuming just the quadrupole component …

➢ Assess the error due to multipole components

Multipole field error effect/ Proposal

Use the multipole field model to estimate the effect

Error on the measured center due to multipole fields

Translating-wire method

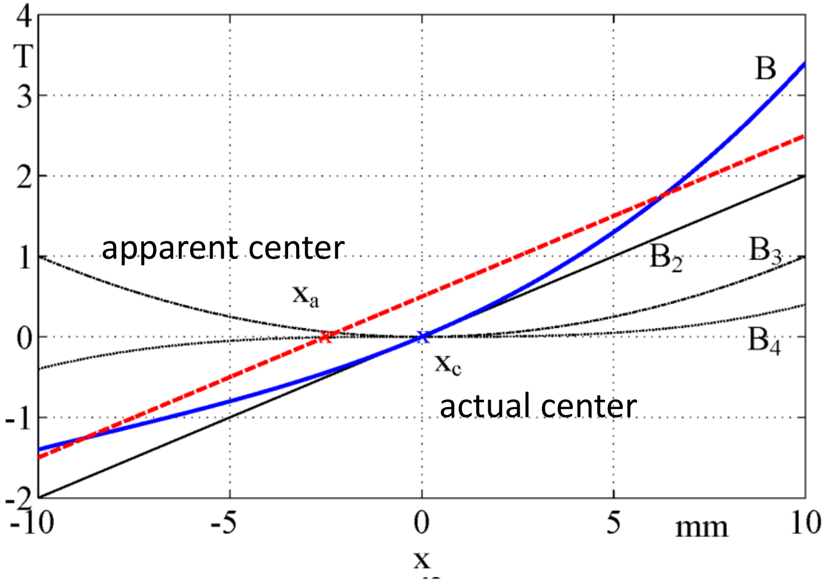
$$x_a = -\frac{x}{2}\frac{\sum_{n=2}^{\infty}\frac{1}{nr_0^{n-1}}\Re\{(B_n+iA_n)\sum_{k=1,3,5,\ldots}^{n}\binom{n}{k}(-x_c-iy_c)^{n-k}x^k)\}}{\sum_{n=2}^{\infty}\frac{1}{nr_0^{n-1}}\Re\{(B_n+iA_n)\sum_{k=2,4,6,\ldots}^{n}\binom{n}{k}(-x_c-iy_c)^{n-k}x^k)\}}$$

Different sensitivity

Vibrating-wire method

$$x_a = -x\frac{\sum_{n=2}^{\infty}\frac{1}{r_0^{n-1}}\Re\{(B_n+iA_n)\sum_{k=0,2,4,\ldots}^{n-1}\binom{n-1}{k}(-x_c-iy_c)^{n-k-1}x^k)\}}{\sum_{n=2}^{\infty}\frac{1}{r_0^{n-1}}\Re\{(B_n+iA_n)\sum_{k=1,3,5,\ldots}^{n-1}\binom{n-1}{k}(-x_c-iy_c)^{n-k-1}x^k)\}}$$

Distortion from multipole fields: calculation example



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Multipole field error correction/ Results

Corrections for translating wire

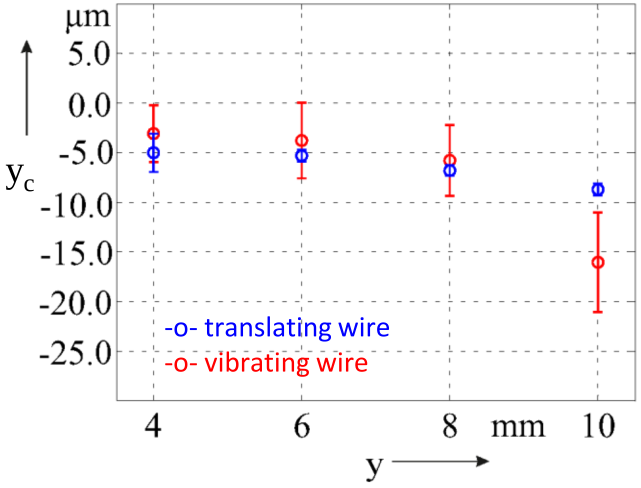
Multipole order	$x_a - x_c$	$y_a - y_c$
3	$b_3 \frac{x^2}{3r_0}$	$-a_3 \frac{y^2}{3r_0}$
4	$(-b_4 x_c + 2a_4 y_c) \frac{x^2}{2r_0^2}$	$(a_4 x_c - 2b_4 y_c) \frac{y^2}{2r_0^2}$
5	$b_5 \frac{x^4}{5r_0^3}$	$-a_5 \frac{y^4}{5r_0^3}$
6	$(-2b_6 x_c + 3a_6 y_c) \frac{x^4}{3r_0^4}$	$(3a_6 x_c - 2b_6 y_c) \frac{y^4}{3r_0^4}$

Corrections for vibrating wire

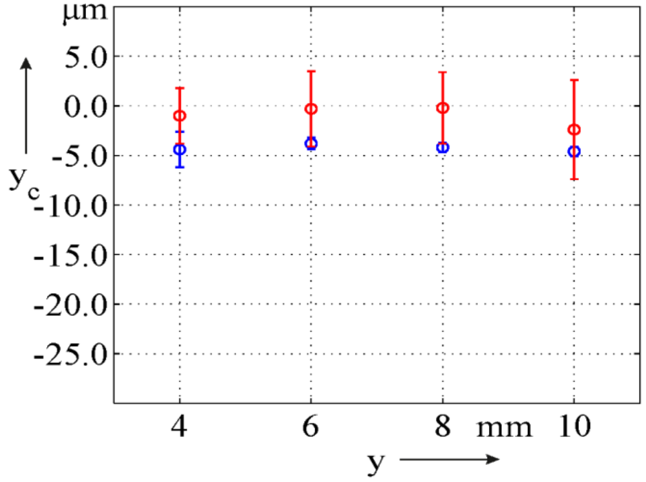
Multipole order	$x_a - x_c$	$y_a - y_c$
3	$b_3 \frac{x^2}{r_0}$	$-a_3 \frac{y^2}{r_0}$
4	$(-2b_4 x_c + 3a_4 y_c) \frac{x^2}{r_0^2}$	$(3a_4 x_c - 2b_4 y_c) \frac{y^2}{r_0^2}$
5	$b_5 \frac{x^4}{r_0^3}$	$-a_5 \frac{y^4}{r_0^3}$
6	$(-4b_6 x_c + 5a_6 y_c) \frac{x^4}{r_0^4}$	$(5a_6 x_c - 4b_6 y_c) \frac{y^4}{r_0^4}$

Experiments on a CLIC quadrupole with 14 units skew sextupole

Before correction



After correction



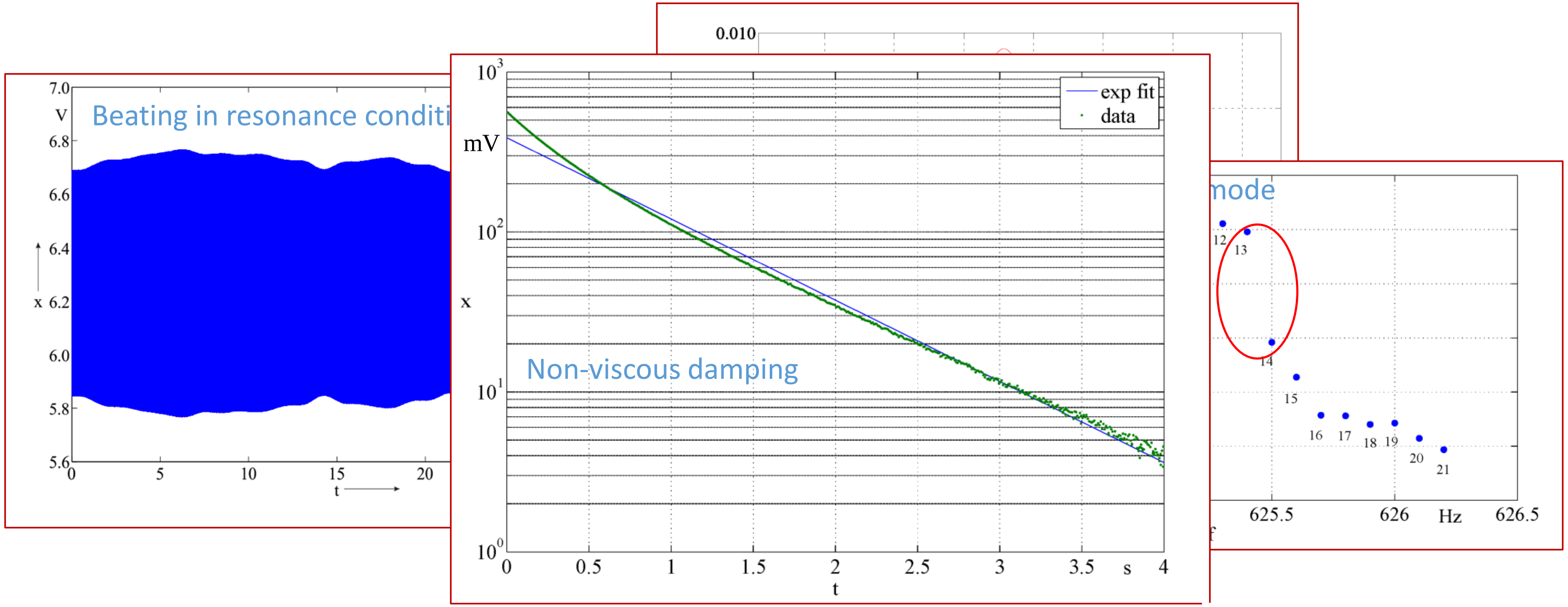
P. Arpaia, D. Caiazza, C. Petrone, S. Russenschuck. "Background and multipole field effects in locating quadrupole magnetic axes". *In review phase.*

PART I - PERFORMANCE ENHANCEMENT FOR ALIGNMENT

Random error, sensitivity, nonlinearity

Random error, sensitivity, nonlinearity
/ Problem

Low frequency resonance peak – fundamental mode



➢ Better working at constant kinematic conditions: decrease the vibration as the frequency increases

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Random error, sensitivity, nonlinearity / Problem formulation & Proposal

Problem:

- Assess the metrological performance as a function of the system configuration (wire length, tension etc.)
- Design improvement aimed at a performance enhancement
  - Best value of a performance index

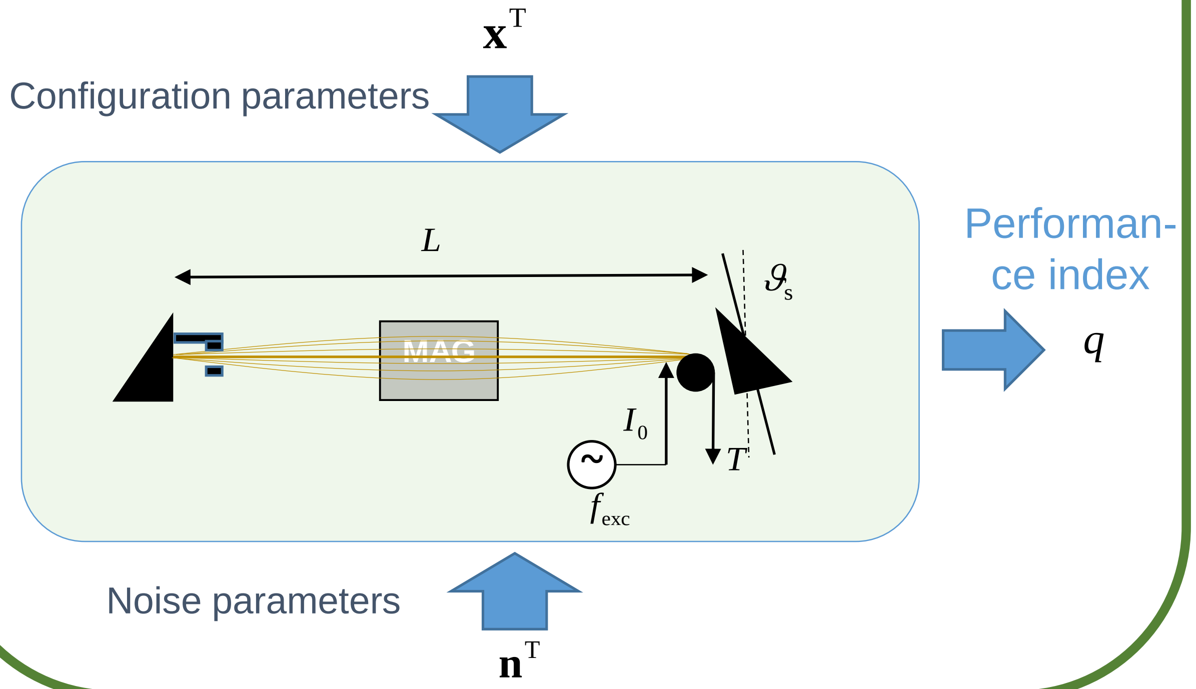
G. Taguchi's Signal-to-Noise ratio

$$q = 10\log_{10}\left(\frac{\theta_S S^2}{\theta_\sigma \sigma_c^2 + \theta_{NL} S_{NL}^2}\right)$$

- *σ_c: 1- σ standard deviation of x_c and y_c*
- *S: the slope of the regression line*
- *σ_{NL}*: squared variance of the linear regression error (residuals)

Proposal

- Sensitivity analysis by experimental design approach
  - extensive measurement campaign at varying the system parameters and analysis by statistical tools



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Random error, sensitivity, nonlinearity

/ Results

Parameter	
Wire length / magnet length	L/L_m
Wire tension	T
Wire current	I_0
Working voltage for optical sensors	V_W
Distance from resonance	Δf_{exc}
Air speed	s_w
Stage angle	ϑ_s

	x	y	m.u.
Predicted performance	38.59	42.06	dB
Observed performance	37.52	41.63	dB
Prediction error variance	20.17	7.64	$(dB)^2$
Confidence interval (2σ)	±8.98	±5.53	dB

- Best value of the performance is associated with
  - ✓ high wire current (50 mA)
  - ✓ High tension (1100 g, 0.125ϕ wire)
  - ✓ Working frequency 1 Hz below resonance
  - ✓ Long wire (L/L_m=20)

Sample experiment: short vs long wire

Repeatability (σ_c)	x_c	y_c	m.u.
1-m wire length	±2.6	±4.6	µm
4-m wire length	±0.9	±1.1	µm

(magnet length 20 cm)

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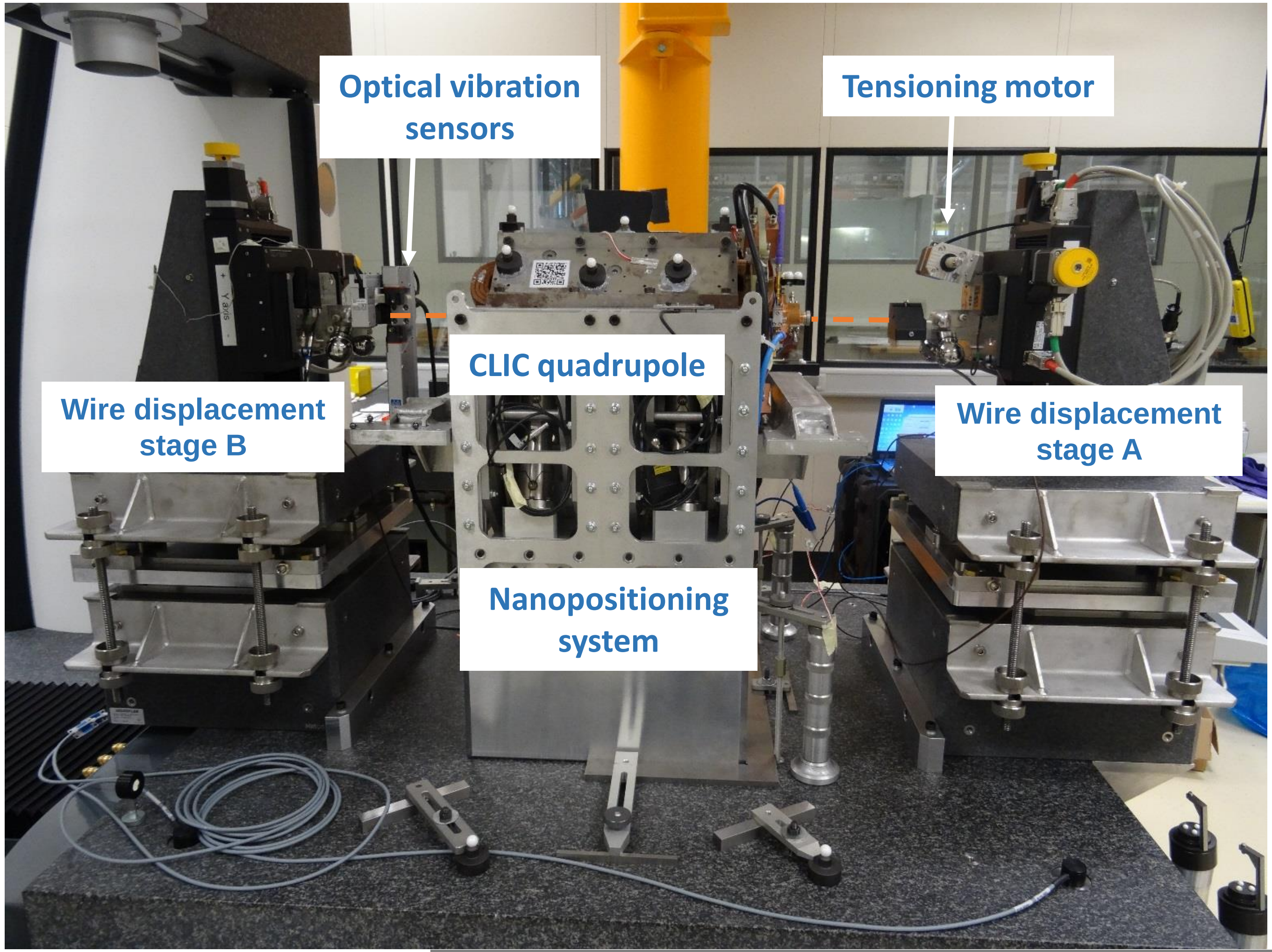
PART I - PERFORMANCE ENHANCEMENT FOR ALIGNMENT

Experimental validation with the PACMAN alignment bench

Experimental validation/ Setup



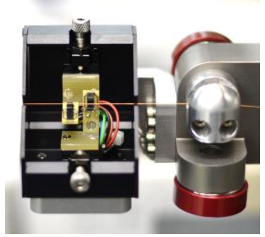
--- **Wire**

Φ0.1 mm conducting wire (Cu-Be alloy)

New location for the tensioning motor to reduce the impact of stray fields

Old sensors

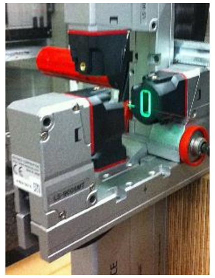
Phototransistors



Range: ~50 µm

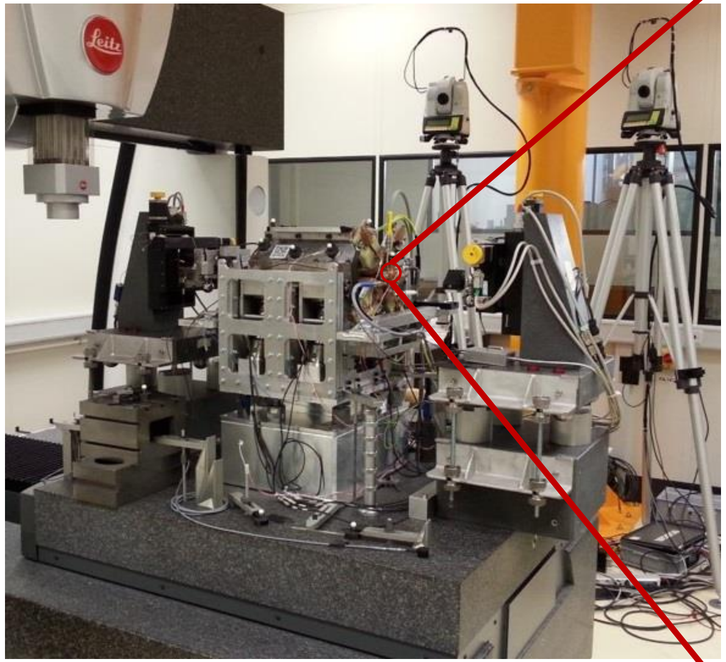
New sensors
Wide linear range

CCD micrometers



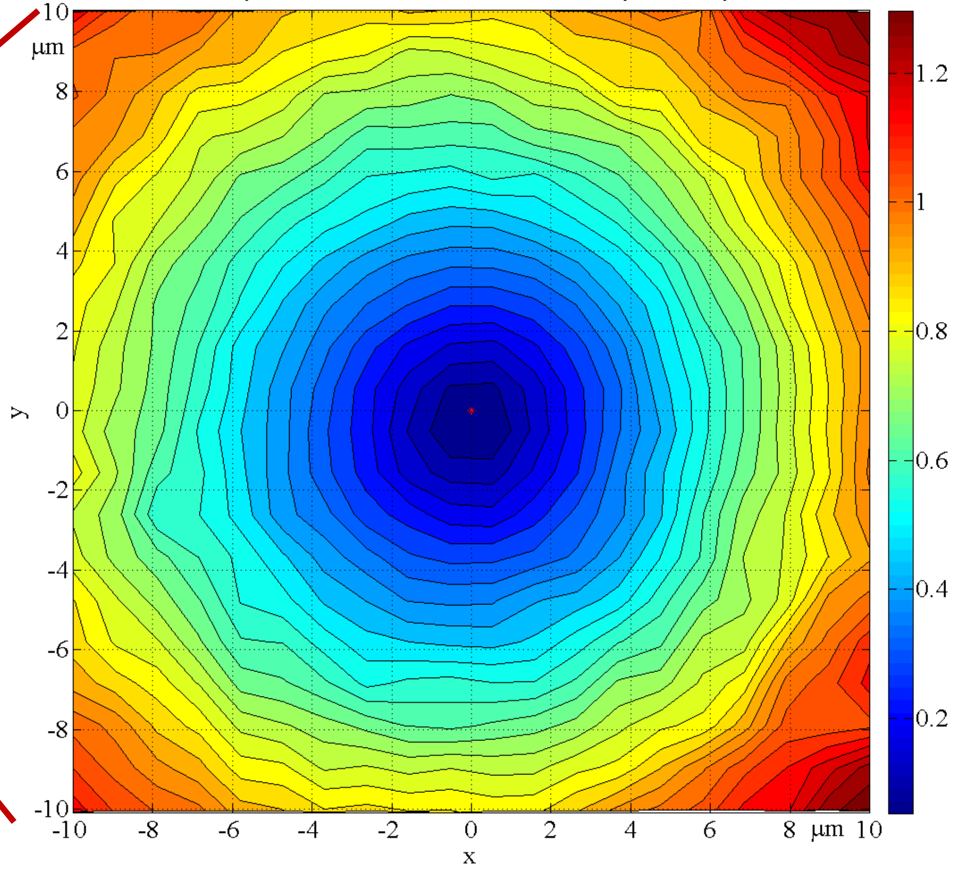
Range: 6 mm

Experimental validation/ Results



Vibrating wire map

(normalized wire vibration amplitudes)



Repeatability in the local frame (wire stages)

x_c	y_c	ϕ_c	θ_c
μm	μm	μrad	μrad
0.06	**0.08**	**1.94**	**0.41**

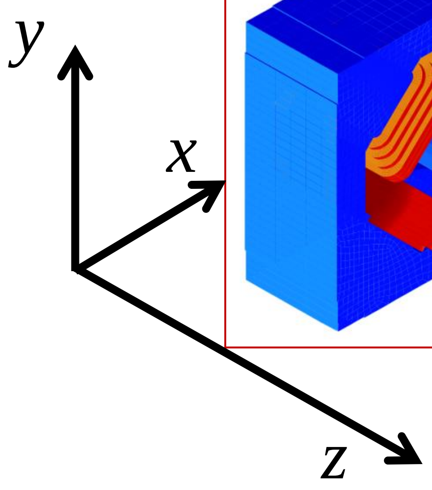
Difference between campaign 1 and 2 in the CMM frame

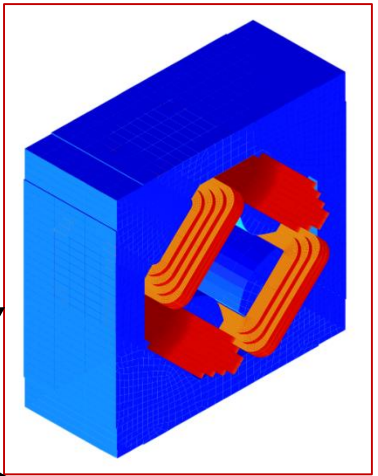
Δx_c	Δy_c	$\Delta \phi_c$	$\Delta \theta_c$
μm	μm	μrad	μrad
0.7	**6.3**	**-77.3**	**139.6**

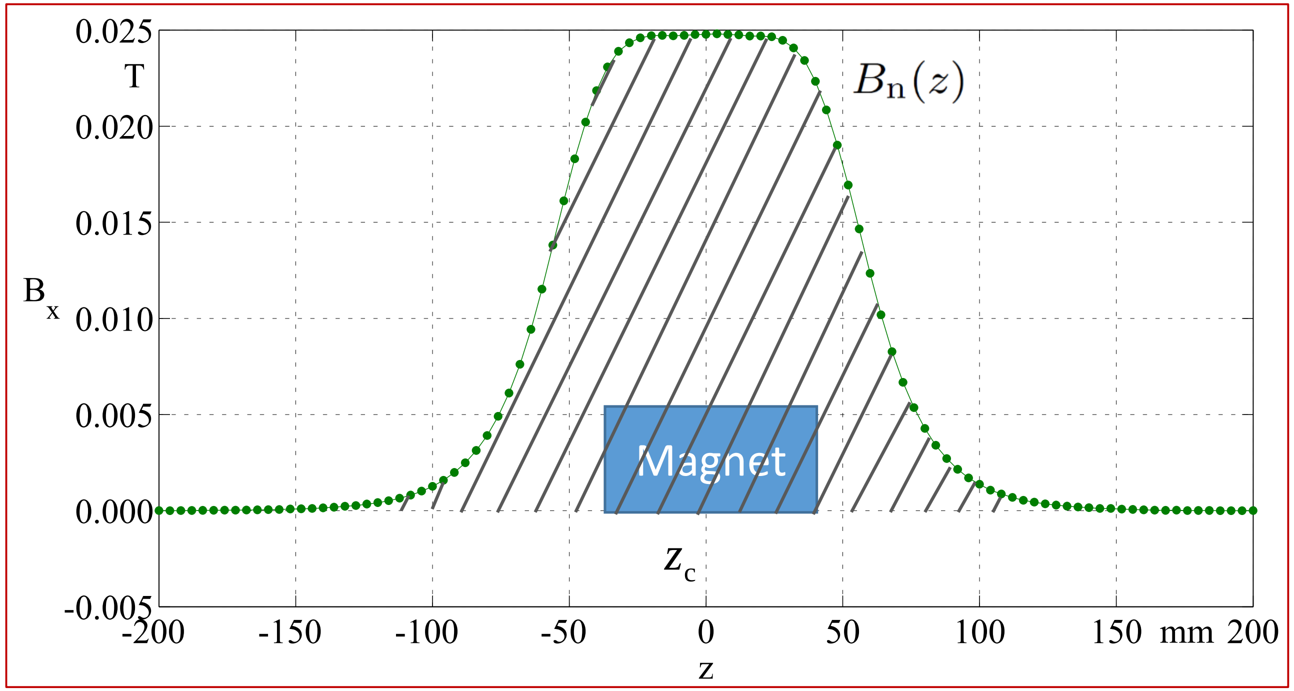
PART II –
FIELD STRENGTH, FIELD PROFILE, FIELD HARMONICS

Hard-edge equivalent/ Problem

- Measure the field integral and magnet location







$$\int B_{\mathrm{n}}(z)\mathrm{d}z$$

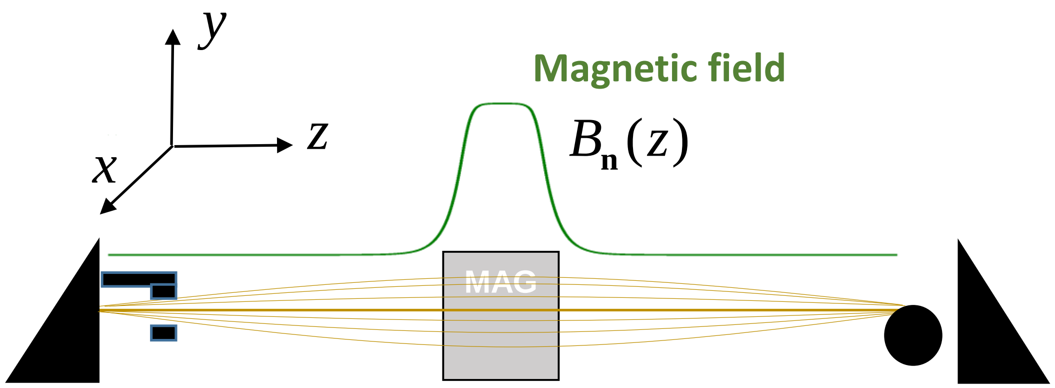
z_c

Translating wire
- ✓ Accurate for field integral
- ✓ Not sensitive to magnet location

Vibrating wire …

Hard-edge equivalent/ More on vibrating wire

Fourier coefficients of the longitudinal field distribution

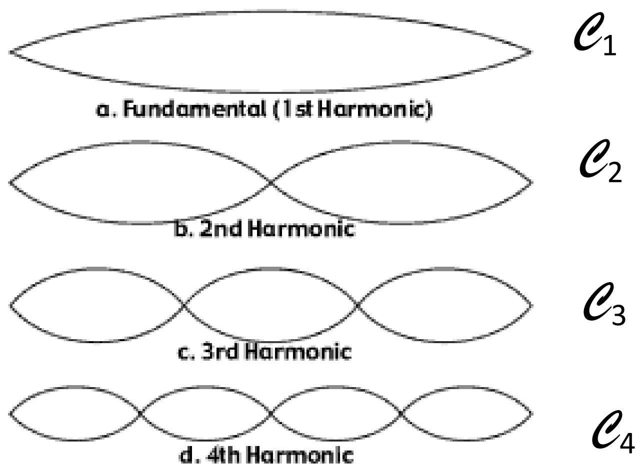


Longitudinal field coefficients

$$\mathcal{C}_m := \frac{2}{L}\int_0^L B_n(z)\sin\left(\frac{m\pi}{L}z\right)\mathrm{d}z$$

Different from harmonics in the cross-section: this is now side view (as a function of *z*)

Exciting several wire vibration modes



$$B_n(z) = \sum_m \mathcal{C}_m \sin\left(\frac{m\pi}{L}z\right)$$

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Hard-edge equivalent / Proposal & results

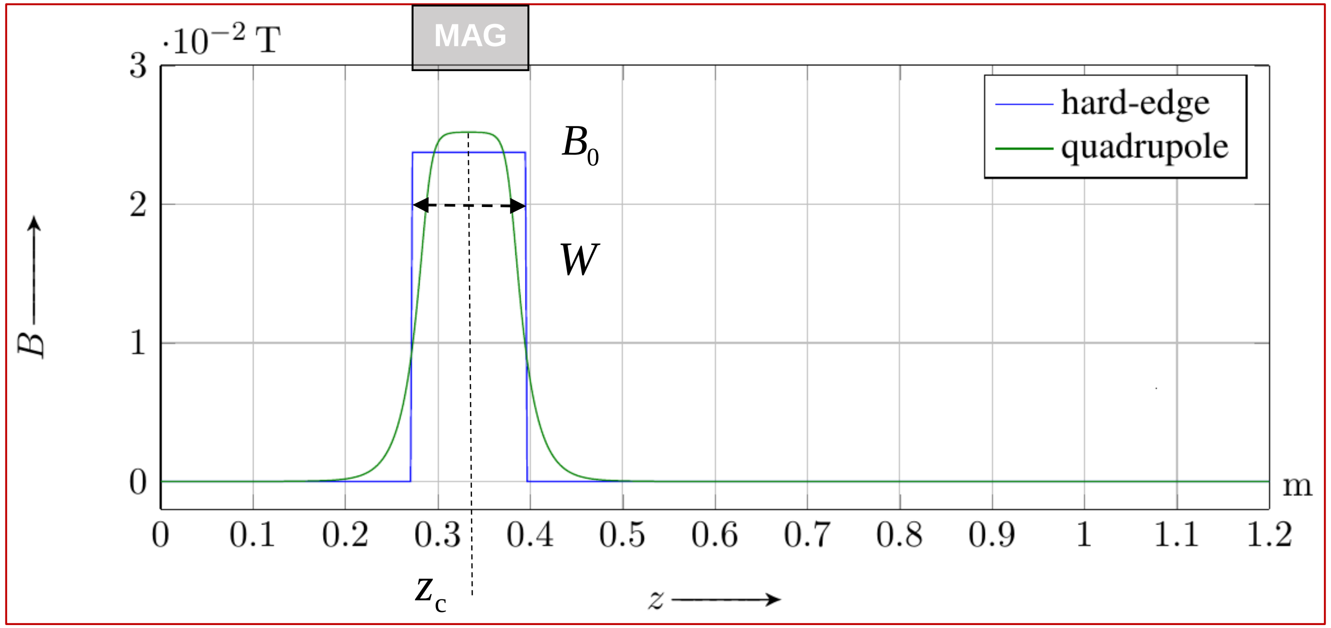
Hard-edge equivalent

$$B_0, W, z_c$$

- Defined by the average magnetic field in the aperture, magnetic length and magnet location
- Extensively used in beam simulation

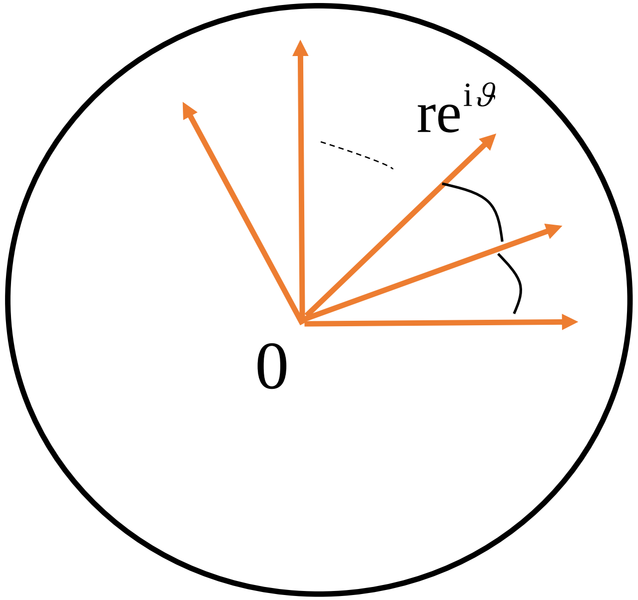
- **Basic idea**:
  - Measure a few Fourier coefficients (3 at least)
  - Least-square fit of Fourier transform

$$B_n(z) = \sum_m \mathcal{C}_m \sin\left(\frac{m\pi}{L} z\right)$$



Achieved performance:
- 1 cm precision on longitudinal location z_c
- 11% error on the integral field B_0W

On the way …/ Field harmonics by translating wire



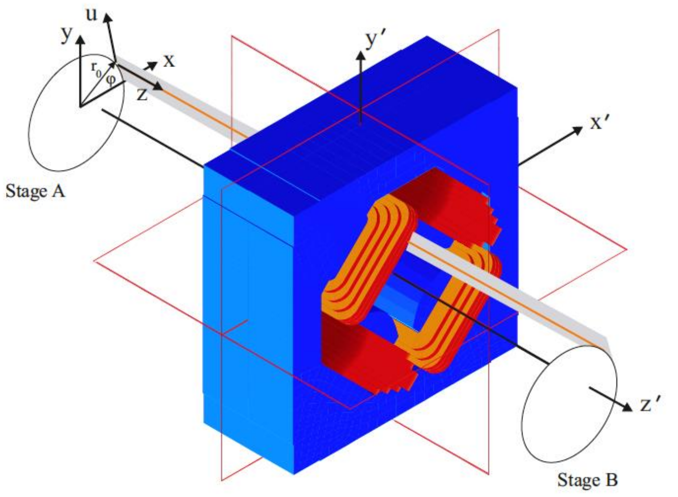
- Measuring the flux intercepted when moving from the origin to an arbitrary point
- Least-square estimation of the multipoles
- Positioning error are taken into account (encoder reading is used)

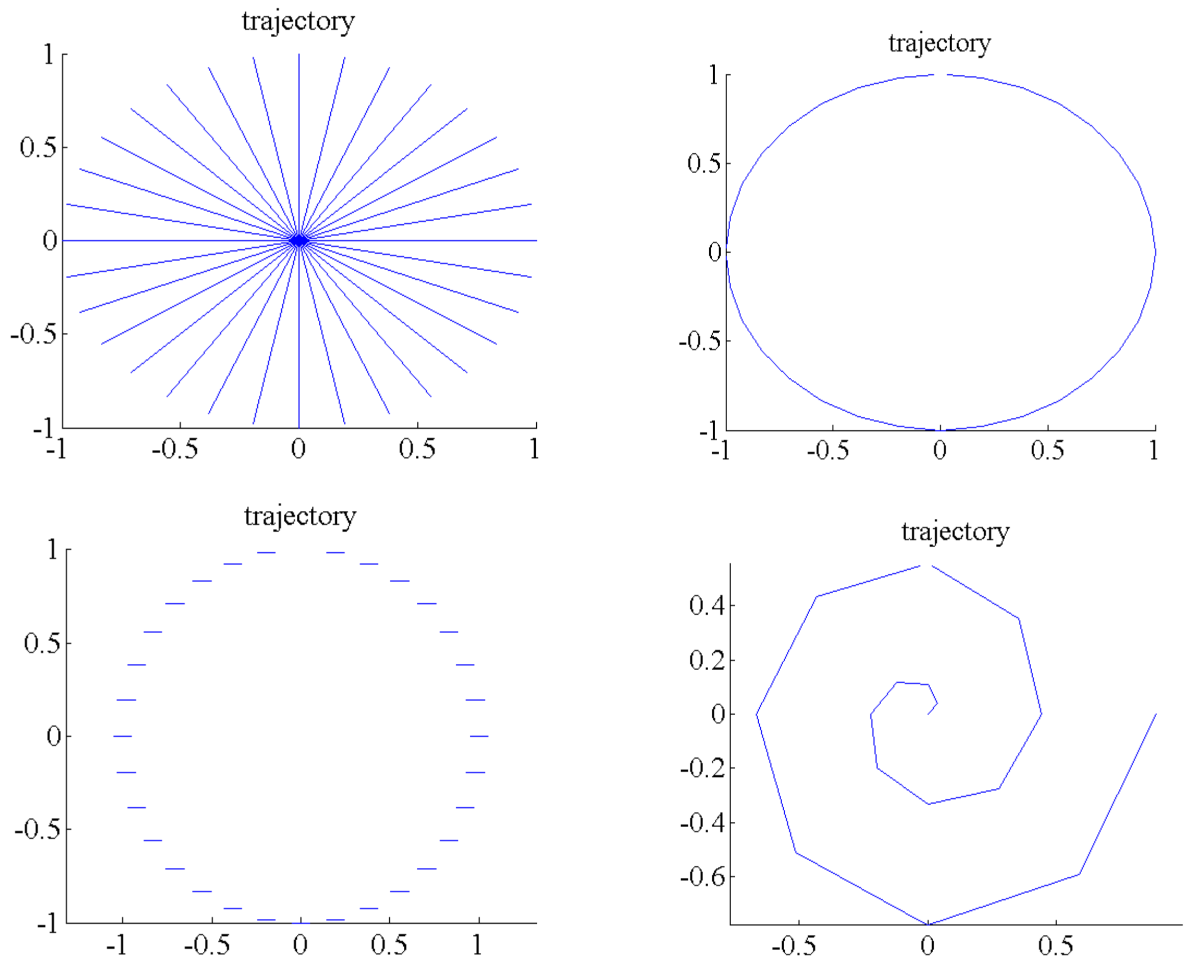
$$\underbrace{\begin{pmatrix} \Delta\phi(0, re^{i\vartheta_1}) \\ \vdots \\ \Delta\phi(0, re^{i\vartheta_{M1}}) \end{pmatrix}}_{M} = L \underbrace{\begin{pmatrix} re^{i\vartheta_1} & \cdots & \frac{r^N}{r_0^{N-1}} e^{iN\vartheta_1} \\ \vdots & \ddots & \vdots \\ re^{i\vartheta_M} & \cdots & \frac{r^N}{r_0^{N-1}} e^{iN\vartheta_M} \end{pmatrix}}_{T} \underbrace{\begin{pmatrix} B_1 + iA_1 \\ \vdots \\ B_N + iA_N \end{pmatrix}}_{P}$$

$$P = (T^t T)^{-1} T^t M$$

On the way …/ Field harmonics by translating wire

- Suitable for any trajectory
- Different trajectories being compared
  - Condition number of *T*, sensitivity
- The method can be extended to the oscillating-wire for field harmonics
- Validation against rotating coil and comparison of stretched and oscillating wire





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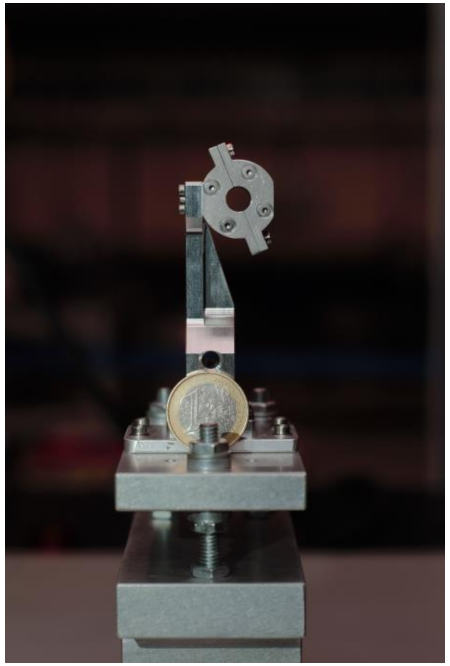
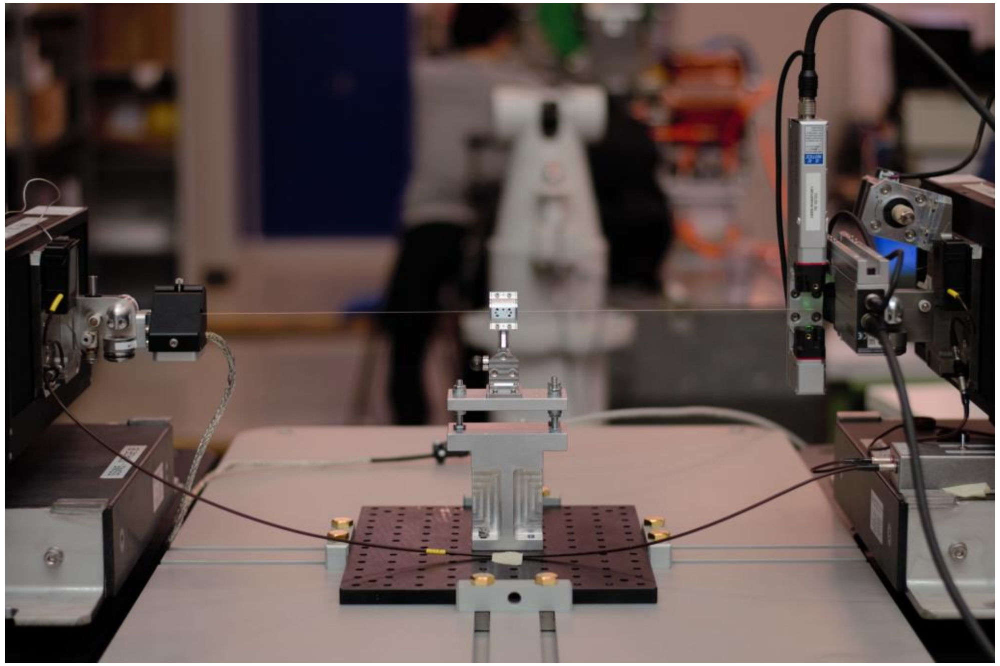
Extrapolation to other projects …/ ADAM

- A small permanent quadrupole for ADAM





- The sunburst trajectory gives more accurate results for the measurement of integrated field strength

- The use of the Keyence micrometers more accurate reliable for the measurement of the field harmonics by oscillating wire …

→ Alberto De Giovanni's talk

Summary

- Metrological performance enhancement of wire methods for alignment
  - Study and reduction of systematic errors (background fields, multipole field errors)
  - Reduction of random errors by experimental design

- A new wire bench developed for
  - Experimental validation of the proposed methods
  - Alignment of a CLIC main beam quadrupole in the frame of PACMAN

- Extension of wire methods for
  - Estimation of hard-edge equivalents
  - Field harmonics with optimized trajectories (on the way …)

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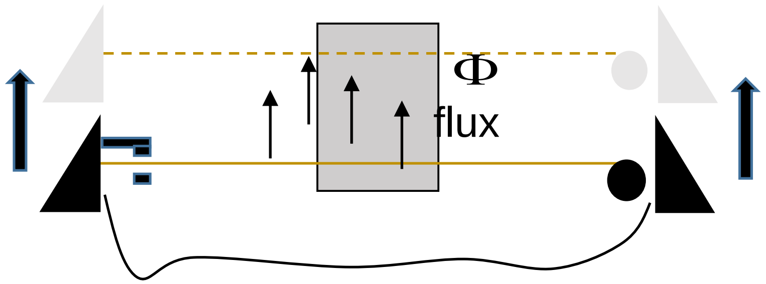
Thanks for your attention

Spares

State of the art /Stretched wire method



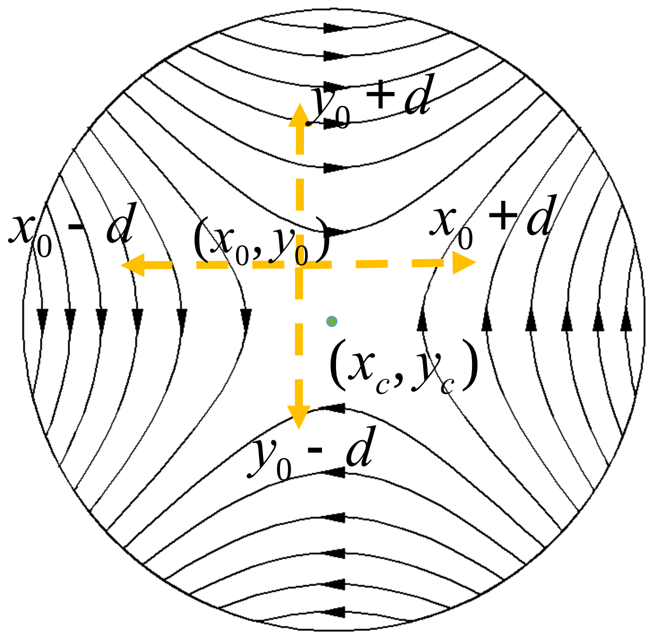
The wire is translating

Measurements for LHC magnets

✓ integrated field strength (10^{-5} uncertainty)

✓ field direction

✓ magnetic axis (50-100 μm)



Example: quadrupole

Integrated field strength

$$\bar{g}L = \frac{\Phi(x_0, x_0 + d) + \Phi(x_0, x_0 - d)}{d^2}$$

$$\bar{g}L = \frac{\Phi(y_0, y_0 + d) + \Phi(y_0, y_0 - d)}{-d^2}$$

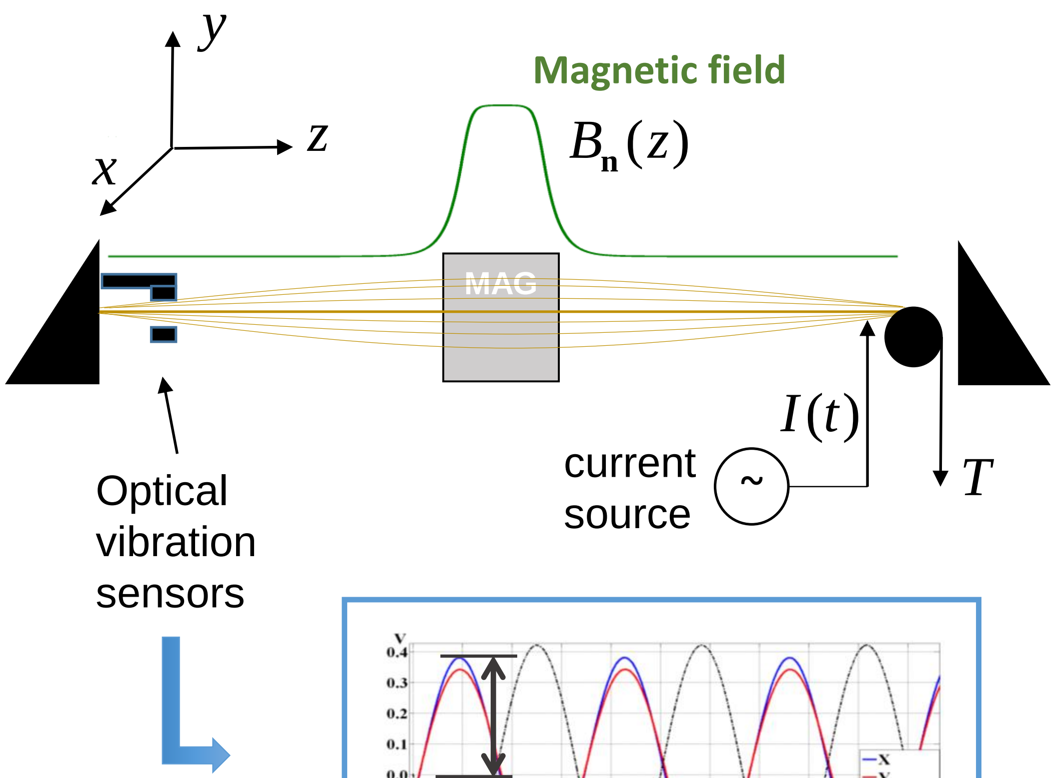
Magnetic center coordinates

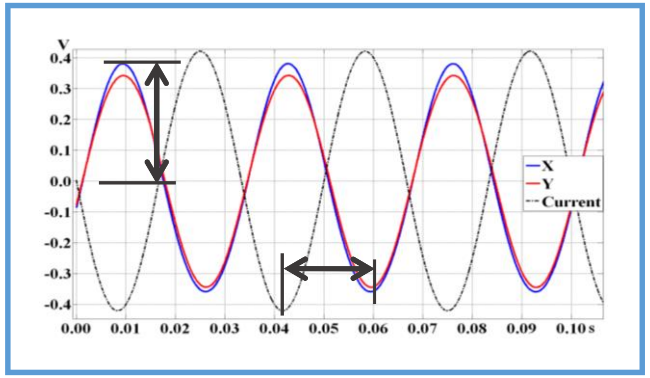
$$x_c = x_0 - \frac{d}{2}\frac{\Phi(x_0, x_0 + d) - \Phi(x_0, x_0 - d)}{\Phi(x_0, x_0 + d) + \Phi(x_0, x_0 - d)}$$

$$y_c = y_0 - \frac{d}{2}\frac{\Phi(y_0, y_0 + d) - \Phi(y_0, y_0 - d)}{\Phi(y_0, y_0 + d) + \Phi(y_0, y_0 - d)}$$

J. Di Marco et al., “Field alignment of quadrupole magnets for the LHC interaction Regions”. *IEEE Transactions on Applied Superconductivity*, 2000.

State of the art / Vibrating/oscillating wire method





Feeding the wire by alternating current (Lorentz force)

⇩

Measure wire vibrations (X and Y components)

⇩

Relate vibrations to magnetic field

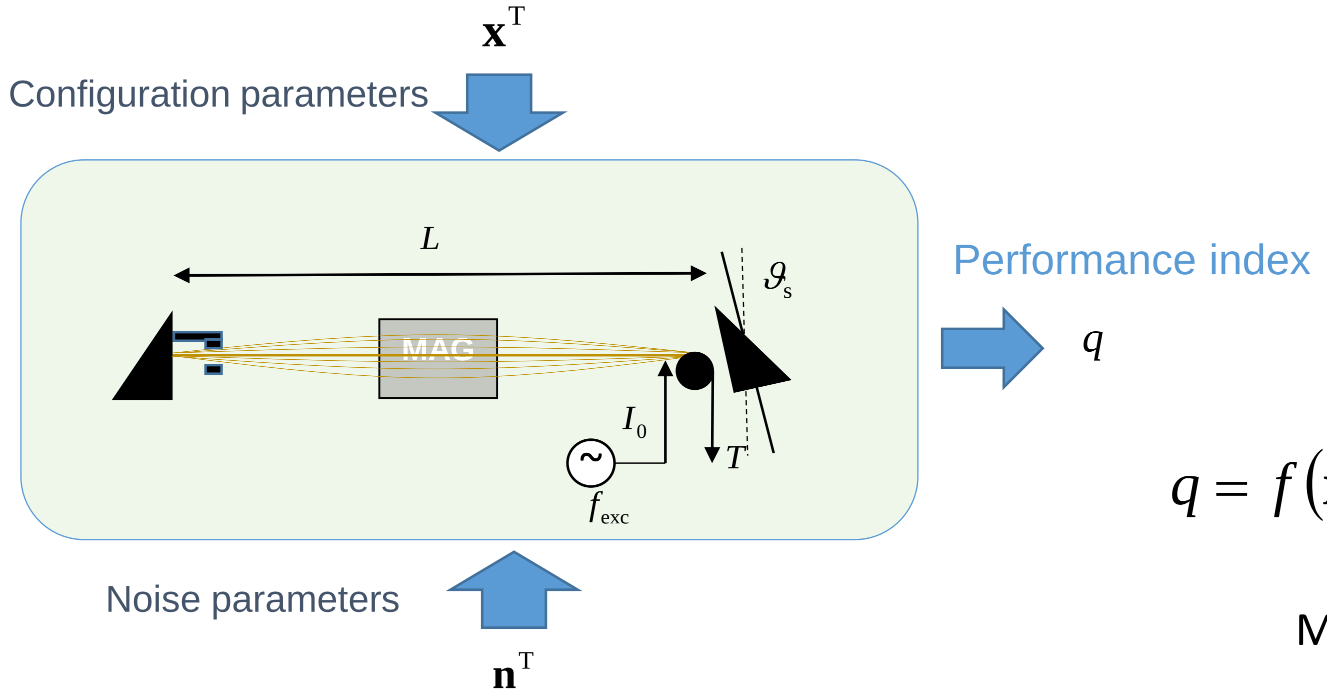
- High sensitivity also for low field and small apertures
- Used in particular for alignment and magnetic field quality (multipoles)

- **A. Temnykh**. “Vibrating wire field-measuring technique”. *Nuclear Instruments and Methods in Physics Research,* **1997**.
- **P. Arpaia, M. Buzio, J. G. Perez, C. Petrone, S. Russenschuck, L. Walckiers**. “Measuring field multipoles in accelerator magnets with small-aperture by an oscillating moved on a circular trajectory”. *JINST – Journal of Instrumentation,* **2012**.

Achievements/ Performance optimization of the vibrating wire for alignment

Studying the metrological performance as a function of the system configuration

- Sensitivity analysis
- Experimental design approach



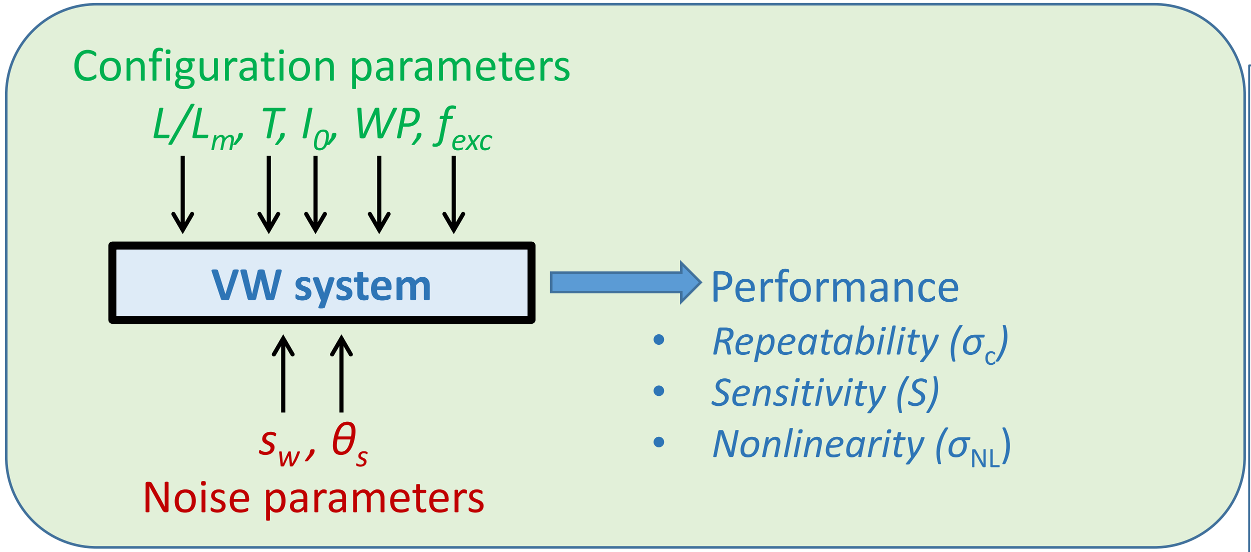
- Hp:
  - orthogonality of the parameters (no interactions)

- The additive statistical model
  - describes the relation between performance and parameters

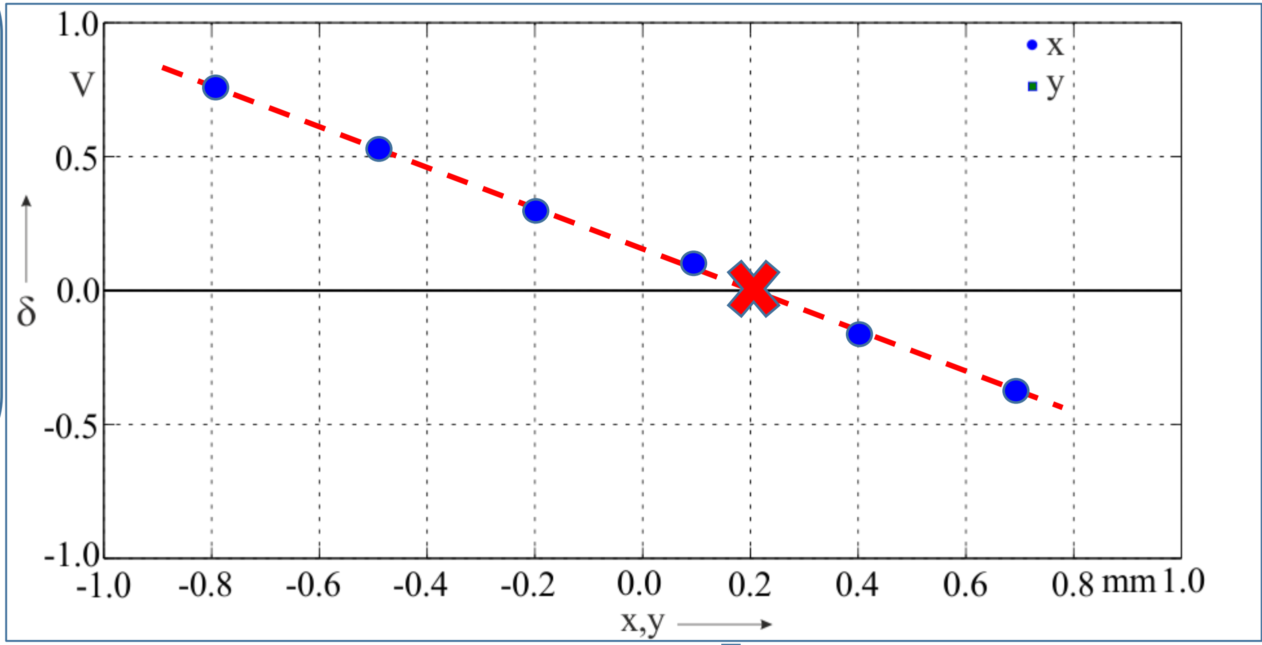
$$q = f(\mathbf{x}, \mathbf{n}) \quad \rightarrow \quad q = \mu + \sum_{k=1}^{K} \tau_k + \varepsilon$$

Mean performance (μ), Parameter effects (τ_k), Model error (ε)

Performance definition



Vibration amplitude with phase change

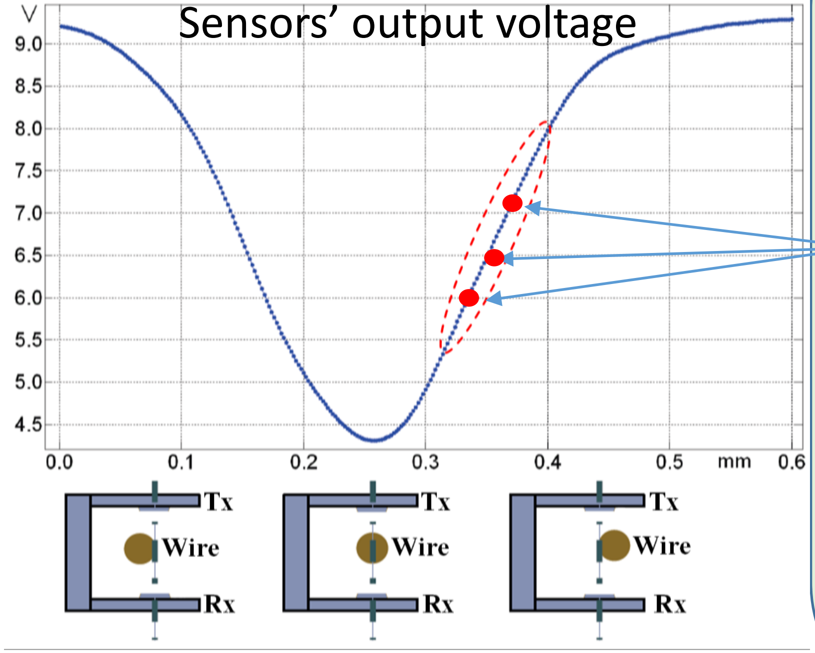


- *σ_c: 1- σ standard deviation of x_c* and y_c
- *S: the slope of the regression line*
- σ_{NL}: squared variance of the linear regression error (residuals)

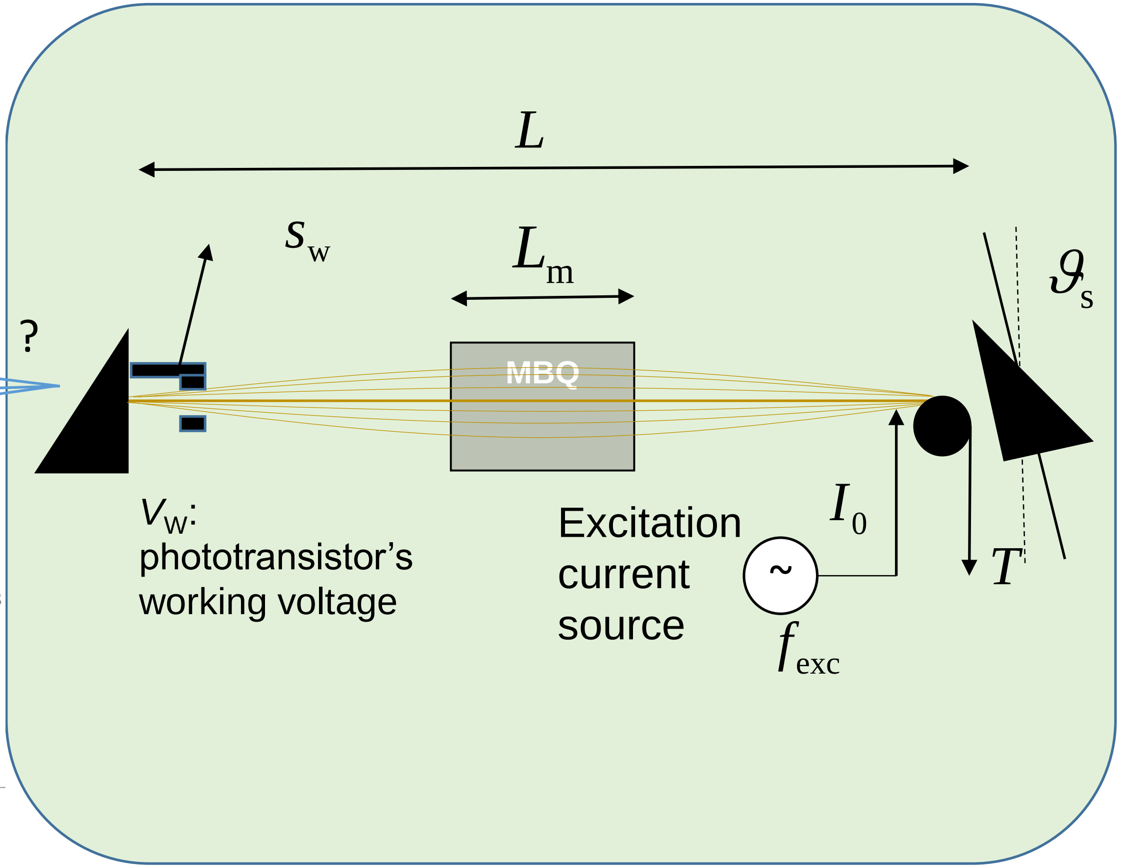
Output: (x_c, y_c)

Combined performance index
$$q = 10\log_{10}\left(\frac{\theta_S S^2}{\theta_\sigma \sigma_c^2 + \theta_{NL} S_{NL}^2}\right)$$
Taguchi's SNR

Environment and configuration



Wire position



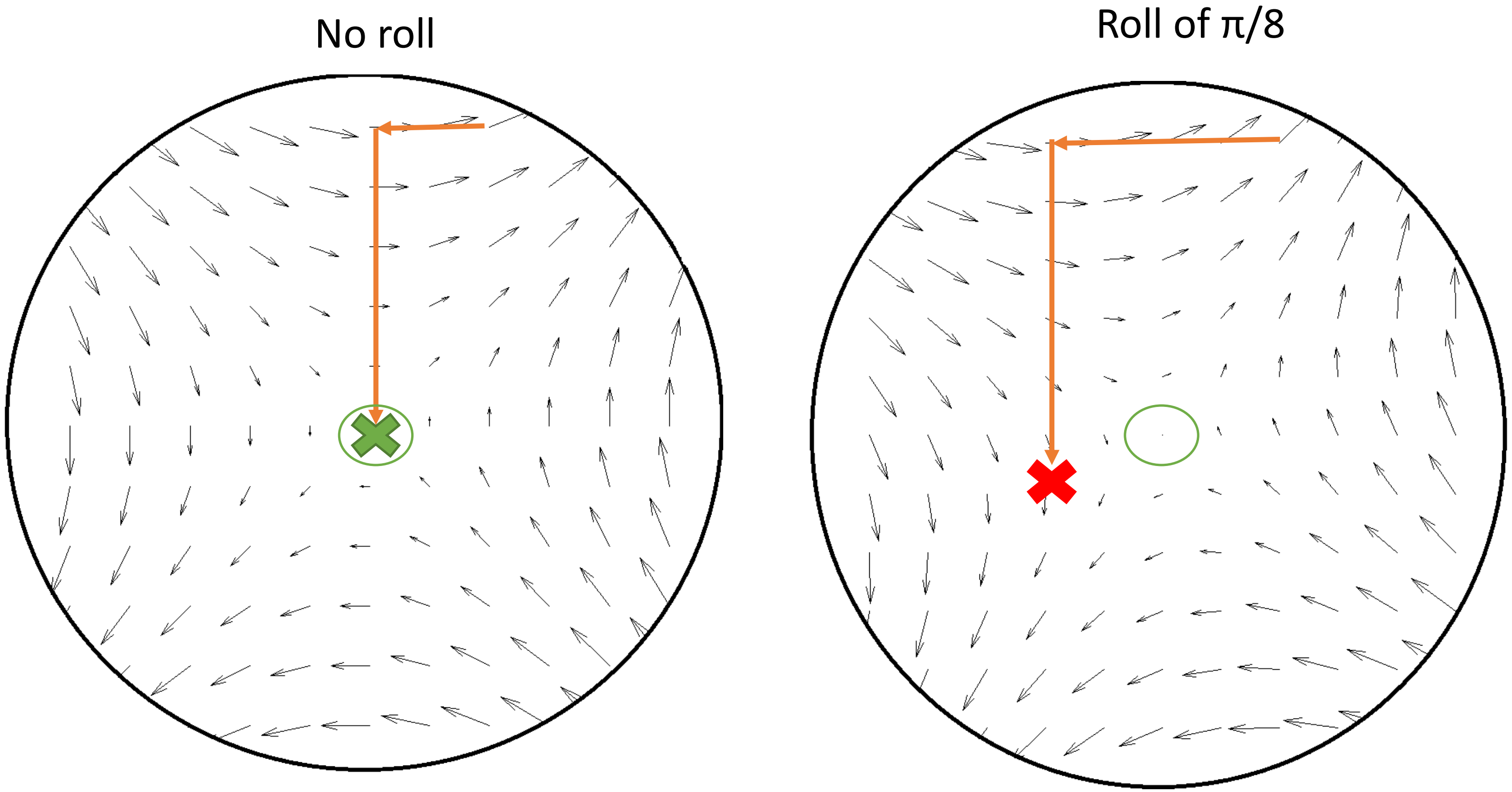
- Magnet length L_m
- Wire length L
- Wire tension T
- Driving current I_0
- Phototransistor's working voltage V_W
- Current excitation frequency: distance from resonance Δf_{exc}
- Stage misalignment θ_s
- Wind speed s_w

Roll angle effect (1/2)

Effect of roll angle

- Standard scan is done by moving the wire horizontally and vertically

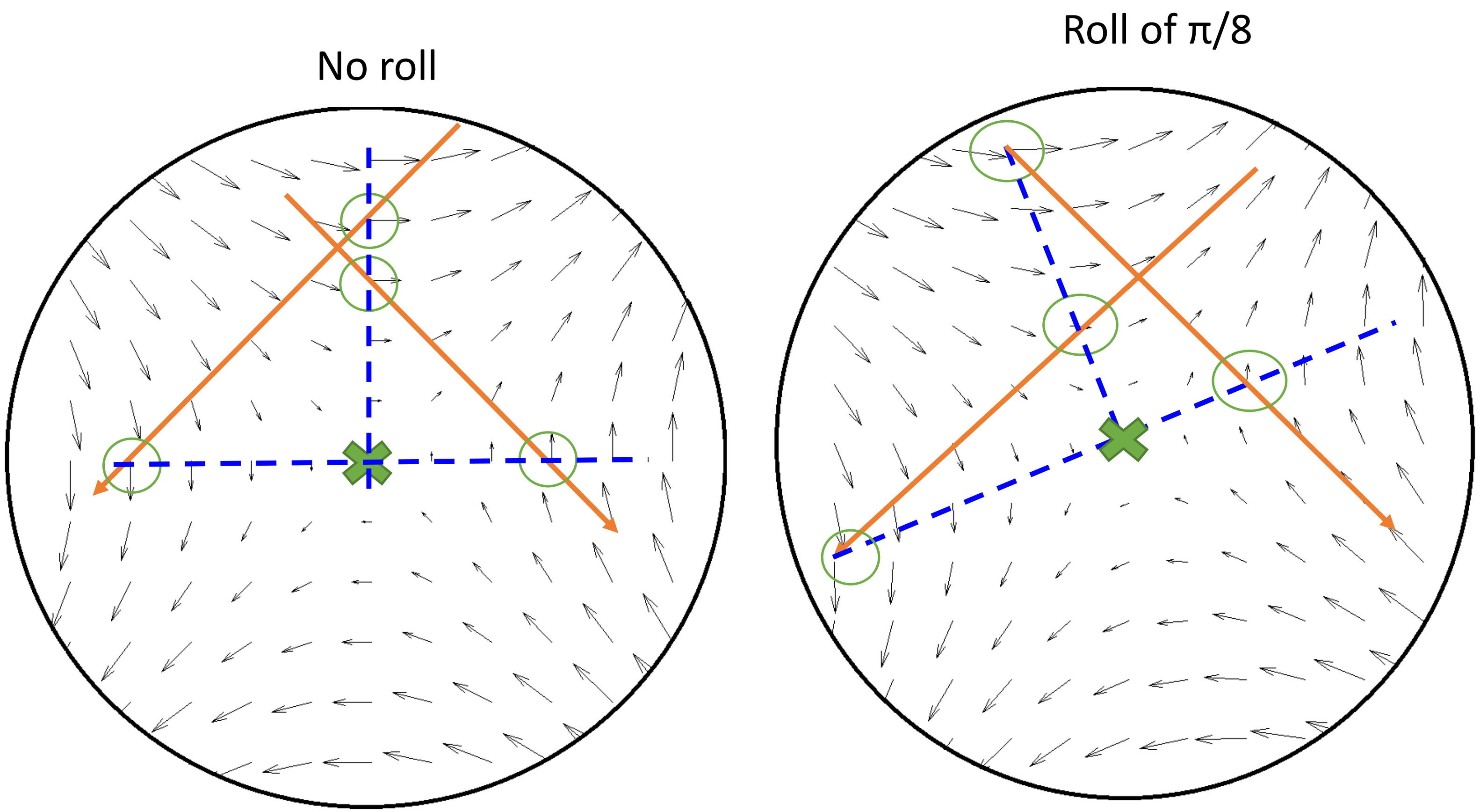
- In the case of non-zero roll, the search does not converge in one iteration



Roll angle effect (2/2)

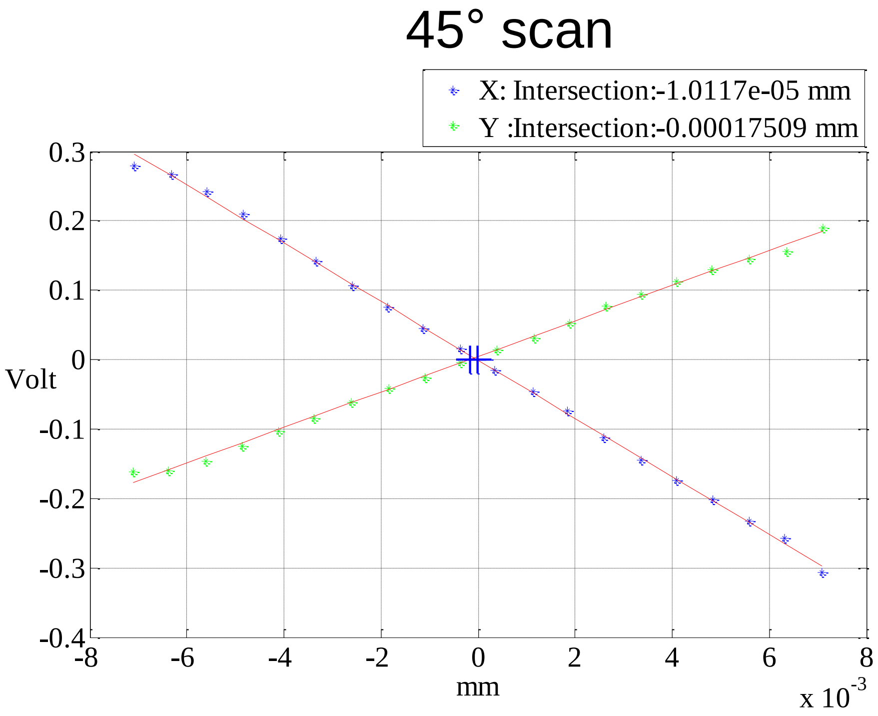
Solution

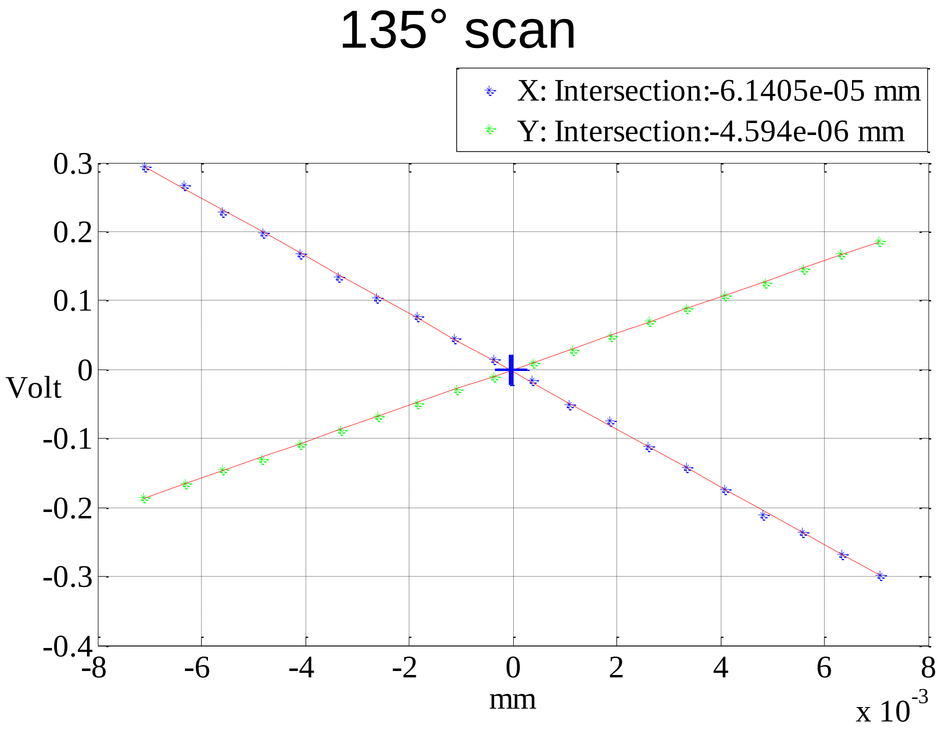
- Two orthogonal scan (with 45° and 135° angle)



Alignment on CMM / Campaign 2

- For each scan *x* and *y* centers are found at the same time





No roll error

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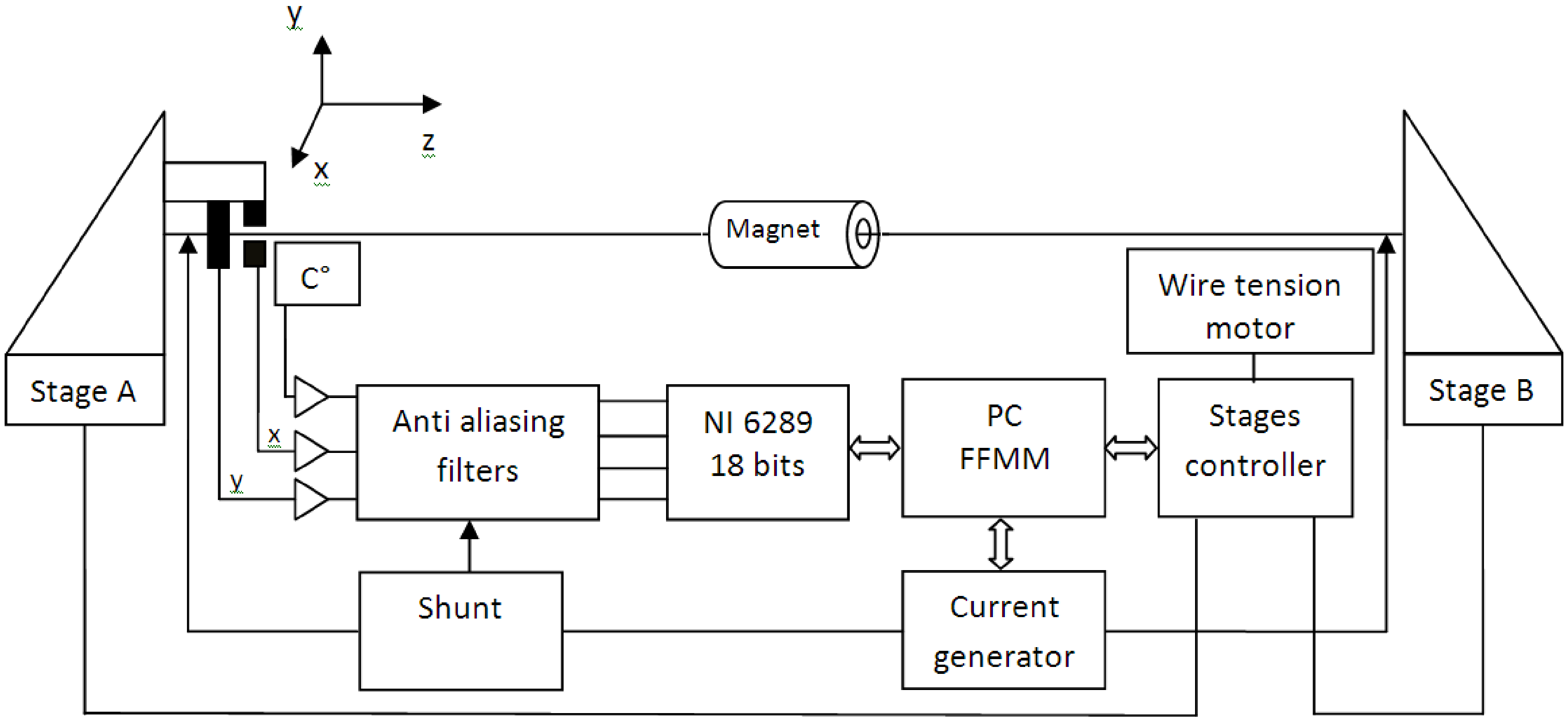
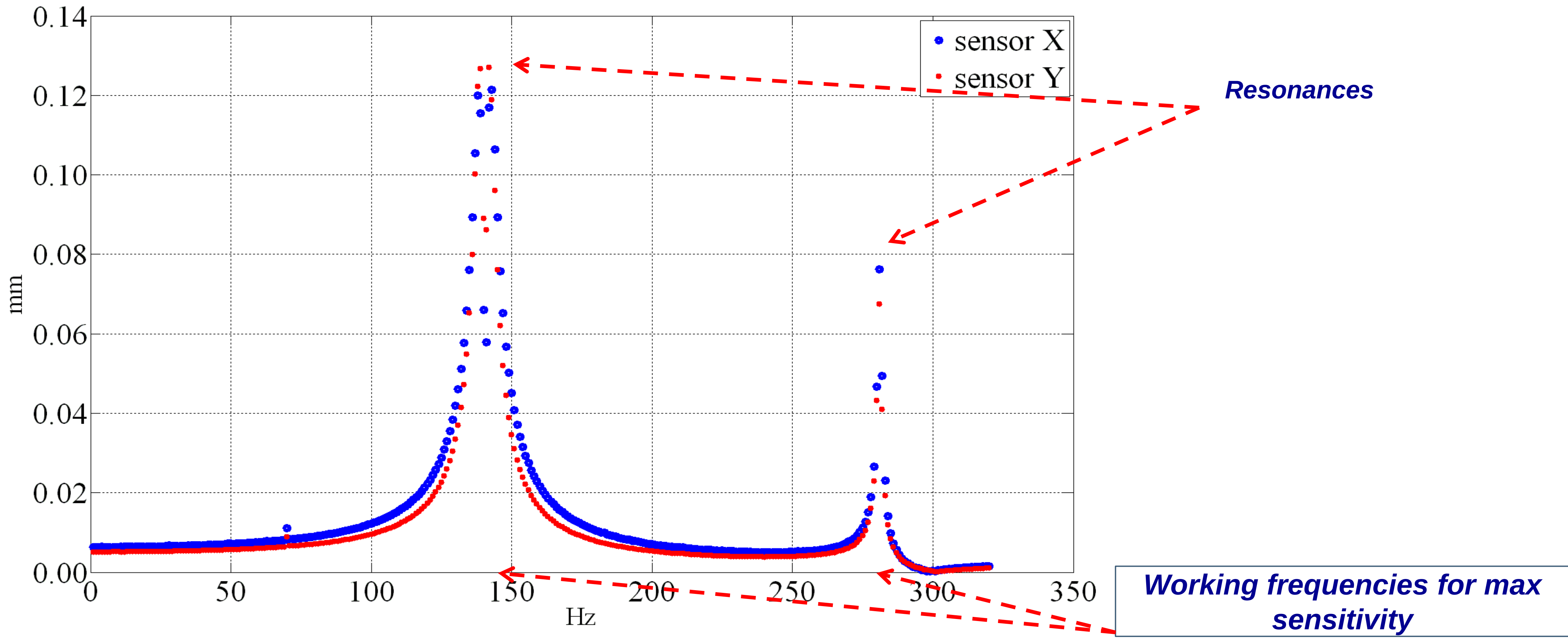
Architecture



P. Arpaia, M. Buzio, J. G. Perez, C. Petrone, S. Russenschuck, L. Walckiers.
"Measuring field multipoles in accelerator magnets with small-apertures by an oscillating wire moved on a circular trajectory", *JINST - Journal of Instrumentation, 2012*

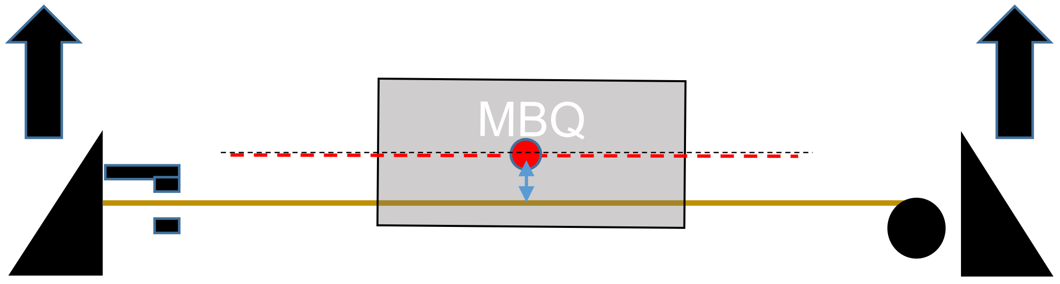
Working conditions

Driving current frequency tuning

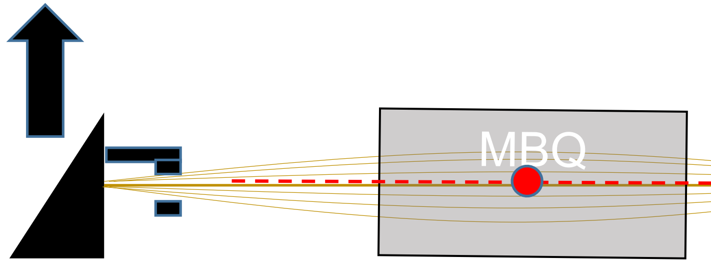


Locating the axis

AVERAGE CENTER

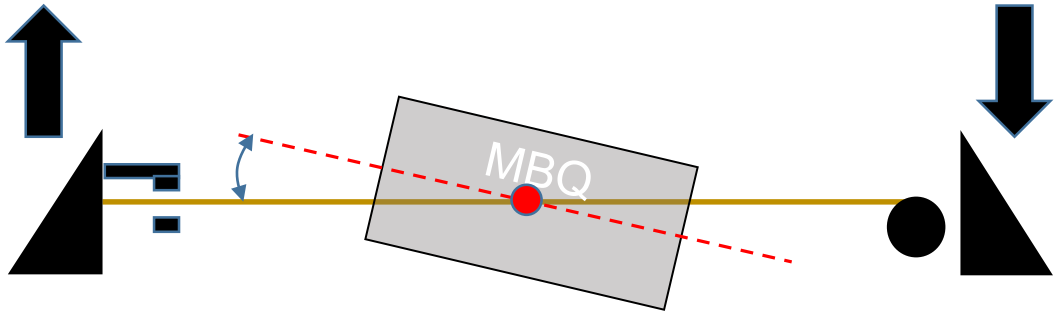


The wire is sensitive to the distance from the average center when the first eigenmode is excited

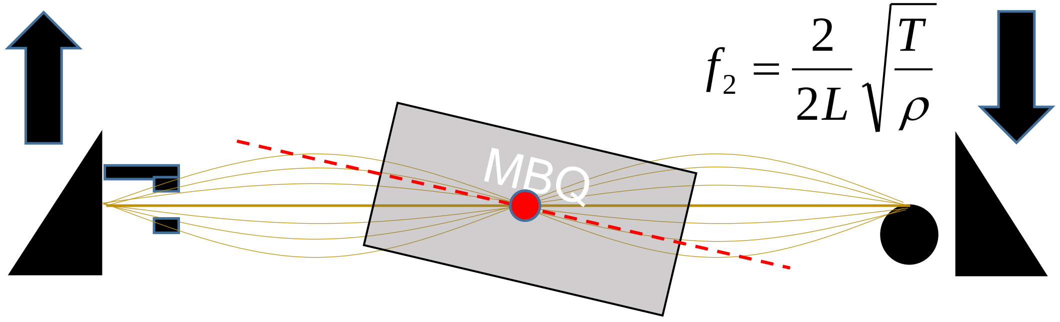


$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\rho}}$$

TILT



The wire is sensitive to the tilt when the second eigenmode is excited



$$f_2 = \frac{2}{2L}\sqrt{\frac{T}{\rho}}$$

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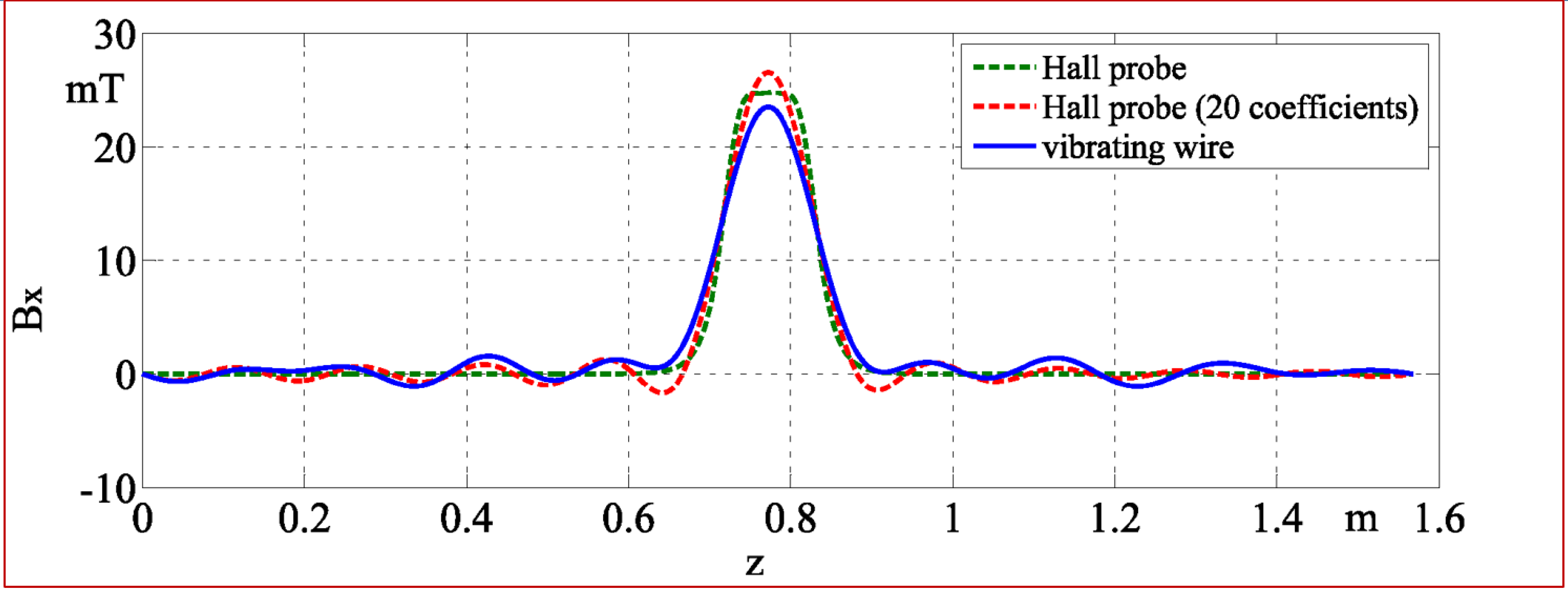
Longitudinal field profile/ Results

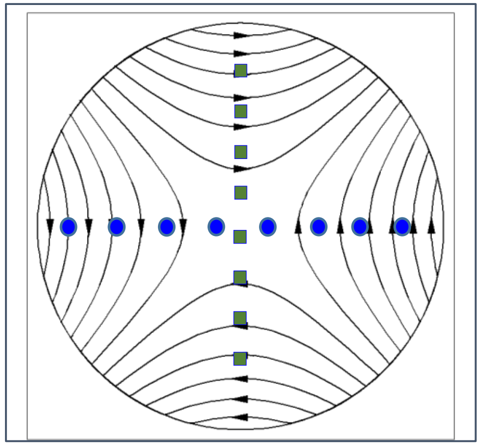


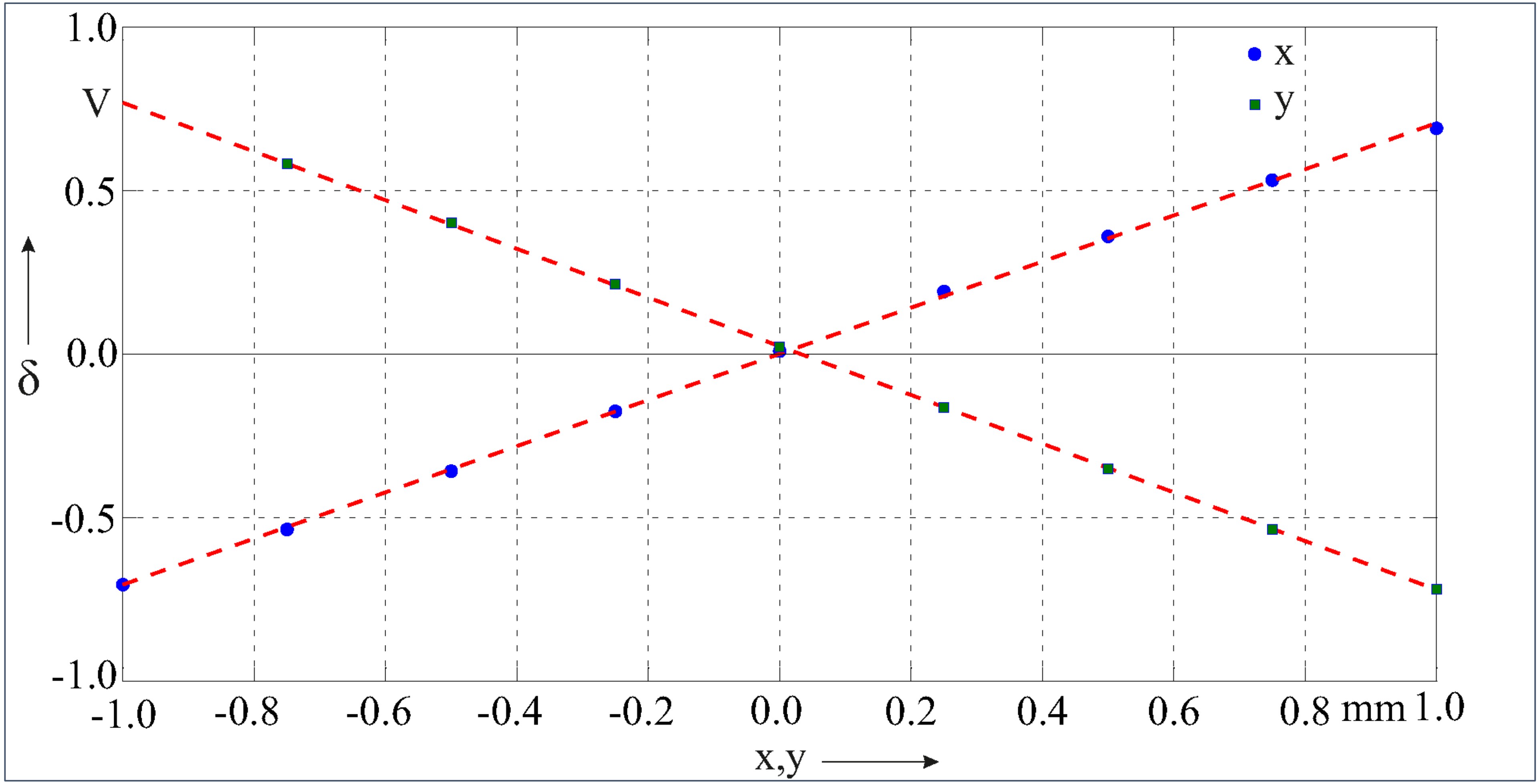
- Reconstruction error 3% of the field peak
- Repeatability 2%
  - RMS difference

- Bandwidth limitation
- Uncertainty sources

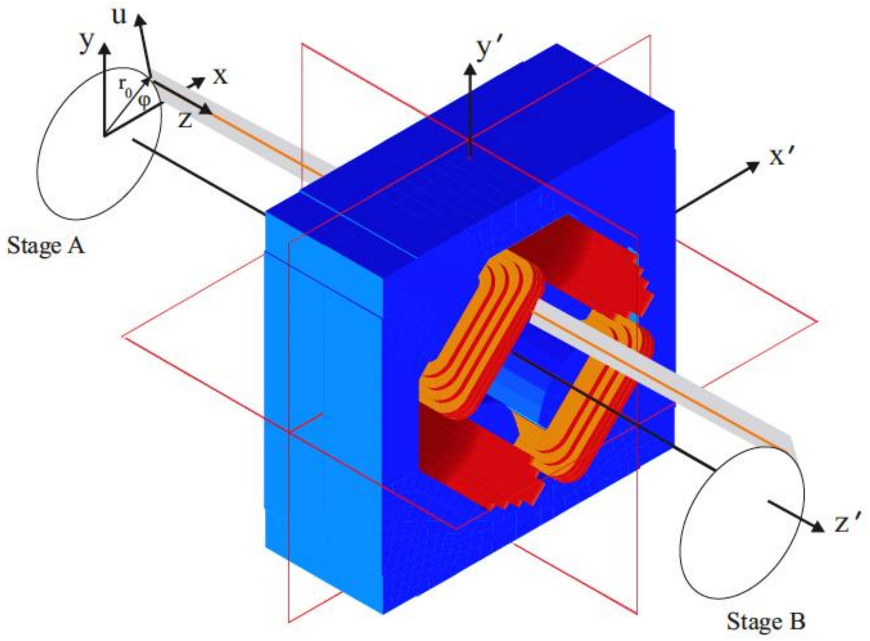
Vibrating-wire zero method

Example





Field quality/ Oscillating wire (1/2)



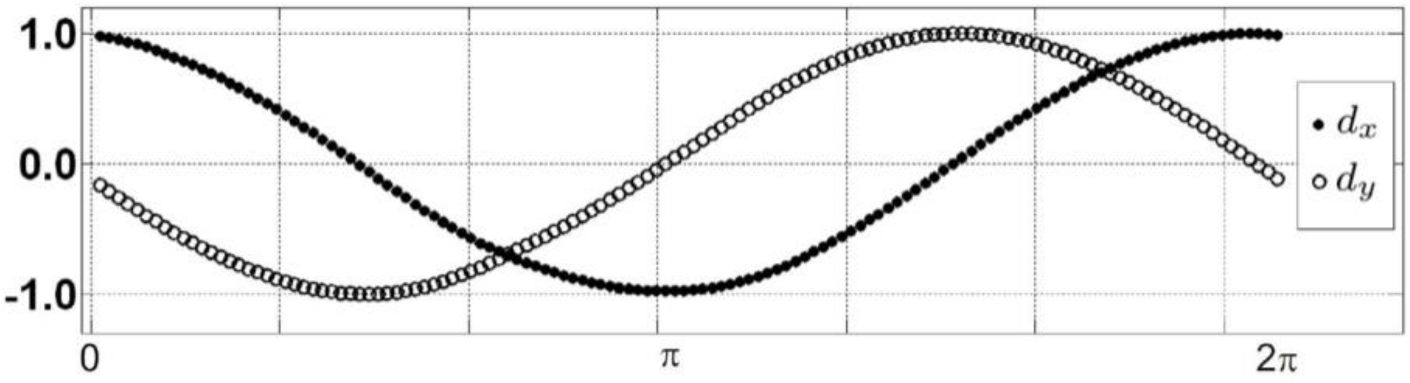
$$d_y^k(r_0) = \lambda_y \int_0^L B_x(r_0, \varphi_k)\mathrm{d}z$$

$$d_x^k(r_0) = \lambda_x \int_0^L B_y(r_0, \varphi_k)\mathrm{d}z\,.$$

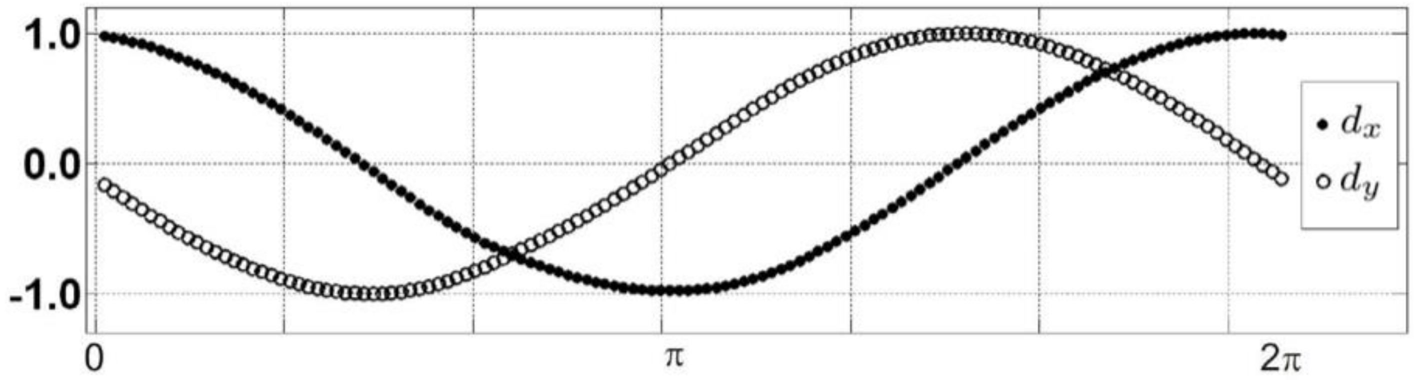
- Quasi-static problem

$$\omega \ll \omega_1$$

- Measure the oscillation amplitudes along a circular trajectory
- Proportionality with respect to the integral field (indepently from azimuthal position)



Field quality/ Oscillating wire (2/2)



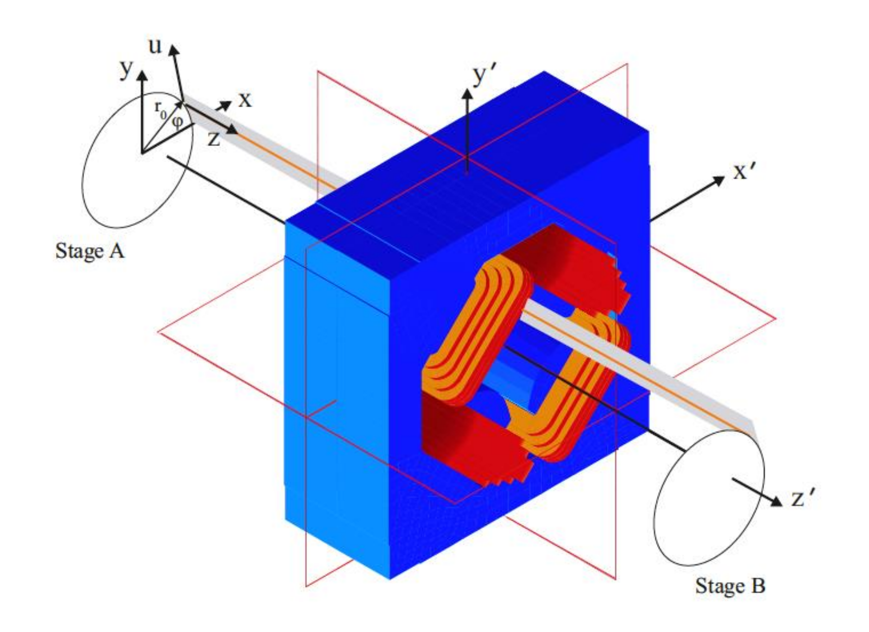
- Relative multipoles are known by harmonic analysis

$$\tilde{A}_n(r_0) = \frac{2}{K}\sum_{k=0}^{K-1} d_y^k(r_0)\cos n\varphi_k\,, \qquad \tilde{B}_n(r_0) = \frac{2}{K}\sum_{k=0}^{K-1} d_y^k(r_0)\sin n\varphi_k\,.$$

$$a_{n+1}(r_0) = \frac{\tilde{A}_n(r_0)}{\tilde{B}_N(r_0)}\,, \qquad b_{n+1}(r_0) = \frac{\tilde{B}_n(r_0)}{\tilde{B}_N(r_0)}\,.$$

Assumptions

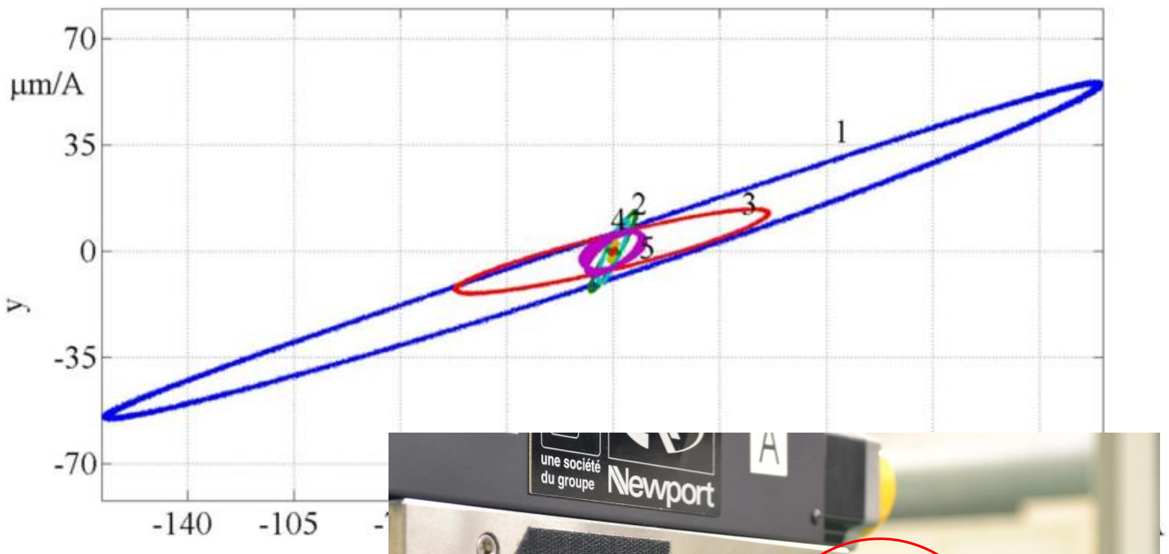
- Linearity
- Plane motion
- Uniform and constant tension
- Small deflections
- Constant length
- Uniform mass distribution

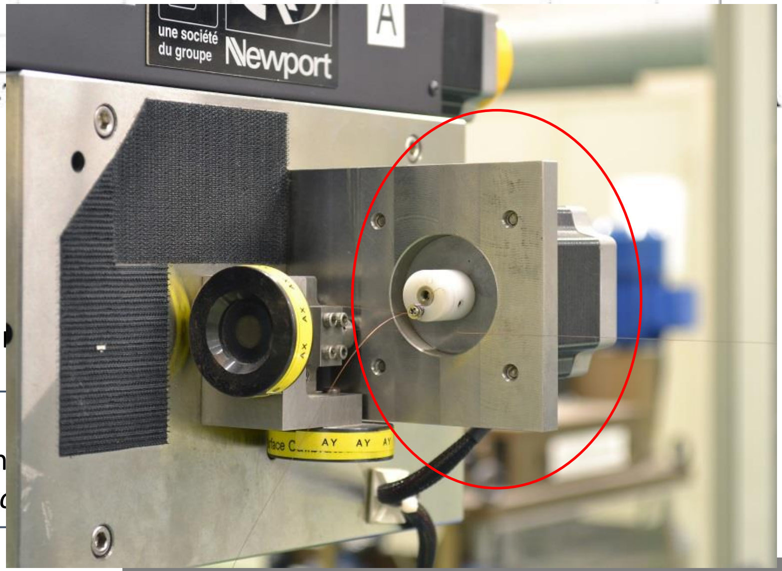


$$u(z,t) = \frac{2I_0}{L} \sum_m \frac{\int_0^L B_n(z) \sin\left(\frac{m\pi}{L} z\right) \mathrm{d}z}{\sqrt{\left[T\left(\frac{m\pi}{L}\right)^2 - \rho\omega^2\right]^2 + (\alpha\omega)^2}} \sin\left(\frac{m\pi}{L} z\right) \sin(\omega t - \varphi_m),$$

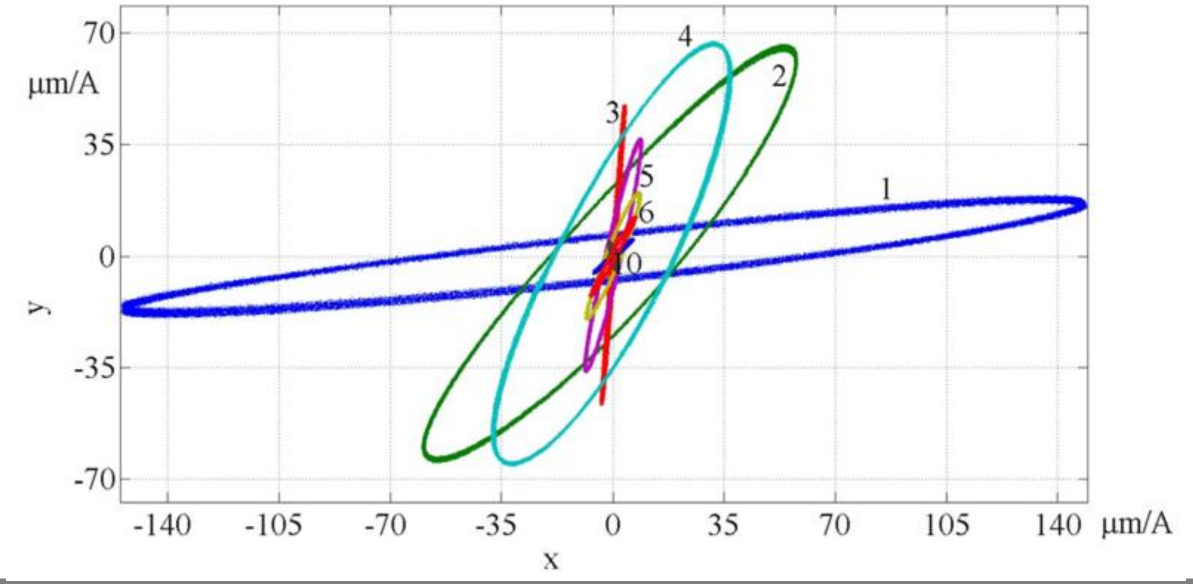
$$\varphi_m = \arctan\left(\frac{\alpha\omega}{-\rho\omega^2 + T\left(\frac{m\pi}{L}\right)^2}\right).$$

Background fields & Plane motion





- No magnet on the measurement station
- Background fields
  - 2% alteration of first harmonic
- Fringe field from equipment (tensioning motor)
  - High order modes amplified

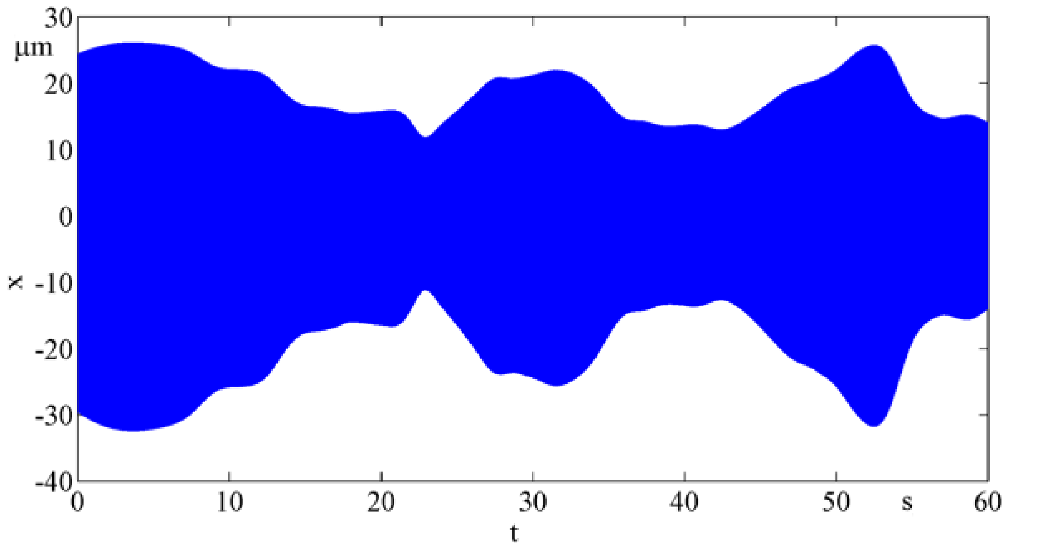


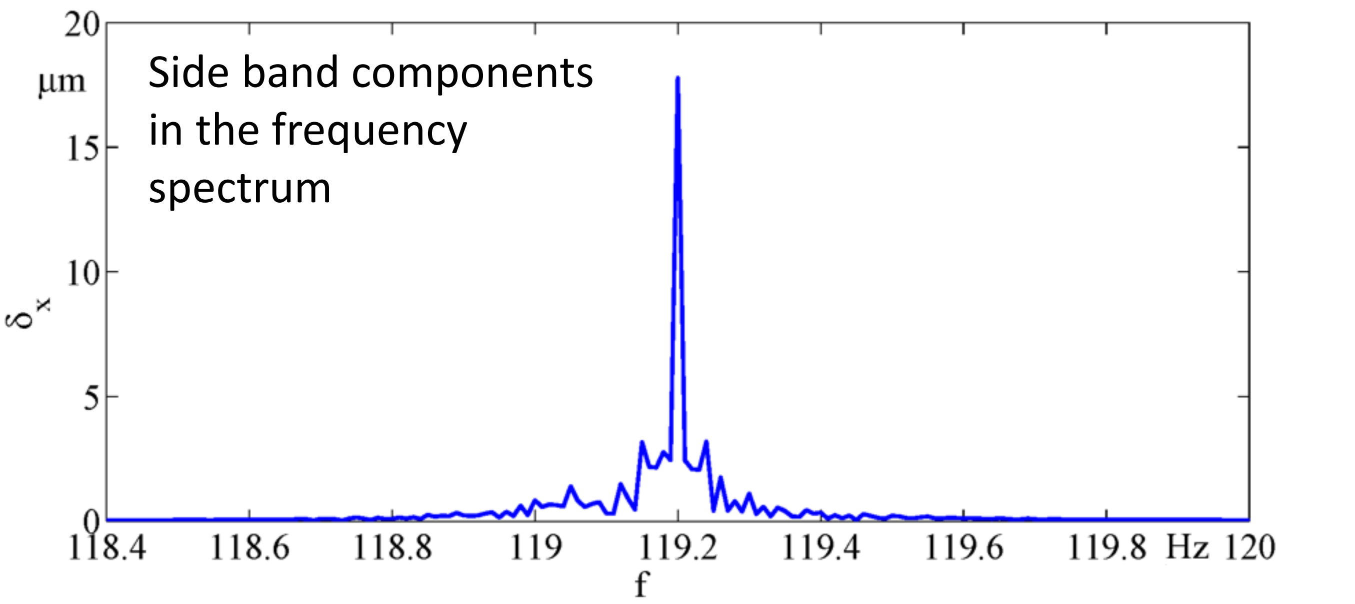
- Elliptically
  - in reso

Predicted in:
J. A. Elliott. "Intrin
American Journal o

Steady state modulation

- Around resonance
  - Non-constant oscillation amplitude!!!
  - Effect depending on the excitation frequency: minimal in resonance condition (5%)





- Possible reasons
  - Non constant length and/or tension
  - Non ideal clamping (friction on the supports)
  - Excluded: coupling with ground vibrations

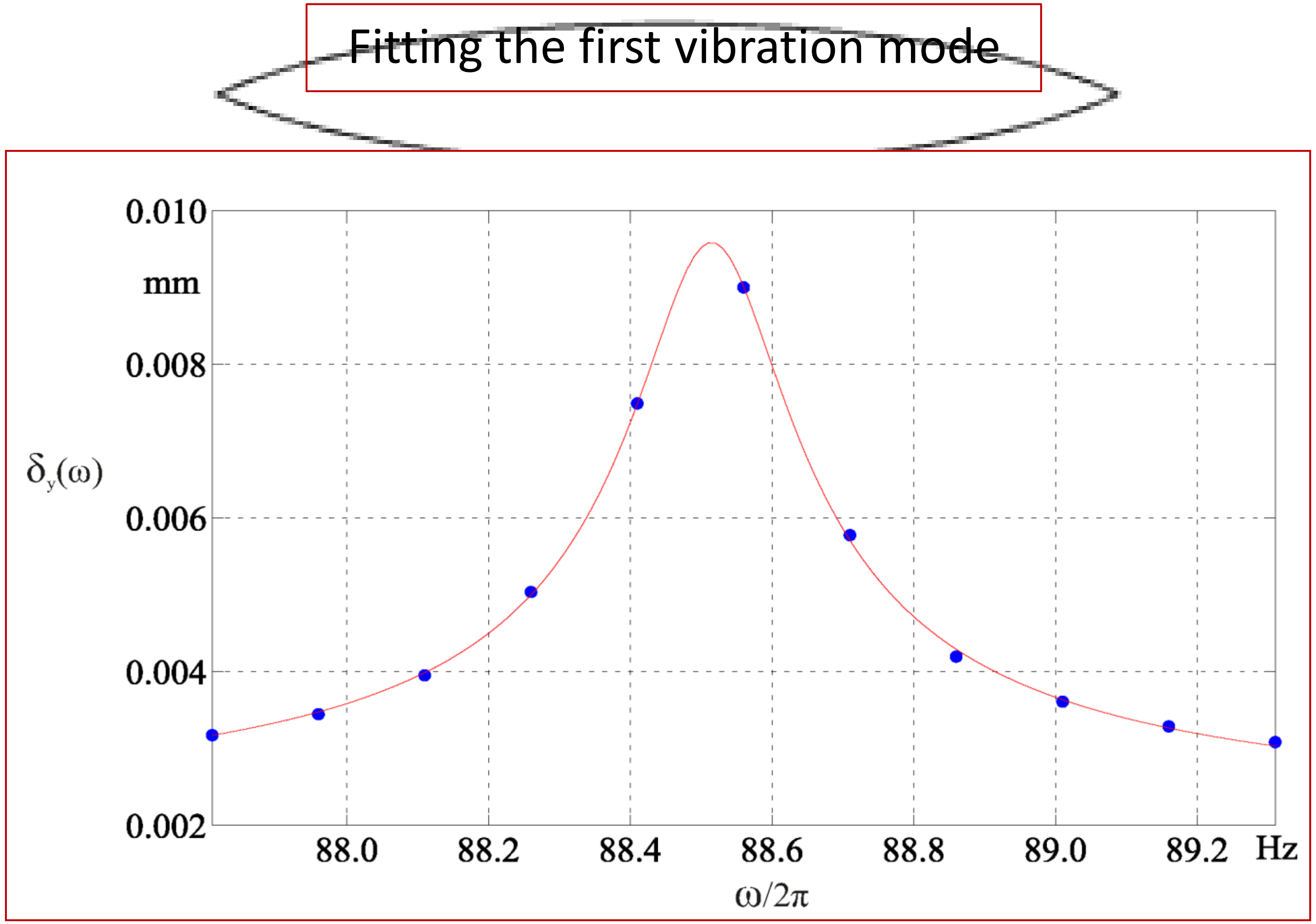
Measurement method

- Measure the frequency response
  - Vibration amplitude and phase
- Fit with the mathematical model
  - Longitudinal field coefficients

$$\mathcal{C}_m := \frac{2}{L}\int_0^L B_\mathrm{n}(z)\sin\left(\frac{m\pi}{L}z\right)\mathrm{d}z$$

- Calculate the longitudinal field profile (by inverse Fourier transform)

$$B_\mathrm{n}(z) = \sum_m \mathcal{C}_m \sin\left(\frac{m\pi}{L}z\right)$$



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