

Precision Jet Production for the LHC

James Currie (IPPP, Durham)

CERN Theory Colloquium, 5th April 2017



European Research Council
Established by the European Commission
**Supporting top researchers
from anywhere in the world**



MC@NNLO

Outline

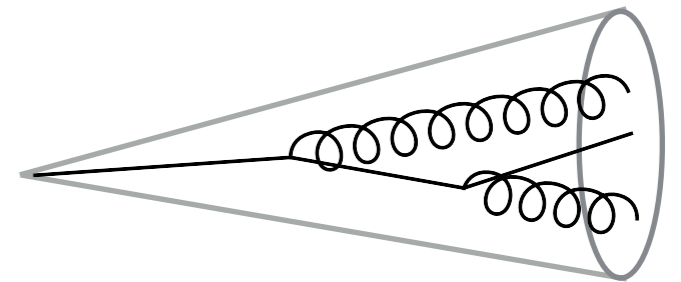
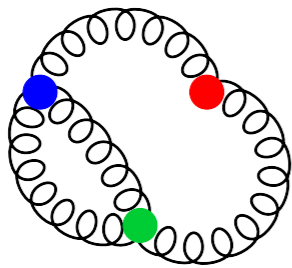
- Jets
- Uses for jets
- Problems when calculating with jets
- Methods for successfully calculating with jets
- Results and future phenomenology

Jets

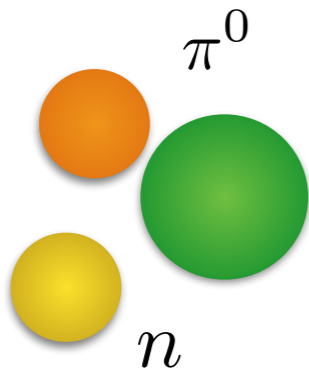
Jets are a rare QCD object common to theory and experiment

$$\mathcal{L} = \bar{\psi}i\not{D}\psi - \frac{1}{4}F \cdot F + \dots$$

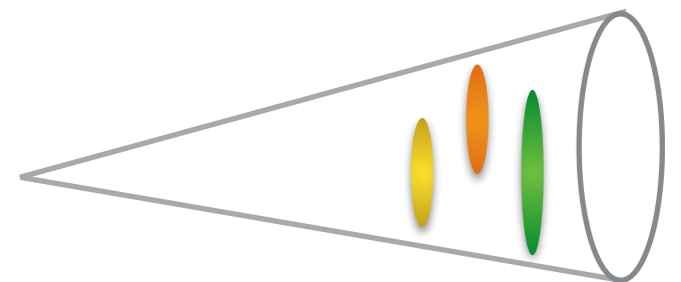
resolution



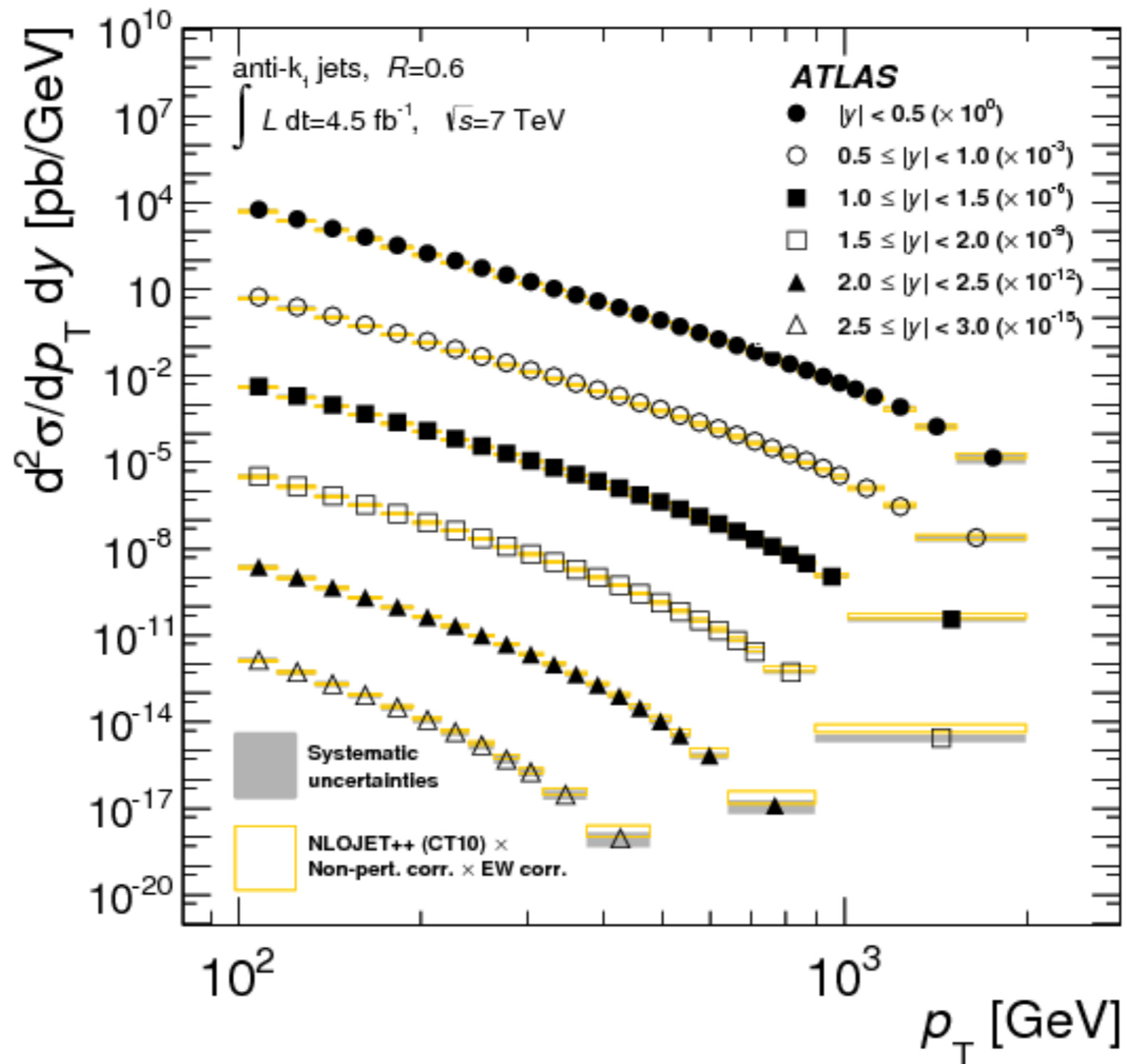
NP correction



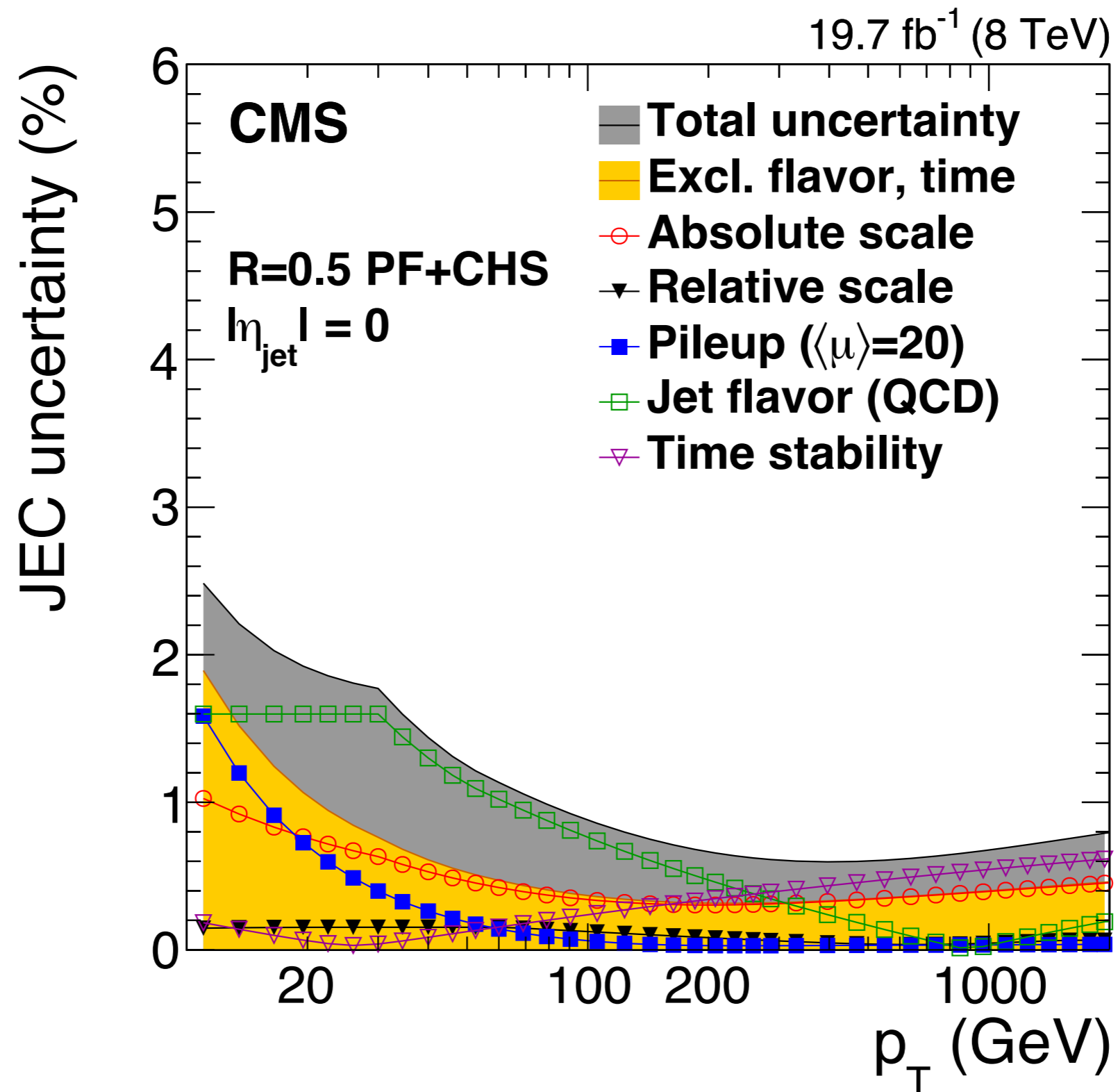
boost to collider
environment



they are produced abundantly at the LHC



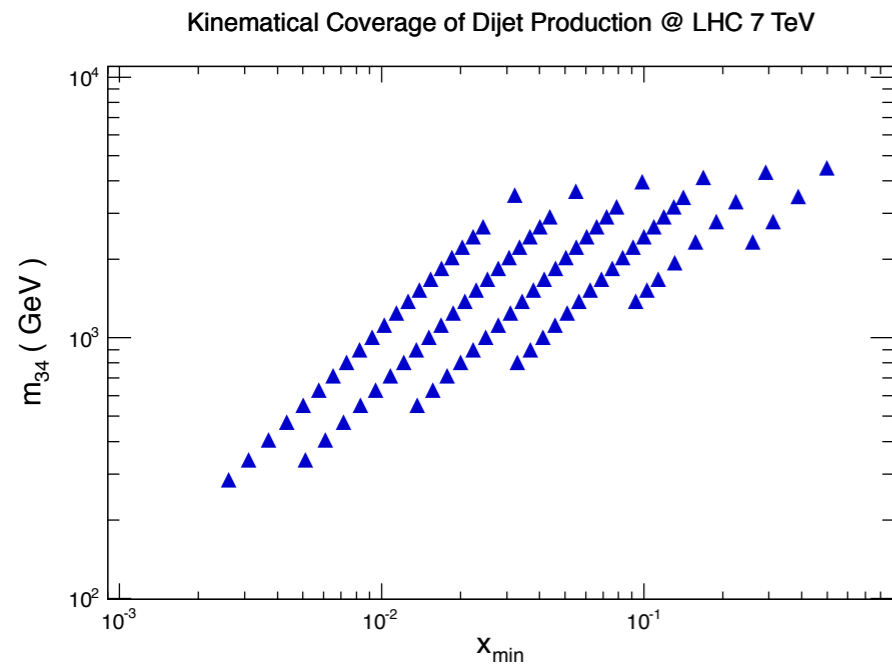
and can be measured very accurately



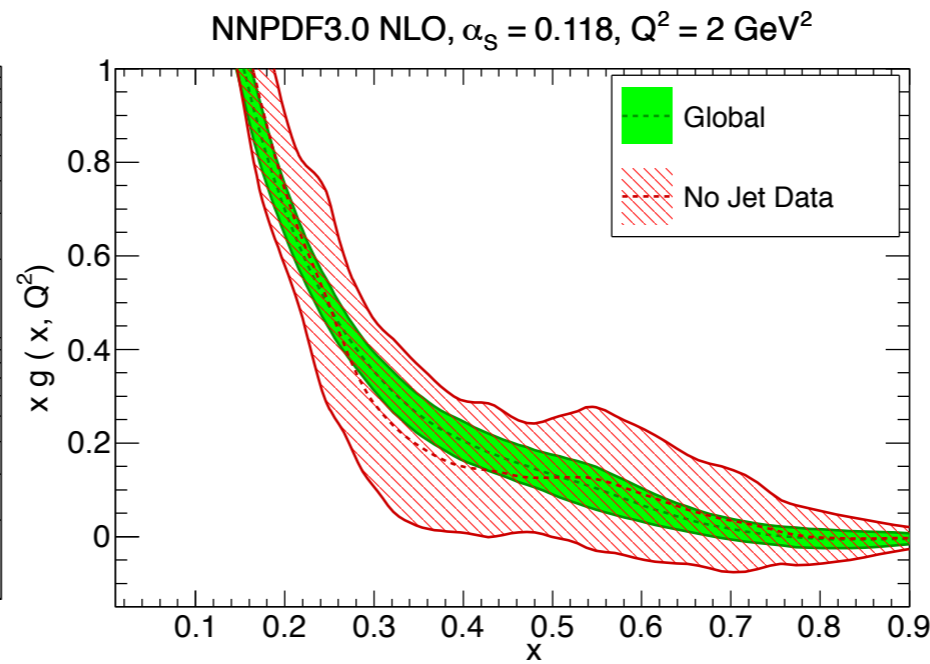
Uses for Jets

LHC is mainly a gluon collider but gluon PDF is not well known:

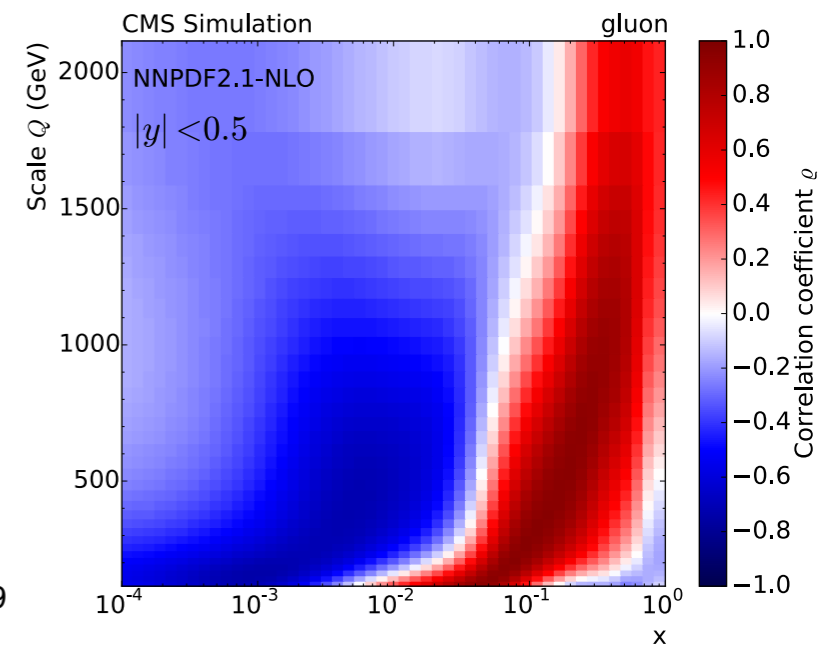
- LHC jets probe a wide range of x
- gluon PDF directly sensitive to jet data, especially at large x
- would like to consistently include jet data in PDF fits



Rojo hep-ph [1410.7728]



NNPDF collaboration hep-ph [1410.8849]

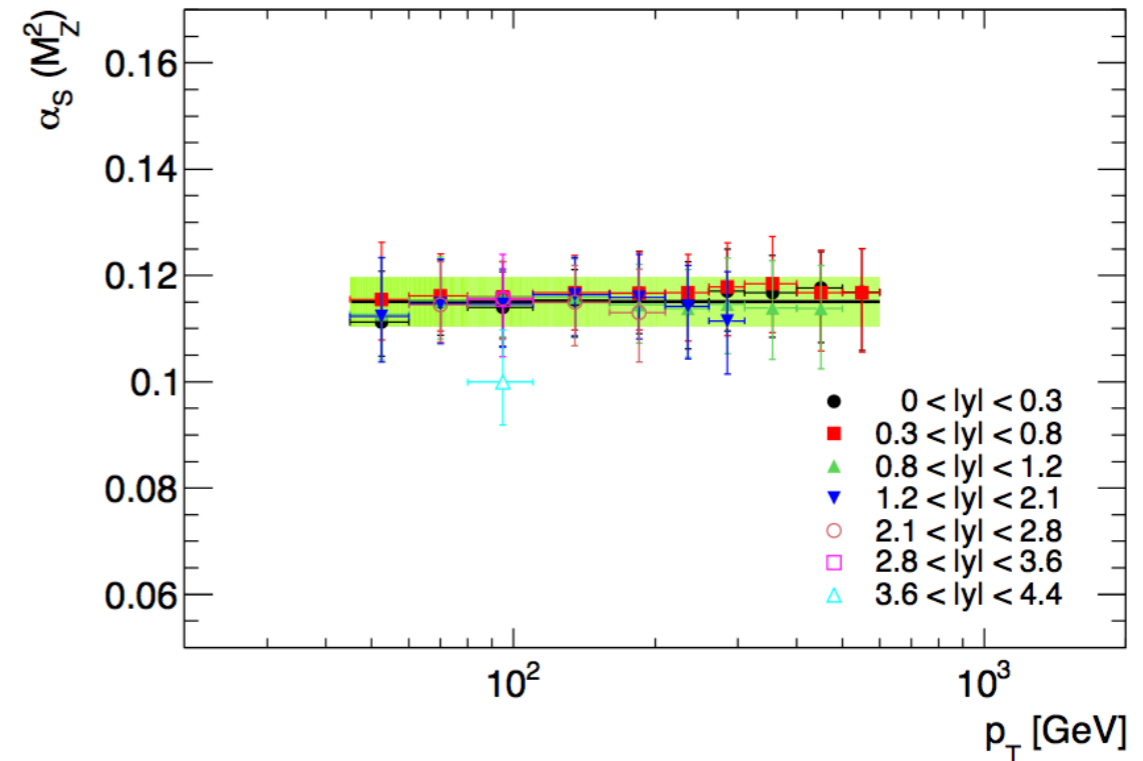
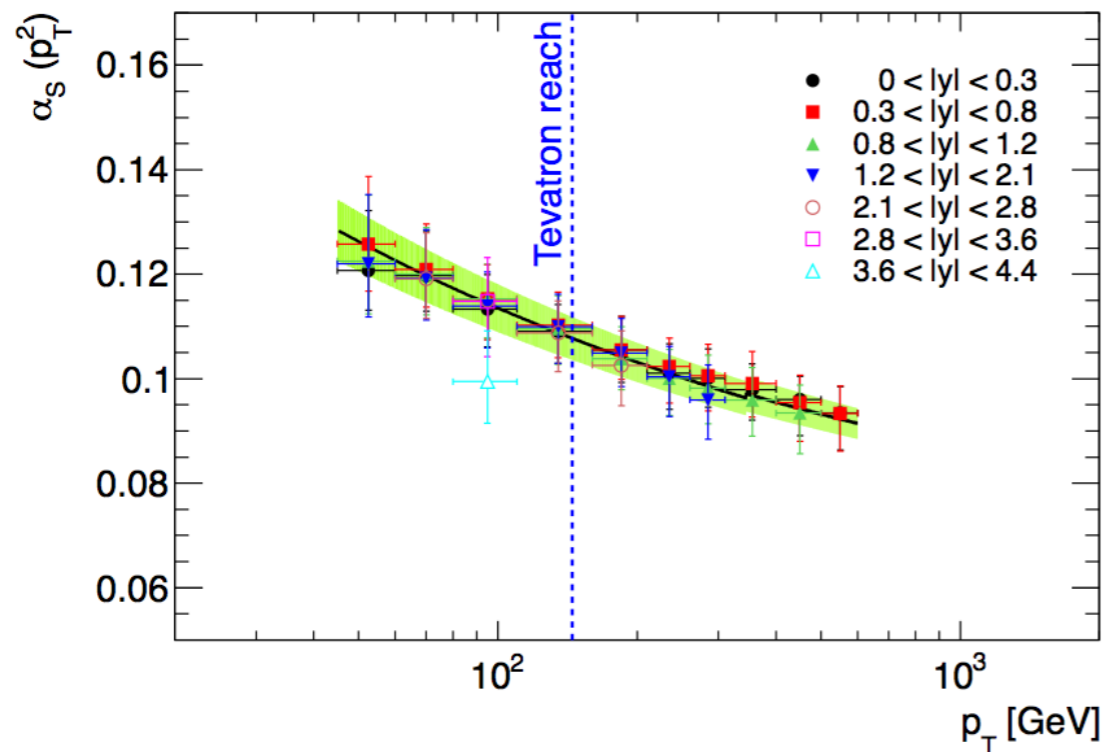


Rojo hep-ph [1410.7728]

Uses for Jets

Can use the single inclusive jet cross section to determine:

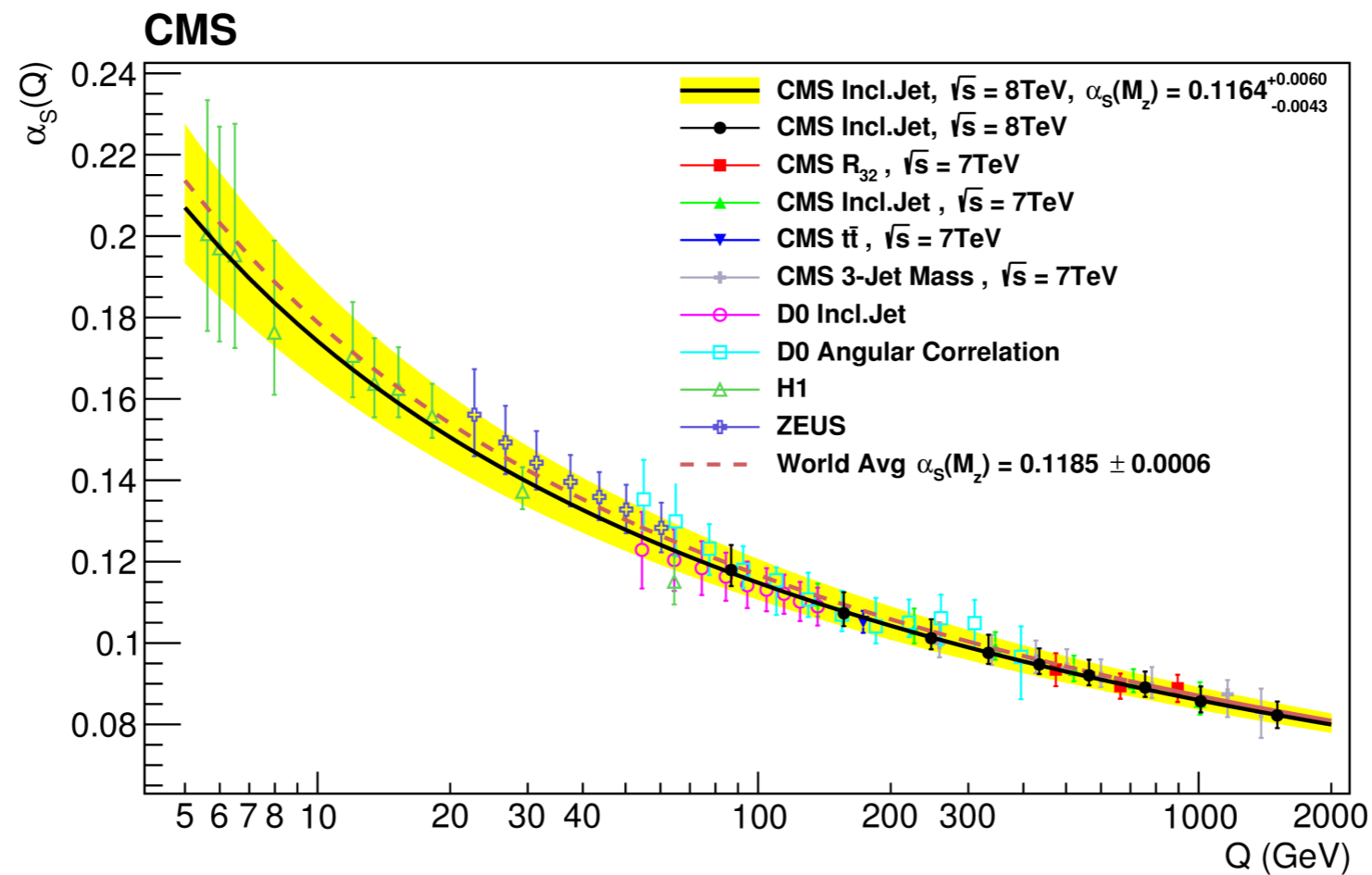
- $\alpha_s(M_Z)$ and running coupling from single experiment
- very satisfying test of QCD and the LHC



Uses for Jets

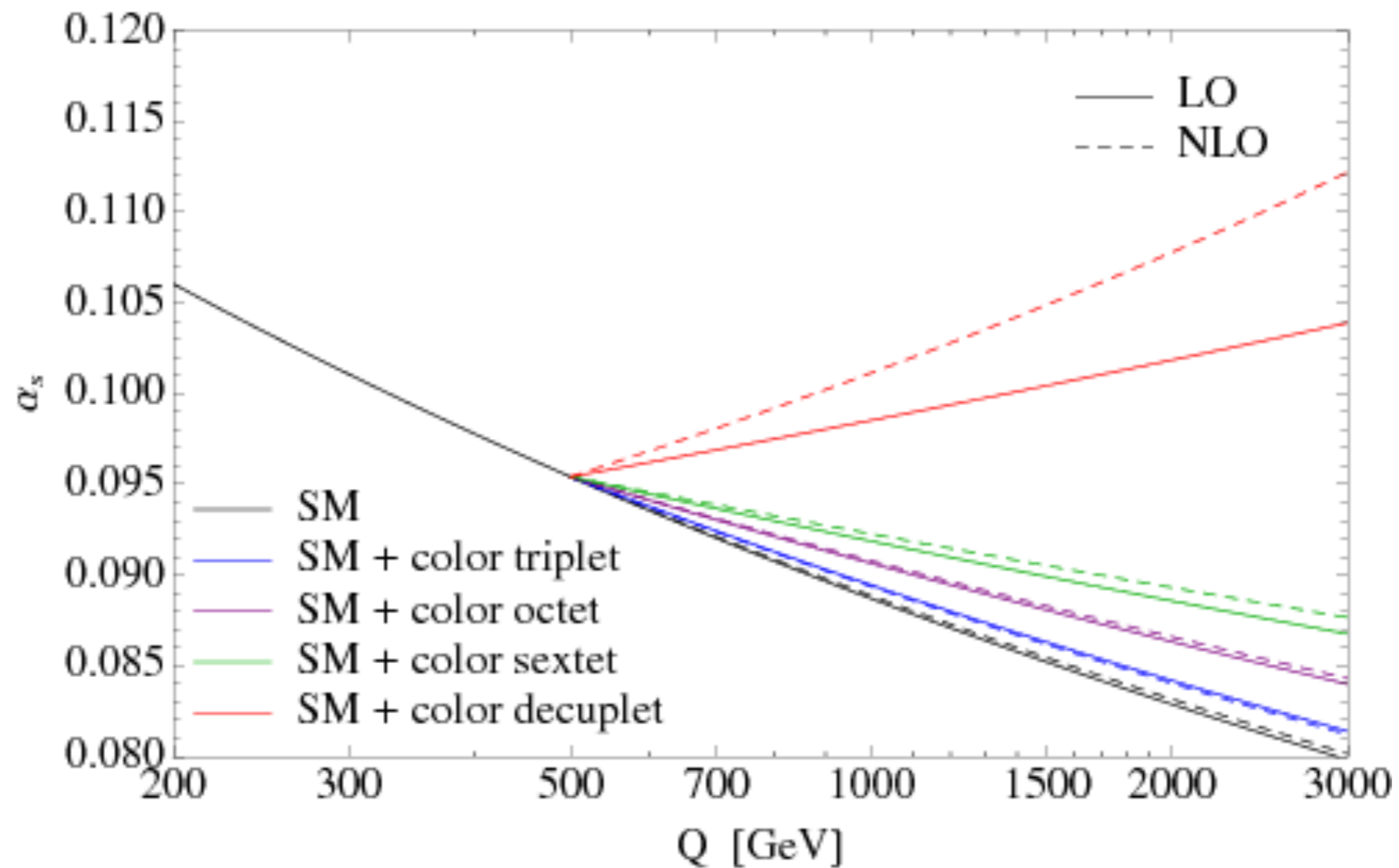
Can use the single inclusive jet cross section to determine:

- $\alpha_s(M_Z)$ and running coupling from single experiment
- very satisfying test of QCD and the LHC



Uses for Jets

- model independent test of (coloured) new physics



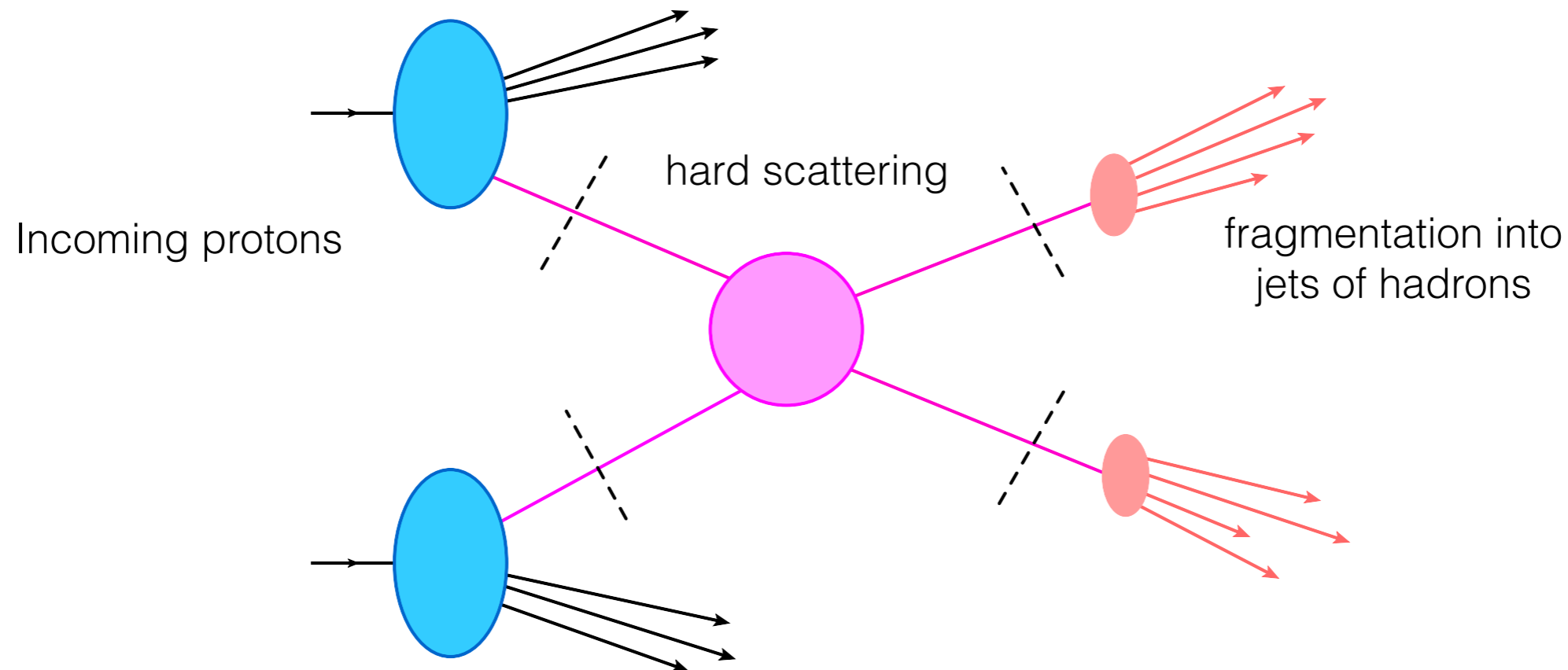
Becciolini, Gillioz, Nardecchia, Sannino, Spannowsky hep-ph [1403.7411]

Calculating things

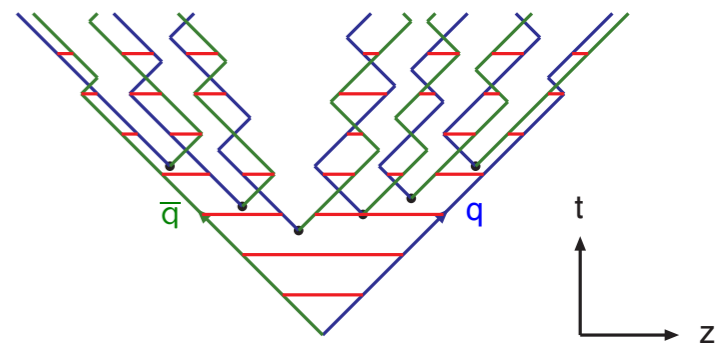
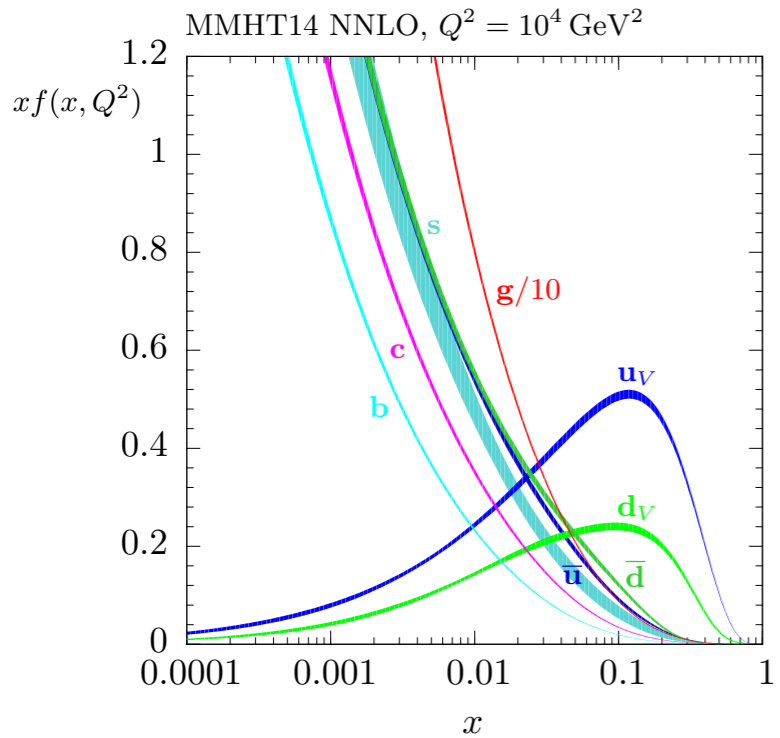
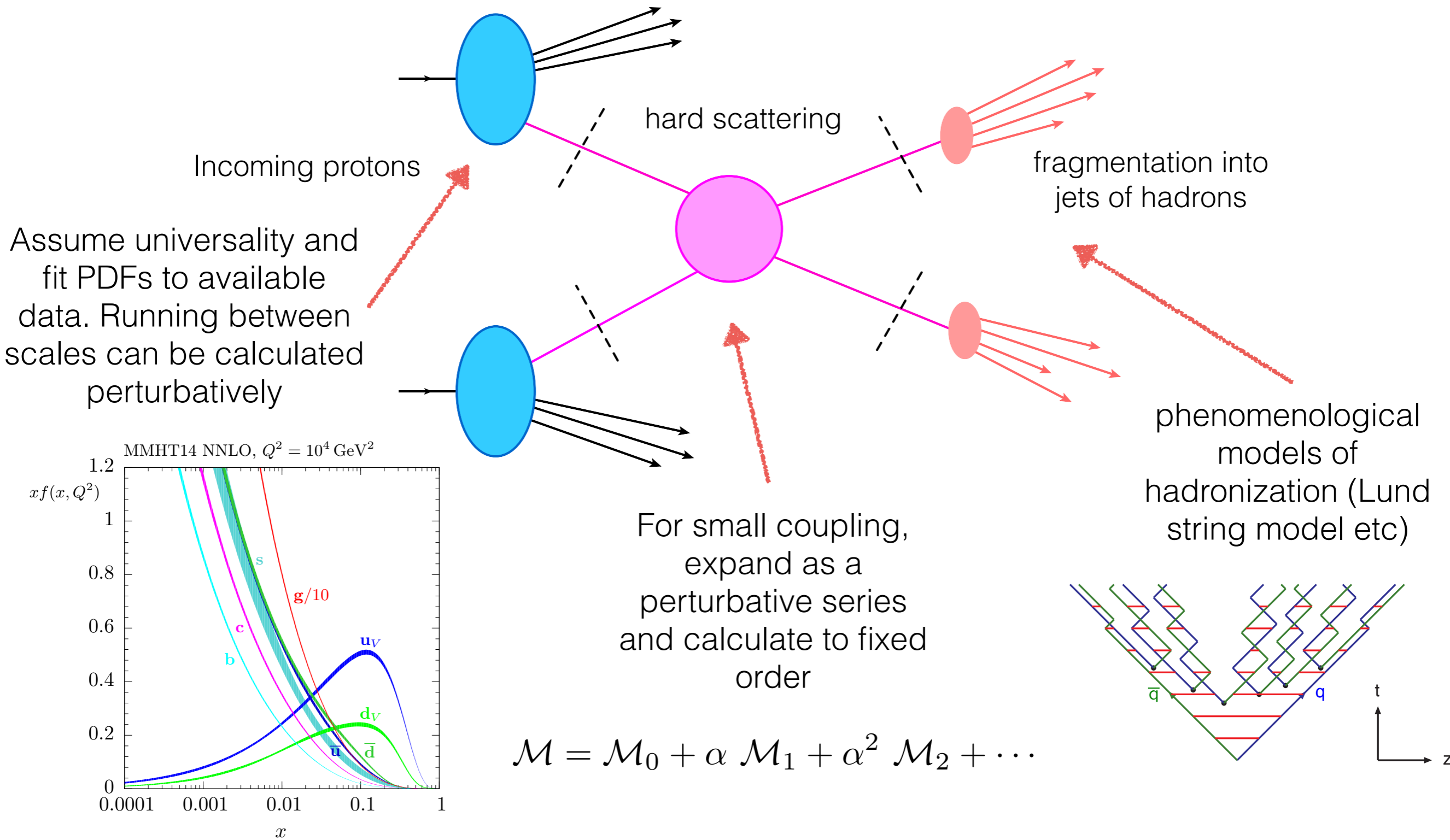
We would like to calculate the transition from colliding protons to outgoing jets:

$$\mathcal{P} \sim |\langle j_1 j_2 \cdots | p_1 p_2 \rangle|^2$$

but we can't calculate this so we simplify (factorize) the problem:



but we can't calculate any of these things either... so we simplify further



Hard scattering

Using vanilla fixed order perturbation theory, can calculate *partonic* cross section:

$$d\sigma = \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a(\xi_1, \mu_F) f_b(\xi_2, \mu_F) d\hat{\sigma}_{ab}(\alpha_s(\mu_R), \mu_R, \mu_F) + \dots$$

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{LO} + d\hat{\sigma}_{ab}^{NLO} + d\hat{\sigma}_{ab}^{NNLO} + \dots$$

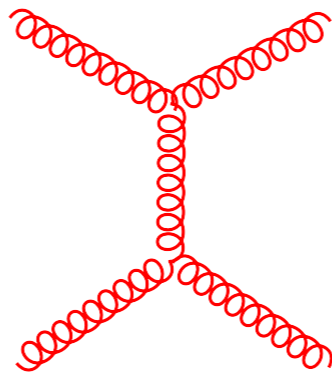
Hard scattering

Using vanilla fixed order perturbation theory, can calculate *partonic* cross section:

$$d\sigma = \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a(\xi_1, \mu_F) f_b(\xi_2, \mu_F) d\hat{\sigma}_{ab}(\alpha_s(\mu_R), \mu_R, \mu_F) + \dots$$

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{LO} + d\hat{\sigma}_{ab}^{NLO} + d\hat{\sigma}_{ab}^{NNLO} + \dots$$

leading order: $\mathcal{O}(\alpha_s^2)$



2 \rightarrow 2
tree-level
“Born”

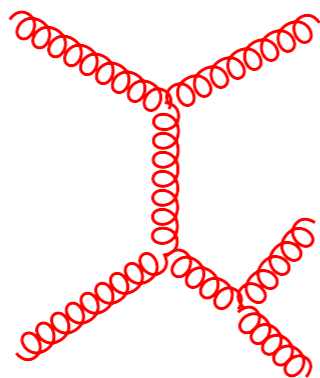
Hard scattering

Using vanilla fixed order perturbation theory, can calculate *partonic* cross section:

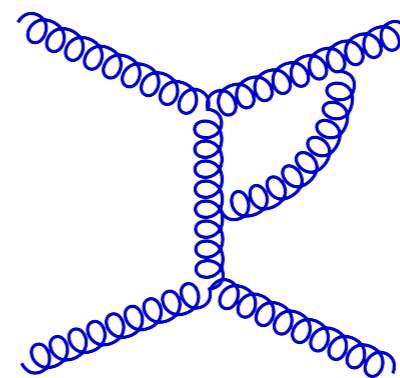
$$d\sigma = \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a(\xi_1, \mu_F) f_b(\xi_2, \mu_F) d\hat{\sigma}_{ab}(\alpha_s(\mu_R), \mu_R, \mu_F) + \dots$$

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{LO} + d\hat{\sigma}_{ab}^{NLO} + d\hat{\sigma}_{ab}^{NNLO} + \dots$$

Next-to-leading order: $\mathcal{O}(\alpha_s^3)$



2 \rightarrow 3
tree-level
“real”



2 \rightarrow 2
one-loop
“virtual”

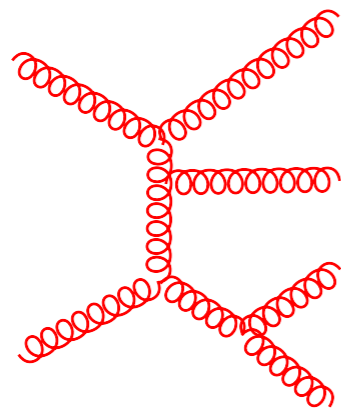
Hard scattering

Using vanilla fixed order perturbation theory, we can calculate the *partonic* cross section:

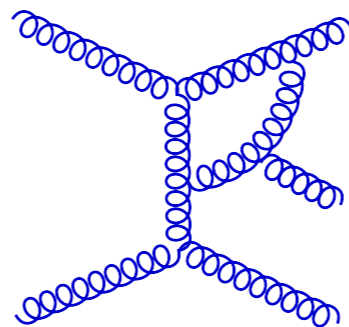
$$d\sigma = \int \frac{d\xi_1}{\xi_1} \frac{d\xi_2}{\xi_2} f_a(\xi_1, \mu_F) f_b(\xi_2, \mu_F) d\hat{\sigma}_{ab}(\alpha_s(\mu_R), \mu_R, \mu_F) + \dots$$

$$d\hat{\sigma}_{ab} = d\hat{\sigma}_{ab}^{LO} + d\hat{\sigma}_{ab}^{NLO} + d\hat{\sigma}_{ab}^{NNLO} + \dots$$

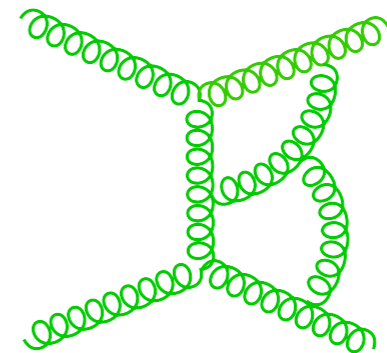
Next-to-next-to-leading order: $\mathcal{O}(\alpha_s^4)$



2 \rightarrow 4
tree-level
“double real”



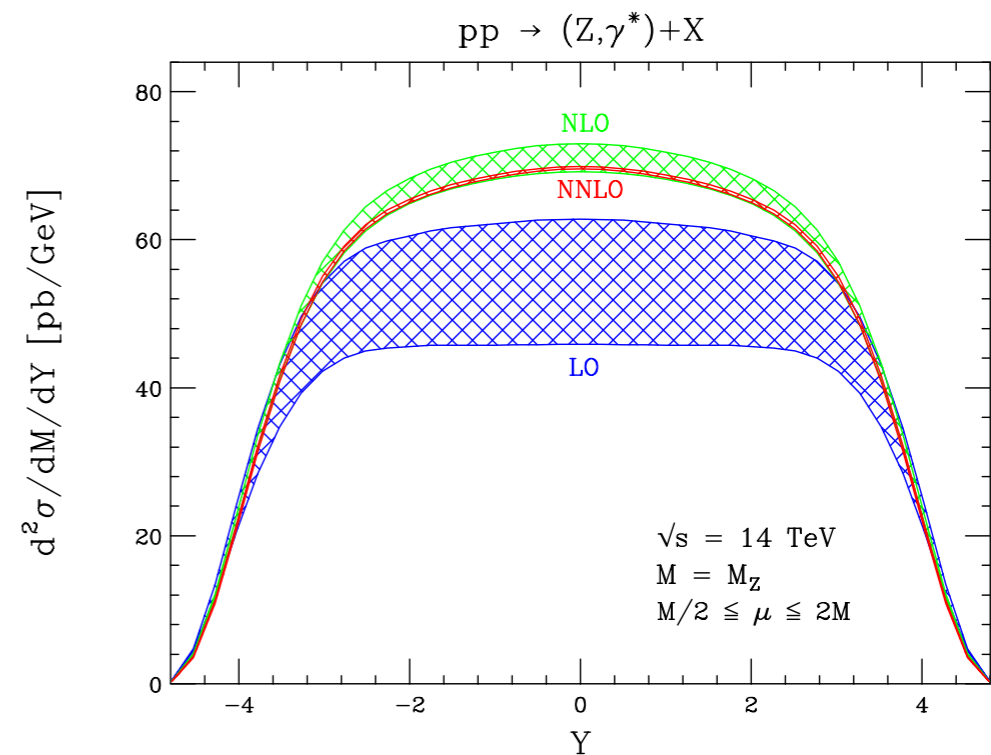
2 \rightarrow 3
one-loop
“real-virtual”



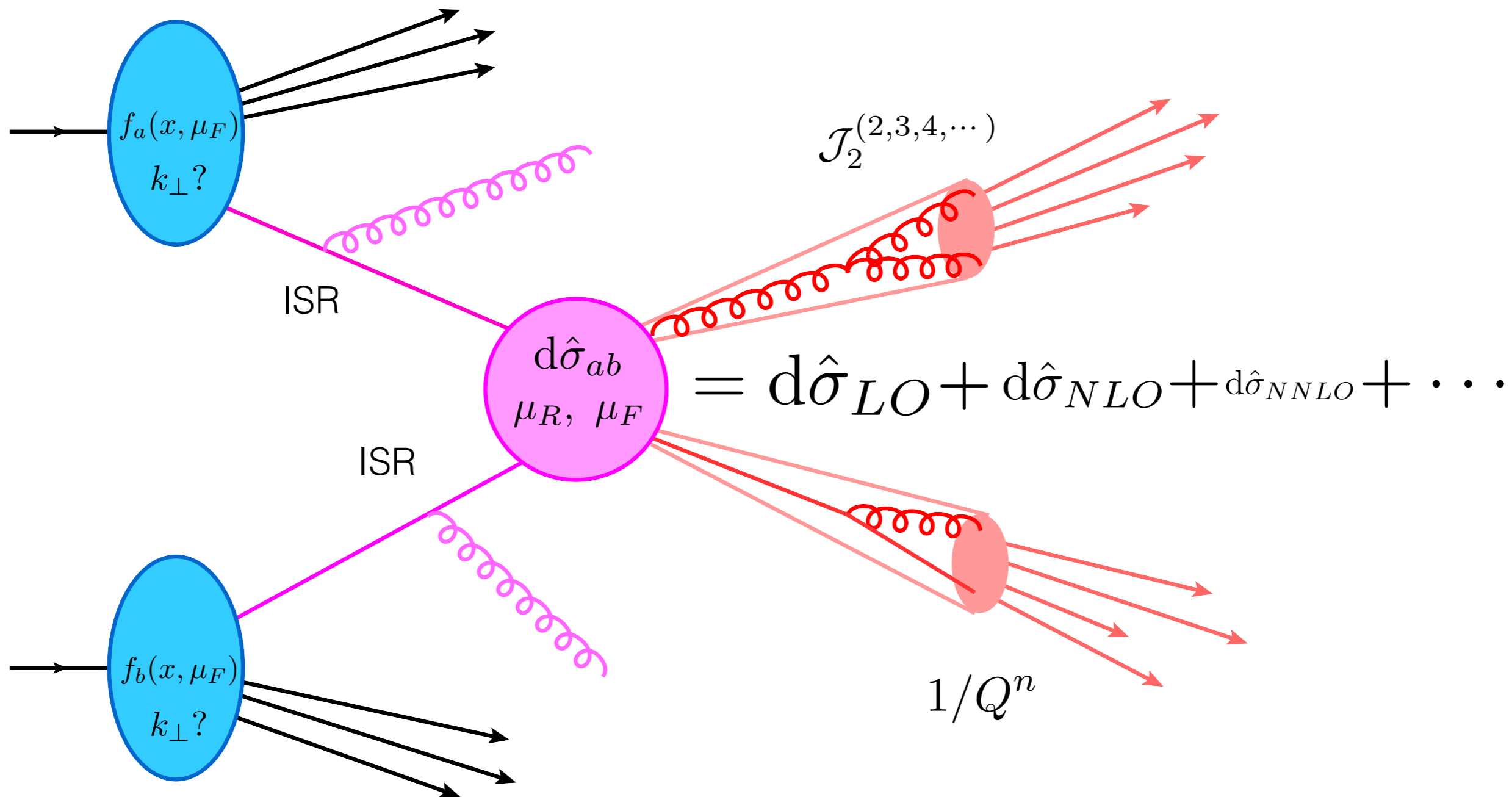
2 \rightarrow 2
two-loop
“double virtual”

Why Higher Orders?

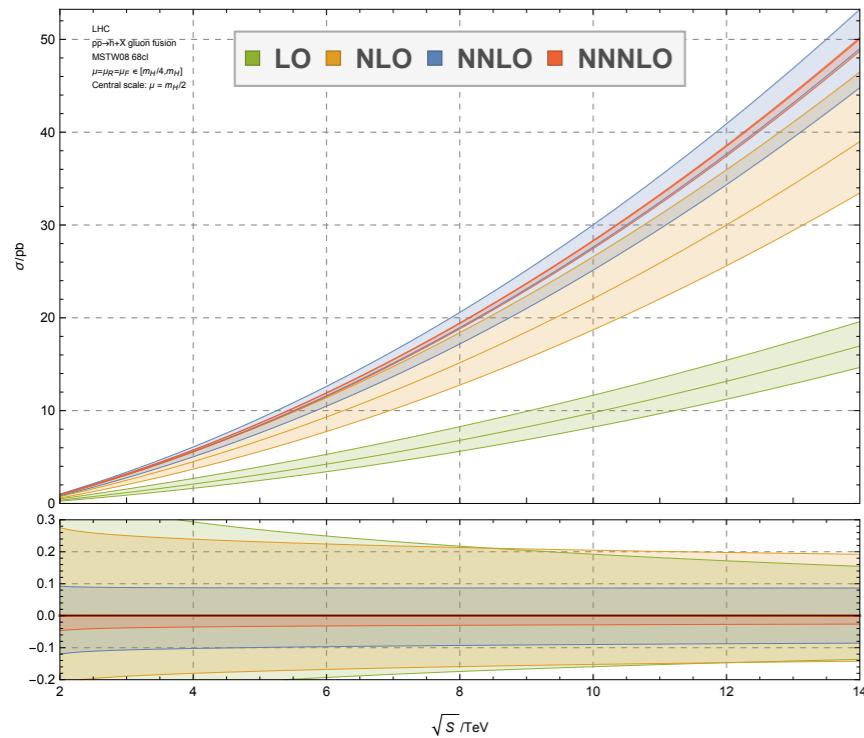
- Theoretical uncertainty typically estimated by dependence on unphysical scales at a given fixed order
- higher orders should reduce this dependence systematically, but also contain physics not captured by scale variation
- NNLO contains all features of calculation
 - initial-state radiation
 - non-trivial jet algorithm
 - all partonic channels
 - non-trivial physical scales
- NLO and NNLO corrections change the normalization and shape of observables



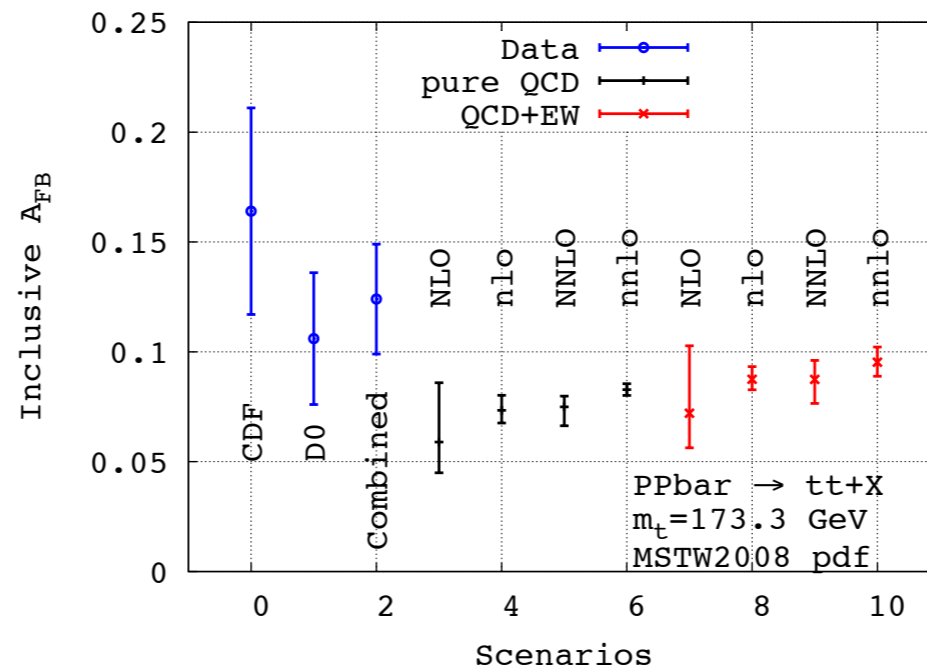
Anastasiou, Dixon, Melnikov, Petriello '04



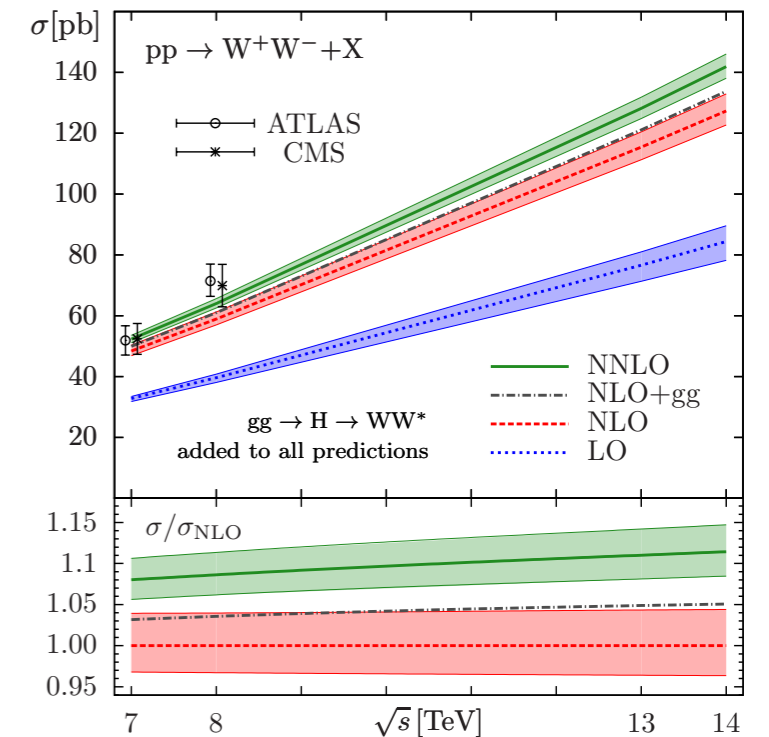
New Physics



Anastasiou, Duhr, Dulat, Herzog,
 Mistlberger



Czakon, Fiedler, Mitov



Gehrmann, Grazzini, Kallweit,
 Maierhöfer, von Manteuffel,
 Pozzorini, Ravlev, Tancredi

No B-SM discovered (yet)... but plenty of B-NLO

Higher orders include physics inaccessible to LO calculations

IR Singularities

Of course life is not that simple:

- phase space integrals over massless states develop IR divergences in soft and collinear limit

for real radiation at NⁿLO $(S + C)^n \sim S^n + S^{n-1}C + \dots + SC^{n-1} + C^n$

- n-loop amplitudes contain singularities in dimensional regularization parameter $\epsilon = (4 - d)/2$

$$\mathcal{M}^n \sim \sum_{m=-2n}^{\infty} c_m \epsilon^m$$

- physical (finite) answer obtained by summing over degenerate intermediate states at the same order in perturbation theory
- need to express singularities in the *same language*

IR Singularities

In unresolved limits the matrix elements factorize:

$$\mathcal{M}_{n+1}(\cdots, p_i, p_j, p_k, \cdots) \xrightarrow{j \text{ unresolved}} U(p_i, p_j, p_k) \mathcal{M}_n(\cdots, p_i, p_k, \cdots)$$

n+1 partons
universal function
n partons

e.g. in single collinear limit:

$$\mathcal{M}_{n+1}(\cdots, p_i, p_j, p_k, \cdots) \xrightarrow{i||j} P_{ij \rightarrow (ij)}(z) \mathcal{M}_n(\cdots, p_{(ij)}, p_k, \cdots)$$

Loop amplitude singularities also factorize, e.g.

$$\mathcal{M}_n^1(p_1 \cdots, p_n) = \mathbf{I}(p_1 \cdots, p_n) \mathcal{M}_n^0(p_1 \cdots, p_n) + \mathcal{O}(\epsilon^0)$$

1-loop
universal singular function
tree-level

factorization and universality are central to all methods for higher order calculations

Methods at NLO

Main problem at NLO is extracting singularities...
many ways to do this:

- Dipole subtraction [Catani, Seymour '96]
- FKS subtraction [Frixione, Kunszt, Signer '95]
- Sector decomposition [Hepp '67; Binoth, Heinrich '00]
- Phase space slicing [Giele, Glover '91]

Methods at NNLO

Main problem at NNLO is disentangling singularities

Most methods basically a generalization of NLO:

- Antenna subtraction [Kosower '03; Gehrmann, Gehrmann-De Ridder, Glover '05]
- CoLorFul subtraction [Del Duca, Somogyi, Trocsanyi '06] (dipoles)
- Projection to Born [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]
- Sector-improved residue subtraction [Czakon '10] (FKS+sectors)
- qT and N-Jettiness subtraction [Catani, Grazzini '07; Gaunt, Stahlhofen, Tackmann, Walsh '15; Boughezal, Focke, Liu, Petriello '15] (slicing)

(not an exhaustive list)

Subtraction at NLO

$$d\sigma_{ab,NLO} = \int_{\Phi_{m+1}} d\sigma_{ab}^R + \int_{\Phi_m} d\sigma_{ab}^V + d\sigma_{ab}^{MF}$$

Reorganize cross section by adding zero

$$d\sigma_{ab,NLO} = \int_{\Phi_{m+1}} \left[d\sigma_{ab}^R - d\sigma_{ab}^S \right] + \int_{\Phi_m} \left[d\sigma_{ab}^V - d\sigma_{ab}^T \right]$$

$$d\sigma_{ab}^T = - \int_1 d\sigma_{ab}^S - d\sigma_{ab}^{MF}$$

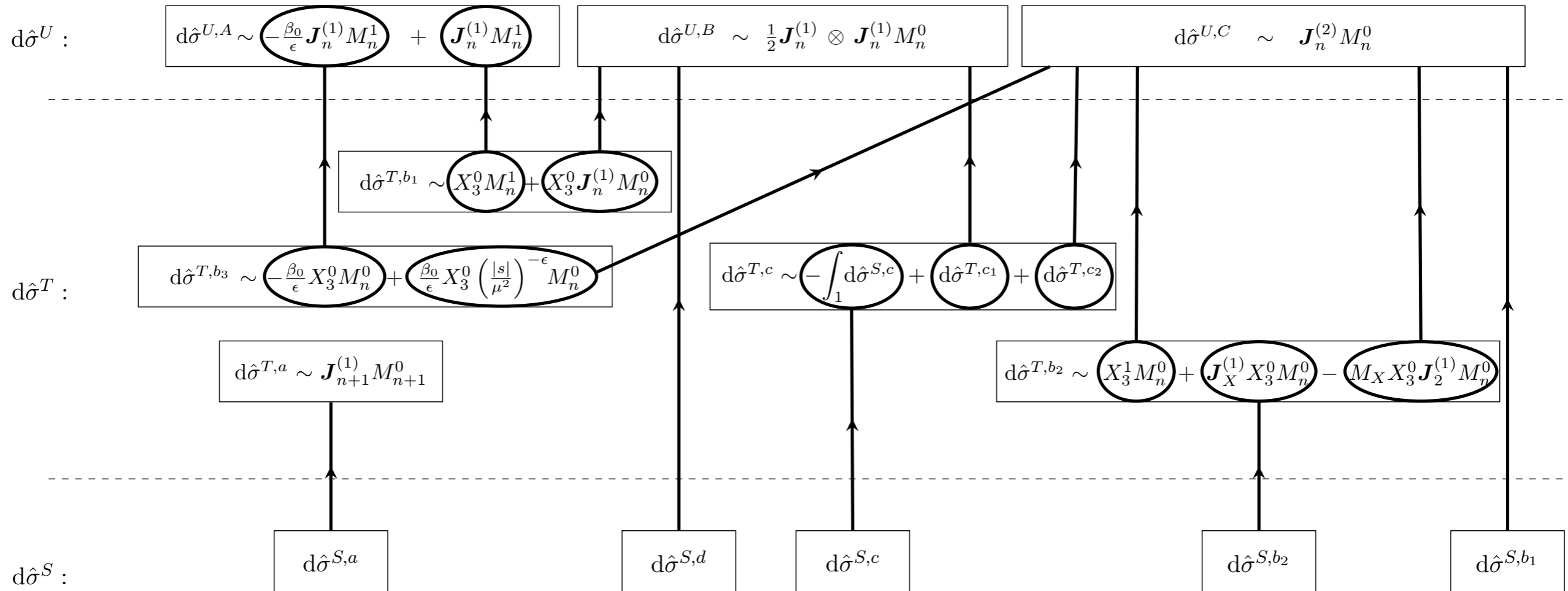
Subtraction at NNLO

At NNLO more terms to regulate

$$\begin{aligned} d\sigma_{ab,NNLO} = & \int_{\Phi_{m+2}} \left[d\sigma_{ab}^{RR} - d\sigma_{ab}^S \right] \\ & + \int_{\Phi_{m+1}} \left[d\sigma_{ab}^{RV} - d\sigma_{ab}^T \right] \\ & + \int_{\Phi_m} \left[d\sigma_{ab}^{VV} - d\sigma_{ab}^U \right] \end{aligned}$$

$$d\sigma_{ab}^T = d\sigma_{ab}^{V,S} - \int_1 d\sigma_{ab}^S - d\sigma_{ab}^{MF,1}$$

$$d\sigma_{ab}^U = - \int_1 d\sigma_{ab}^{V,S} - \int_2 d\sigma_{ab}^S - d\sigma_{ab}^{MF,2}$$



Antenna Subtraction

Basic idea:



construct a counterterm that mimics the matrix element
in all singular regions of phase space

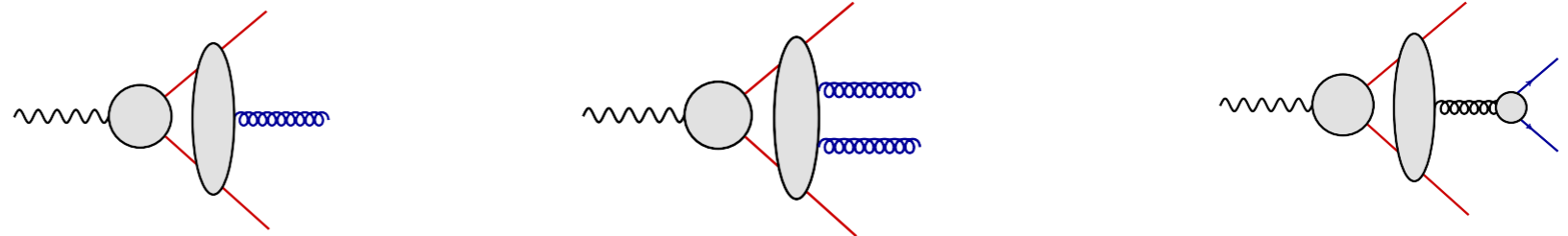
Antennae

Antenna functions built from matrix elements:

$$X_3^0(i, j, k) \sim \frac{|\mathcal{M}_3^0(i, j, k)|^2}{|\mathcal{M}_2^0(I, K)|^2}, \quad X_4^0(i, j, k, l) \sim \frac{|\mathcal{M}_4^0(i, j, k, l)|^2}{|\mathcal{M}_2^0(I, L)|^2}$$

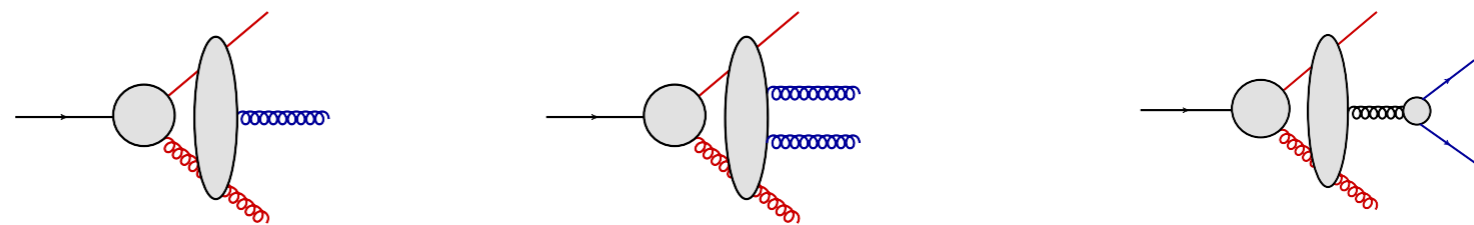
Quark-antiquark:

$$\gamma^* \rightarrow q\bar{q} + \dots$$



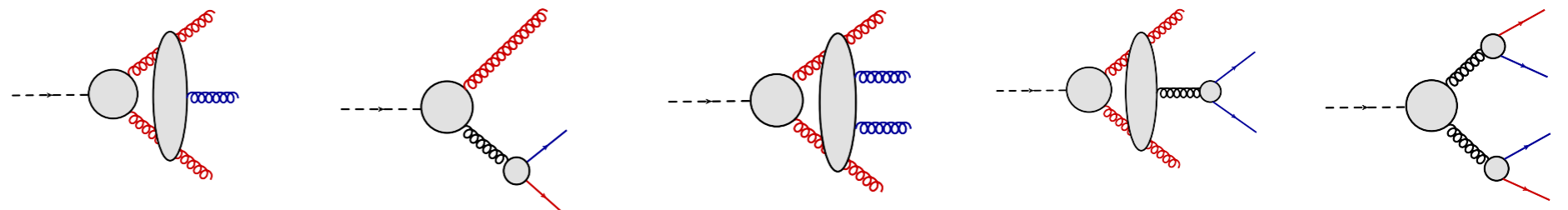
Quark-gluon:

$$\bar{\chi}^0 \rightarrow \tilde{g}g + \dots$$

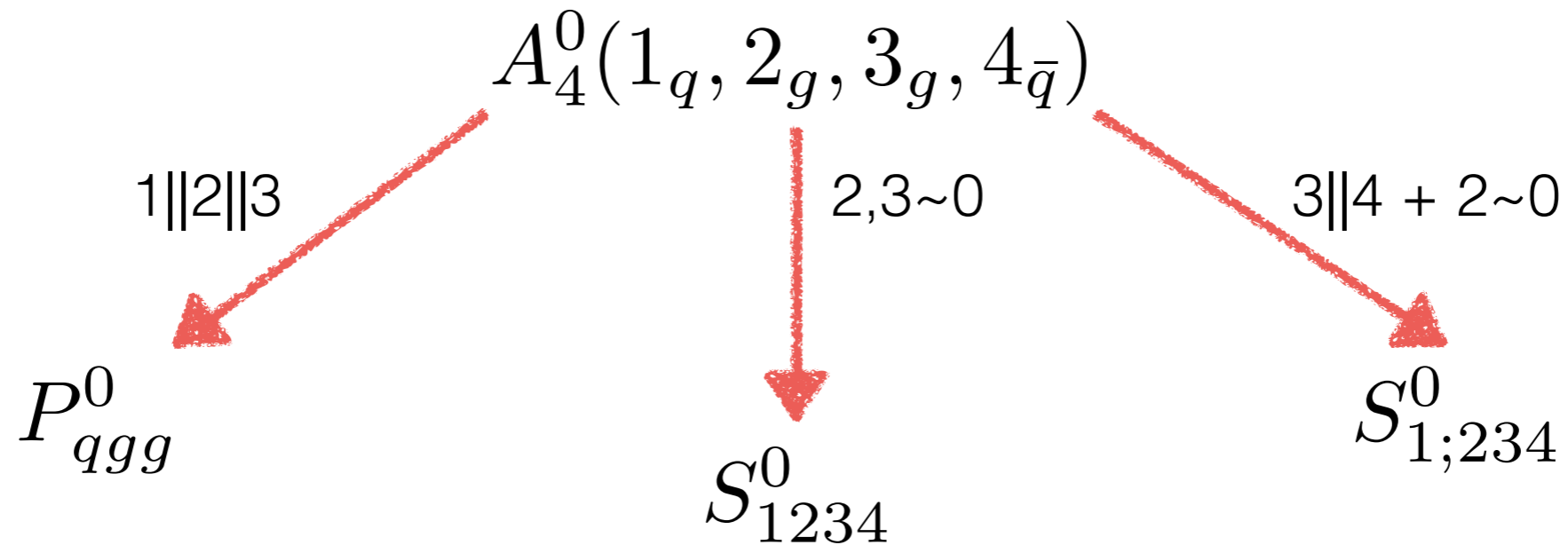


Gluon-gluon:

$$H \rightarrow gg + \dots$$



Antenna mimics all singularities of QCD



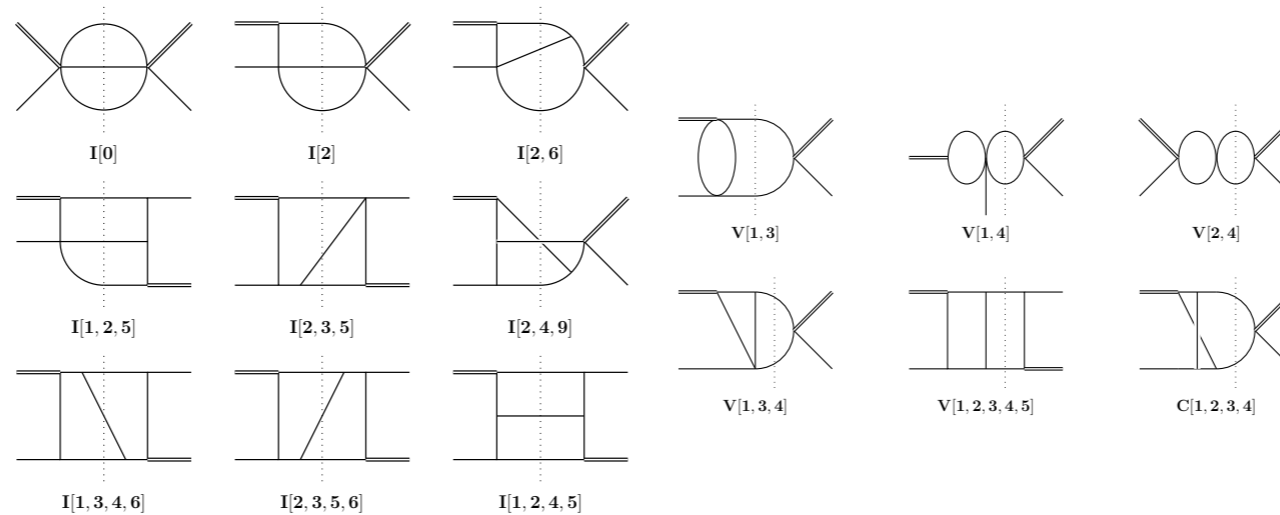
Phase space map smoothly interpolates momenta for reduced matrix element between limits

$$\widetilde{(123)} = xp_1 + r_1p_2 + r_2p_3 + zp_4$$

$$\widetilde{(234)} = (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4$$

Integrating the Antennae

- Relate phase space integrals to multiloop integrals via optical theorem
- apply well developed techniques IBP, LI to masters

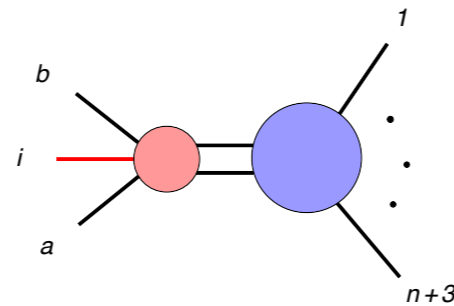


- all antennae in all crossings now successfully integrated:
 - Final-Final [Gehrman, Gehrmann-De Ridder, Glover '04, '05]
 - Initial-Final [Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni '10]
 - Initial-Initial [Gehrmann, Monni '11; Boughezal, Gehrmann-De Ridder, Ritzmann '11; Gehrmann, Ritzmann '12]

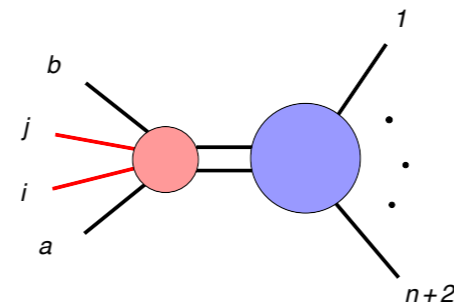
Double Real

Subtraction term constructed to remove:

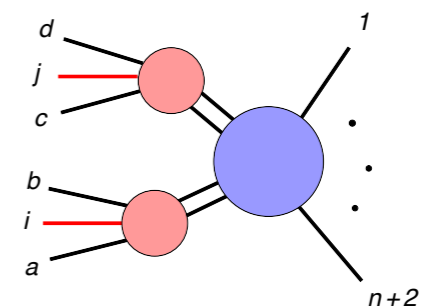
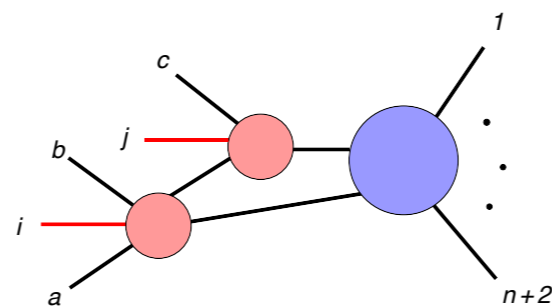
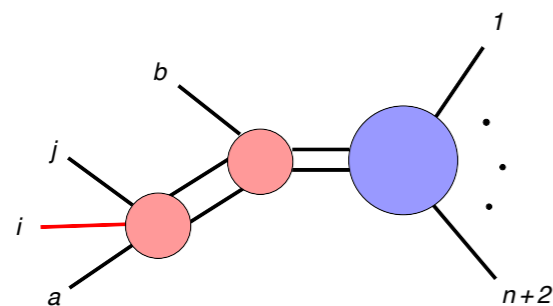
- single unresolved



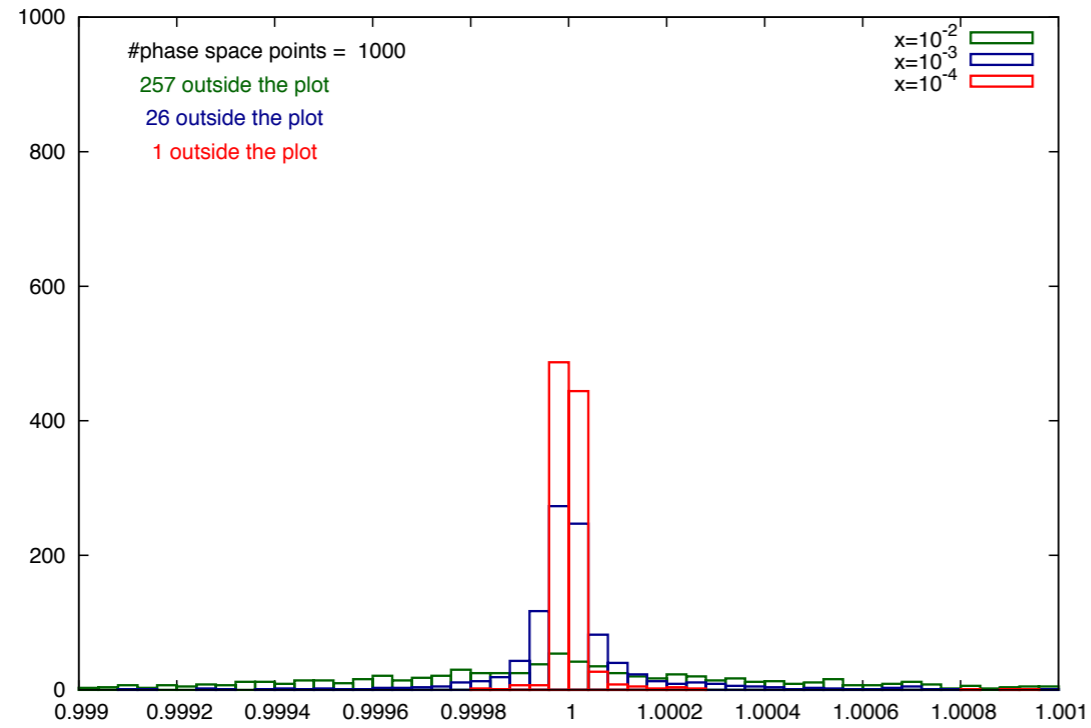
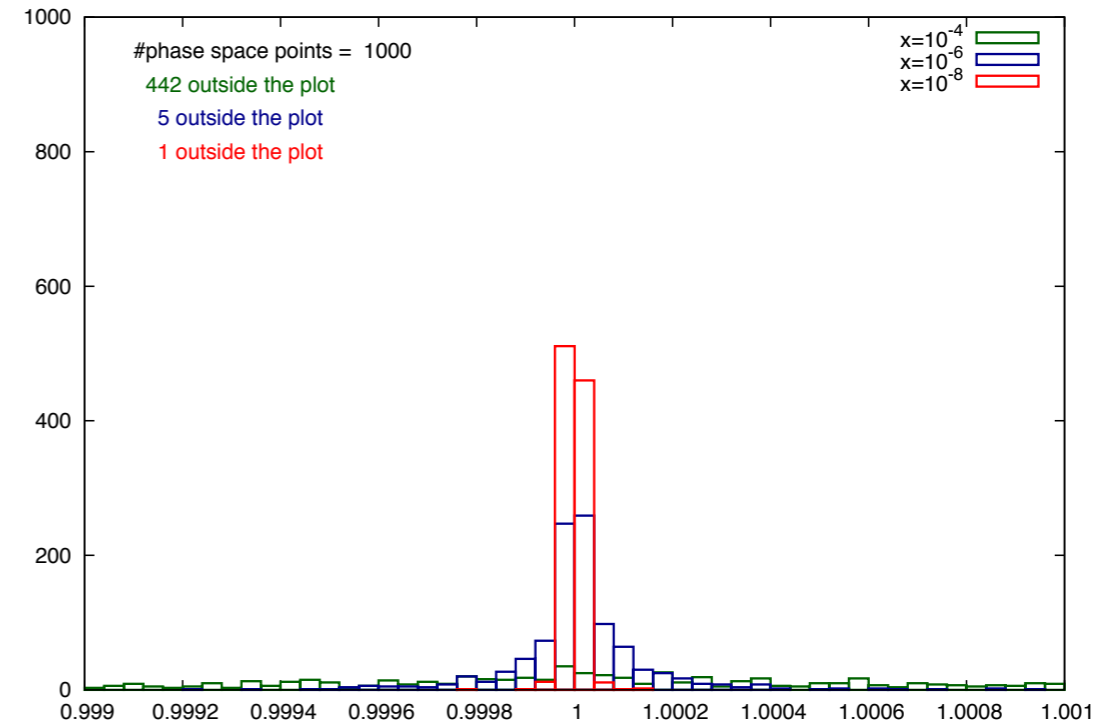
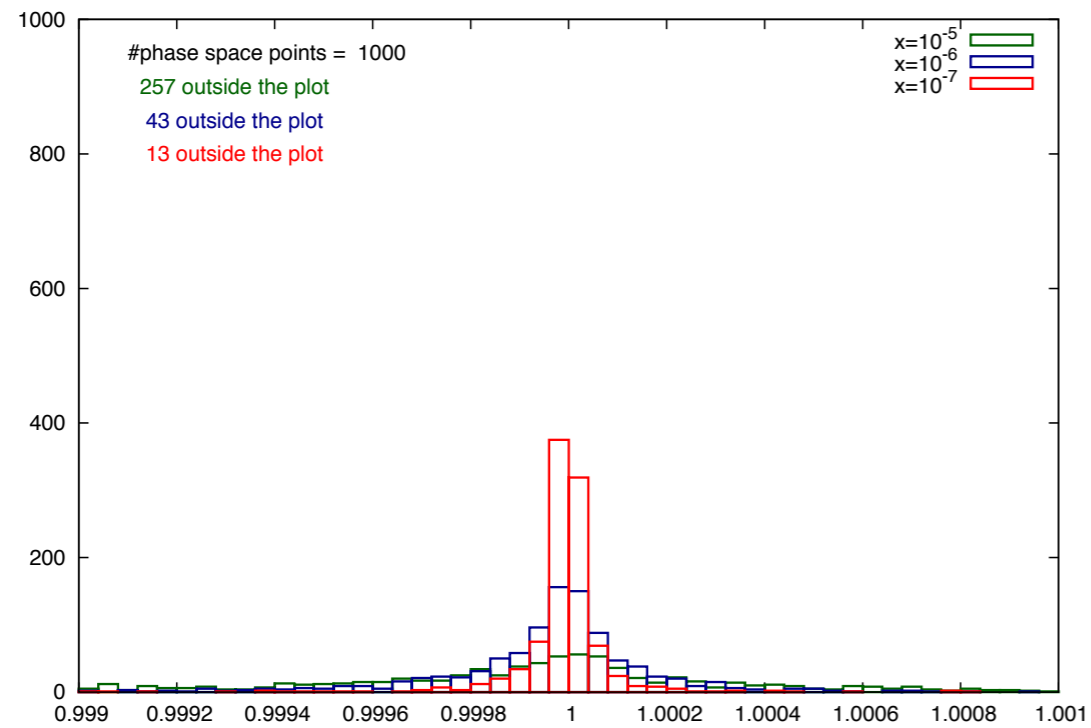
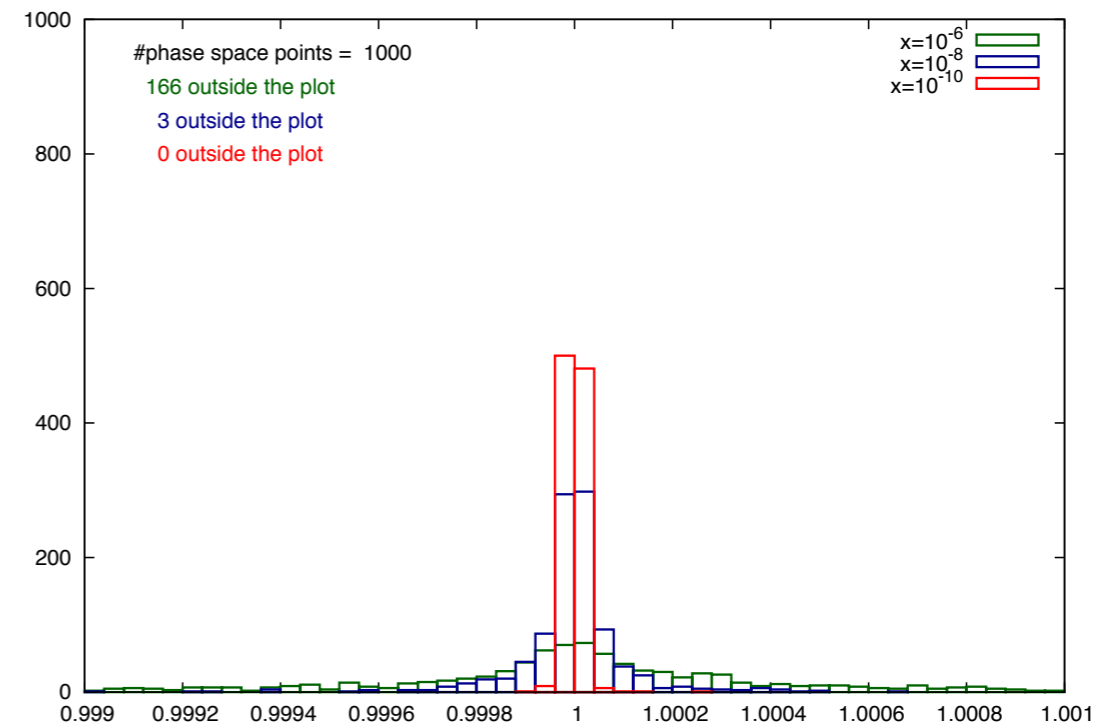
- colour connected double unresolved



- over-subtraction in single and double unresolved limits



7.Double soft

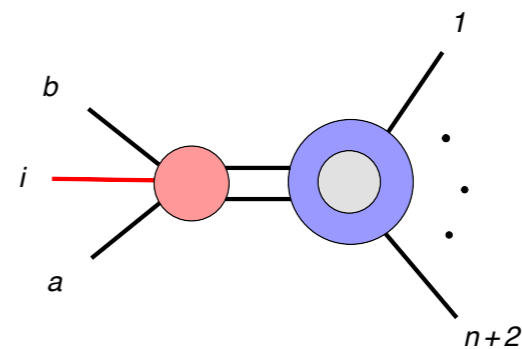
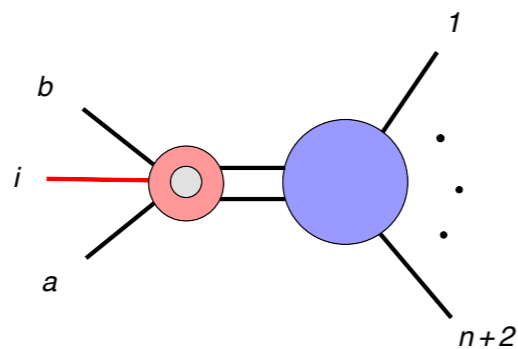
11.IF triple collinear $g|g|g$ 20.Soft + FF collinear $q|g$ 14.IF-FF double collinear $q|g-g|g$ 

Real Virtual

1. Analytic pole cancellation against 1-loop matrix element

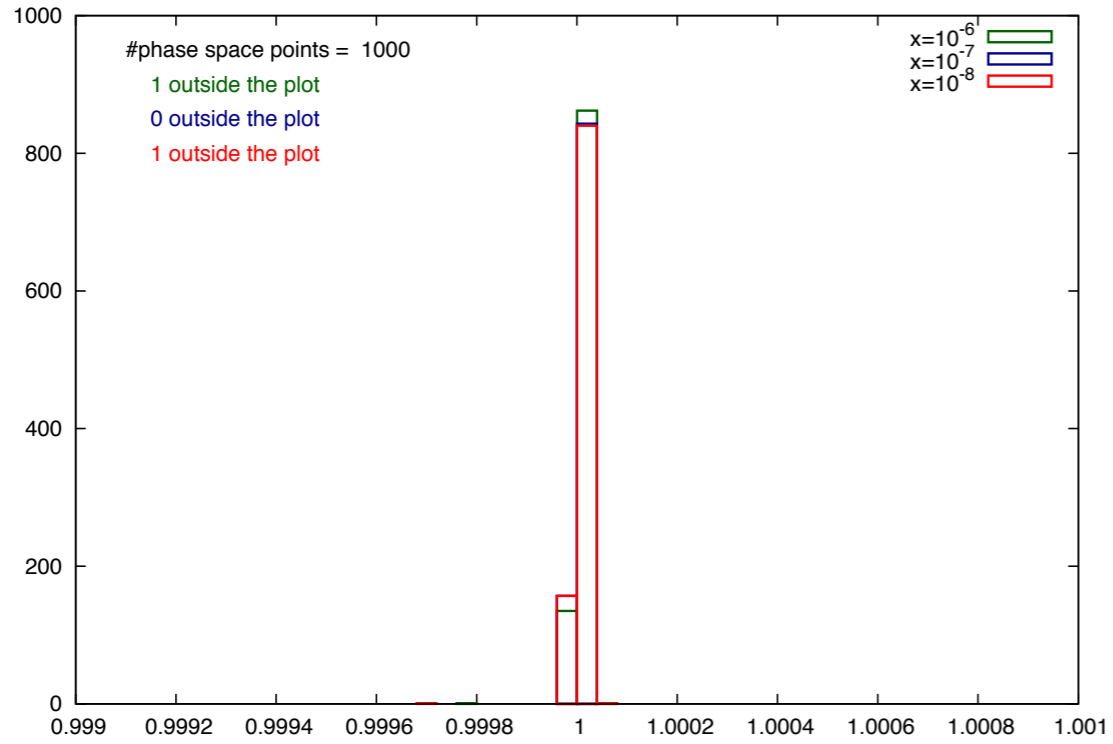
$$2\text{Re}\langle \mathcal{M}_{n+3}^0 | \mathcal{M}_{n+3}^1 \rangle + \mathbf{J}_{n+3}^{(1)}(1, \dots, n+3; \epsilon) \langle \mathcal{M}_{n+3}^0 | \mathcal{M}_{n+3}^0 \rangle = \mathcal{O}(\epsilon^0)$$

2. Only single unresolved limits

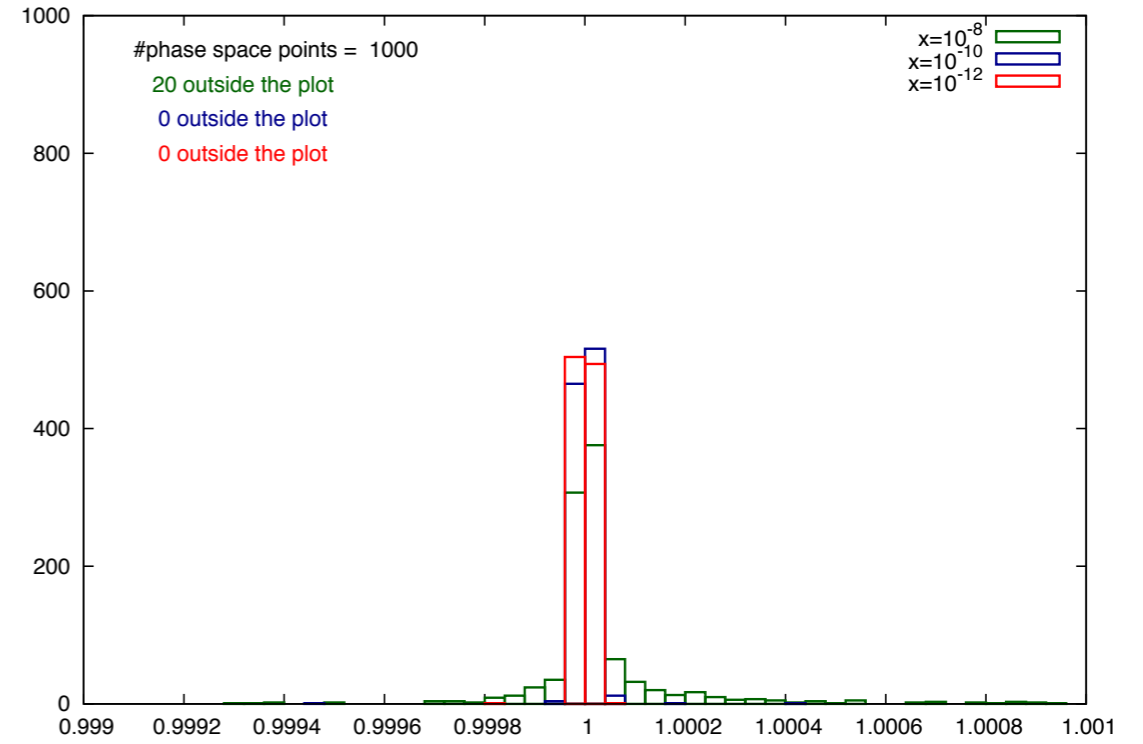


Single unresolved of (1) and poles of (2) also subtracted

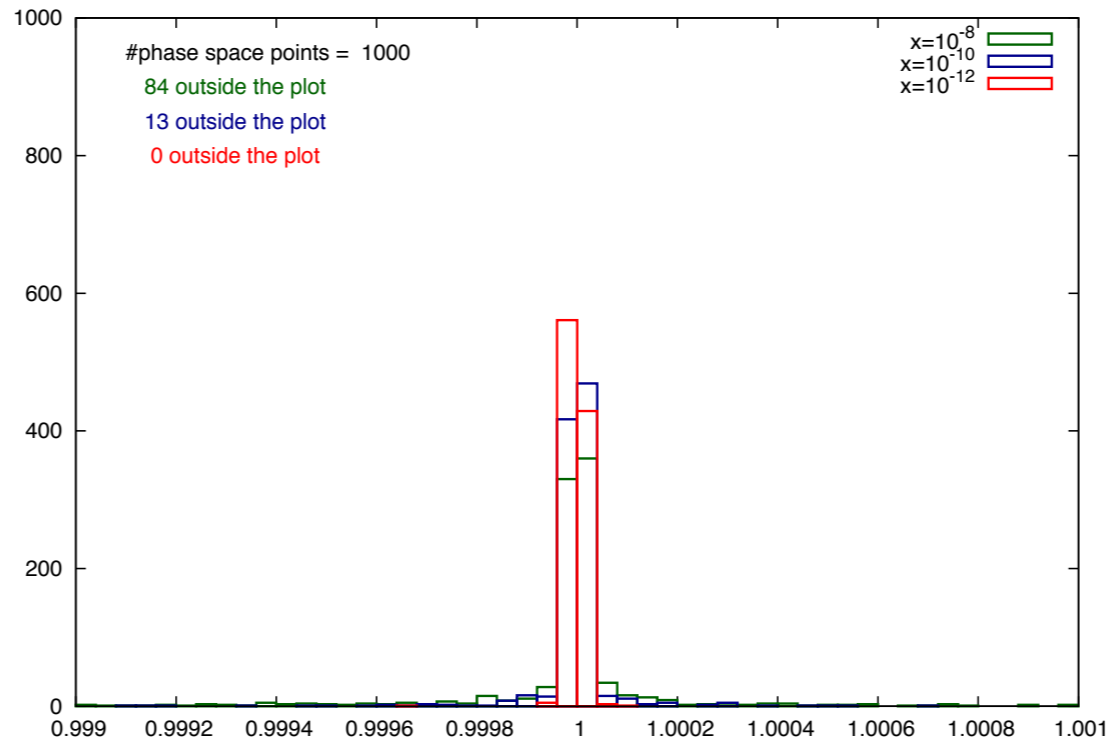
Single soft - 3



Single collinear - 3/4



Single collinear - 1/3



Double Virtual

Analytic pole cancellation against 2-loop and (1-loop)² matrix element

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoA4g2XU.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:02:57 2016
#-
```

```
poles = 0;
```

```
17.51 sec out of 17.53 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoA4g2YU.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:03:24 2016
#-
```

```
poles = 0;
```

```
6.55 sec out of 6.55 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoAh4g2XU.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:03:36 2016
#-
```

```
poles = 0;
```

```
8.48 sec out of 8.48 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoAh4g2YU.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:03:49 2016
#-
```

```
poles = 0;
```

```
4.90 sec out of 4.91 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoggB2g2U1.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:04:14 2016
#-
```

```
poles = 0;
```

```
17.61 sec out of 17.64 sec
```

```
James@Jamess-MacBook-Pro-4:~/hepforge/maple/process/jet$ form autoqgB2g2XU.frm
FORM 4.1 (Mar 13 2014) 64-bits                               Run: Wed Nov 16 15:04:43 2016
#-
```

```
poles = 0;
```

Single jet inclusive cross section

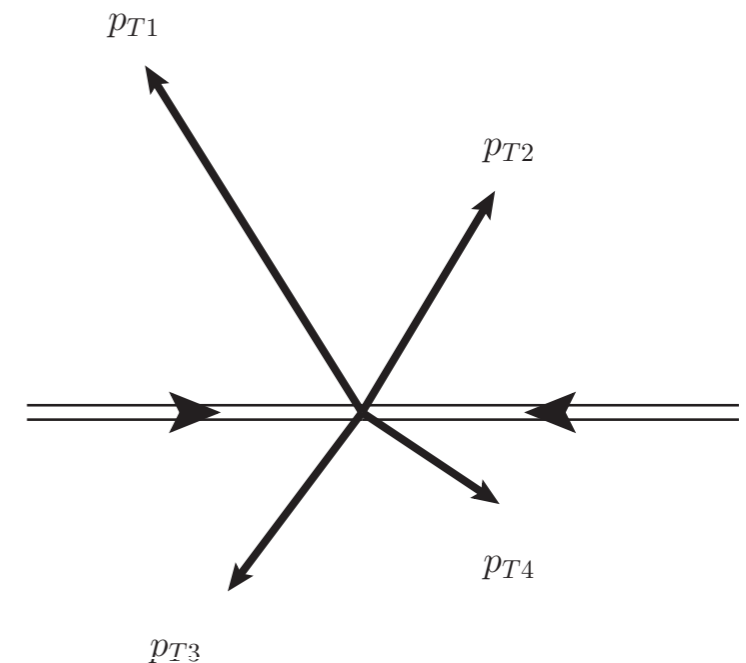
The basic QCD scattering process at the LHC

Experimentally:

- bin all jets inclusively within fiducial cuts
- many entries from the same event

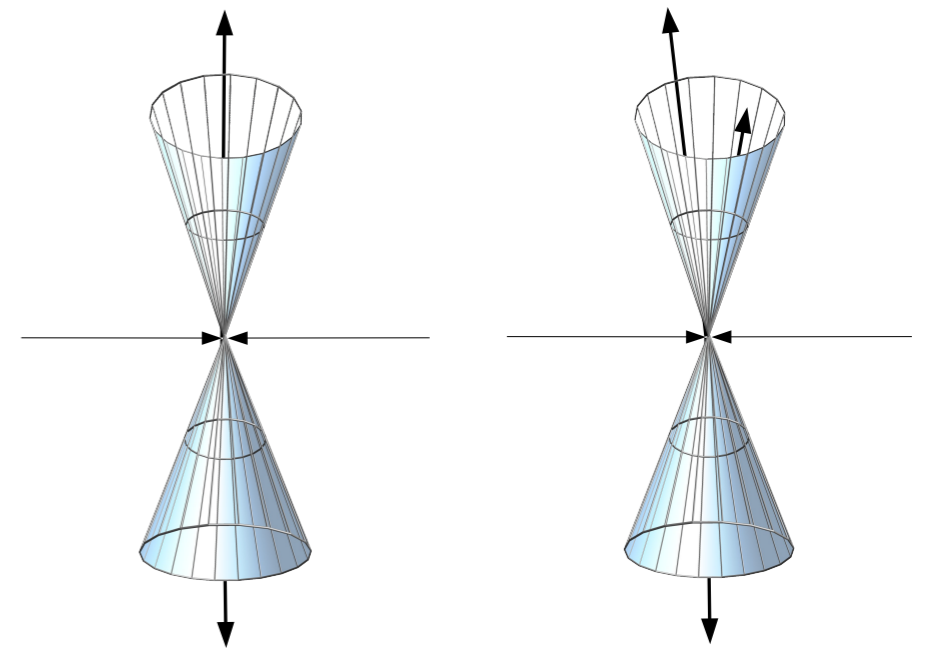
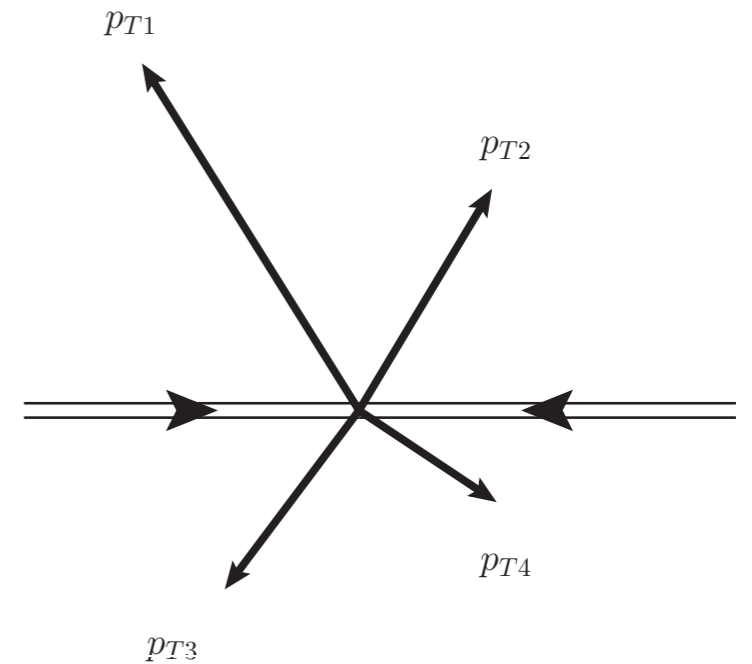
Theoretically:

- sample the relevant phase space (parton momenta)
- cluster partons into jets using the jet algorithm
- each jet enters the distribution as a *weighted event*
- weights depend on PDFs, α_s and thus the theoretical scales μ_F , μ_R



Canonical scale choices

- no fixed hard scale for jet production
- two widely used theoretical scale choices:
 - leading jet p_{T1} for all jets in an event
 - individual jet p_T
- smaller scale changes PDFs and α_s
- no difference for back-to-back jet configurations (only arises at higher orders)

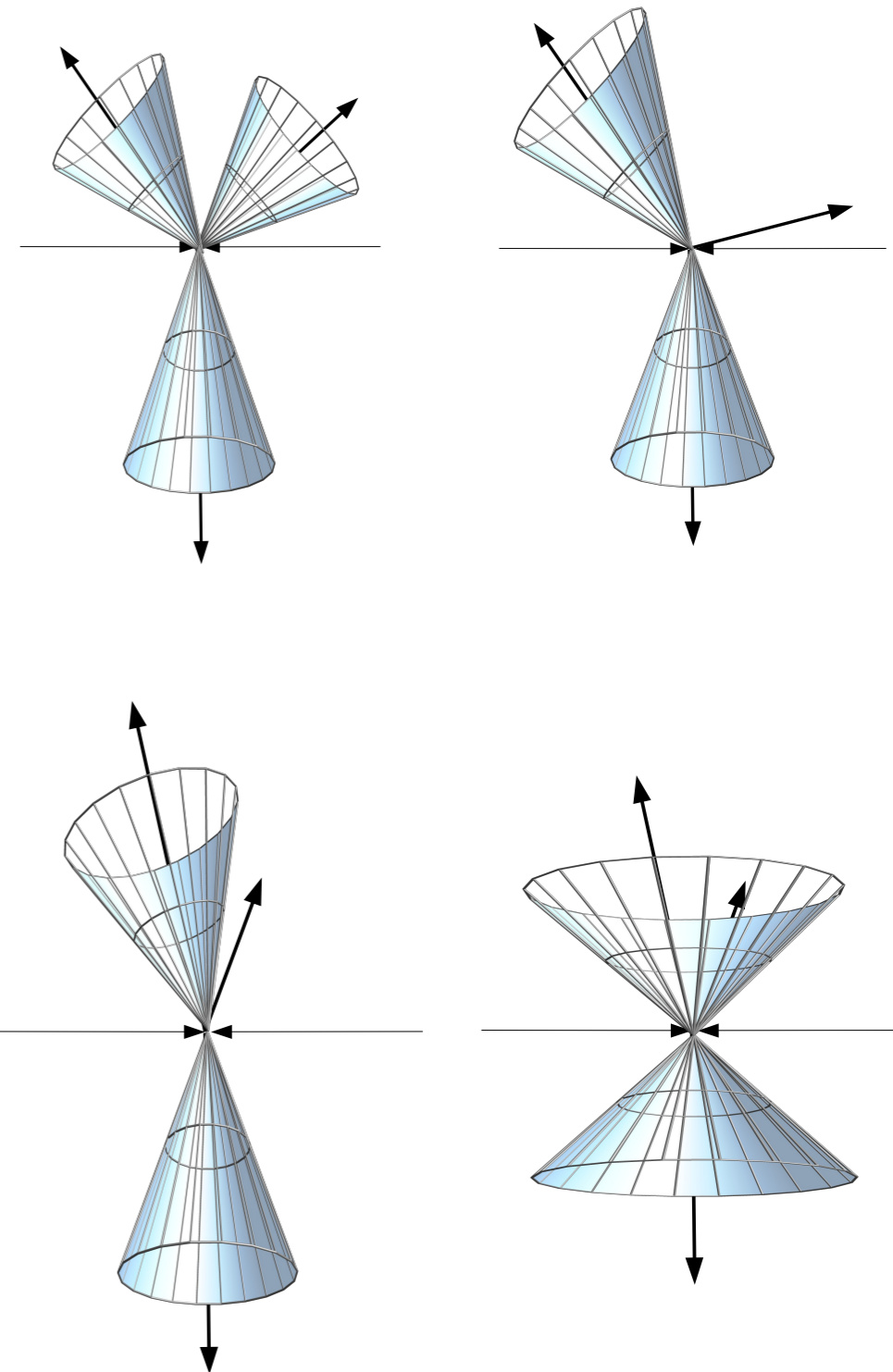


At NLO, $p_T \neq p_{T1}$ for:

- 3-jet rate (small rate)
- 2-jet rate (3rd parton falls outside jet, fails cuts)

Changing R has an effect on the cross section, but also on the scale choice:

- p_{T1} scale has no R -dependence at NLO, unlike p_T
- at NNLO even p_{T1} scale choice has R -dependence in some four-parton configurations



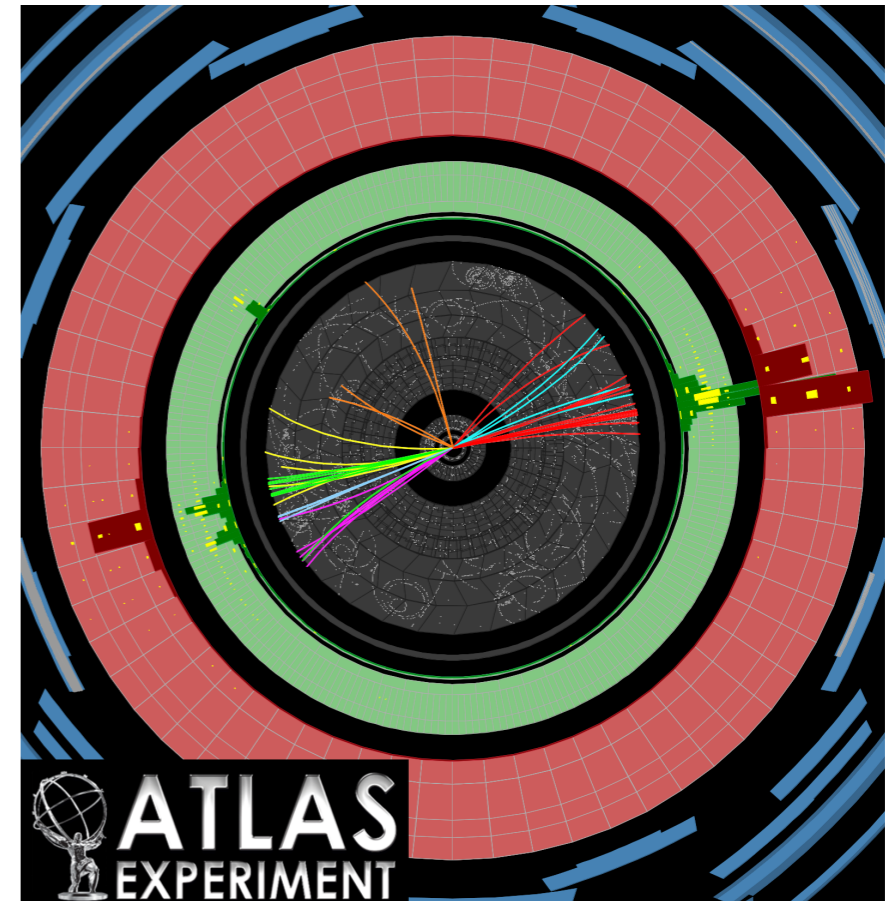
Setup

Theory setup:

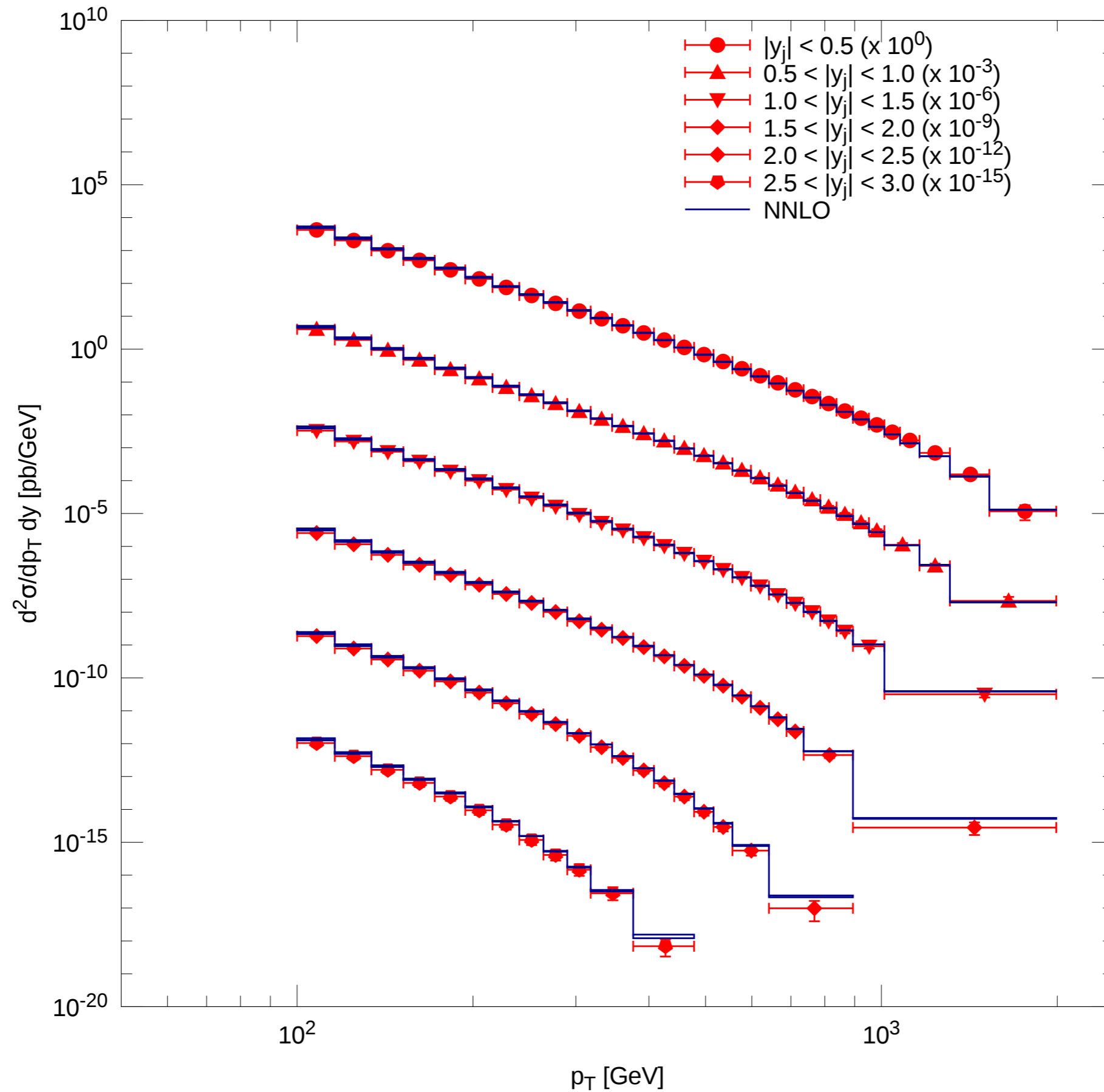
- NNPDF3.0_NNLO
- anti- k_T jet algorithm
- scale choices $\mu_R = \mu_F = \{p_{T_1}, p_T\}$
- vary up and down by factors of 2

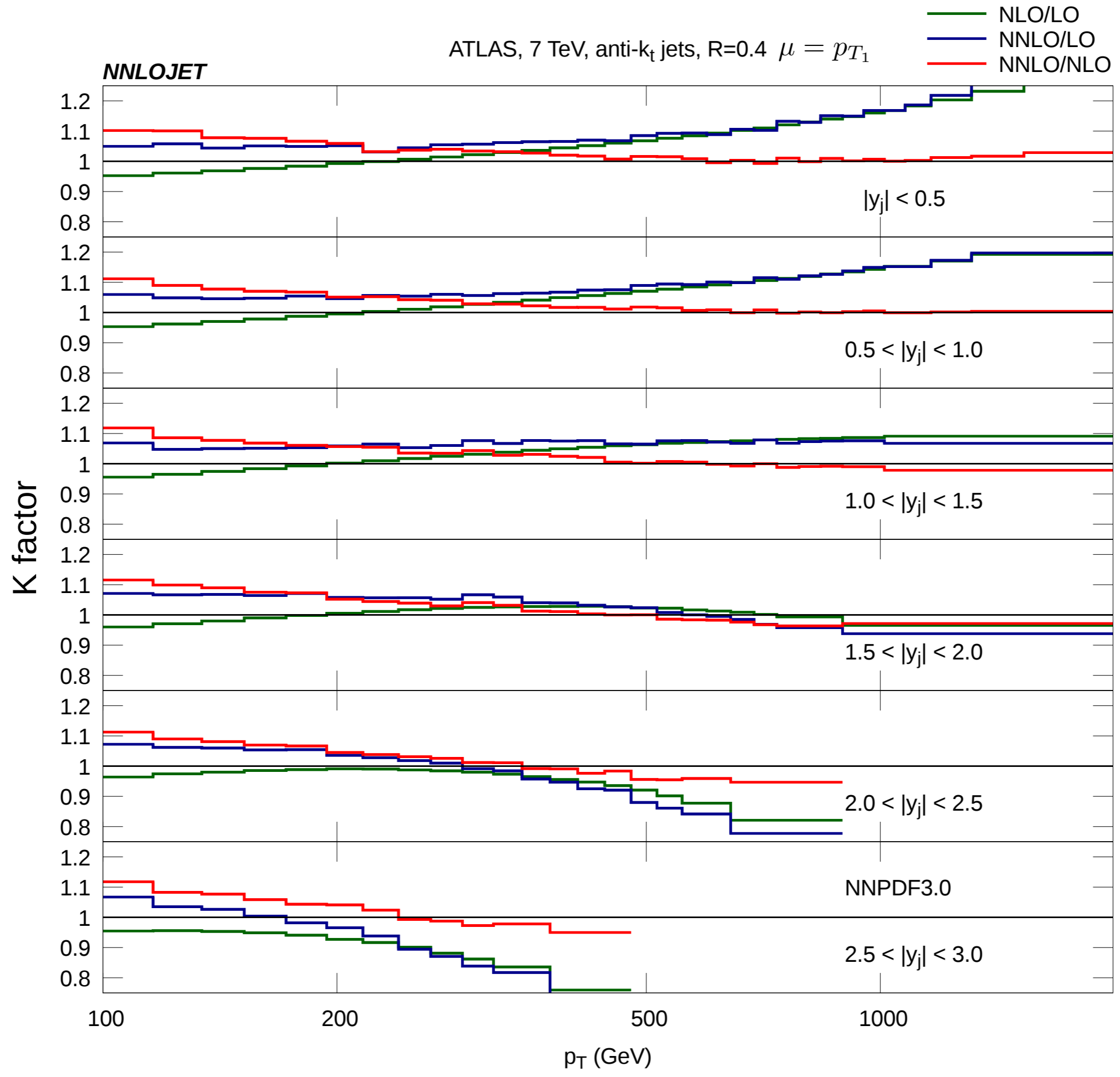
Comparison to data:

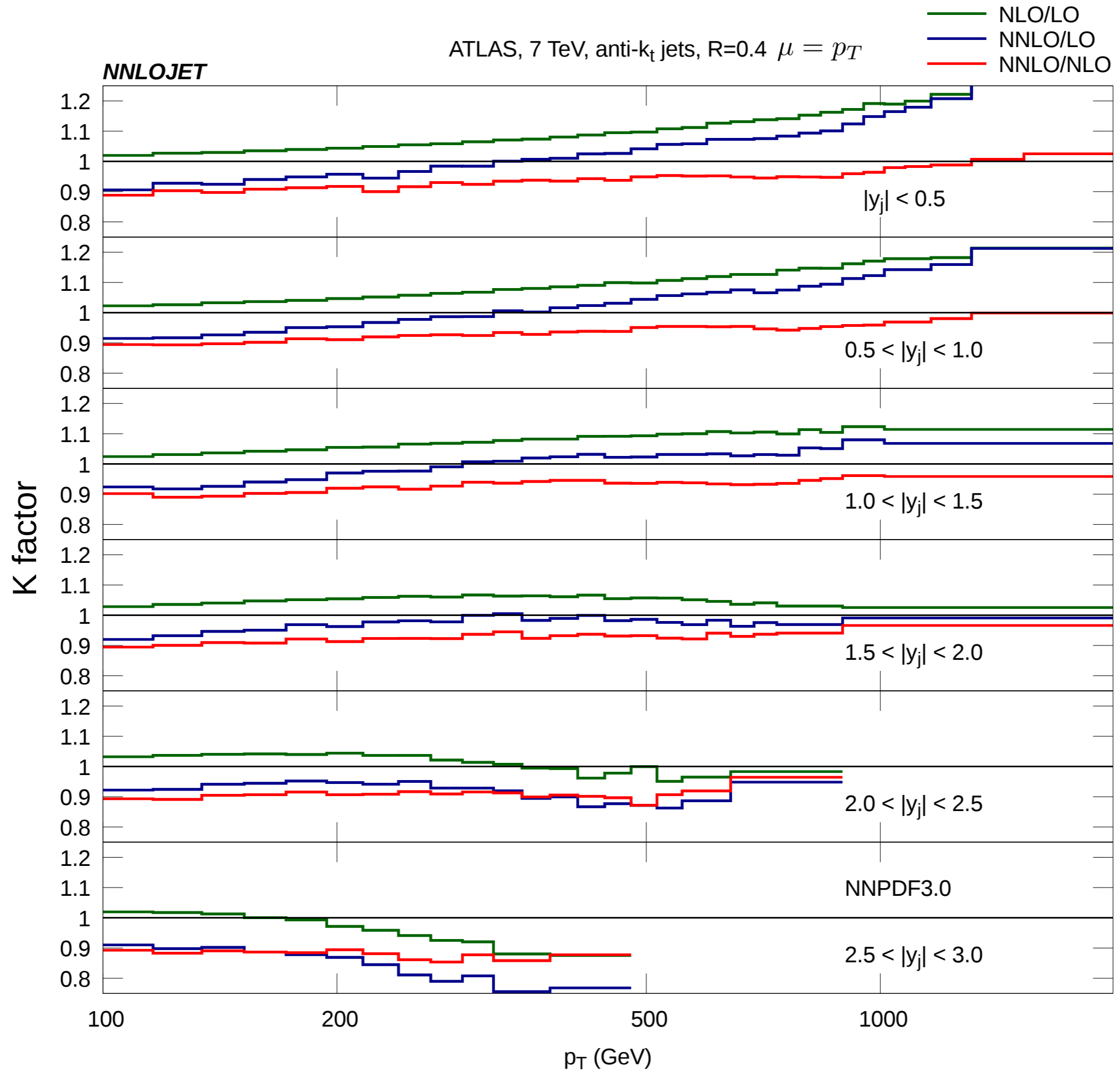
- ATLAS 7 TeV 4.5 fb^{-1} , $p_T > 100 \text{ GeV}$, $|y| < 3.0$, $R=0.4, 0.6$
- CMS 7 TeV 5.0 fb^{-1} , $p_T > 56 \text{ GeV}$, $|y| < 3.0$, $R=0.5, 0.7$



ATLAS, 7 TeV, anti- k_t jets, R=0.4







Scale variation

Writing out all the renormalization scale dependent terms and varying about a starting scale μ_0 , where $L_R = \log(\mu_R/\mu_0)$

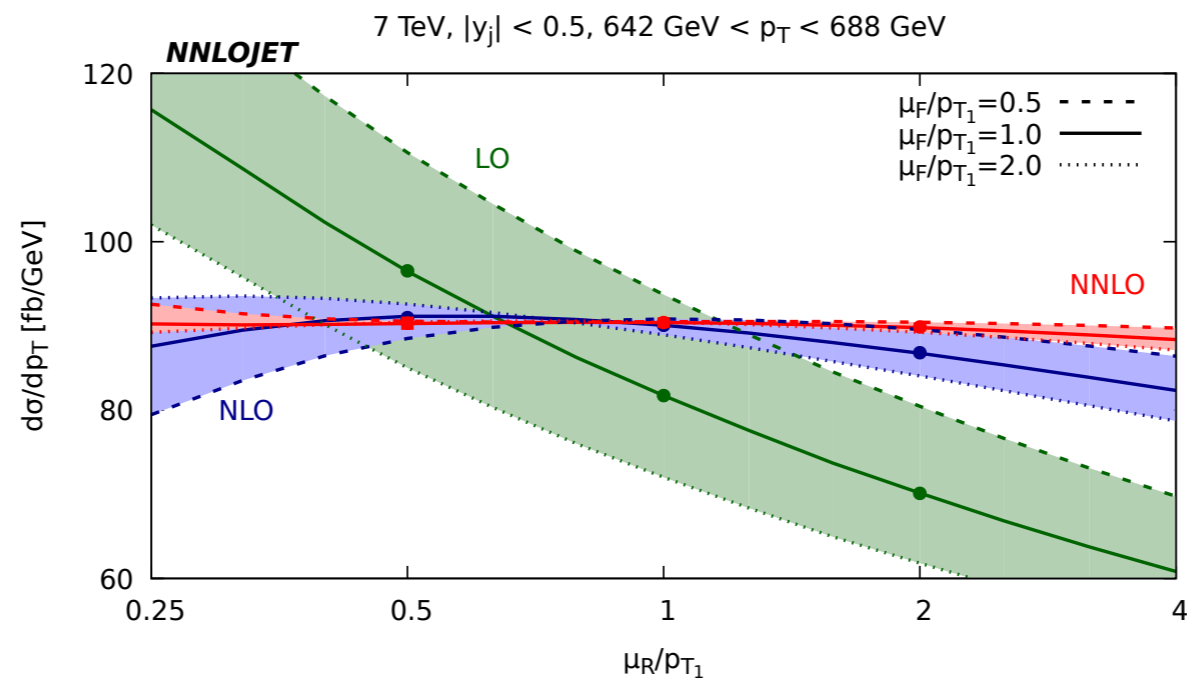
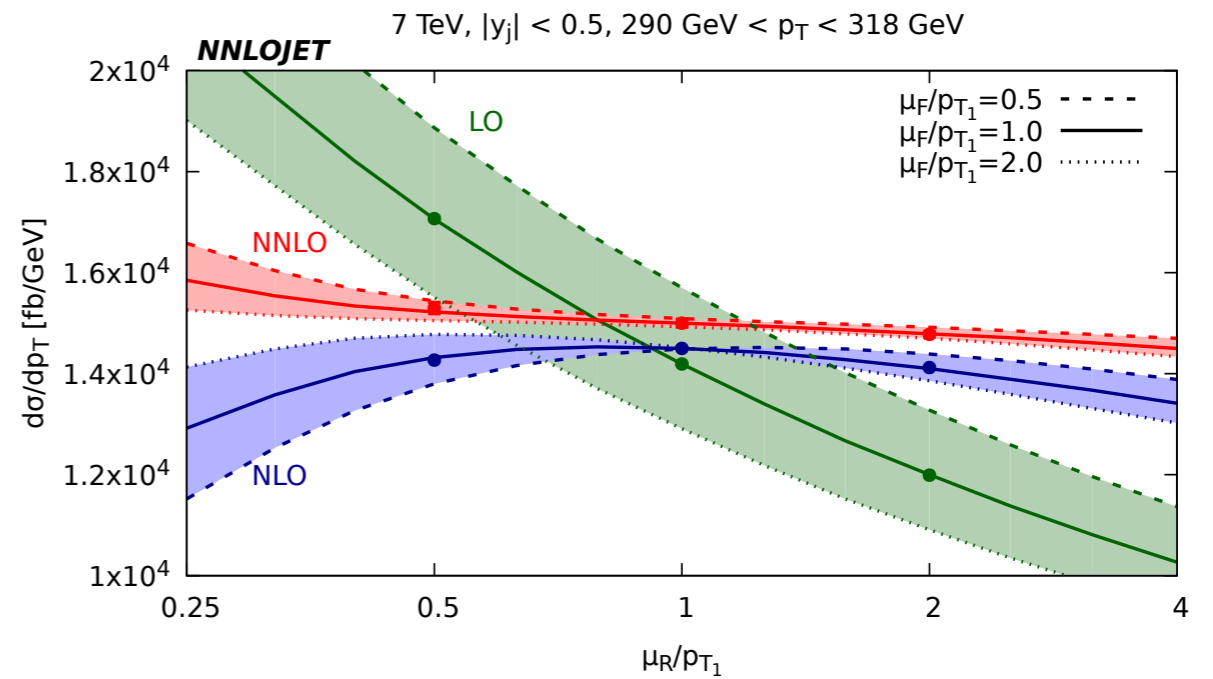
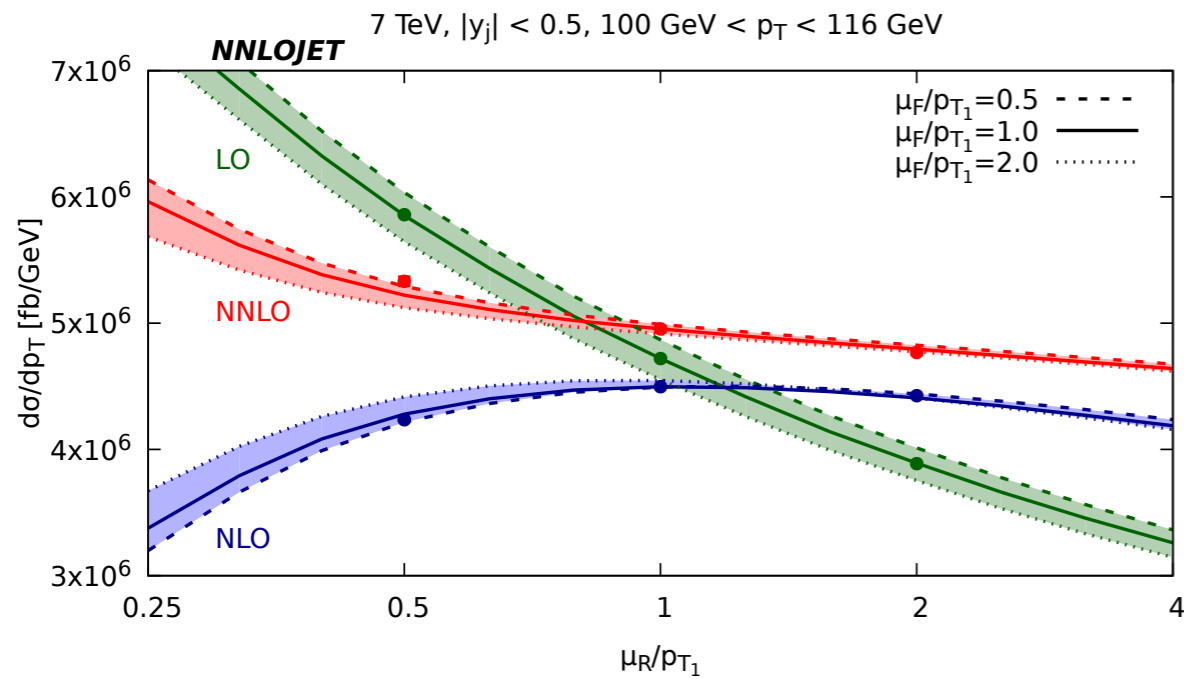
$$\begin{aligned}
 \sigma(\mu, \alpha_s(\mu^2), \mu_0) = & \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^n \sigma_0 \\
 & + \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^{n+1} \left[\sigma_1 + n\beta_0 L_R \sigma_0 \right] \\
 & + \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^{n+2} \left[\sigma_2 + (n+1)\beta_0 L_R \sigma_1 + n\beta_1 L_R \sigma_0 + \frac{1}{2}n(n+1)\beta_0^2 L_R^2 \sigma_0 \right] \\
 & + \left(\frac{\alpha_s(\mu_R)}{2\pi} \right)^{n+3} \left[\dots \right]
 \end{aligned}$$

running coupling

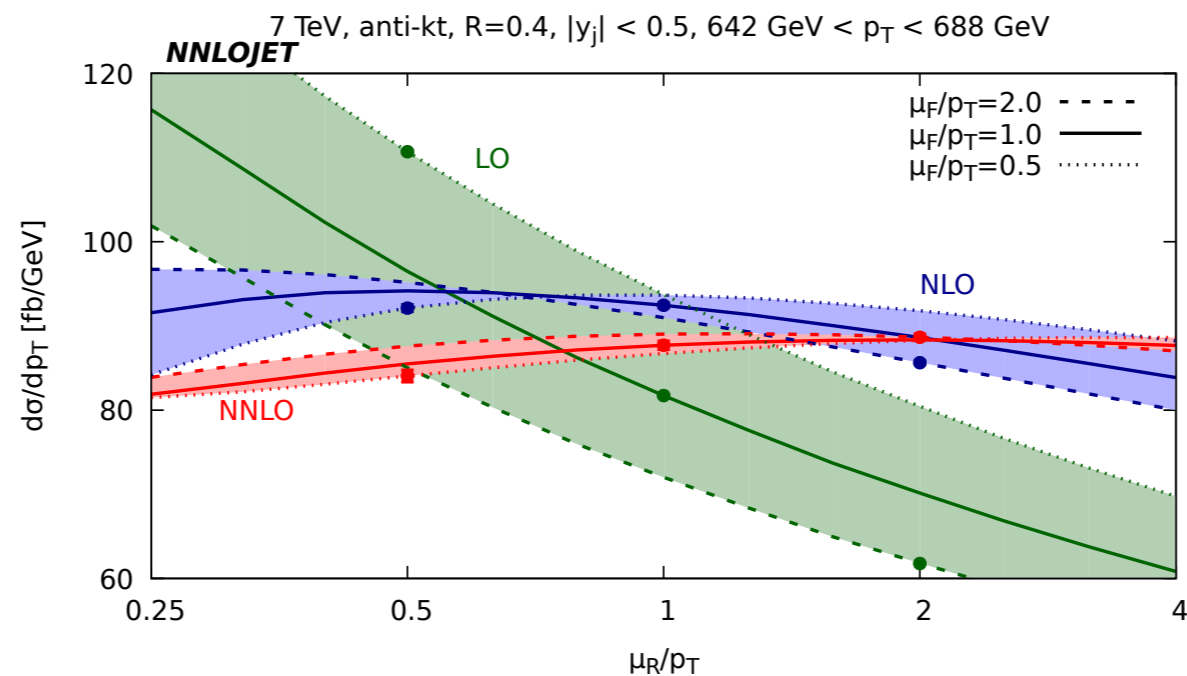
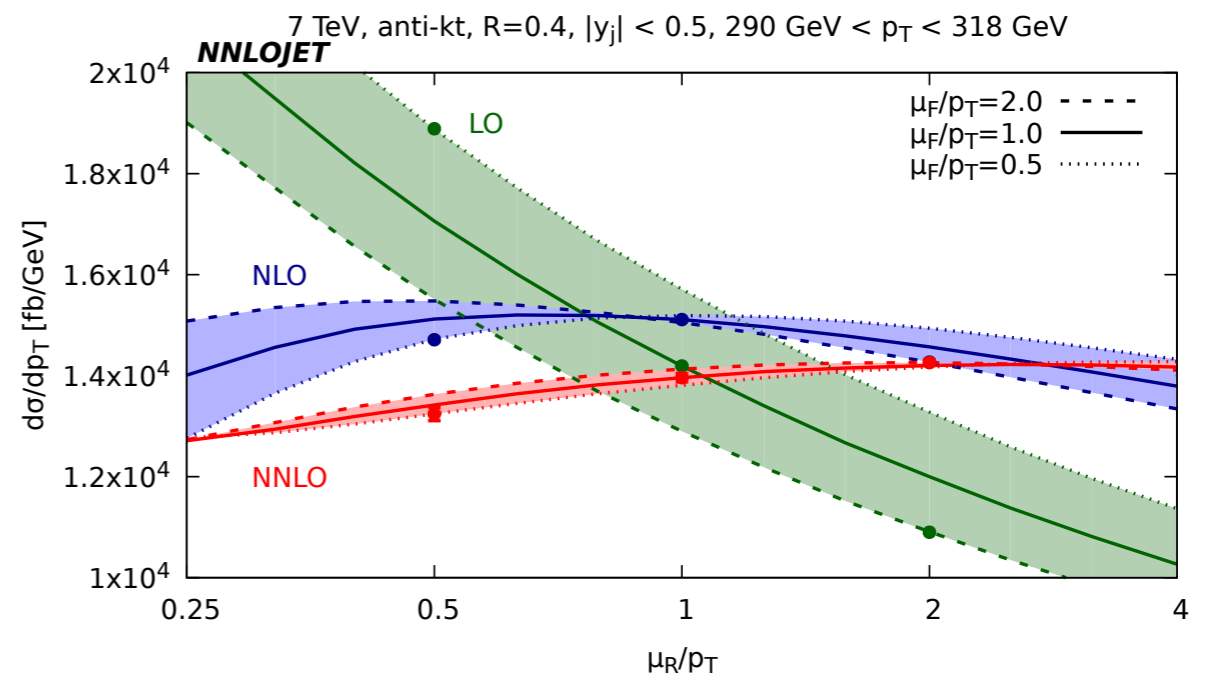
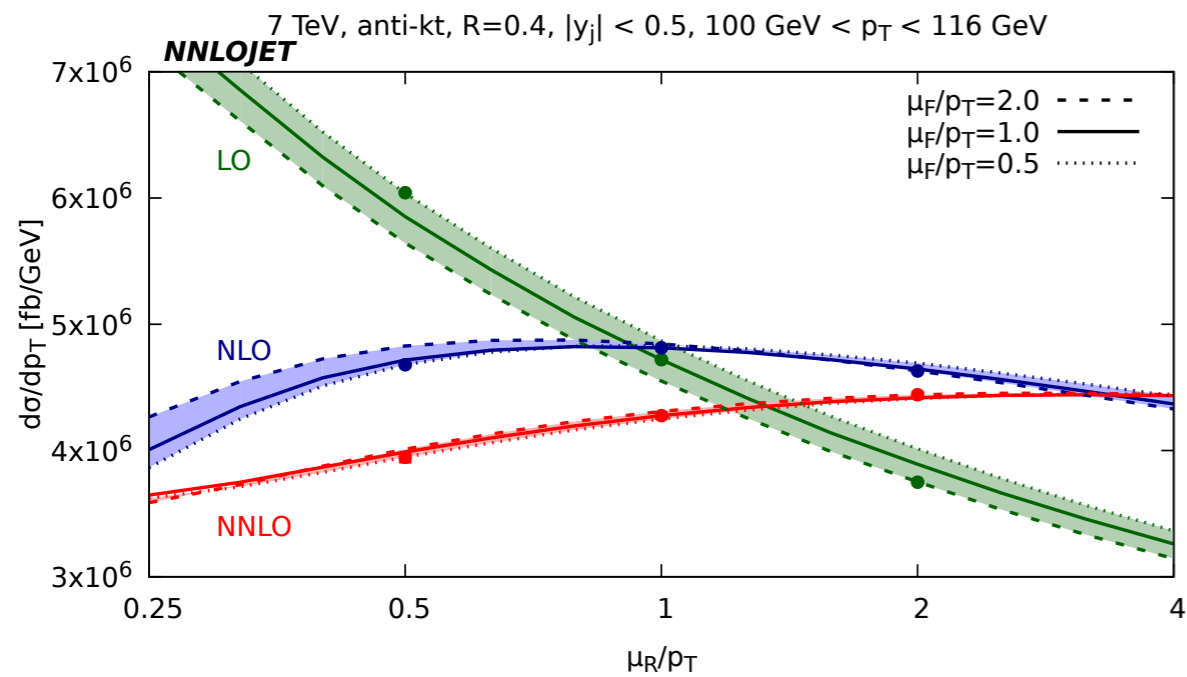
negative for $\mu_R < p_T, p_{T_1}$

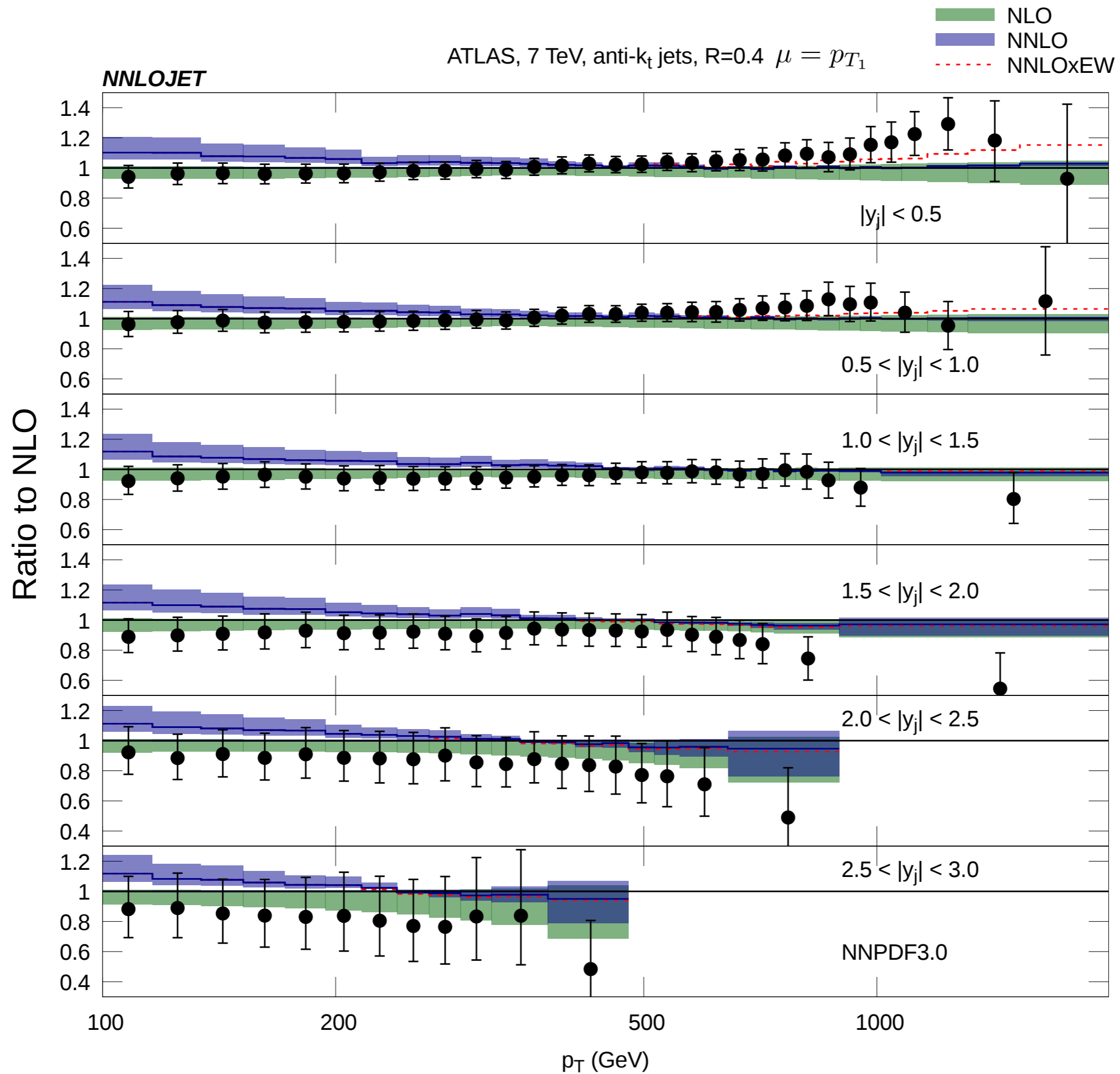
- scale variation given by varying up and down by a factor of 2
- variation due to central scale choice a parametric uncertainty

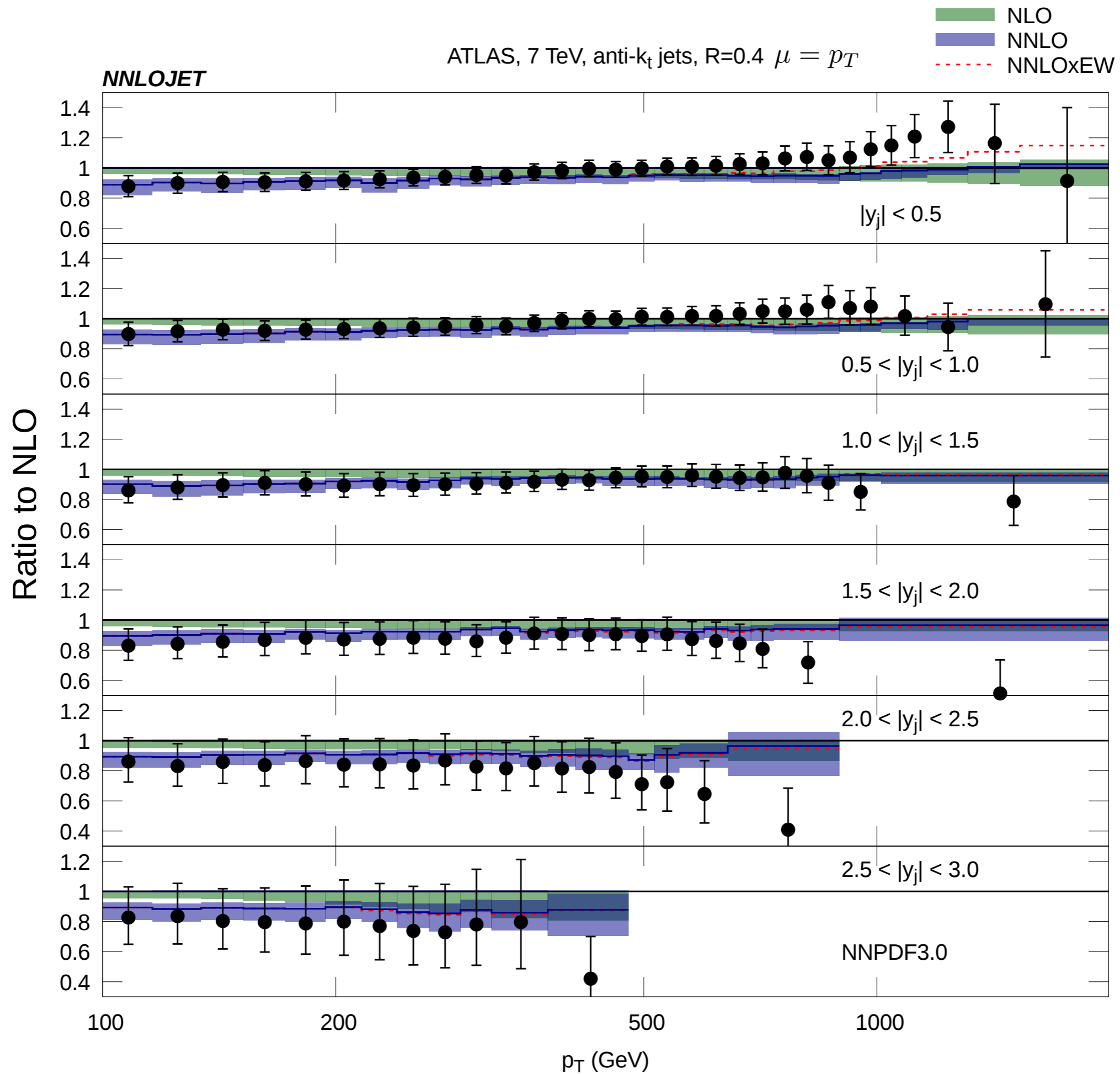
Scale variation p_{T1}



Scale variation p_T





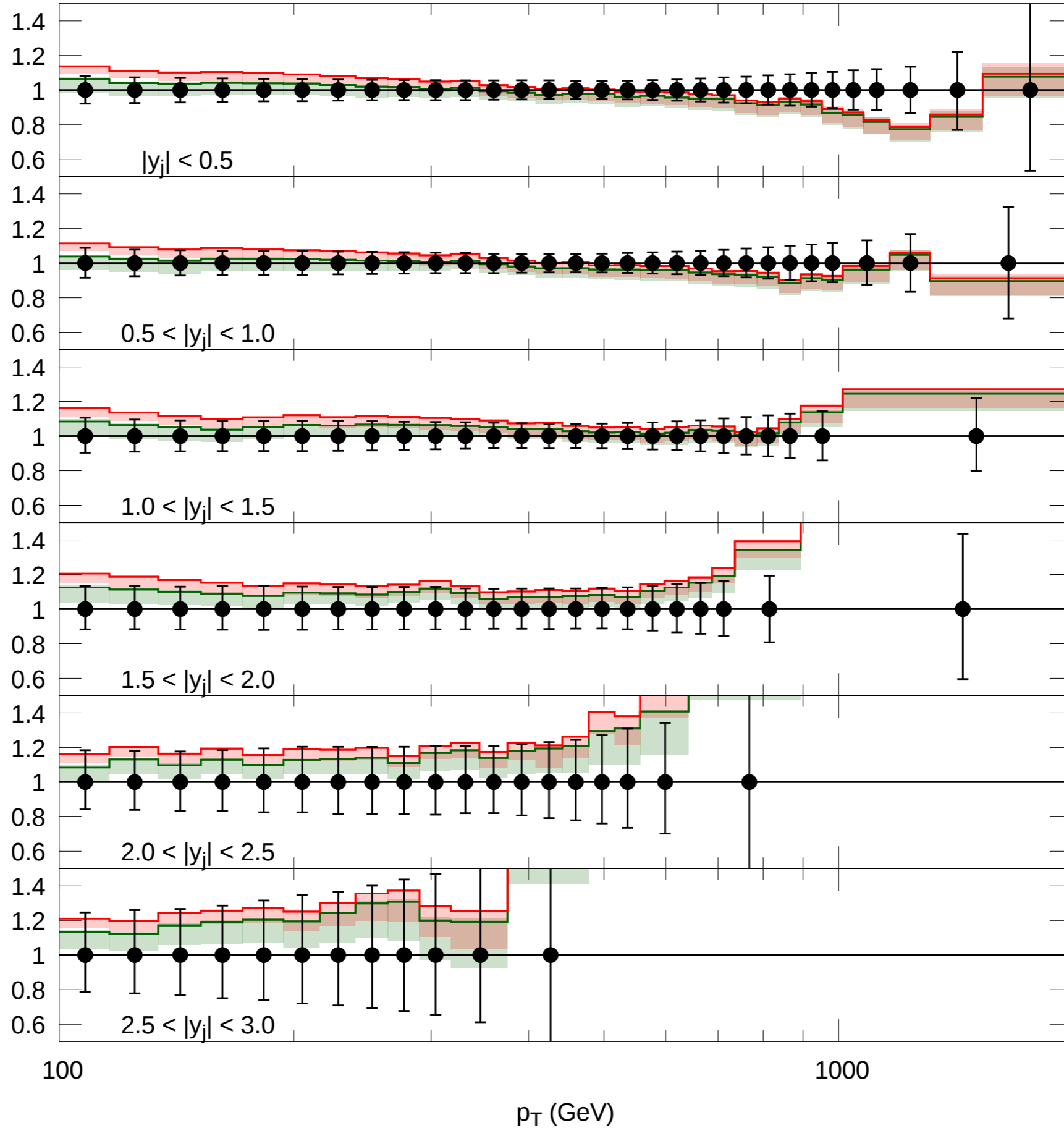


ATLAS, 7 TeV, anti- k_t jets, R=0.4, NNPDF3.0

$\mu=p_{T1}$
 $\mu=p_T$

NNLOJET

NLO Ratio to data

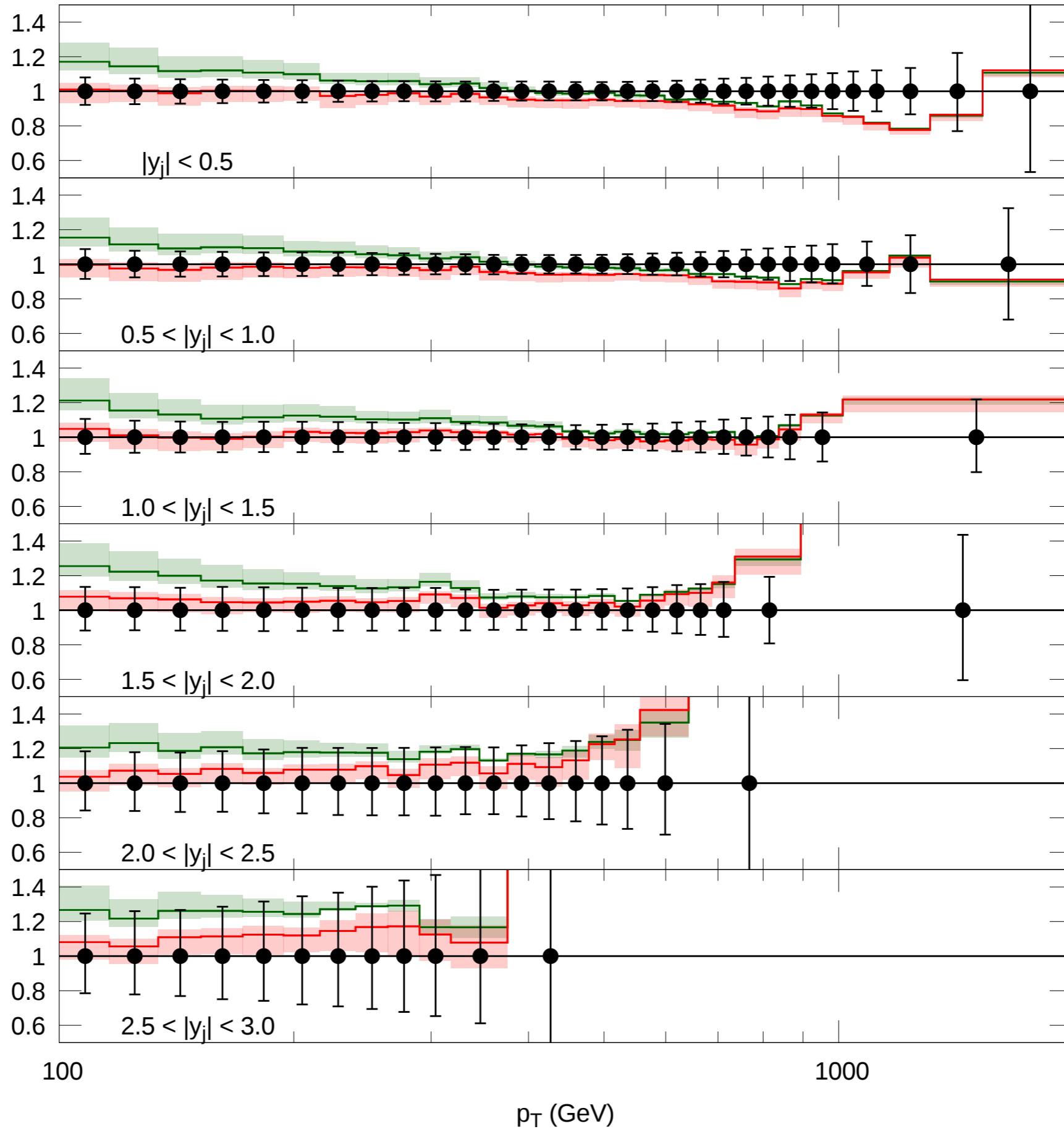


ATLAS, 7 TeV, anti- k_t jets, $R=0.4$, NNPDF3.0

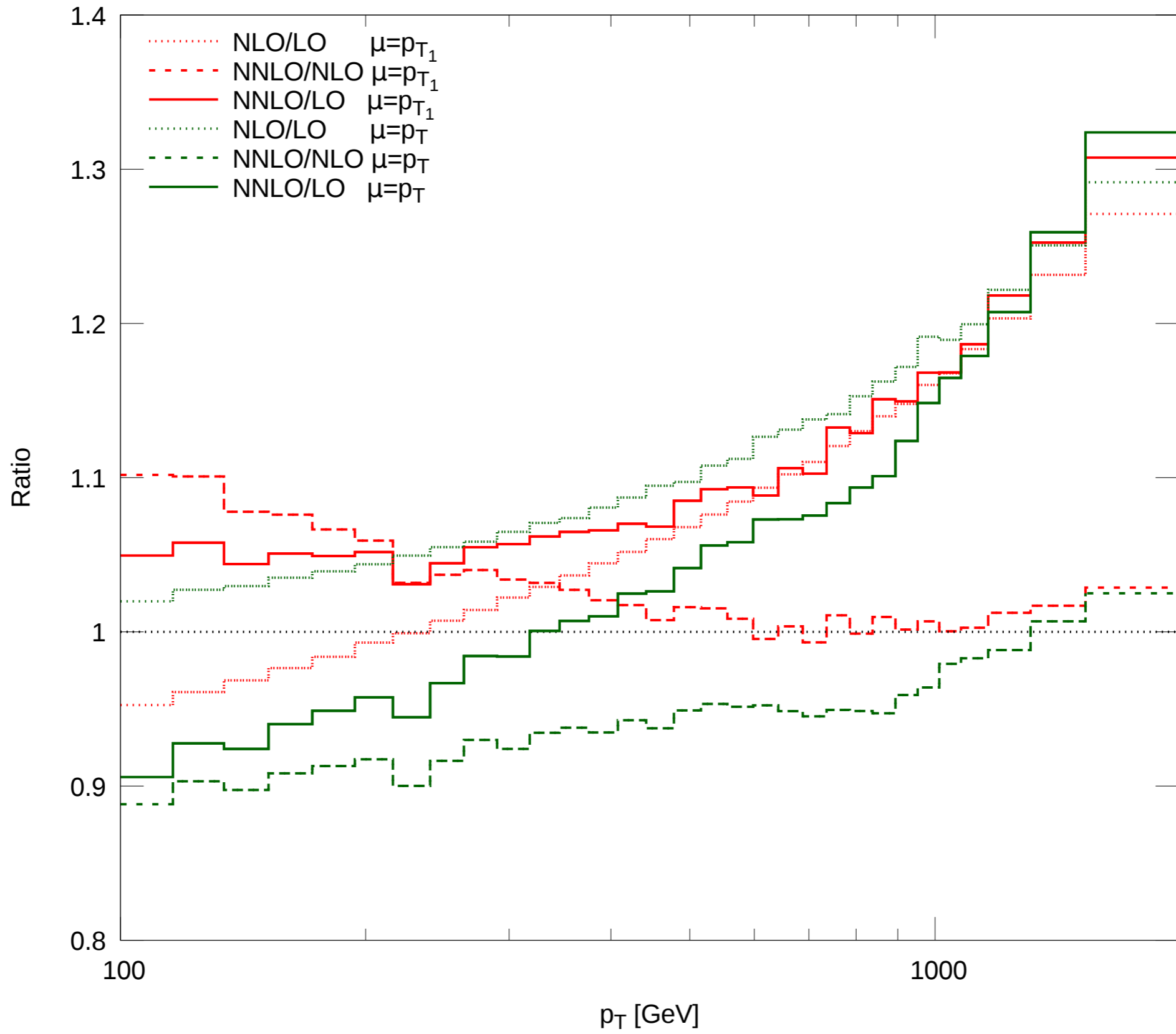
$\mu=p_{T1}$
 $\mu=p_T$

NNLOJET

NNLO Ratio to data

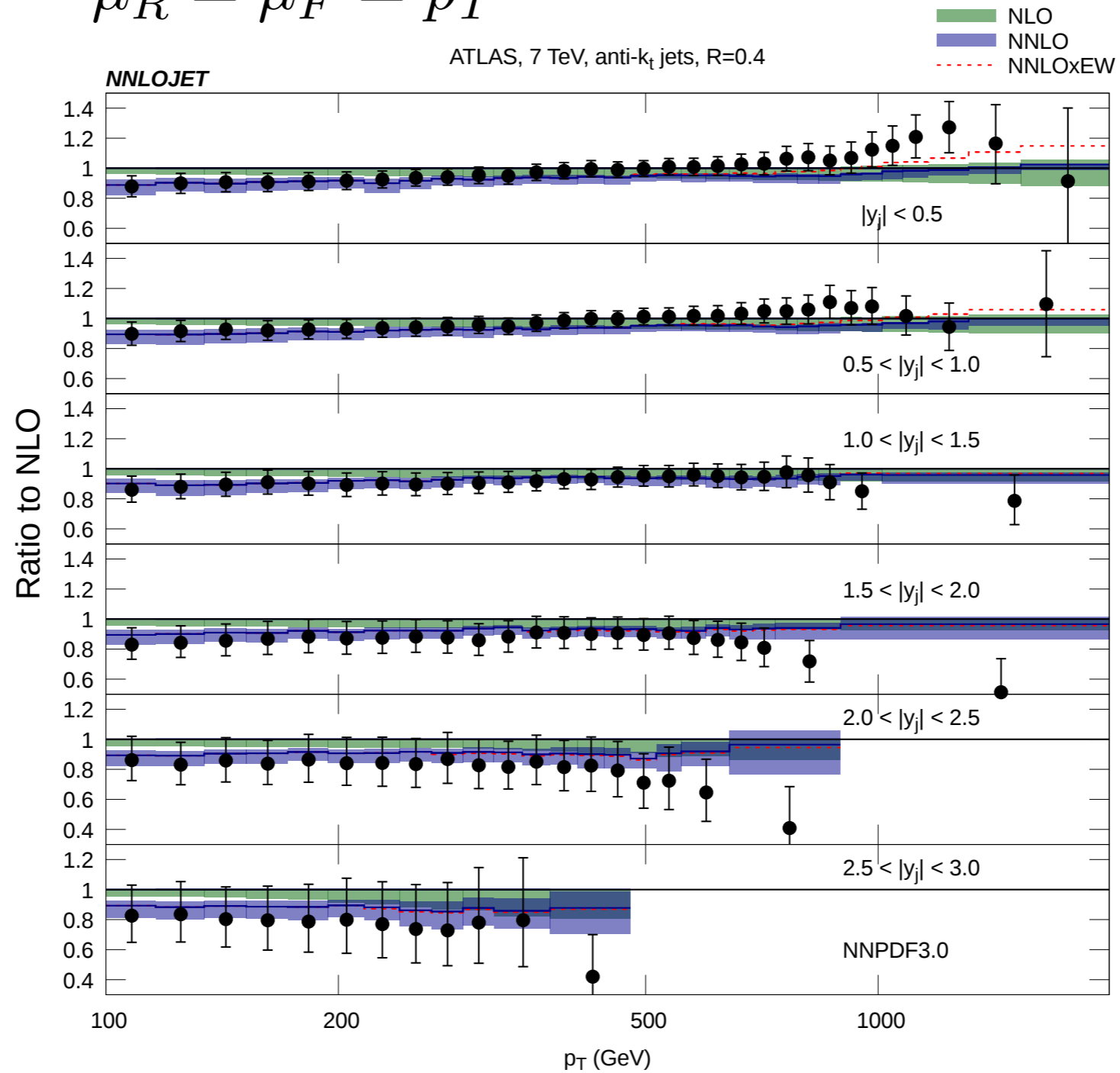


ATLAS, 7 TeV, anti- k_t jets, $R=0.4$, NNPDF3.0, TOT, $|y_j| < 0.5$



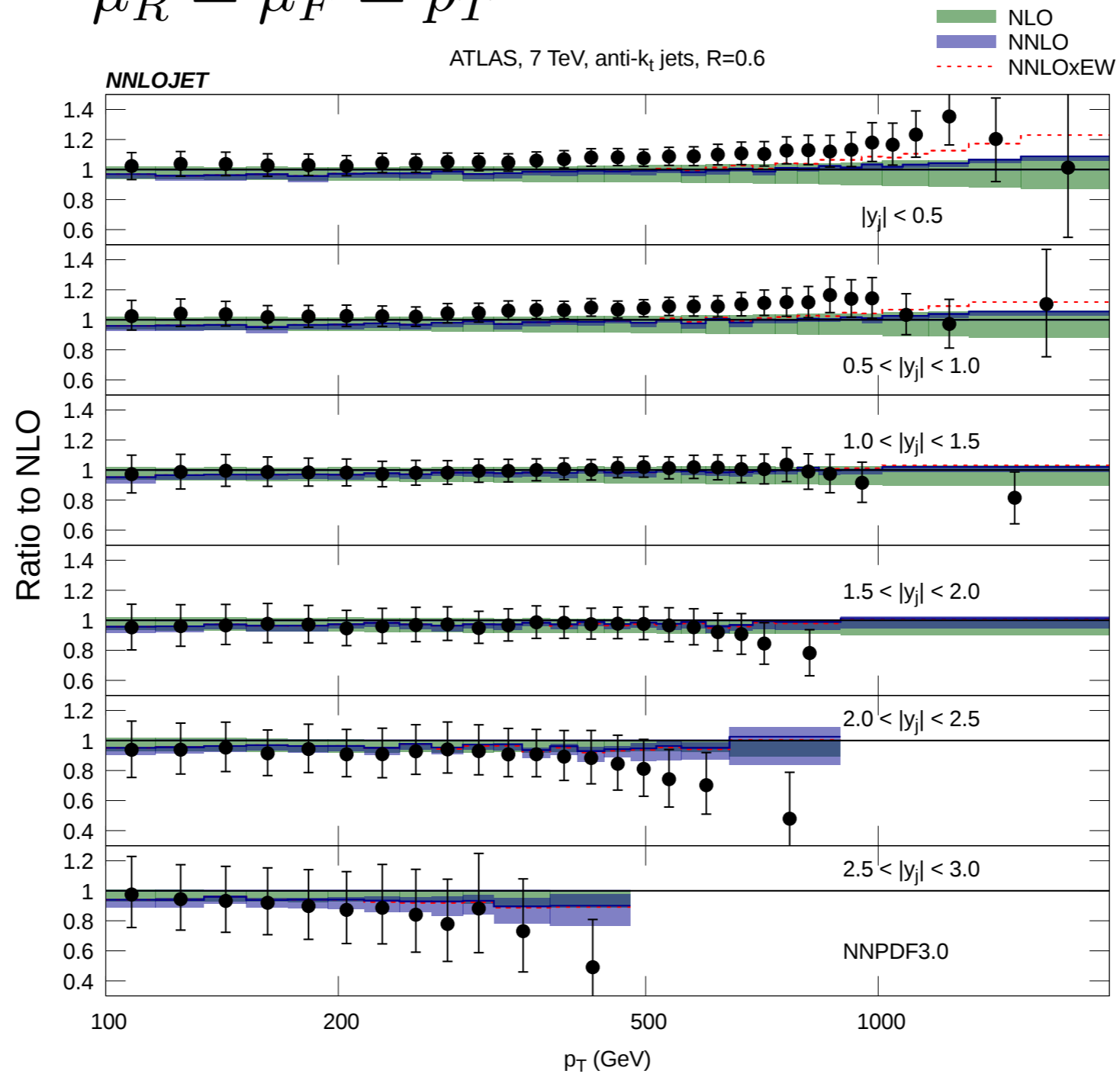
R=0.4 ATLAS

$$\mu_R = \mu_F = p_T$$



R=0.6 ATLAS

$$\mu_R = \mu_F = p_T$$

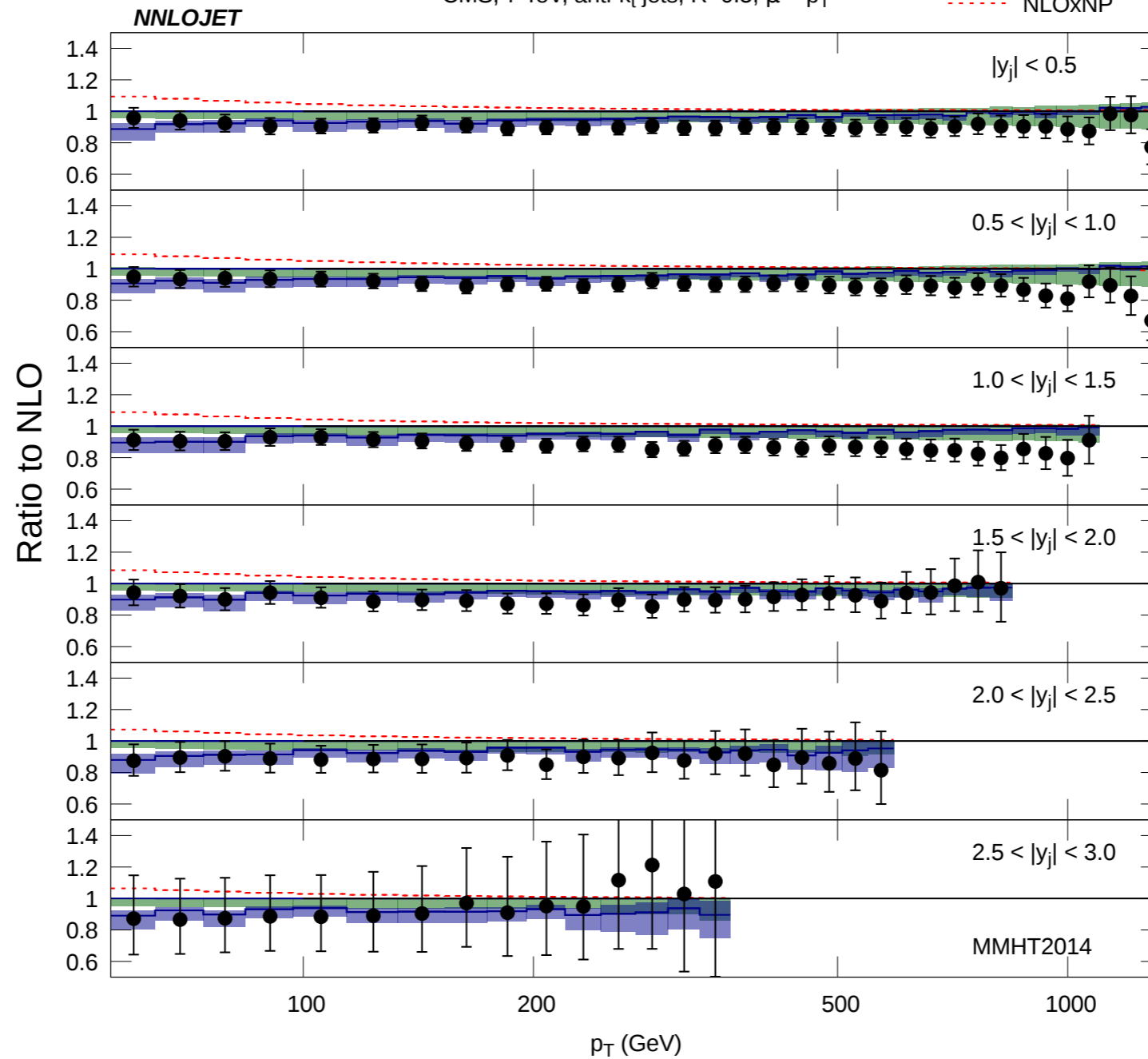


R=0.5 CMS

$$\mu_R = \mu_F = p_T$$

CMS, 7 TeV, anti- k_t jets, R=0.5, $\mu = p_T$

- NLO
- NNLO
- NLOxNP

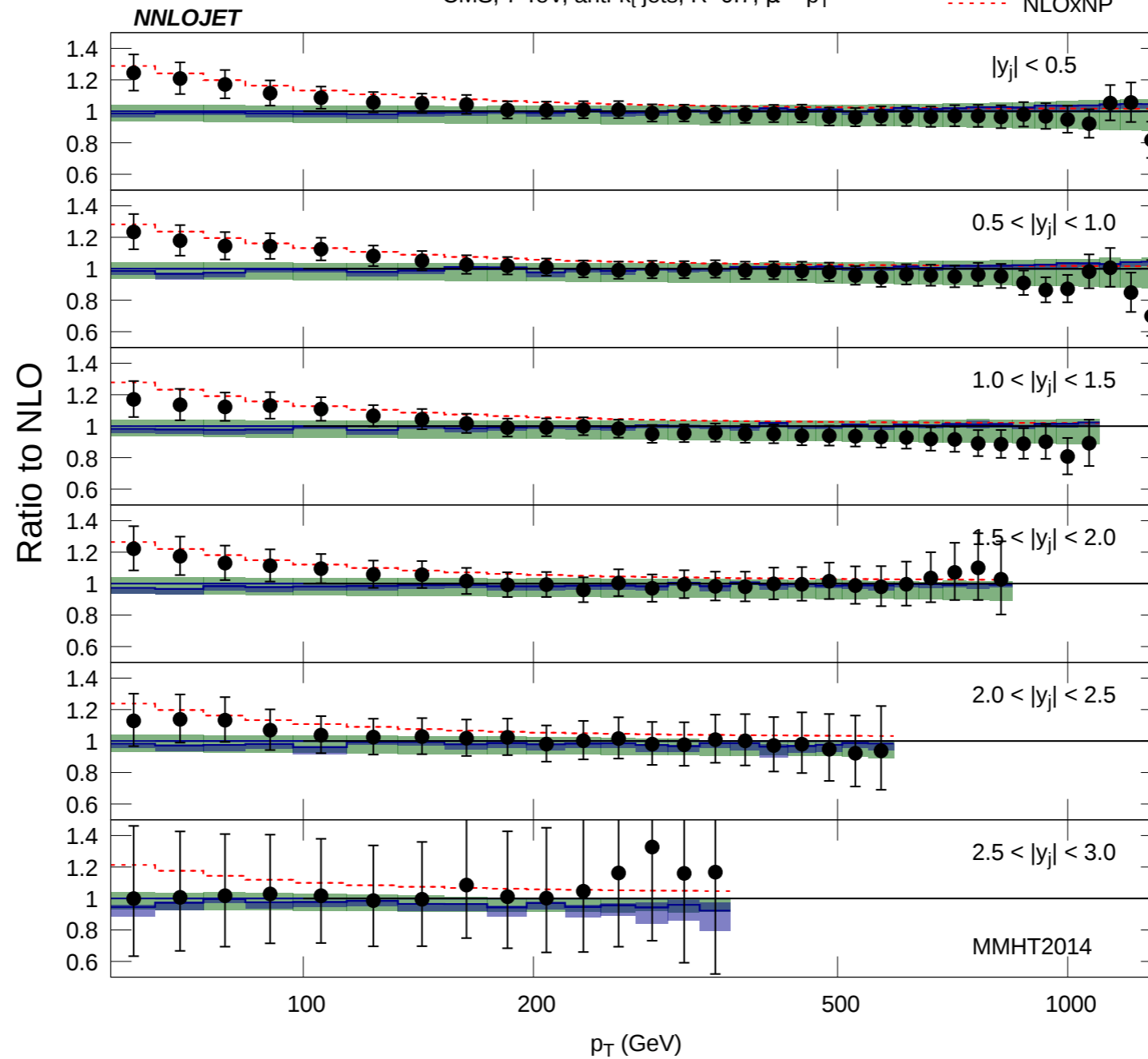


R=0.7 CMS

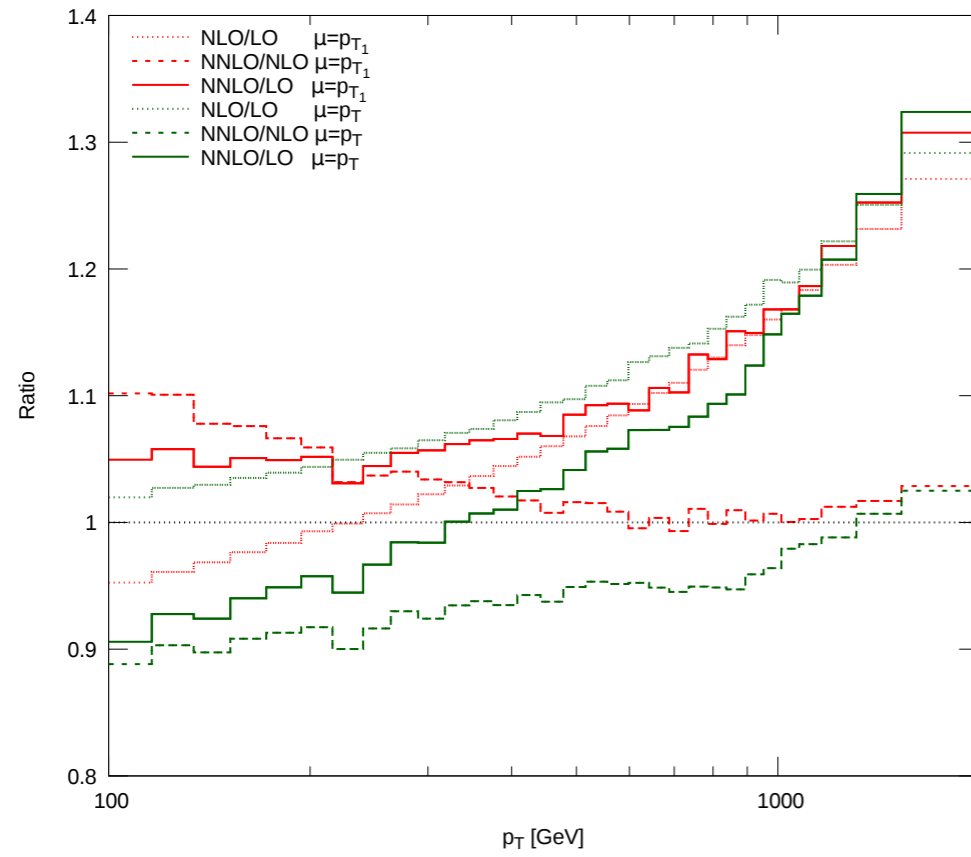
$$\mu_R = \mu_F = p_T$$

CMS, 7 TeV, anti- k_t jets, R=0.7, $\mu = p_T$

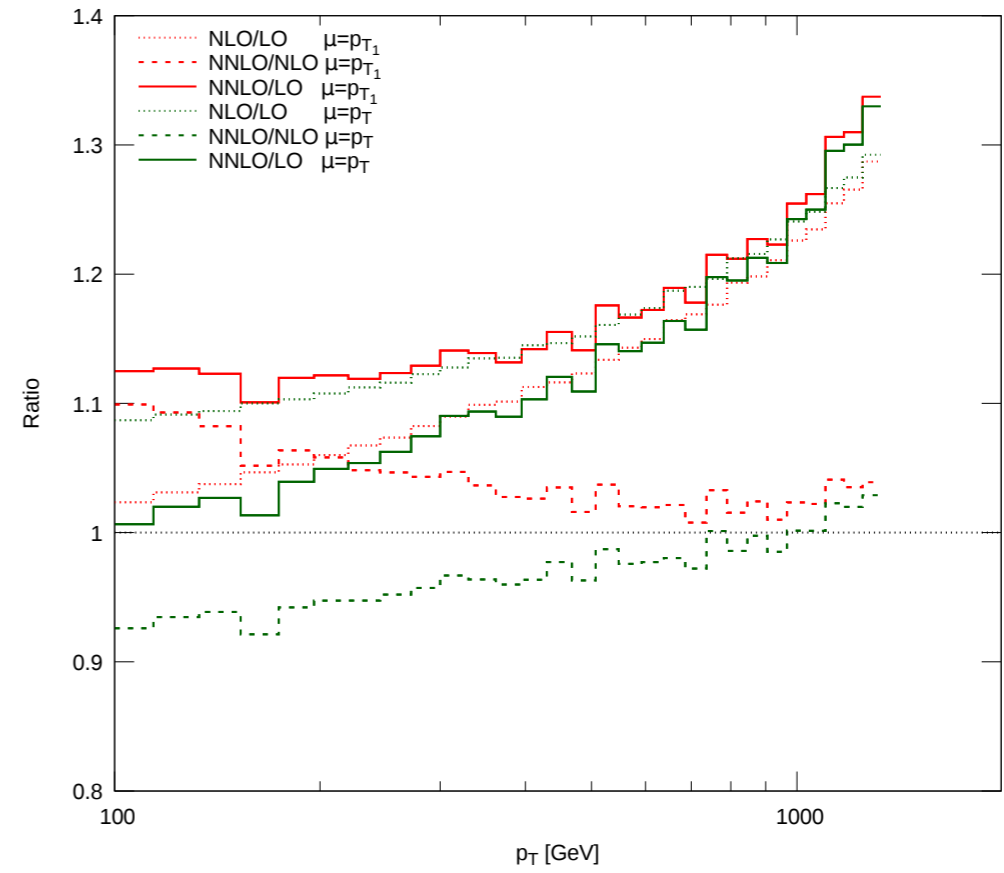
- NLO
- NNLO
- NLOxNP



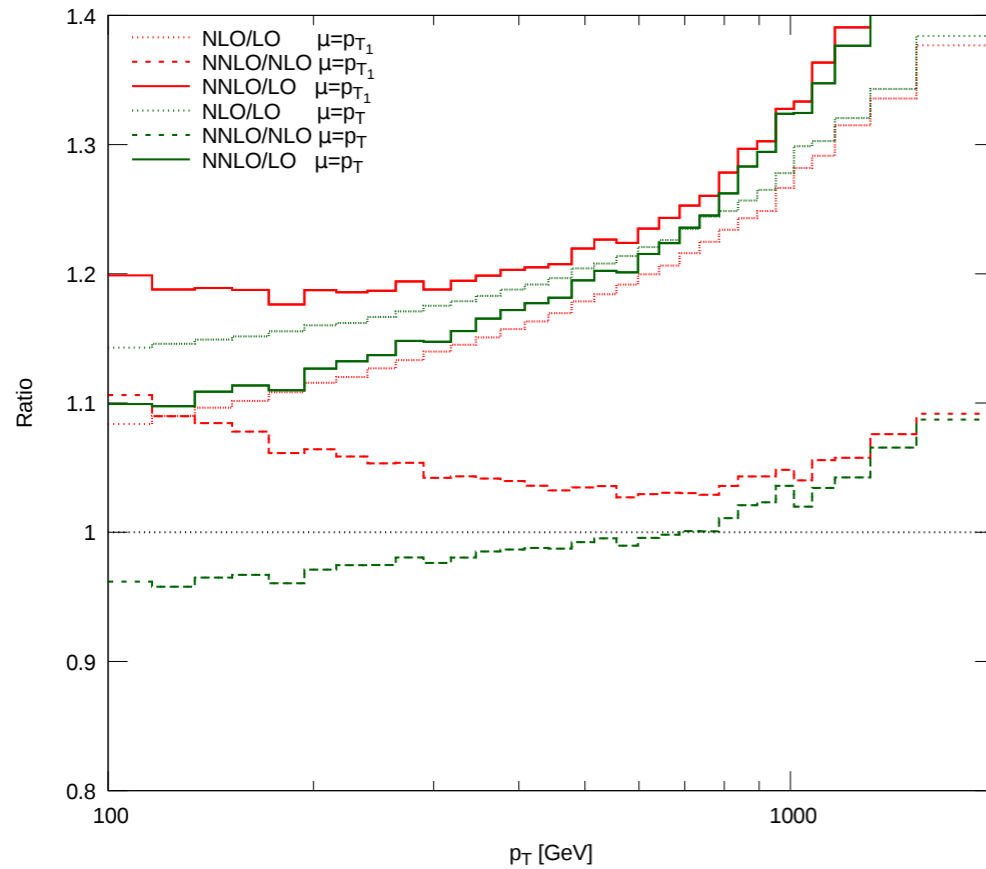
ATLAS, 7 TeV, anti- k_t jets, $R=0.4$, NNPDF3.0, TOT, $|y_j| < 0.5$



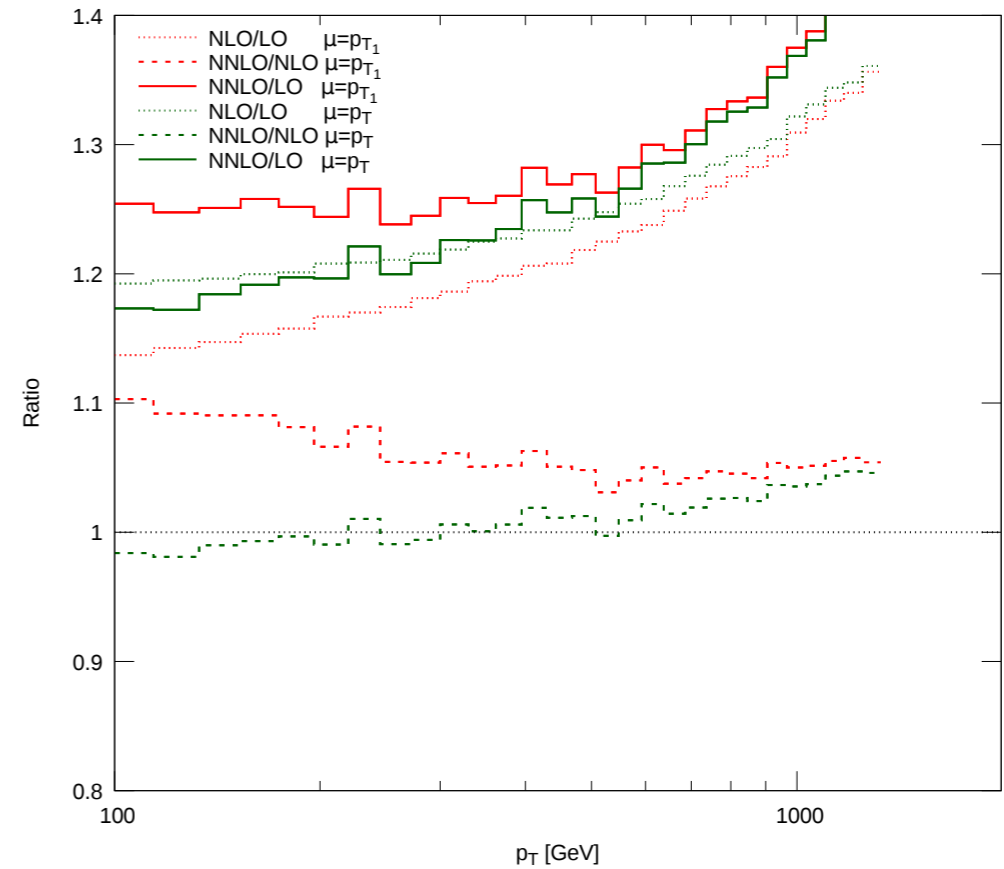
CMS, 7 TeV, anti- k_t jets, $R=0.5$, MMHT2014, TOT, $|y_j| < 0.5$



ATLAS, 7 TeV, anti- k_t jets, $R=0.6$, NNPDF3.0, TOT, $|y_j| < 0.5$



CMS, 7 TeV, anti- k_t jets, $R=0.7$, MMHT2014, TOT, $|y_j| < 0.5$



Future phenomenology

Jet phenomenology at NNLO really just starting, many more processes to consider:

- more exclusive searches like dijet mass distribution
- ratios of cross sections, good for systematics: centre of mass energy, R-values etc
- PDF fits: will be interesting to see the impact of jet K-factors on NNLO PDFs and test consistency with top data
- interface to AppleGrid for detailed phenomenology with strong coupling and PDFs
- any suggestions welcome!

Summary

Jets are a key ingredient for testing QCD at the LHC

- provide a bridge between perturbative theory and precision experiment
- sensitive to important SM parameters and a powerful probe of BSM physics

Calculating higher order corrections to jet production:

- is necessary to capture all key features of the process
- allows us to assess the theoretical error in our calculation
- complicated by intricate IR singularity structure

Nevertheless this has recently been achieved at NNLO:

- first results are out, opening the gateway to NNLO jet phenomenology