An introduction to POWHEG

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Important references:


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Outlines

• Brief motivation: the quest of precision

• Monte Carlo (MC) event generators

• Parton shower vs fixed order calculations

• How to merge NLO+PS: the POWHEG master formula

• Phenomenological impact

• Conclusions and outlook
Motivation: the quest for precision
New Physics at the LHC?

- After Higgs boson discovery focus on its properties and on hints for BSM physics
  - So far no sign of new physics at the TeV scale from direct searches
- Possible hits for new physics could come from very precise measurements e.g. in the Higgs sector
  - Very accurate predictions are needed, both
    - in shape
    - in normalization

Precise description of scattering process at all stages!
Monte Carlo Event Generators

What they are and what they do
PDF’s

**Hard scattering**
\[ \Lambda_{QCD} \ll \mu \approx Q \]

**Parton shower**
\[ \Lambda_{QCD} < \mu < Q \]

**Beam remnants**

**Hadronization and Hadron decay**
\[ \mu \approx \Lambda_{QCD} \]

**Multiple interactions**
PDF’s

Determined from data

Hard scattering

\[ \Lambda_{QCD} \ll \mu \approx Q \]

Hard process computed using fixed order (FO) perturbation theory at LO, NLO, NNLO, N^3LO,...

Parton shower

\[ \Lambda_{QCD} < \mu < Q \]

Hierarchy of scales appearing as large logarithms in the calculation.
Computed using resummation or MC parton showers (PS)

Beam remnants

Hadronization and Hadron decay

\[ \mu \approx \Lambda_{QCD} \]

Cannot be computed directly: non-perturbative models tuned to e^+e^- data

Multiple interactions

\[
\sigma_{h_1h_2 \to X} = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1/a}(x_1, \mu_F^2) f_{h_2/b}(x_2, \mu_F^2) \times \hat{\sigma}_{a,b \to X} \left( x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right) \quad + \mathcal{O} \left( \frac{1}{Q^2} \right)
\]

PDFs \quad \text{partonic cross section} \quad \text{np corrections}
**PDF’s**

- Determined from data

**Hard scattering**

\[ \Lambda_{QCD} \ll \mu \approx Q \]

**Parton shower**

\[ \Lambda_{QCD} < \mu < Q \]

**Beam remnants**

- Hadronization and Hadron decay
  \[ \mu \approx \Lambda_{QCD} \]

**Multiple interactions**

- Cannot be computed directly: non-perturbative models tuned to e^+e^- data

**Theorists’ calculation**

- Hard process computed using fixed order (FO) perturbation theory at LO, NLO, NNLO, N^3LO, ...
- Hierarchy of scales appearing as large logarithms in the calculation. Computed using resummation or MC parton showers (PS)
- FO + PS matching: different orders ↔ different schemes

**Theorists’ calculation, Pythia, Herwig, ...**

\[ \sigma_{h_1h_2 \rightarrow X} = \sum_{a, b} \int_0^1 dx_1 dx_2 f_{h_1/a}(x_1, \mu^2_F) f_{h_2/b}(x_2, \mu^2_F) \times \hat{\sigma}_{a,b \rightarrow X} \left( x_1, x_2, \alpha_s(\mu^2_R), \frac{Q^2}{\mu^2_F}, \frac{Q^2}{\mu^2_R} \right) + \mathcal{O} \left( \frac{1}{Q^2} \right) \]

- PDFs
- Partonic cross section
- np corrections

**PDF set**

- Theorists’ calculation
Monte Carlo Event Generators try to give the best full description of a collision combining theoretical predictions for the different stages of an event and providing a fully exclusive final state in terms of hadrons and leptons which is as close as possible to what is measured in a real experiment.

Can be fed to a detector-simulation software to determine efficiencies.
Parton showers and fixed order

Two complementary approach for parton level computations
Basics on parton showers

- Describe **evolution** from hard process at scale $Q$ to low energy (long distance) non-perturbative physics at scale $\Lambda_{QCD}$

- Proceeds via successive quark & gluon production
  - Start from a low multiplicity process at high $Q$
  - Color-charged partons like to radiate
Basics on parton showers

- Describe evolution from hard process at scale $Q$ to low energy (long distance) non-perturbative physics at scale $\Lambda_{QCD}$

- Proceeds via successive quark & gluon production
  - Start from a low multiplicity process at high $Q$
  - Color-charged partons like to radiate
  - Leading contribution from soft/collinear emissions (propagators become small)

\[
\frac{1}{(k_2 + k_3)^2} = \frac{1}{2E_2E_3(1 - \cos \theta_{23})} \rightarrow \infty \quad \text{if} \quad E_3 \rightarrow 0 \quad \text{or} \quad \theta_{23} \rightarrow 0.
\]
Basics on parton showers

- In this limit QCD amplitudes factorize:

\[ d\Phi_{n+1} = d\Phi_n \, d\Phi_r \quad \text{and} \quad d\Phi_r \approx dt \, dz \, d\varphi \]

- Factorization of phase space:

\[ |M_{n+1}|^2 d\Phi_{n+1} \rightarrow |M_n|^2 d\Phi_n \]

\[ \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{qg,qg}(z) \, dz \, \frac{d\varphi}{2\pi} \]

- Factorization of matrix element:

\[ t = (k+l)^2, \quad p_T^2, \quad E^2 \theta^2 \ldots \]

\[ z = \frac{k^0}{(k^0 + l^0)} \quad \text{energy (or } p_\parallel \text{ or } p^+ \text{) fraction of quark} \]

\[ P_{qg,qg}(z) = C_F \frac{1 + z^2}{1 - z} \quad \text{Altarelli-Parisi splitting function} \]

(ignore \( z \rightarrow 1 \) IR divergence for now)

- For successive emissions iterate this formula:

\[ |M_{n+1}|^2 d\Phi_{n+1} \rightarrow |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{qg,qg}(z') \, dz' \, \frac{d\varphi'}{2\pi} \]

\[ \times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{qg,qg}(z) \, dz \, \frac{d\varphi}{2\pi} \theta(t' - t) \]

\[ \theta', \theta \rightarrow 0 \text{ with } \theta' > \theta \]
Basics on parton showers

- Dominant contribution comes from strongly ordered emissions

\[ t_1 > t_2 > \ldots > t_n \]

- Virtual corrections are also taken into account in the same approximation (they have same structure but opposite sign) leading to a Sudakov form factor \( \Delta(t_1, t_2) \):

\[
\Delta(t_1, t_2) \equiv \exp \left\{ - \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz \ P_{i,j,k}(z) \right\}
\]

- Corresponds to the probability of having no resolved emission between \( t_1 \) and \( t_2 \)

- Therefore, Shower Monte Carlo (SMC) cross section for first emission given by \( (d\Phi_r = dt \ dz \ d\varphi) \)

\[
\langle O \rangle = \int d\Phi_n \ B(\Phi_n) \left\{ O(\Phi_n) \Delta_0 + \int_{t_0}^{t} \frac{dt}{t} \ dz \ d\varphi \ O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \right\}
\]
Basics on parton showers: final recipe

Consider all Born graphs

At each vertex: generate radiation variables and include factor

Include a factor \( \Delta_i(t, t_0) \) on final lines (\( t_0 \) is IR cutoff)

- This allows to resum the leading logarithmic contributions to all orders
  - Probabilistic formulation of shower evolution
  - Can change shapes of distributions but not the overall normalization: at the inclusive level the cross section remains unchanged (unitarity of PS)
  - Attaching this to a LO matrix element allows to reach LO+LL accuracy
Partonic cross section: for hard processes computed as series exp. in the strong coupling $\alpha_s \sim 0.1$

$$\hat{\sigma}_{a,b \rightarrow X} = \alpha_s^n \left[ \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \mathcal{O}(\alpha_s^4) \right]$$

- **LO:**
  - Predicts only the order of magnitude:
    - scale in coupling is not defined
    - 1 parton $\leftrightarrow$ 1 jet

- **NLO:**
  - First reliable predictions:
    - scale choices can be made
    - first description of jet substructure

- **NNLO:**
  - Possible to quantify uncertainties:
    - convergence can be checked
    - richer jet substructure
Importance of NLO calculations

• To control the theoretical uncertainties in the computation of LHC processes NLO predictions are needed:

Consequences of a bad (wrong) scale choice!
Sometimes not even NLO is enough

- In some processes NLO corrections are still large: need to compute cross section at even higher accuracy to obtain a precise theoretical prediction

- Canonical example: Higgs boson production

![Diagram showing cross section at various orders (LO, NLO, NNLO, N3LO) for Higgs boson production with various uncertainties.]

- Huge NLO \([O(100\%)]\) and NNLO \([O(20\%)]\) effects!

- Now known at N\(^3\)LO:

<table>
<thead>
<tr>
<th>Order</th>
<th>Value (\pm Uncertainty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>15.05 ± 14.8%</td>
</tr>
<tr>
<td>NLO</td>
<td>38.2 ± 16.6%</td>
</tr>
<tr>
<td>NNLO</td>
<td>45.1 ± 8.8%</td>
</tr>
<tr>
<td>N3LO</td>
<td>45.2 ± 1.9%</td>
</tr>
</tbody>
</table>

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger, ‘15–’16]
Structure of a NLO calculation

- NLO calculation for an observable $O$ consists of different ingredients:

\[
\langle O \rangle = \int O \, d\sigma = \int d\Phi_n \, O(\Phi_n) \left[ B(\Phi_n) + V_b(\Phi_n) \right] + \int d\Phi_n \, d\Phi_r \, O(\Phi_n, \Phi_r) \, R(\Phi_n, \Phi_r)
\]

- Parametrized $(n+1)$-body phase space $\Phi_{n+1}$ in terms of Born and radiation $\Phi_{n+1} = \{\Phi_n, \Phi_r\}$
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$$

• Parametrized (n+1)-body phase space $\Phi_{n+1}$ in terms of Born and radiation $\Phi_{n+1} = \{\Phi_n, \Phi_r\}$

• Divergent parts can be regulated e.g. with a subtraction scheme:

$$
\langle O \rangle = \int d\Phi_n \, O(\Phi_n) \left[ B(\Phi_n) + V_b(\Phi_n) + \int d\Phi_r \, C(\Phi_n, \Phi_r) \right] \\
+ \int d\Phi_n \, d\Phi_r \left[ O(\Phi_n, \Phi_r) \, R(\Phi_n, \Phi_r) - O(\Phi_n) \, C(\Phi_n, \Phi_r) \right]
$$

• Defining:

$$
V(\Phi_n) = V_b(\Phi_n) + \int d\Phi_r \, C(\Phi_n, \Phi_r) \quad \Leftarrow \text{finite}
$$

$$
\langle O \rangle = \int d\Phi_n \, O(\Phi_n) \left[ B(\Phi_n) + V(\Phi_n) \right] + \int d\Phi_n \, d\Phi_r \left[ O(\Phi_n, \Phi_r) \, R(\Phi_n, \Phi_r) - O(\Phi_n) \, C(\Phi_n, \Phi_r) \right]
$$
# PS vs NLO: pro and cons

<table>
<thead>
<tr>
<th>NLO</th>
<th>PS</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ Normalization accurate at NLO</td>
<td>✓ Sudakov suppression at small $p_T$</td>
</tr>
<tr>
<td>✓ reduced scale uncertainties</td>
<td>✓ Realistic &amp; flexible (hadron level)</td>
</tr>
<tr>
<td>✓ Accurate shapes at high $p_T$</td>
<td>✗ Limited precision (LO uncertainty)</td>
</tr>
<tr>
<td>✗ Limited multiplicity</td>
<td>✗ Bad description at high $p_T$</td>
</tr>
</tbody>
</table>

Two approaches are very **complementary**: ideal to **merge** them trying to keep the good features of both!

**PROBLEM**: avoid **double counting**! First extra emission is accounted for in both..

Several ways exist: **MC@NLO** [Frixione, Webber 2001], **POWHEG** [Nason 2004] and more recently also: KrKNLO, Vincia, Geneva
NLO+PS: the POWHEG method
NLO+PS combination: SMC accuracy vs NLO

• Let’s consider again the SMC cross section for first emission:

\[
\langle O \rangle = \int d\Phi_n \, B(\Phi_n) \left\{ O(\Phi_n) \Delta t_0 + \int_{t_0}^{\frac{t}{t'}} dt \, dz \, d\varphi \, O(\Phi_n, \Phi_r) \Delta t \, \frac{\alpha_s}{2\pi} \, P(z) \right\}
\]

with \( \Delta_t = \exp \left[ -\int_t^{t'} \frac{dt'}{t'} \, dz' \, d\varphi' \, \frac{\alpha_s}{2\pi} \, P(z') \right] \)

• Expand it in \( \alpha_s \) to get NLO\text{SMC}:

\[
\langle O \rangle = \int d\Phi_n \, B(\Phi_n) \left\{ O(\Phi_n) + \int_{t_0}^{\frac{t}{t'}} dt \, dz \, d\varphi \left[ O(\Phi_n, \Phi_r) - O(\Phi_n) \right] \frac{\alpha_s}{2\pi} \, P(z) \right\}
\]

• This is the inexact NLO implemented in parton showers, as compared to the exact NLO:

\[
\langle O \rangle = \int d\Phi_n \, O(\Phi_n) \left[ B(\Phi_n) + V(\Phi_n) \right] + \int d\Phi_n \, d\Phi_r \left[ O(\Phi_n, \Phi_r) \, R(\Phi_n, \Phi_r) - O(\Phi_n) \, C(\Phi_n, \Phi_r) \right]
\]

How do we reach exact NLO accuracy retaining shower resummation?
\[ \langle O \rangle = \int d\Phi_n O(\Phi_n) \left[ B(\Phi_n) + V(\Phi_n) \right] \]
\[ + \int d\Phi_n d\Phi_r \left[ O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r) \right] \]
\[ = \int d\Phi_n O(\Phi_n) \left\{ B(\Phi_n) + V(\Phi_n) + \int d\Phi_r \left[ R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right] \right\} \]
\[ + \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) \left[ O(\Phi_n, \Phi_r) - O(\Phi_n) \right] \]

Define: \( \overline{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r \left[ R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r) \right] \)

And rewrite this as:

\[ \langle O \rangle = \int d\Phi_n O(\Phi_n) \overline{B}(\Phi_n) + \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) \left[ O(\Phi_n, \Phi_r) - O(\Phi_n) \right] \]

Let’s compare it again to the NLO_{SMC}:

\[ \langle O \rangle = \int d\Phi_n O(\Phi_n) B(\Phi_n) + \int d\Phi_n d\Phi_r B(\Phi_n) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \left[ O(\Phi_n, \Phi_r) - O(\Phi_n) \right] \]

Very similar structure, but upper one accurate at NLO.
In summary: \( \text{NLO}_{\text{SMC}} \leftrightarrow \text{NLO} : \quad B(\Phi_n) \leftrightarrow \overline{B}(\Phi_n) \quad \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \leftrightarrow R(\Phi_n, \Phi_r) \)

Back to all order: the emission probability in SMC was

\[
\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta t_0 + \int_{t_0} T d\Phi_r O(\Phi_n, \Phi_r) \Delta t \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \right\}
\]

\[
\text{with} \quad \Delta t = \exp \left[ - \int d\Phi_r' \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right]
\]

All order emission probability in POWHEG:

\[
\langle O \rangle = \int d\Phi_n \overline{B}(\Phi_n) \left\{ O(\Phi_n) \Delta t_0 + \int d\Phi_r O(\Phi_n, \Phi_r) \Delta t \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \right\}
\]

\[
\Delta t = \exp \left[ - \int d\Phi_r' \frac{R(\Phi_n, \Phi_r')}{B(\Phi_n)} \theta(t' - t) \right]
\]

with \( t = k_T(\Phi_n, \Phi_r) \) and \( \overline{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] \)

**POSITIVE** if \( \overline{B} \) is positive (i.e. NLO < LO).
Phenomenological impact

• What is the impact of PS corrections to NLO?
• What should stay unchanged, what could/should instead change?

• Consider most recent POWHEG implementation: HH-production
  [Heinrich, Jones, Kerner, G.L., Vryonidou to appear Wednesday]

➢ Parton shower cannot affect observables inclusive in the radiation:
Phenomenological impact

- Parton shower should instead modify the shape of observable sensitive to QCD (QED) radiation.
Conclusions and Outlook

• Fixed order computation and parton showers are two complementary approaches to compute a cross section

  ➢ Describe two different regimes in a particle scattering

• Best solution is to combine (match) them together!
• POWHEG is one possible matching scheme

• NLO+PS has been extended to merge different multiplicity calculations at NLO and match them to PS:
  MiNLO  MEPS@NLO  FxFx  UNLOPS

• In meanwhile first schemes exist for NNLO+PS matching: this is the future!