

# An introduction to POWHEG

Gionata Luisoni

gionata.luisoni@cern.ch

CERN

27.03.2017

**Important references:**

**hep-ph/0409146, arXiv:0709.2092, arXiv:1002.2581**

CREDITS: Thanks to E.Re and C.Oleari for a lot of the material appearing in the slides



# Outline

- Brief motivation: the quest of precision
- Monte Carlo (MC) event generators
- Parton shower vs fixed order calculations
- How to merge NLO+PS: the POWHEG master formula
- Phenomenological impact
- Conclusions and outlook

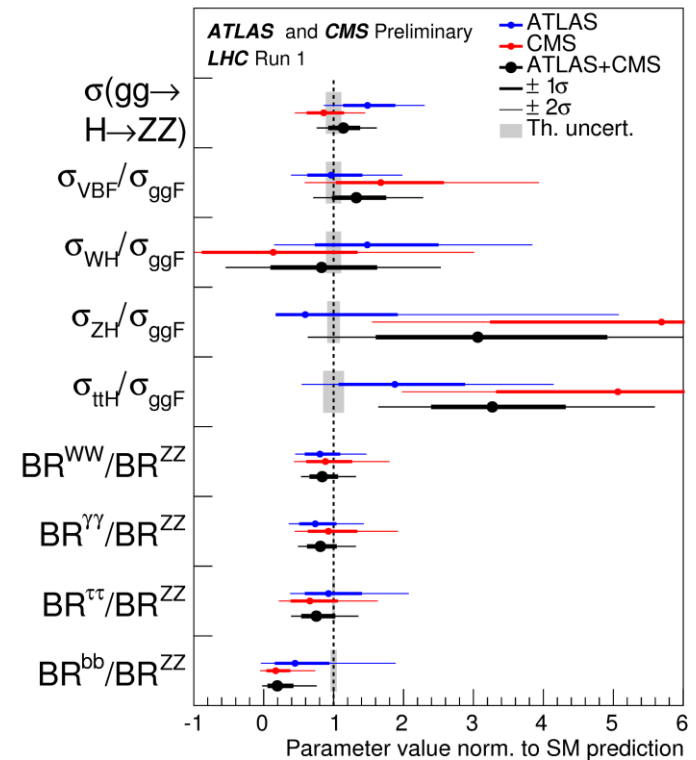
• • • Motivation: the quest for precision

# New Physics at the LHC?

- After Higgs boson discovery focus on its properties and on **hints for BSM** physics
  - So far no sign of new physics at the TeV scale from direct searches
- Possible hits for new physics could come from **very precise measurements** e.g. in the Higgs sector

- Very accurate predictions are needed, both
  - in shape
  - in normalization

Precise description of scattering process at all stages!



# Monte Carlo Event Generators

What they are and what they do

PDF's

Hard scattering

$$\Lambda_{QCD} \ll \mu \approx Q$$

Parton shower

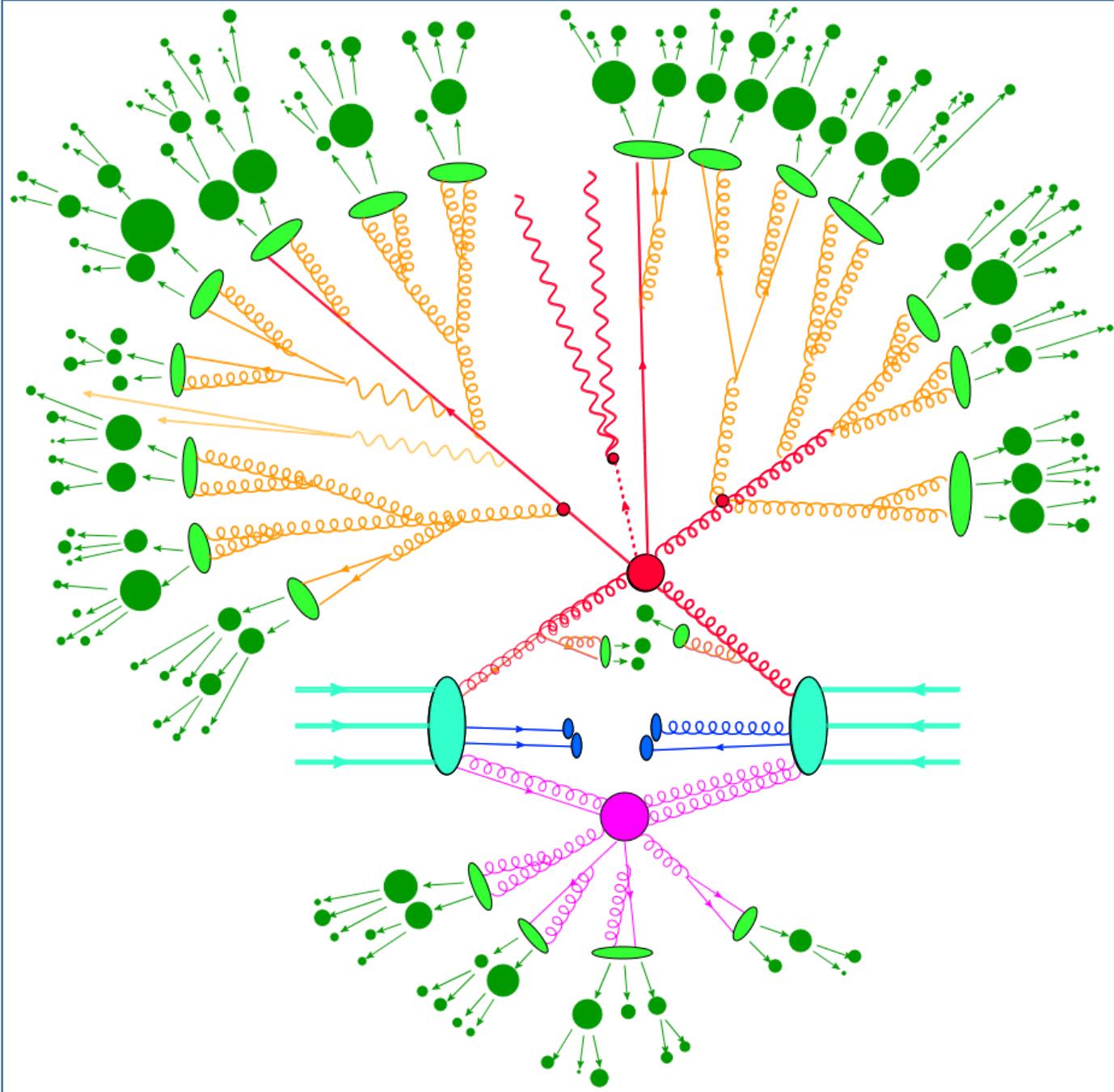
$$\Lambda_{QCD} < \mu < Q$$

Beam remnants

Hadronization and  
Hadron decay

$$\mu \approx \Lambda_{QCD}$$

Multiple interactions



PDF's

Determined from data

Hard scattering

$\Lambda_{QCD} \ll \mu \approx Q$

Hard process computed using fixed order (FO) perturbation theory at LO, NLO, NNLO, N<sup>3</sup>LO,...

Parton shower

$\Lambda_{QCD} < \mu < Q$

Hierarchy of scales appearing as large logarithms in the calculation.

Computed using resummation or MC parton showers (PS)

Beam remnants

Hadronization and  
Hadron decay

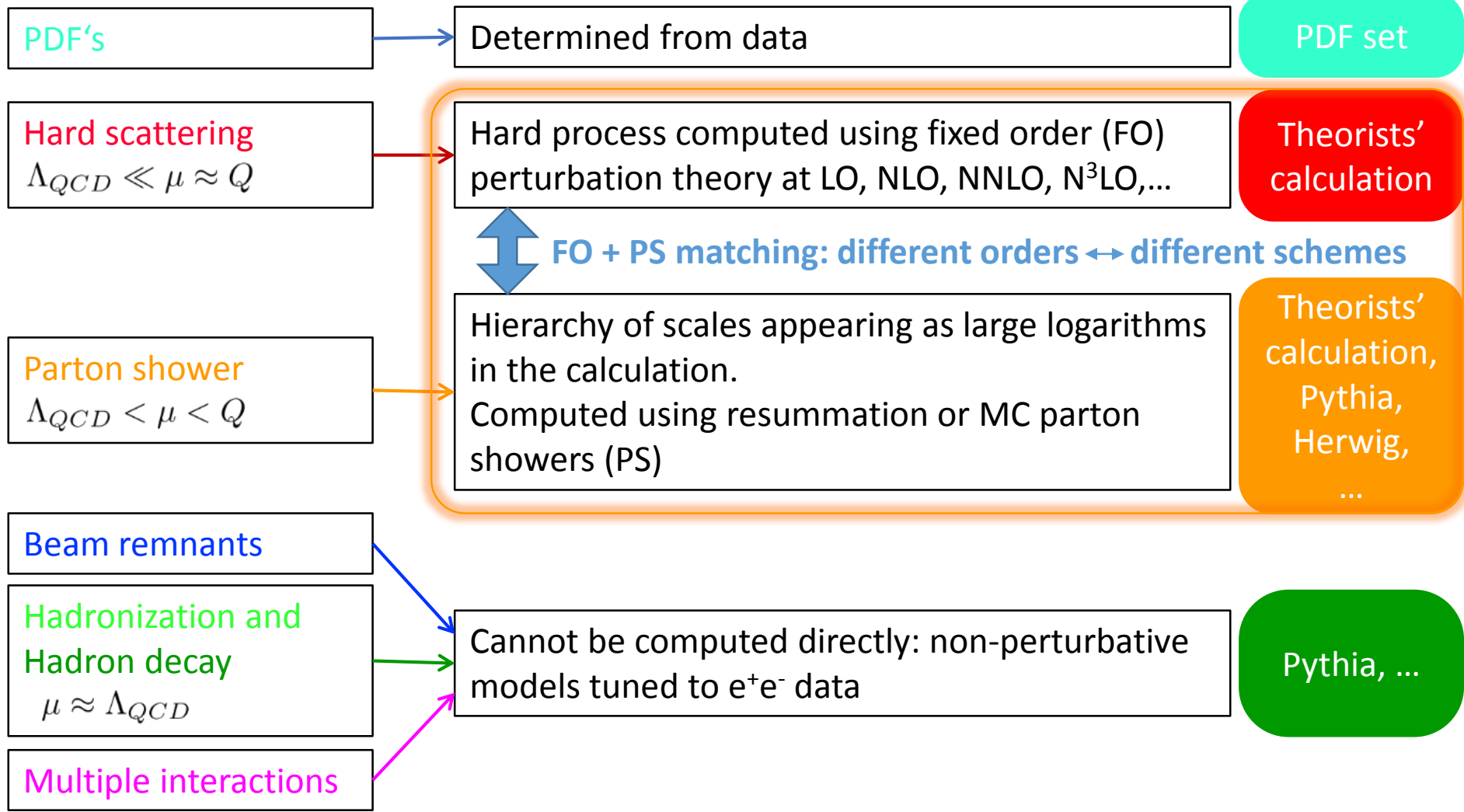
$\mu \approx \Lambda_{QCD}$

Multiple interactions

Cannot be computed directly: non-perturbative models tuned to e<sup>+</sup>e<sup>-</sup> data

$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{a,b} \int_0^1 dx_1 dx_2 \underbrace{f_{h_1/a}(x_1, \mu_F^2) f_{h_2/b}(x_2, \mu_F^2)}_{\text{PDFs}} \times \underbrace{\hat{\sigma}_{a,b \rightarrow X} \left( x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2} \right)}_{\text{partonic cross section}} \left[ + \mathcal{O} \left( \frac{1}{Q^2} \right) \right]_{\text{np corrections}}$$





$$\sigma_{h_1 h_2 \rightarrow X} = \sum_{a,b} \int_0^1 dx_1 dx_2 \underbrace{f_{h_1/a}(x_1, \mu_F^2) f_{h_2/b}(x_2, \mu_F^2)}_{\text{PDFs}} \times \underbrace{\hat{\sigma}_{a,b \rightarrow X}(x_1, x_2, \alpha_s(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})}_{\text{partonic cross section}} \left[ + \mathcal{O}\left(\frac{1}{Q^2}\right) \right]_{\text{np corrections}}$$





- Monte Carlo Event Generators try to give the **best full description** of a collision combining theoretical predictions for the **different stages** of an event and providing a **fully exclusive** final state in terms of **hadrons and leptons** which is as close as possible to what is measured in a real experiment

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS	V-X	V-Y	V-Z	V-C*T
30	NU_E	12	1	28	23	0	0	64.30	25.12	-1194.4	1196.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
31	E+	-11	1	29	23	0	0	-22.36	6.19	-234.2	235.4	0.00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
230	PI0	111	1	155	24	0	0	0.31	0.38	0.9	1.0	0.13	4.209E-11	6.148E-11	-3.341E-11	5.192E-10
231	RHO+	213	197	155	24	317	318	-0.06	0.07	0.1	0.8	0.77	4.183E-11	6.130E-11	-3.365E-11	5.189E-10
232	P	2212	1	156	24	0	0	0.40	0.78	1.0	1.6	0.94	4.156E-11	6.029E-11	-4.205E-11	5.250E-10
233	NBAR	-2112	1	156	24	0	0	-0.13	-0.35	-0.9	1.3	0.94	4.168E-11	6.021E-11	-4.217E-11	5.249E-10
234	PI-	-211	1	157	9	0	0	0.14	0.34	286.9	286.9	0.14	4.660E-13	8.237E-12	1.748E-09	1.749E-09
235	PI+	211	1	157	9	0	0	-0.14	-0.34	624.5	624.5	0.14	4.056E-13	8.532E-12	2.462E-09	2.462E-09
236	P	2212	1	158	9	0	0	-1.23	-0.26	0.9	1.8	0.94	-4.815E-11	1.893E-11	7.520E-12	3.252E-10
237	DLTABR--	-2224	197	158	9	319	320	0.94	0.35	1.6	2.2	1.23	-4.817E-11	1.900E-11	7.482E-12	3.252E-10
238	PI0	111	1	159	9	0	0	0.74	-0.31	-27.9	27.9	0.13	-1.889E-10	9.893E-11	-2.123E-09	2.157E-09
239	RHO0	113	197	159	9	321	322	0.73	-0.88	-19.5	19.5	0.77	-1.888E-10	9.859E-11	-2.129E-09	2.163E-09
240	K+	321	1	160	9	0	0	0.58	0.02	-11.0	11.0	0.49	-1.890E-10	9.873E-11	-2.135E-09	2.169E-09
241	KL_1-	-10323	197	160	9	323	324	1.23	-1.50	-50.2	50.2	1.57	-1.890E-10	9.879E-11	-2.132E-09	2.166E-09
242	K-	-321	1	161	24	0	0	0.01	0.22	1.3	1.4	0.49	4.250E-11	6.333E-11	-2.746E-11	5.211E-10
243	PI0	111	1	161	24	0	0	0.31	0.38	0.2	0.6	0.13	4.301E-11	6.282E-11	-2.751E-11	5.210E-10

- Can be fed to a detector-simulation software to determine efficiencies

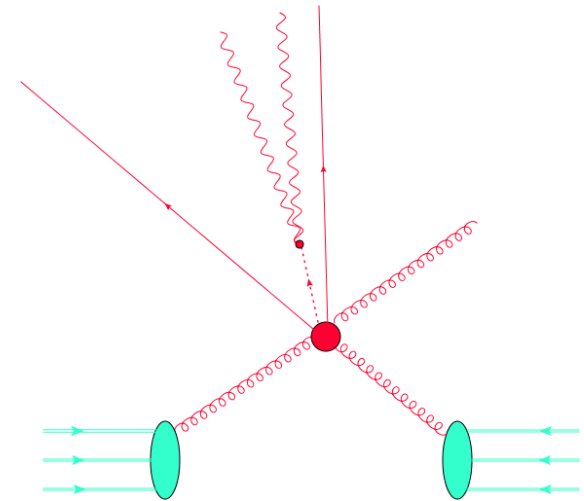


# Parton showers and fixed order

Two complementary approach for parton level computations

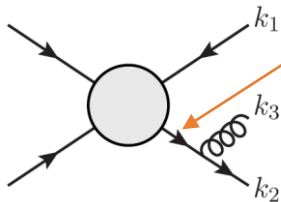
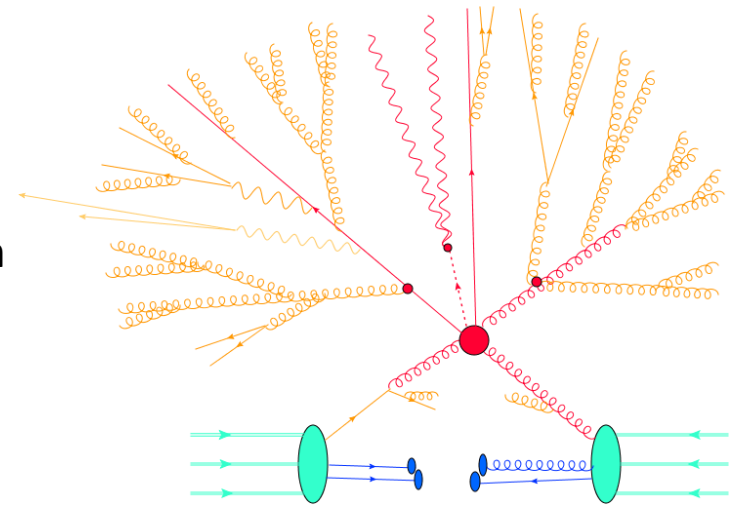
# Basics on parton showers

- Describe **evolution** from hard process at scale  $Q$  to low energy (long distance) non-perturbative physics at scale  $\Lambda_{QCD}$
- Proceeds via successive quark & gluon production
  - Start from a low multiplicity process at high  $Q$
  - Color-charged partons like to radiate



# Basics on parton showers

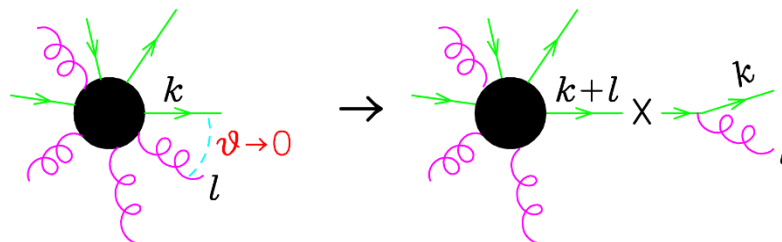
- Describe **evolution** from hard process at scale  $Q$  to low energy (long distance) non-perturbative physics at scale  $\Lambda_{QCD}$
- Proceeds via successive quark & gluon production
  - Start from a low multiplicity process at high  $Q$
  - Color-charged partons like to radiate
  - Leading contribution from soft/collinear emissions (propagators become small)



$$\frac{1}{(k_2 + k_3)^2} = \frac{1}{2E_2 E_3 (1 - \cos \theta_{23})} \rightarrow \infty \text{ if } E_3 \rightarrow 0 \text{ or } \theta_{23} \rightarrow 0.$$

# Basics on parton showers

➤ In this limit QCD amplitudes factorize:



➤ Factorization of phase space:  $d\Phi_{n+1} = d\Phi_n d\Phi_r$   $d\Phi_r \div dt dz d\varphi$

➤ Factorization of matrix element:

$$|M_{n+1}|^2 d\Phi_{n+1} \implies |M_n|^2 d\Phi_n \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi}$$

$$\left\{ \begin{array}{l} \frac{dt}{t} \approx \frac{d\theta}{\theta} \quad \text{collinear singularity} \\ \frac{dz}{1-z} \approx \frac{dE_g}{E_g} \quad \text{soft singularity} \end{array} \right.$$

$$t : (k+l)^2, p_T^2, E^2\theta^2 \dots$$

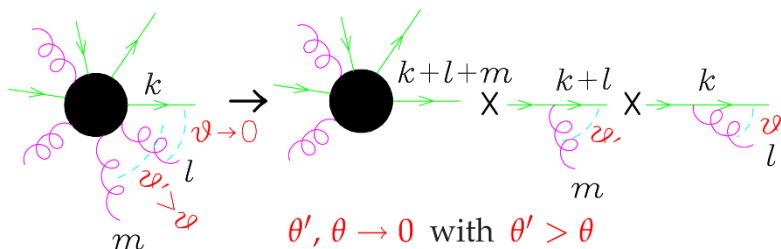
$$z = k^0 / (k^0 + l^0) : \text{energy (or } p_{||} \text{ or } p^+) \text{ fraction of quark}$$

$$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z} : \text{Altarelli-Parisi splitting function}$$

(ignore  $z \rightarrow 1$  IR divergence for now)

Gives rise to double logarithmic structure  
 $\alpha_s L^2$

➤ For successive emissions **iterate** this formula:

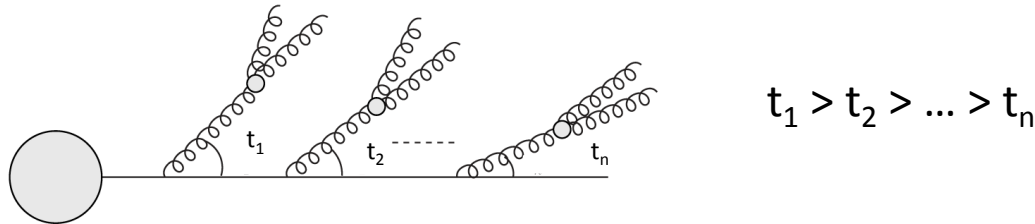


$\theta', \theta \rightarrow 0$  with  $\theta' > \theta$

$$|M_{n+1}|^2 d\Phi_{n+1} \implies |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z') dz' \frac{d\varphi'}{2\pi} \times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) dz \frac{d\varphi}{2\pi} \theta(t' - t)$$

# Basics on parton showers

- Dominant contribution comes from **strongly ordered** emissions



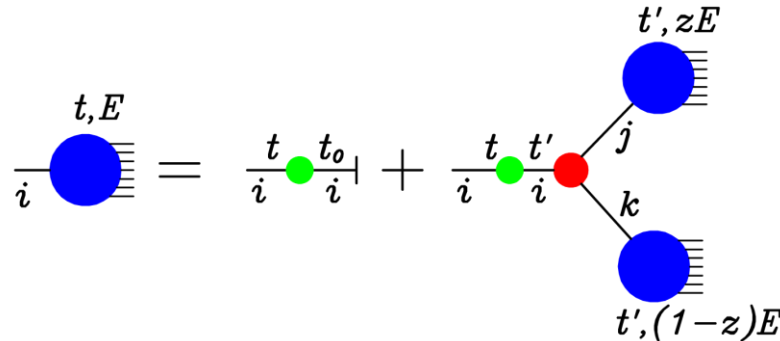
- Virtual corrections are also taken into account in the same approximation (they have same structure but opposite sign) leading to a **Sudakov form factor**  $\Delta(t_1, t_2)$ :

$$\Delta(t_1, t_2) \equiv \exp \left\{ - \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \int dz P_{i,jk}(z) \right\}$$

- Corresponds to the **probability** of having **no resolved emission** between  $t_1$  and  $t_2$
- Therefore, Shower Monte Carlo (SMC) cross section for first emission given by  $(d\Phi_r = dt dz d\varphi)$

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int_{t_0} \frac{dt}{t} dz d\varphi O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \right\}$$

# Basics on parton showers: final recipe



$$\mathcal{S}_i(t, E) = \Delta_i(t, t_0) \mathbb{1} + \sum_{(jk)} \int_{t_0}^t \frac{\alpha_S(t')}{2\pi} \frac{dt'}{t'} \int dz \int \frac{d\varphi}{2\pi} \Delta_i(t, t') P_{i,jk}(z) \mathcal{S}_j(t', zE) \mathcal{S}_k(t', (1-z)E)$$

- Consider all Born graphs
  - At each vertex: generate radiation variables and include factor  $\frac{dt}{t} dz \frac{\alpha_s(t)}{2\pi} P_{i,jk}(z) \frac{d\varphi}{2\pi}$
  - Include a factor  $\Delta_i(t_1, t_2)$  to each internal parton i from hardness t1 to hardness t2
  - Include a factor  $\Delta_i(t, t_0)$  on final lines ( $t_0$  is IR cutoff)
- This allows to resum the leading logarithmic contributions to all orders
    - Probabilistic formulation of shower evolution
    - Can change shapes of distributions but not the overall normalization: at the inclusive level the cross section remains unchanged (unitarity of PS)
    - Attaching this to a LO matrix element allows to reach LO+LL accuracy

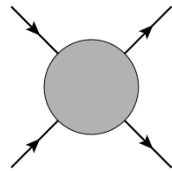
# Fixed order calculations

- **Partonic cross section:** for hard processes computed as series exp. in the strong coupling  $\alpha_s \sim 0.1$

$$\hat{\sigma}_{a,b \rightarrow X} = \alpha_s^n \left[ \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \alpha_s^3 \sigma_3 + \mathcal{O}(\alpha_s^4) \right]$$

LO | NLO | NNLO | N<sup>3</sup>LO

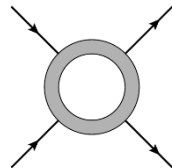
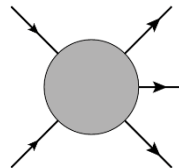
- LO:



Predicts only the order of magnitude:

- scale in coupling is not defined
- 1 parton  $\longleftrightarrow$  1 jet

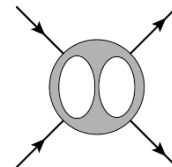
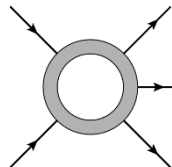
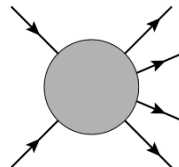
- NLO:



First reliable predictions:

- scale choices can be made
- first description of jet substructure

- NNLO:



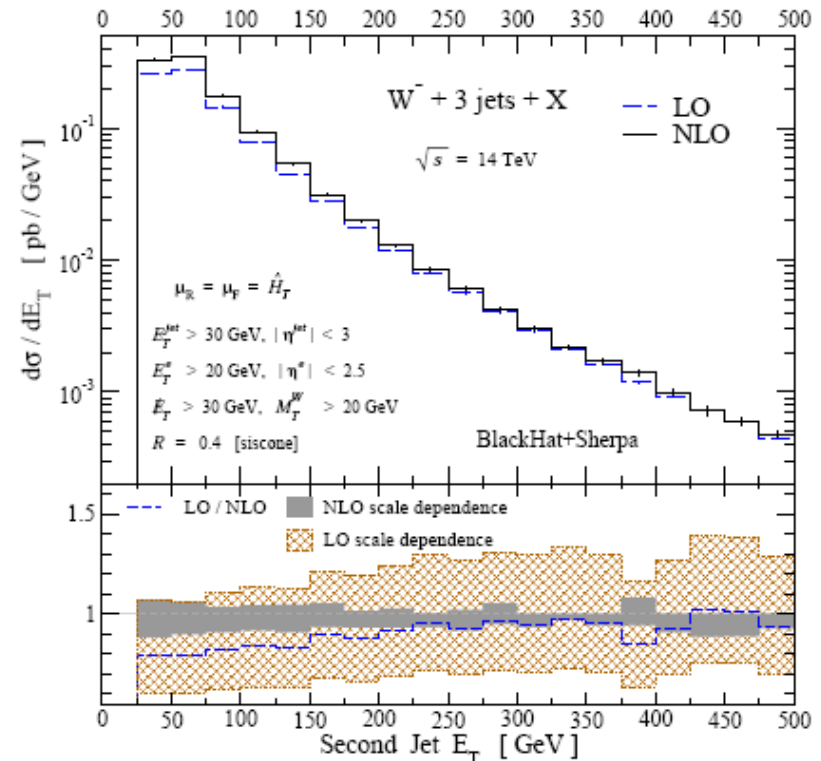
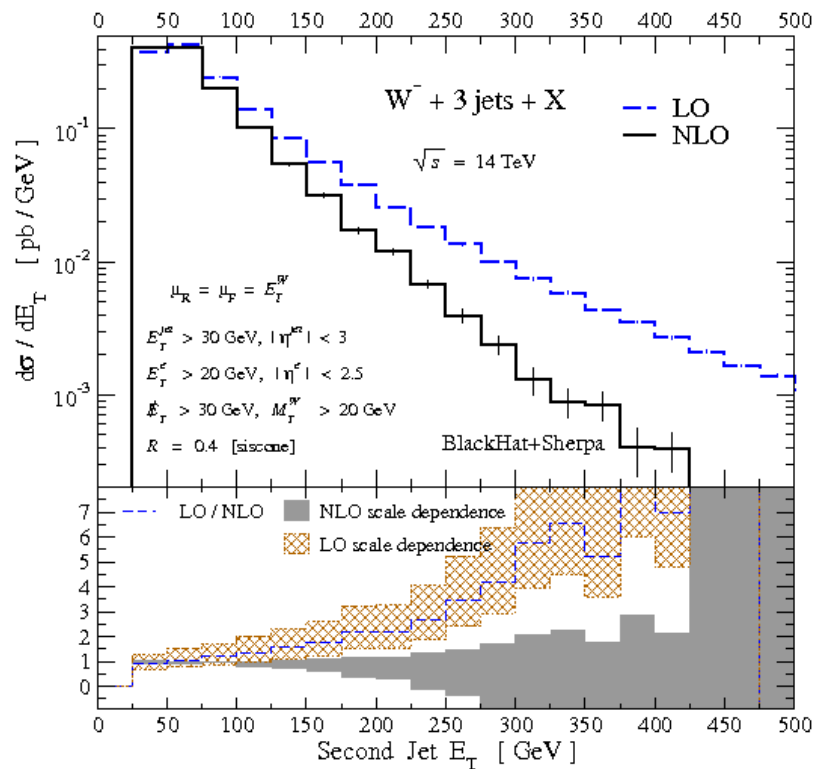
Possible to quantify uncertainties:

- convergence can be checked
- richer jet substructure



# Importance of NLO calculations

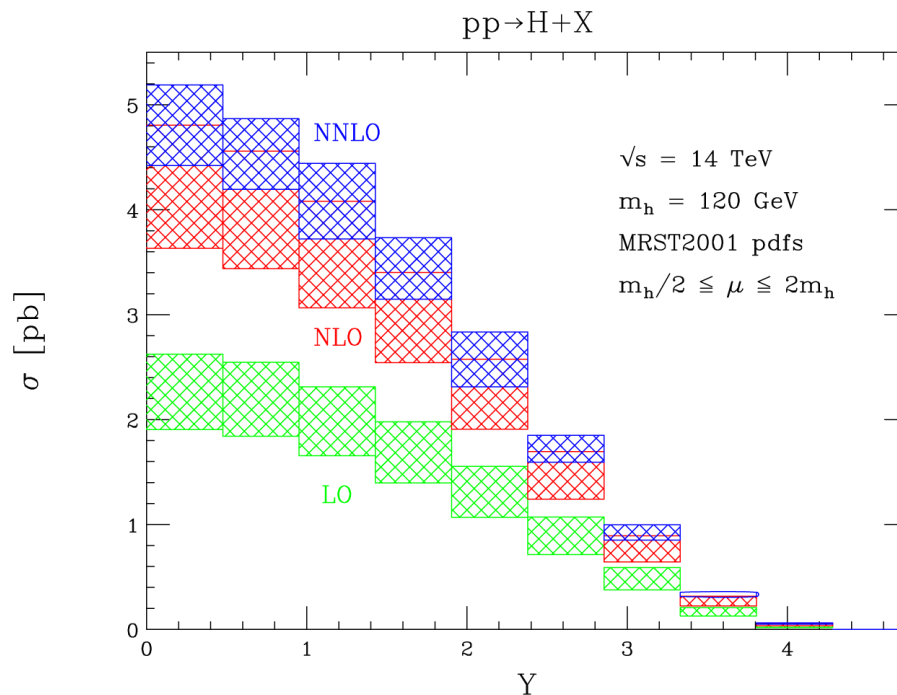
- To control the theoretical uncertainties in the computation of LHC processes NLO predictions are needed:



Consequences of a bad (wrong) scale choice!

# Sometimes not even NLO is enough

- In some processes NLO corrections are still large: need to compute cross section at even higher accuracy to obtain a precise theoretical prediction
- Canonical example: Higgs boson production



[Anastasiou, Melnikov, Petriello, '05]

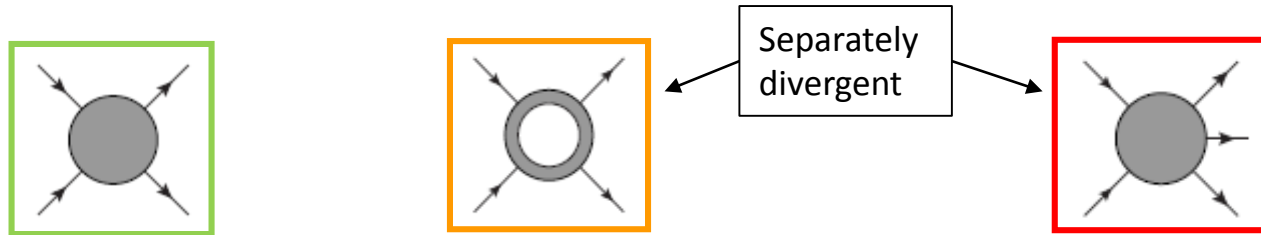
- Huge NLO [ $\mathcal{O}(100\%)$ ] and NNLO [ $\mathcal{O}(20\%)$ ] effects!
- Now known at N<sup>3</sup>LO:

LO	$15.05 \pm 14.8\%$
NLO	$38.2 \pm 16.6\%$
NNLO	$45.1 \pm 8.8\%$
N <sup>3</sup> LO	$45.2 \pm 1.9\%$

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger, '15-'16]

# Structure of a NLO calculation

- NLO calculation for an observable  $O$  consists of different ingredients:

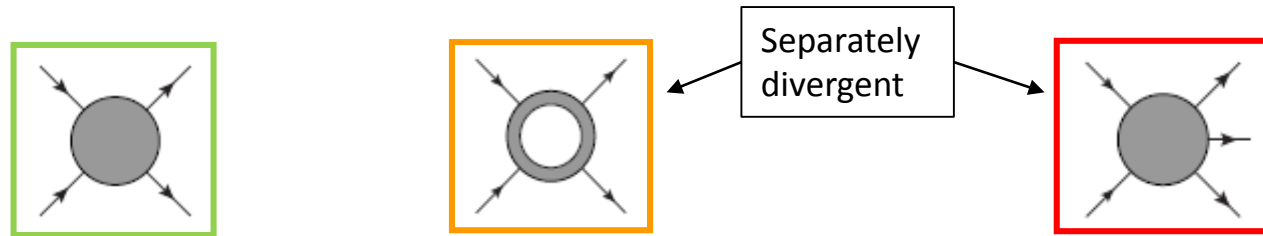


$$\langle O \rangle = \int O d\sigma = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V_b(\Phi_n)] + \int d\Phi_n d\Phi_r O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r)$$

- [Parametrized (n+1)-body phase space  $\Phi_{n+1}$  in terms of Born and radiation  $\Phi_{n+1} = \{\Phi_n, \Phi_r\}$ ]

# Structure of a NLO calculation

- NLO calculation for an observable  $O$  consists of different ingredients:



$$\langle O \rangle = \int O d\sigma = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V_b(\Phi_n)] + \int d\Phi_n d\Phi_r O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r)$$

- [Parametrized (n+1)-body phase space  $\Phi_{n+1}$  in terms of Born and radiation  $\Phi_{n+1} = \{\Phi_n, \Phi_r\}$ ]
- Divergent parts can be regulated e.g. with a subtraction scheme:

$$\begin{aligned} \langle O \rangle &= \int d\Phi_n O(\Phi_n) \left[ B(\Phi_n) + V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \right] \\ &+ \int d\Phi_n d\Phi_r \underbrace{\left[ O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r) \right]}_{\text{finite}} \end{aligned}$$

- Defining:  $V(\Phi_n) = V_b(\Phi_n) + \int d\Phi_r C(\Phi_n, \Phi_r) \iff \text{finite}$

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] + \int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)]$$

# PS vs NLO: pro and cons

## NLO

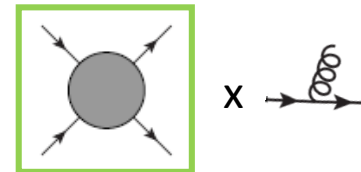
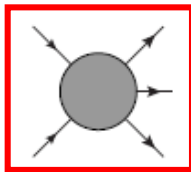
- ✓ Normalization accurate at NLO
- ✓ reduced scale uncertainties
- ✓ Accurate shapes at high  $p_T$
- ✗ Limited multiplicity
- ✗ Fails on resummation regions

## PS

- ✓ Sudakov suppression at small  $p_T$
- ✓ Realistic & flexible (hadron level)
- ✗ Limited precision (LO uncertainty)
- ✗ Bad description at high  $p_T$

➔ Two approaches are very **complementary**: ideal to **merge** them trying to keep the **good features of both**!

PROBLEM: avoid **double counting**! First extra emission is accounted for in both..



Several ways exist: MC@NLO [Frixione, Webber 2001], POWHEG [Nason 2004]  
and more recently also: KrKNLO, Vincia, Geneva

# ... NLO+PS: the POWHEG method

# NLO+PS combination: SMC accuracy vs NLO

- Let's consider again the SMC cross section for first emission:

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int_{t_0}^t \frac{dt}{t} dz d\varphi O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \right\}$$

$$\text{with } \Delta_t = \exp \left[ - \int_t^t \frac{dt'}{t'} dz' d\varphi' \frac{\alpha_s}{2\pi} P(z') \right]$$

- Expand it in  $\alpha_s$  to get  $\text{NLO}_{\text{SMC}}$ :

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) + \int_{t_0}^t \frac{dt}{t} dz d\varphi \left[ O(\Phi_n, \Phi_r) - O(\Phi_n) \right] \frac{\alpha_s}{2\pi} P(z) \right\}$$

- This is the **inexact** NLO implemented in parton showers, as compared to the exact NLO:

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] + \int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)]$$

How do we reach exact NLO accuracy retaining shower resummation?

# NLO+PS combination: towards NLO accuracy

$$\begin{aligned}
 \langle O \rangle &= \int d\Phi_n O(\Phi_n) [B(\Phi_n) + V(\Phi_n)] \\
 &+ \int d\Phi_n d\Phi_r [O(\Phi_n, \Phi_r) R(\Phi_n, \Phi_r) - O(\Phi_n) C(\Phi_n, \Phi_r)] \\
 &= \int d\Phi_n O(\Phi_n) \left\{ B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)] \right\} \\
 &+ \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)]
 \end{aligned}$$

Define:  $\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$

And rewrite this as:

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) \bar{B}(\Phi_n) + \int d\Phi_n d\Phi_r R(\Phi_n, \Phi_r) [O(\Phi_n, \Phi_r) - O(\Phi_n)]$$

Very similar structure, but upper one accurate at NLO

Let's compare it again to the  $\text{NLO}_{\text{SMC}}$ :

$$\langle O \rangle = \int d\Phi_n O(\Phi_n) B(\Phi_n) + \int d\Phi_n d\Phi_r B(\Phi_n) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} [O(\Phi_n, \Phi_r) - O(\Phi_n)]$$



# NLO+PS combination: POWHEG

In summary:  $\text{NLO}_{\text{SMC}} \leftrightarrow \text{NLO}$  :  $B(\Phi_n) \leftrightarrow \bar{B}(\Phi_n)$   $B(\Phi_n) \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \leftrightarrow R(\Phi_n, \Phi_r)$

Back to all order: the emission probability in SMC was

$$\langle O \rangle = \int d\Phi_n B(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int_{t_0} d\Phi_r O(\Phi_n, \Phi_r) \Delta_t \frac{\alpha_s}{2\pi} P(z) \frac{1}{t} \right\}$$

$$\text{with } \Delta_t = \exp \left[ - \int d\Phi'_r \frac{\alpha_s}{2\pi} P(z') \frac{1}{t'} \theta(t' - t) \right]$$

All order emission probability in POWHEG:

$$\langle O \rangle = \int d\Phi_n \bar{B}(\Phi_n) \left\{ O(\Phi_n) \Delta_{t_0} + \int d\Phi_r O(\Phi_n, \Phi_r) \Delta_t \frac{R(\Phi_n, \Phi_r)}{B(\Phi_n)} \right\}$$

$$\Delta_t = \exp \left[ - \int d\Phi'_r \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \theta(t' - t) \right]$$

with  $t = k_T(\Phi_n, \Phi_r)$  and  $\bar{B}(\Phi_n) = B(\Phi_n) + V(\Phi_n) + \int d\Phi_r [R(\Phi_n, \Phi_r) - C(\Phi_n, \Phi_r)]$

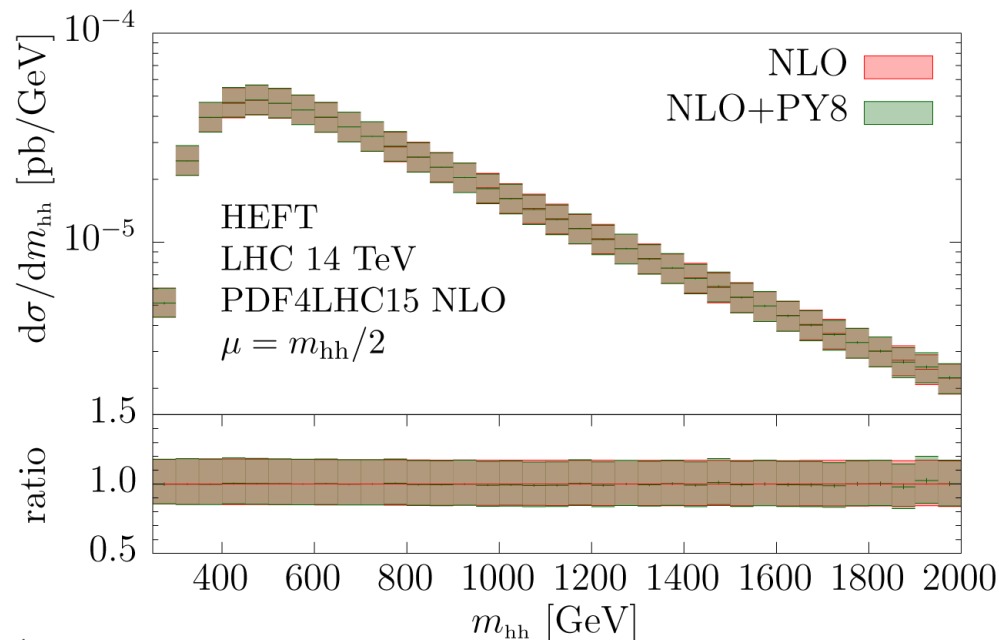
**POSITIVE** if  $\bar{B}$  is positive (i.e. NLO < LO).

# Phenomenological impact

- What is the impact of PS corrections to NLO?
- What should stay unchanged, what could/should instead change?
- Consider most recent POWHEG implementation: HH-production

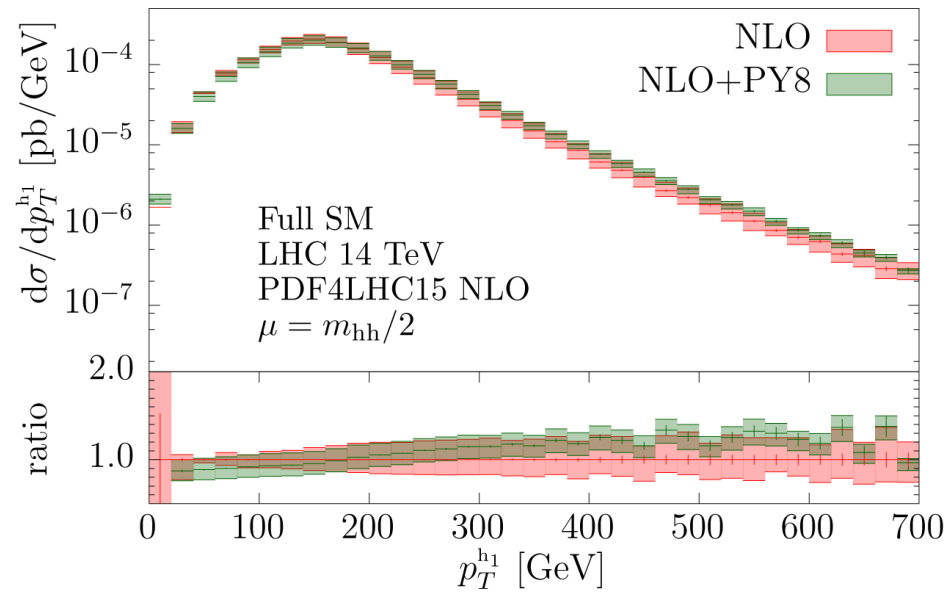
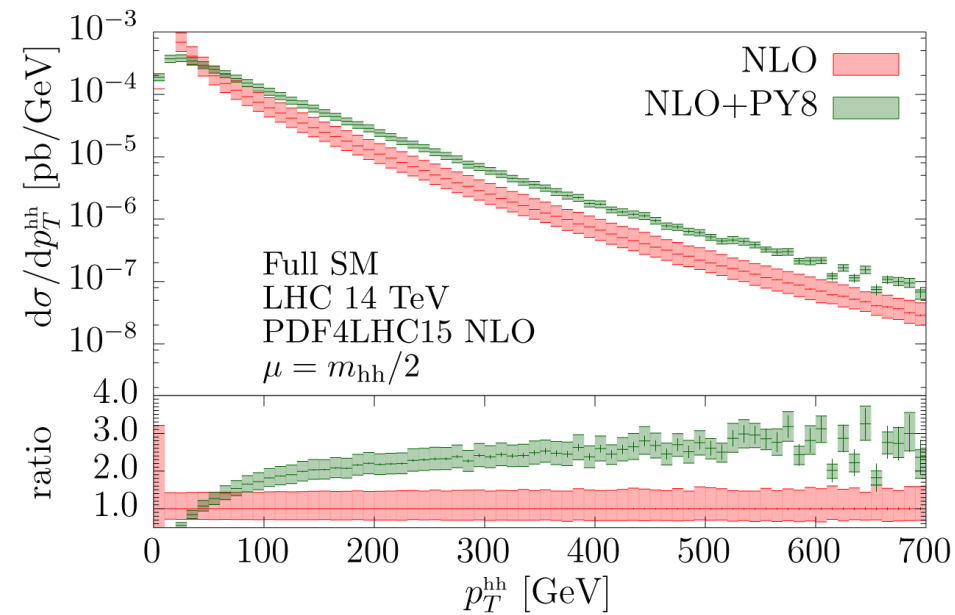
[Heinrich, Jones, Kerner, G.L., Vryonidou to appear Wednesday]

➤ Parton shower **cannot** affect observables inclusive in the radiation:



# Phenomenological impact

- Parton shower **should** instead modify the shape of observable sensitive to QCD (QED) radiation



# Conclusions and Outlook

- Fixed order computation and parton showers are two complementary approaches to compute a cross section
  - Describe two different regimes in a particle scattering
- Best solution is to combine (match) them together!
- POWHEG is one possible matching scheme
  
- NLO+PS has been extended to merge different multiplicity calculations at NLO and match them to PS:
  - MiNLO
  - MEPS@NLO
  - FxFx
  - UNLOPS
- In meanwhile first schemes exist for NNLO+PS matching: this is the future!