Little Hierarchy in Minimally Specified MSSM

PIKIO Spring 2017

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March 4, 2017

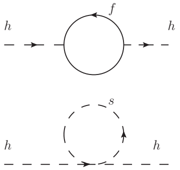
Outline

Naturalness and the MSSM

Little Hierarchy from Minimal Specification

SUSY and the EW scale

- SUSY postulates the existence of scalar partners to SM fermions as a means to stabilize the large hierarchy between the EW and Planck scales



$\Rightarrow \delta M_h^2 \sim (\lambda_s - \lambda_f^2)\Lambda^2$

- Requiring EW symmetry breaking

$$\Rightarrow M_Z^2 = \frac{|m_{H_d}^2(M_Z) - m_{H_u}^2(M_Z)|}{\sqrt{1-\sin^2(2\beta)}} - m_{H_u}^2(M_Z) - m_{H_d}^2(M_Z) - 2|\mu|^2(M_Z)$$

$$\Rightarrow M_Z^2 \approx -2|\mu|^2 - 2m_{H_u,0}^2 - 2\Delta m_{H_u}^2 \ * \qquad *(\tan\beta \geqslant 5)$$

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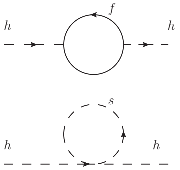

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$$\Rightarrow M_Z^2 = -1.9\mu_0^2 + 5.9M_{3,0}^2 - 1.2m_{H_u,0}^2 + 1.5m_{\tilde{t},0}^2 - 0.8A_t M_{3,0} + 0.2A_{t,0}^2 + \ldots$$

⋆ stops needed $\sim$ 10 TeV for Higgs mass

⋆ Experimental limits pushing SUSY into TeV range

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$\star$ **SUSY two orders above M_Z requires large fine tuning and is improbable**$\star$

$$m_{\tilde{t}} \sim 10^4 \text{ GeV} \Rightarrow \Delta \sim 10^4 \Rightarrow \text{tuning of 1 part in } 10^4$$

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Minimally Specified MSSM

⋆ Different approach⋆

- Scan for outcomes of M_Z
  - Minimally specify parameters up to one significant figure. $\Rightarrow$ Automatically avoids outcomes resulting from carefully chosen parameters.
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"Sufficient" complexity:

$$M_Z^2 = -1.9\mu_0^2 + 5.9M_{3,0}^2 - 1.2m_{H_u,0}^2 + 1.5m_{\tilde{t},0}^2 - 0.8A_t M_{3,0} + 0.2A_{t,0}^2 + \ldots$$

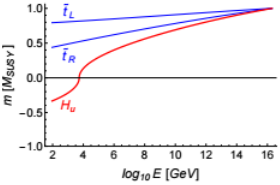
⋆ Numerous parameters

⋆ parameters contribute at various orders of magnitude

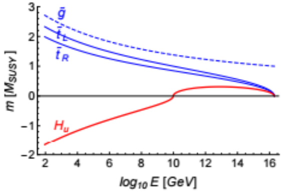
⋆ Leading order contributions are a cancellation of comparable numbers

Minimal Specification (cont.)

$$\frac{M_Z^2}{2} \approx -|\mu|^2 - m_{H_u,0}^2 - \Delta m_{H_u}^2$$



(a) $M_{SUSY} = m_0$, $M_{1/2} = 0$.



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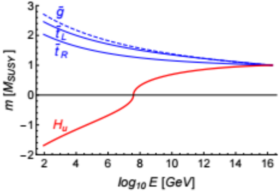

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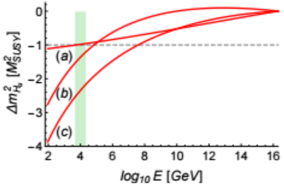

(d)

Toy Model

$$\frac{M_Z^2}{2} \approx \left(- |\mu|^2 - m_{H_u,0}^2 \right) - \Delta m_{H_u}$$

$$X \approx A - B$$

- Consider an observable X whose value, to leading order, depends on a difference of two parameters

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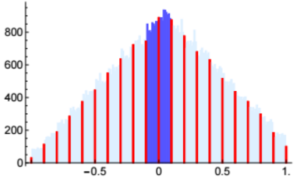
- Consider an observable X whose value, to leading order, depends on a difference of two parameters
- Vary A and B, specifying one significant figure $\pm 50\%$ of central values i.e. $A + A\{0, \pm 0.1, \pm 0.2, .. \pm 0.5\}$



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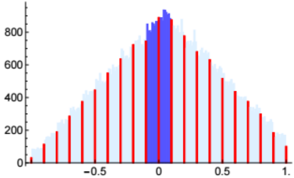

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⋆ ⋆ ⋆ No matter how the distribution is shifted by different central values, gap ~ 0.1 will remain. $\Rightarrow X \sim 0.1$ is the smallest naturally outcome

Toy Model (cont.)

$$\Delta m_{H_u} = .. + 0.2 A_{t,0}^2 \ \ + ..$$

- Add parameter contributing at the next order of magnitude to X, and vary specifying one sig. fig. $C \pm 0.5C$ ($A = B = C = 1$)

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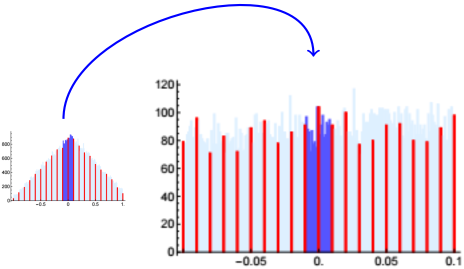

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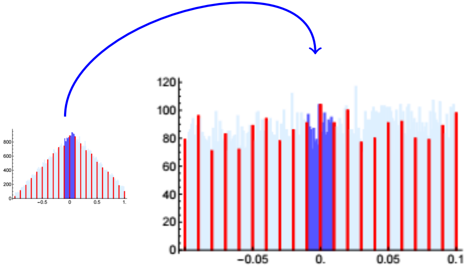

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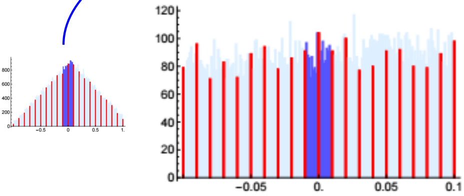

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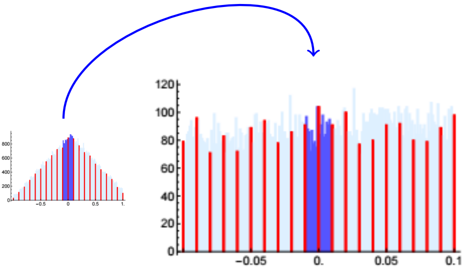

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⋆ **Largest gap size is taken as smallest natural outcome**⋆

CMSSM ($A_0 = 0$)

- $M_{1/2}$, m_0, $\mu \sim M_{SUSY} \pm 50\%$ specifying one significant figure
  $M_{1/2} = 1.2 M_{SUSY}\,,\ m_0 = .6 M_{SUSY}\,,\ \mu = 1.5 M_{SUSY}$
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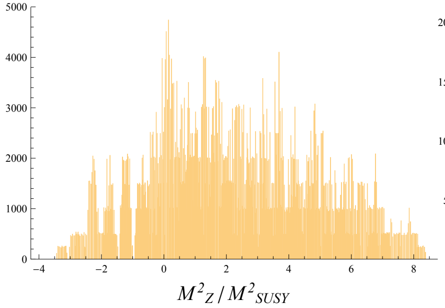

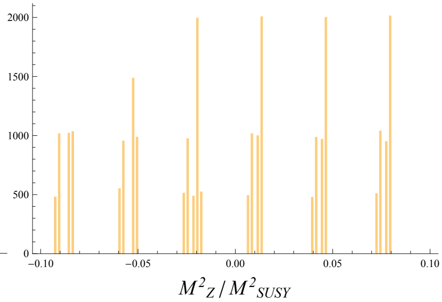

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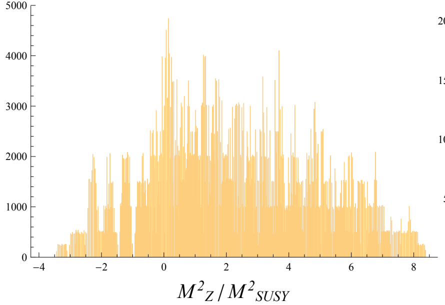

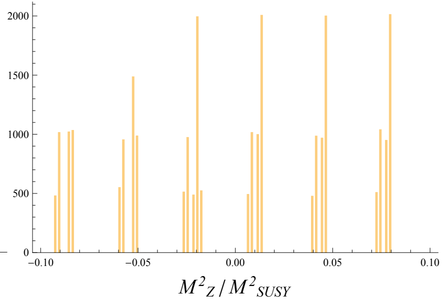

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CMSSM

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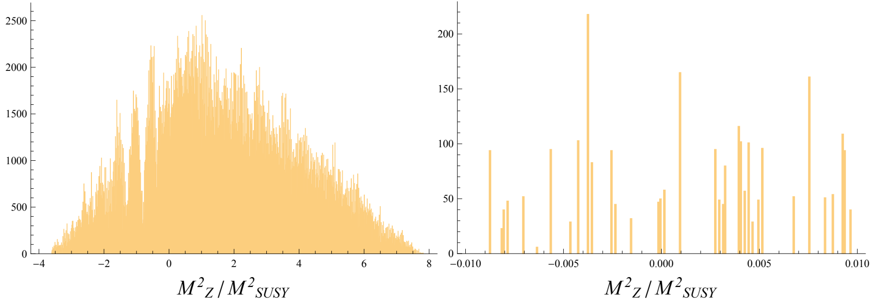
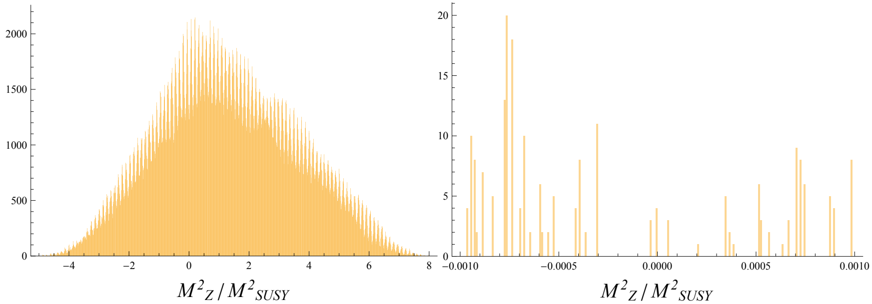


Largest gap size $\sim 0.002 \quad \Rightarrow M_{SUSY} \leqslant 20 M_Z$

MSSM: non-universal Higgs masses

- $M_{1/2}$, m_0, m_{H_u}, m_{H_d}, $\mu \sim M_{SUSY} \pm 50\%$, one significant figure, $A_0 = 0$



Largest gap size $\sim 0.0003 \quad \Rightarrow M_{SUSY} \leqslant 60 M_Z$

$$M_Z^2 \supset -1.2 m_{H_u}^2 - 0.053 m_{H_d}^2 + \ldots$$

Conclusions

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  - In the MSSM, parameters contributing at orders continuously down to 10^{-6} in masses squared $\Rightarrow M_{SUSY} \leqslant 10^3 M_Z$ (work in progress)

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