

Lepton flavor violating quarkonium decays

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PIKIO workshop

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Outline

- Introduction
 - Motivation
 - Effective Lagrangian
- Two-body quarkonium decays
 - Vector
 - Pseudoscalar
 - Scalar
- Three-body vector quarkonium decays
 - Resonant transitions
 - Nonresonant transitions
- Summary

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- ***Single operator dominance:*** The assumption that only one effective operator dictates the result.
- Model independence: Any NP scenario involving LFV can be matched to L_{eff} .

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- Calculate quarkonia to $\tau \mu, \tau e, \mu e$

Effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_D + \mathcal{L}_{\ell q} + \mathcal{L}_G + \dots ^*$$

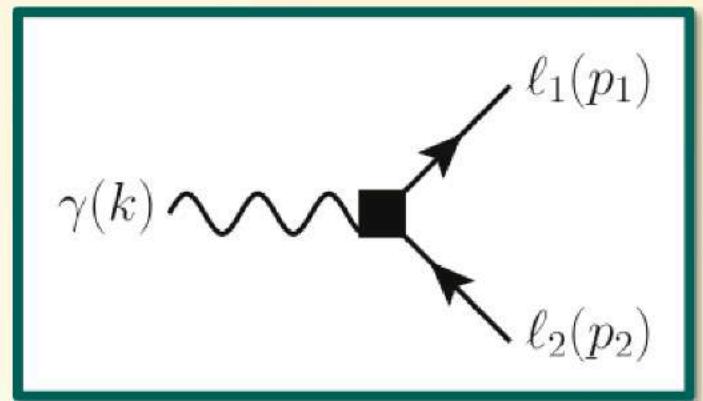
*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

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Dipole Lagrangian

$$\mathcal{L}_D = -\frac{m_2}{\Lambda^2} \left[(C_{DR}^{\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{DL}^{\ell_1 \ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2) F_{\mu\nu} + h.c. \right]^*$$



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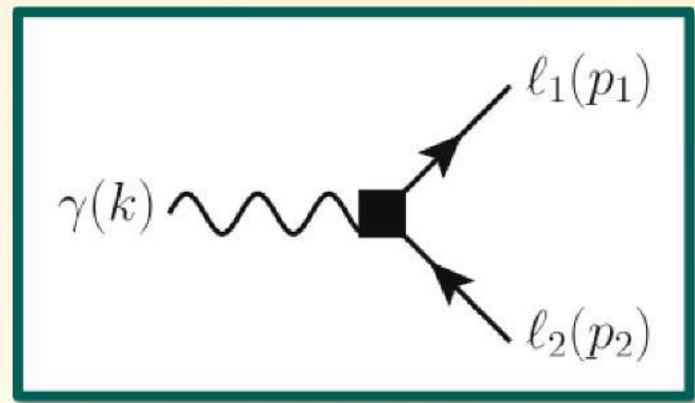
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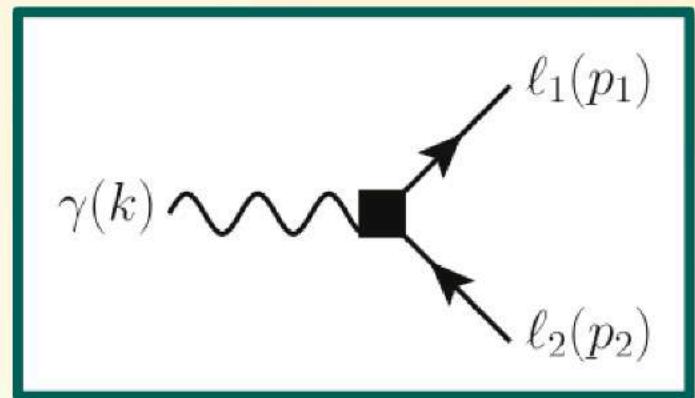
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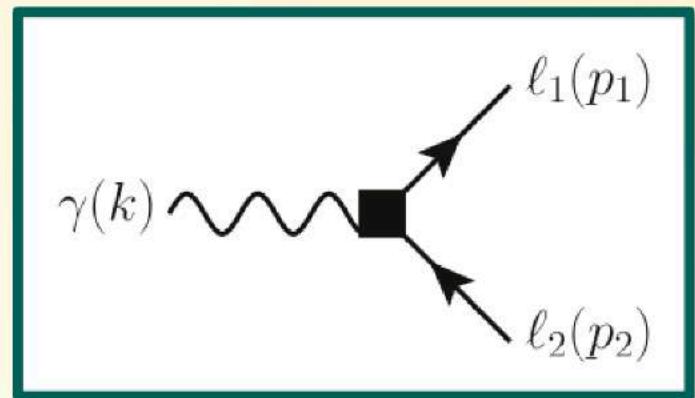
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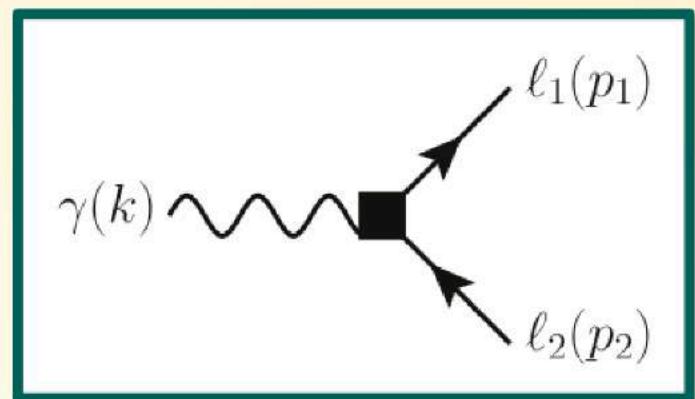
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$P_{R(L)} = (1 \pm \gamma^5)/2$



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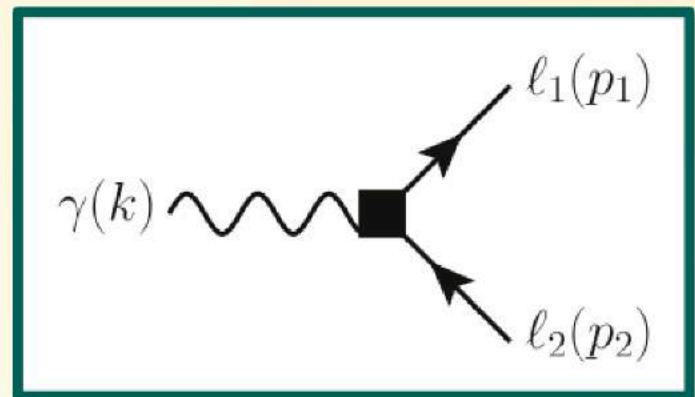
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Dipole operators are selected by the quantum numbers of the 2-body vector quarkonium decays.

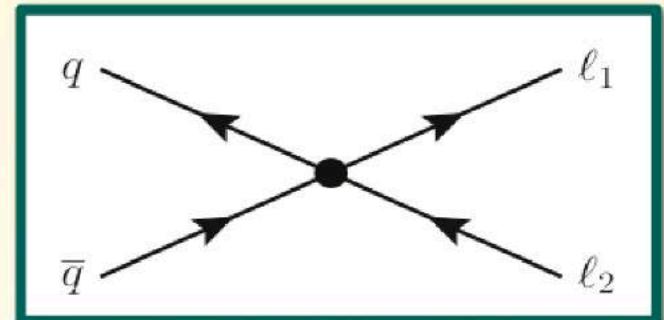


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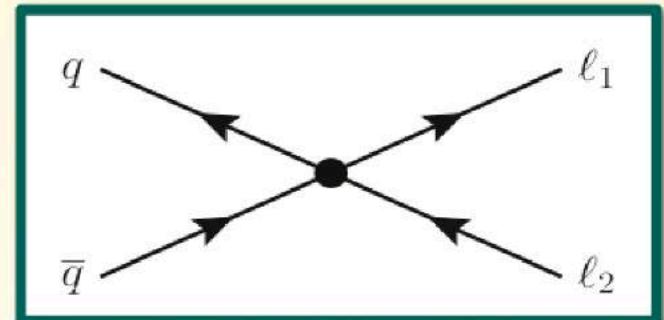


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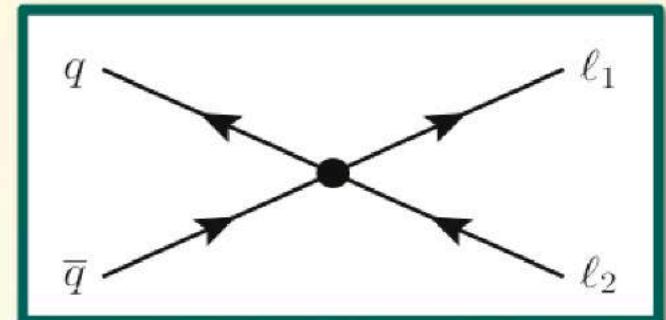


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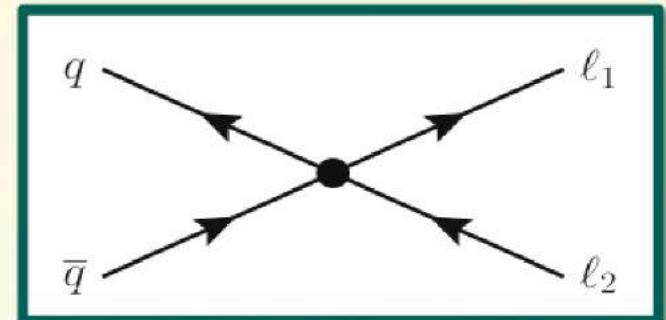


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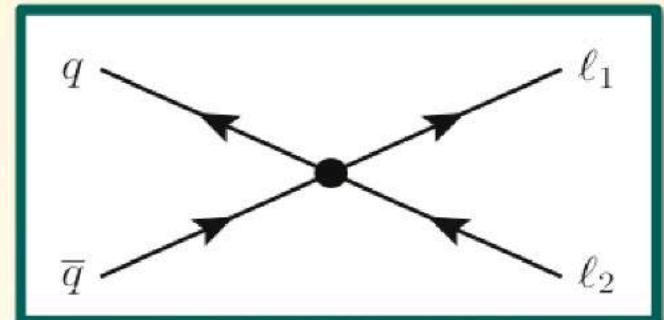


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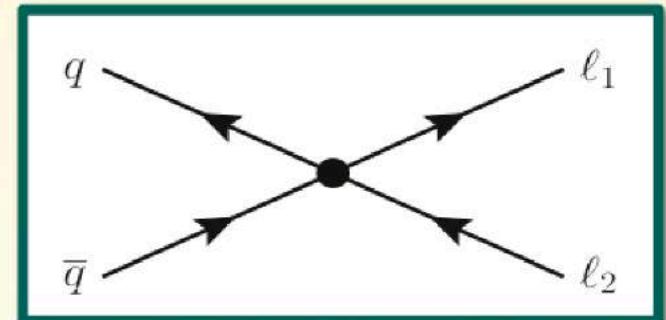
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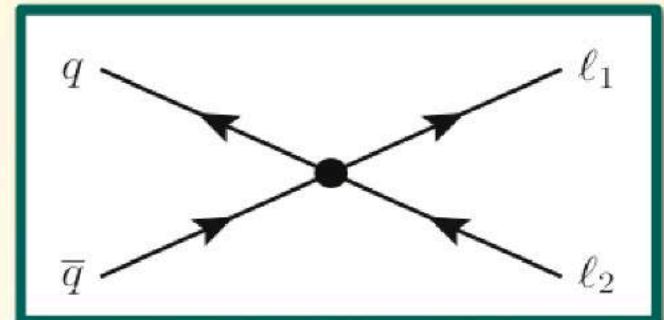
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$$\begin{aligned}\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q & \left[\left(C_{VR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu q \right. \\ & + \left(C_{AR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu \gamma_5 q \\ & + m_q G_F \left(C_{SR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q \\ & + m_q G_F \left(C_{PR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} \gamma_5 q \\ & \left. + m_q G_F \left(C_{TR}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q} \sigma_{\mu\nu} q + h.c. \right]^*\end{aligned}$$

m_q = MS bar quark mass



*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

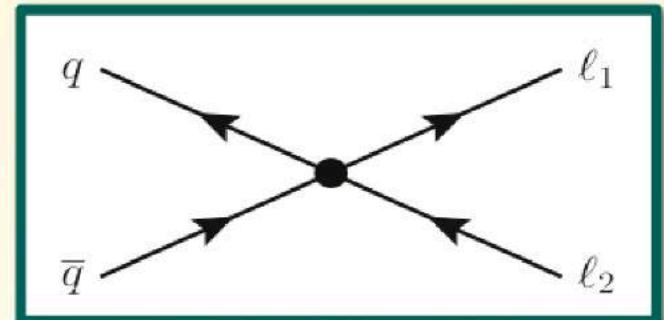
Effective Lagrangian

Four fermion Lagrangian

$$\begin{aligned}\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q & \left[\left(C_{VR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu q \right. \\ & + \left(C_{AR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu \gamma_5 q \\ & + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q \\ & + m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} \gamma_5 q \\ & \left. + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q} \sigma_{\mu\nu} q + h.c. \right]^*\end{aligned}$$

m_q = MS bar quark mass

G_F = Fermi constant



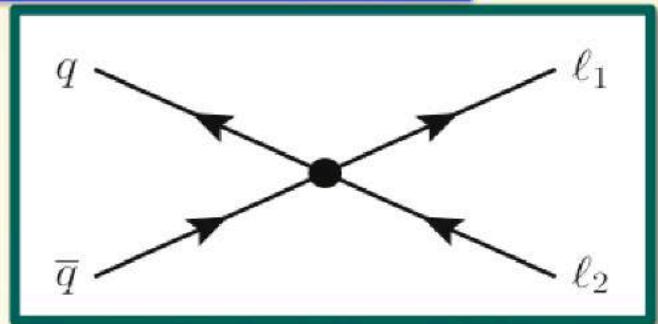
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Effective Lagrangian

Four fermion Lagrangian

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Vector and tensor operators are selected by the quantum numbers of the 2-body vector quarkonium decays.



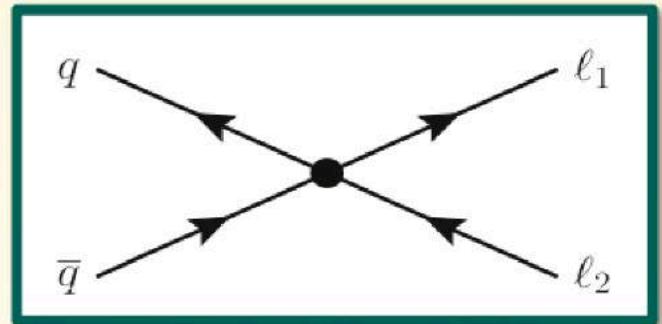
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Effective Lagrangian

Four fermion Lagrangian

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Axial and pseudoscalar operators are selected by the quantum numbers of the 2-body pseudoscalar quarkonium decays.



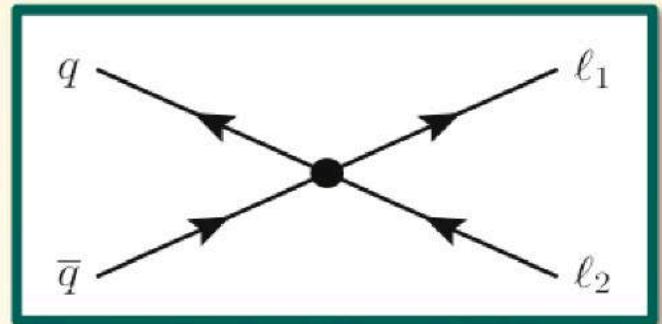
*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

Effective Lagrangian

Four fermion Lagrangian

$$\begin{aligned}\mathcal{L}_{\ell q} = -\frac{1}{\Lambda^2} \sum_q & \left[\left(C_{VR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{VL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu q \right. \\ & + \left(C_{AR}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_R \ell_2 + C_{AL}^{q\ell_1\ell_2} \bar{\ell}_1 \gamma^\mu P_L \ell_2 \right) \bar{q} \gamma_\mu \gamma_5 q \\ & + m_2 m_q G_F \left(C_{SR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{SL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} q \boxed{+ m_2 m_q G_F \left(C_{PR}^{q\ell_1\ell_2} \bar{\ell}_1 P_L \ell_2 + C_{PL}^{q\ell_1\ell_2} \bar{\ell}_1 P_R \ell_2 \right) \bar{q} \gamma_5 q} \\ & + m_2 m_q G_F \left(C_{TR}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_L \ell_2 + C_{TL}^{q\ell_1\ell_2} \bar{\ell}_1 \sigma^{\mu\nu} P_R \ell_2 \right) \bar{q} \sigma_{\mu\nu} q + h.c. \left. \right]^*\end{aligned}$$

Scalar operators are selected by the quantum numbers of the 2-body scalar quarkonium decays.

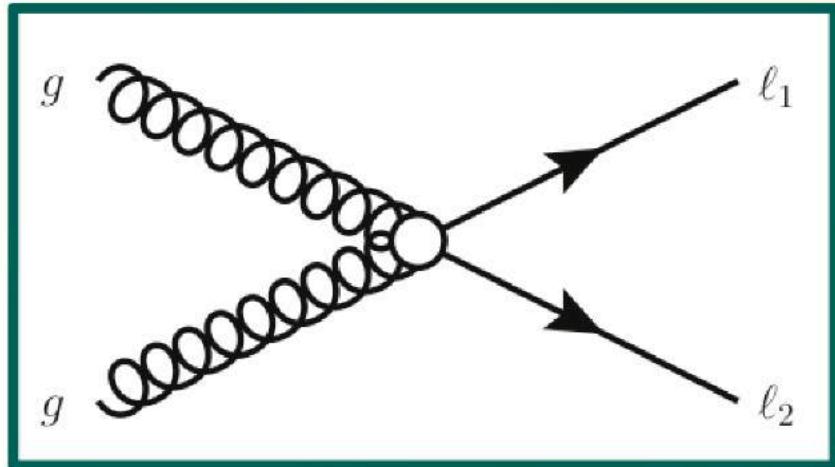


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Effective Lagrangian

Gluonic Lagrangian

$$\begin{aligned}\mathcal{L}_G = & -\frac{m_2 G_F}{\Lambda^2} \frac{\beta_L}{4\alpha_s} \left[\left(C_{GR}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{GL}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a G^{a\mu\nu} \right. \\ & \left. + \left(C_{\bar{G}R}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{\bar{G}L}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + h.c. \right]^*\end{aligned}$$

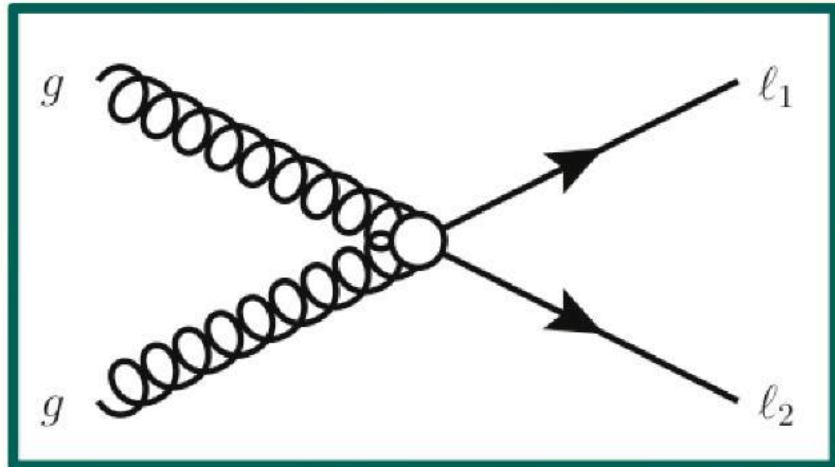


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Effective Lagrangian

Gluonic Lagrangian

$$\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2} \frac{\beta_L}{4\alpha_s} \left[\left(C_{GR}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{GL}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a G^{a\mu\nu} + \left(C_{\bar{G}R}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{\bar{G}L}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + h.c. \right]^*$$



1 loop beta function defined for the number of light active flavors, L, relevant to the scale of the process ($\mu=2\text{GeV}$).

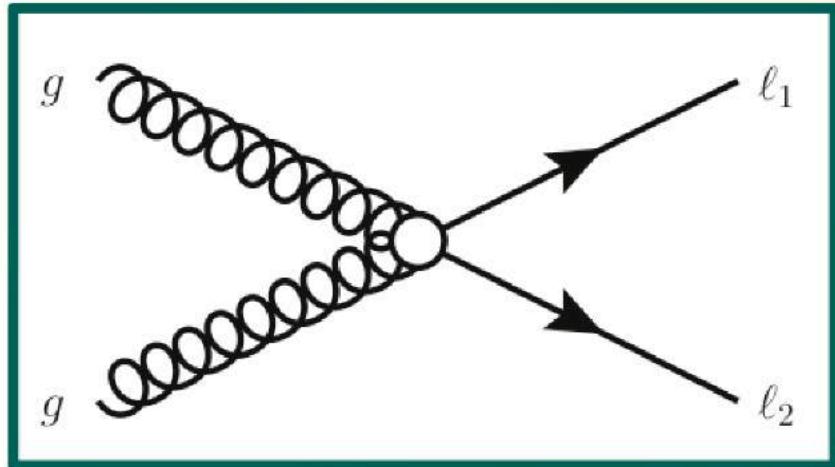
$$\beta_L = -9\alpha_s^2/(2\pi)$$

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Effective Lagrangian

Gluonic Lagrangian

$$\mathcal{L}_G = -\frac{m_2 G_F}{\Lambda^2} \frac{\beta_L}{4\alpha_s} \left[\left(C_{GR}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{GL}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a G^{a\mu\nu} \right. \\ \left. + \left(C_{\bar{G}R}^{\ell_1 \ell_2} \bar{\ell}_1 P_L \ell_2 + C_{\bar{G}L}^{\ell_1 \ell_2} \bar{\ell}_1 P_R \ell_2 \right) G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + h.c. \right]^*$$



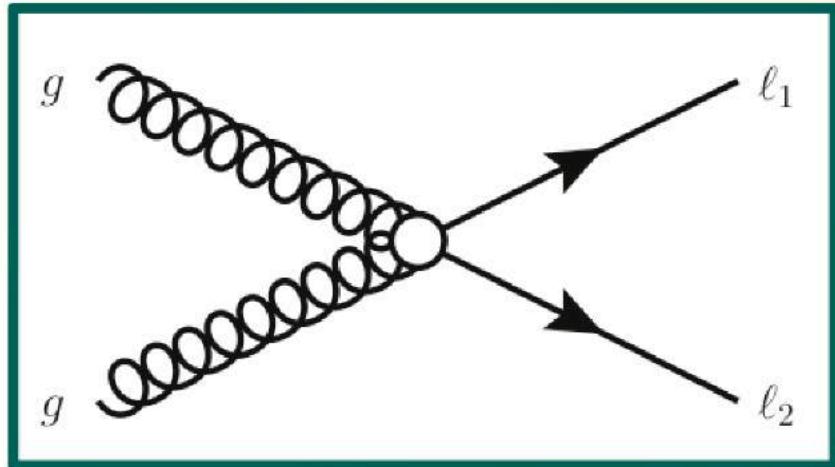
α_s = QCD coupling constant

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Effective Lagrangian

Gluonic Lagrangian

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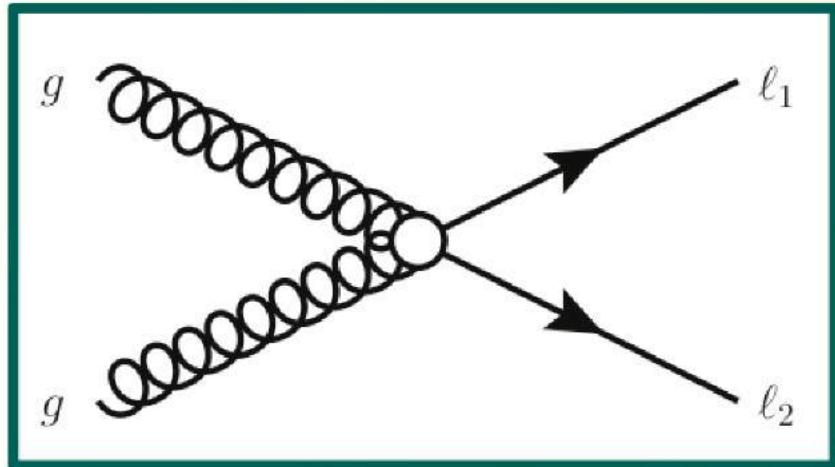
Gluonic Wilson coefficients

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Effective Lagrangian

Gluonic Lagrangian

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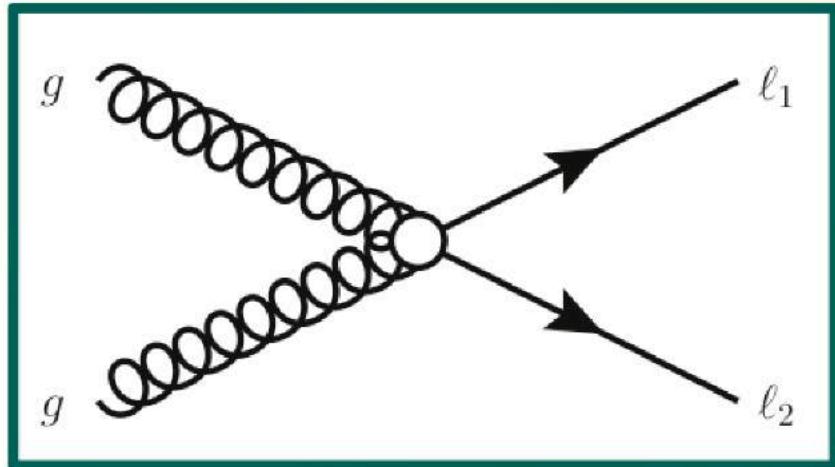
Gluonic operators selected by the 2-body scalar quarkonium decays.

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Effective Lagrangian

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Gluonic operators selected by the 2-body pseudoscalar quarkonium decays.

*A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89, no. 9, 095014 (2014)

2-body vector quarkonium decays

$$\langle 0 | \bar{q} \gamma^\mu q | V(p) \rangle = f_V m_V \epsilon^\mu(p), \quad \text{Decay constant}$$

$$\langle 0 | \bar{q} \sigma^{\mu\nu} q | V(p) \rangle = i f_V^T (\epsilon^\mu p^\nu - p^\mu \epsilon^\nu) \quad \text{Tensor decay constant}$$

2-body vector quarkonium decays

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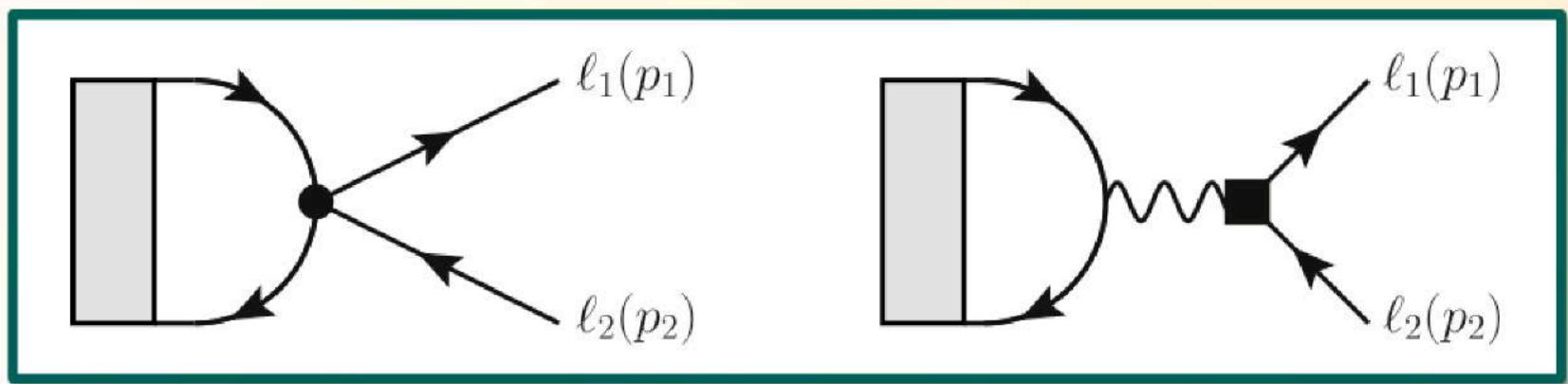
$$\langle 0 | \bar{q} \sigma^{\mu\nu} q | V(p) \rangle = i f_V^T (\epsilon^\mu p^\nu - p^\mu \epsilon^\nu) \quad \text{Tensor decay constant}$$

V = any quarkonium with quantum numbers 1^- i.e. Y, J/ ψ , Φ , ρ , ω

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V = any quarkonium with quantum numbers 1⁻ i.e. Υ, J/ψ, Φ, ρ, ω



Amplitude

$$\mathcal{A}(V \rightarrow \ell_1 \bar{\ell}_2) = \bar{u}(p_1, s_1) \left[A_V^{\ell_1 \ell_2} \gamma_\mu + B_V^{\ell_1 \ell_2} \gamma_\mu \gamma_5 + \frac{C_V^{\ell_1 \ell_2}}{m_V} (p_2 - p_1)_\mu + \frac{i D_V^{\ell_1 \ell_2}}{m_V} (p_2 - p_1)_\mu \gamma_5 \right] v(p_2, s_2) \epsilon^\mu(p)$$

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

m_V = vector meson mass

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$

$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

$$C_V^{\ell_1 \ell_2} = \frac{f_V m}{\Lambda^2} \left[y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right] \right]$$

$$D_V^{\ell_1 \ell_2} = i \frac{f_V m}{\Lambda^2} \left[y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right] \right]$$

m_V = vector meson mass

$y = m_2/m_V$

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha Q_q} y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha Q_q} y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha Q_q} (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha Q_q} (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

m_V = vector meson mass

Q_q = quark charge (2/3, -1/3)

$y = m_2/m_V$

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

m_V = vector meson mass

$y = m_2/m_V$

Q_q = quark charge (2/3, -1/3)

α = fine structure constant

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \boxed{\kappa_V} \left(C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2} \right) + 2y^2 \boxed{\kappa_V} \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \boxed{\kappa_V} \left(C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2} \right) - 2y^2 \boxed{\kappa_V} \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \boxed{2\kappa_V} \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \boxed{2\kappa_V} \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

m_V = vector meson mass

Q_q = quark charge (2/3, -1/3)

$y = m_2/m_V$

α = fine structure constant

$\kappa_V = 1/2$ (constant for pure $q\bar{q}$ states)

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

Dipole operator dependence

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V \left(C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2} \right) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V \left(C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2} \right) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

Dipole operator dependence

Vector operator dependence

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

Dipole operator dependence

Vector operator dependence

Tensor operator dependence

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi \alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V n_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V n_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V n_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V n_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

f_V cancels out ...

Two-body vector quarkonium decays

Branching ratio

$$\frac{\mathcal{B}(V \rightarrow \ell_1 \bar{\ell}_2)}{\mathcal{B}(V \rightarrow e^+ e^-)} = \left(\frac{m_V (1 - y^2)}{4\pi\alpha f_V Q_q} \right)^2 \left[\left(|A_V^{\ell_1 \ell_2}|^2 + |B_V^{\ell_1 \ell_2}|^2 \right) + \frac{1}{2} (1 - 2y^2) \left(|C_V^{\ell_1 \ell_2}|^2 + |D_V^{\ell_1 \ell_2}|^2 \right) + y \operatorname{Re} (A_V^{\ell_1 \ell_2} C_V^{\ell_1 \ell_2*} + i B_V^{\ell_1 \ell_2} D_V^{\ell_1 \ell_2*}) \right]$$

Dimensionless Constants

$$A_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + \kappa_V (C_{VL}^{q\ell_1 \ell_2} + C_{VR}^{q\ell_1 \ell_2}) + 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$B_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} \left[-\sqrt{4\pi\alpha} Q_q y^2 (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - \kappa_V (C_{VL}^{q\ell_1 \ell_2} - C_{VR}^{q\ell_1 \ell_2}) - 2y^2 \kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$C_V^{\ell_1 \ell_2} = \frac{f_V m_V}{\Lambda^2} y \left[\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} + C_{DR}^{\ell_1 \ell_2}) + 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} + C_{TR}^{q\ell_1 \ell_2}) \right]$$
$$D_V^{\ell_1 \ell_2} = i \frac{f_V m_V}{\Lambda^2} y \left[-\sqrt{4\pi\alpha} Q_q (C_{DL}^{\ell_1 \ell_2} - C_{DR}^{\ell_1 \ell_2}) - 2\kappa_V \frac{f_V^T}{f_V} G_F m_V m_q (C_{TL}^{q\ell_1 \ell_2} - C_{TR}^{q\ell_1 \ell_2}) \right]$$

f_V cancels out ...
except for the ratio of f_V^T/f_V

Two-body vector quarkonium decays

Decay constants used to constrain Wilson coefficients

State	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	J/ψ	$\psi(2S)$	ϕ	$\rho(\omega)$
$f_V, \text{ MeV}$	649 ± 31	481 ± 39	539 ± 84	418 ± 9	294 ± 5	241 ± 18	209.4 ± 1.5

Two-body vector quarkonium decays

Decay constants used to constrain Wilson coefficients

State	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	J/ψ	$\psi(2S)$	ϕ	$\rho(\omega)$
$f_V, \text{ MeV}$	649 ± 31	481 ± 39	539 ± 84	418 ± 9	294 ± 5	241 ± 18	209.4 ± 1.5

- Tensor decay constants are not well known

Two-body vector quarkonium decays

Decay constants used to constrain Wilson coefficients

State	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	J/ψ	$\psi(2S)$	ϕ	$\rho(\omega)$
$f_V, \text{ MeV}$	649 ± 31	481 ± 39	539 ± 84	418 ± 9	294 ± 5	241 ± 18	209.4 ± 1.5

- Tensor decay constants are not well known
- Assume $f_V^T = f_V$

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- Except $f_{J/\psi}^T = 410 \pm 10 \text{ MeV}^*$

*D. Becirevic, et al. Nucl. Phys. B. 883, 306 (2014).

Two-body vector quarkonium decays

Experimental upper limits of BR for 2-body vector decays and radiative lepton decays from the PDG.

$\ell_1 \ell_2$	$\mu\tau$	$e\tau$	$e\mu$
$\mathcal{B}(\Upsilon(1S) \rightarrow \ell_1 \ell_2)$	6.0×10^{-6}	—	—
$\mathcal{B}(\Upsilon(2S) \rightarrow \ell_1 \ell_2)$	3.3×10^{-6}	3.2×10^{-6}	—
$\mathcal{B}(\Upsilon(3S) \rightarrow \ell_1 \ell_2)$	3.1×10^{-6}	4.2×10^{-6}	—
$\mathcal{B}(J/\psi \rightarrow \ell_1 \ell_2)$	2.0×10^{-6}	8.3×10^{-6}	1.6×10^{-7}
$\mathcal{B}(\phi \rightarrow \ell_1 \ell_2)$	n/a	n/a	4.1×10^{-6}
$\mathcal{B}(\ell_2 \rightarrow \ell_1 \gamma)$	4.4×10^{-8}	3.3×10^{-8}	5.7×10^{-13}

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$$\mathcal{B}(\mu \text{ to } e\gamma)_{vSM} \sim 10^{-54}$$

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Dashes = no data

Two-body vector quarkonium decays

Constraints on Wilson coefficients of 4-fermion operators.

Wilson coefficient (GeV^{-2})	Leptons		Initial state (quark)				
	$\ell_1 \ell_2$		$\Upsilon(1S)$ (b)	$\Upsilon(2S)$ (b)	$\Upsilon(3S)$ (b)	J/ψ (c)	ϕ (s)
$ C_{VL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	n/a
	$e\tau$	—	—	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	n/a
	$e\mu$	—	—	—	—	1.0×10^{-5}	2×10^{-3}
$ C_{VR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	n/a
	$e\tau$	—	—	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	n/a
	$e\mu$	—	—	—	—	1.0×10^{-5}	2×10^{-3}
$ C_{TL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	n/a
	$e\tau$	—	—	3.3×10^{-2}	3.2×10^{-2}	2.4	n/a
	$e\mu$	—	—	—	—	4.8	1×10^4
$ C_{TR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	n/a
	$e\tau$	—	—	3.3×10^{-2}	3.2×10^{-2}	2.4	n/a
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$\mathcal{B}(\Upsilon(3S) \rightarrow \ell_1 \ell_2)$	3.1×10^{-6}	4.2×10^{-6}	—
$\mathcal{B}(J/\psi \rightarrow \ell_1 \ell_2)$	2.0×10^{-6}	8.3×10^{-6}	1.6×10^{-7}
$\mathcal{B}(\phi \rightarrow \ell_1 \ell_2)$	n/a	n/a	4.1×10^{-6}
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Dashes = no data

n/a = forbidden by phase space

Two-body vector quarkonium decays

Constraints on Wilson coefficients of 4-fermion operators.

Wilson coefficient (GeV^{-2})	Leptons		Initial state (quark)				
	$\ell_1 \ell_2$		$\Upsilon(1S)$ (b)	$\Upsilon(2S)$ (b)	$\Upsilon(3S)$ (b)	J/ψ (c)	ϕ (s)
$ C_{VL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	n/a
	$e\tau$	—		4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	n/a
	$e\mu$	—	—	—	—	1.0×10^{-5}	2×10^{-3}
$ C_{VR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	n/a
	$e\tau$	—		4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	n/a
	$e\mu$	—	—	—	—	1.0×10^{-5}	2×10^{-3}
$ C_{TL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$		4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	n/a
	$e\tau$	—		3.3×10^{-2}	3.2×10^{-2}	2.4	n/a
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	$\ell_1 \ell_2$	$\Upsilon(1S) (b)$	$\Upsilon(2S) (b)$	$\Upsilon(3S) (b)$	$J/\psi (c)$	$\phi (s)$	
$\mu\tau$		5.6×10^{-6}	4.1×10^{-6}	3.5×10^{-6}	5.5×10^{-5}	n/a	
$ C_{VL}^{q\ell_1\ell_2}/\Lambda^2 $	$e\tau$	—	4.1×10^{-6}	4.1×10^{-6}	1.1×10^{-4}	n/a	
	$e\mu$	—	—	—	1.0×10^{-5}	2×10^{-3}	

- Not a breakdown of the EFT!
- Existing data doesn't allow for strong constraints

$ C_{TL}^{q\ell_1\ell_2}/\Lambda^2 $	$e\mu$	—	—	—	1.0×10^{-3}	2×10^{-3}
	$\mu\tau$	4.4×10^{-2}	3.2×10^{-2}	2.8×10^{-2}	1.2	n/a
$ C_{TR}^{q\ell_1\ell_2}/\Lambda^2 $	$e\tau$	—	3.3×10^{-2}	3.2×10^{-2}	2.4	n/a
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	$e\mu$	—	—	—	4.8	1×10^4

Two-body vector quarkonium decays

Constraints on Wilson coefficients of dipole operators.

Dipole Wilson coefficient (GeV^{-2})	Leptons	Initial state					
$ C_{DL}^{\ell_1\ell_2}/\Lambda^2 $	$\ell_1\ell_2$	$\Upsilon(1S)$ (b)	$\Upsilon(2S)$ (b)	$\Upsilon(3S)$ (b)	J/ψ (c)	$\phi(s)$	$\ell_2 \rightarrow \ell_1\gamma$
	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	n/a	2.6×10^{-10}
	$e\tau$	—	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	n/a	2.7×10^{-10}
$ C_{DR}^{q\ell_1\ell_2}/\Lambda^2 $	$e\mu$	—	—	—	1.1×10^{-3}	0.2	3.1×10^{-7}
	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	n/a	2.6×10^{-10}
	$e\tau$	—	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	n/a	2.7×10^{-10}
	$e\mu$	—	—	—	1.1×10^{-3}	0.2	3.1×10^{-7}

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	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	n/a	2.6×10^{-10}	
	$e\tau$	—	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	n/a	2.7×10^{-10}	
$ C_{DR}^{q\ell_1\ell_2}/\Lambda^2 $	$e\mu$	—	—	—	—	1.1×10^{-3}	0.2	
	$\mu\tau$	2.0×10^{-4}	1.6×10^{-4}	1.4×10^{-4}	2.5×10^{-4}	n/a	2.6×10^{-10}	
	$e\tau$	—	1.6×10^{-4}	1.6×10^{-4}	5.3×10^{-4}	n/a	2.7×10^{-10}	
		$e\mu$	—	—	—	1.1×10^{-3}	0.2	3.1×10^{-7}

- Radiative lepton decays give much stronger constraints
- Vector decay constraints are complimentary

2-body pseudoscalar quarkonium decays

$$\langle 0 | \bar{q} \gamma^\mu \gamma_5 q | P(p) \rangle = -i f_P p^\mu \quad \text{Decay constant}$$

$$\langle 0 | \frac{\alpha_s}{4\pi} G^{a\mu\nu} \tilde{G}_{\mu\nu}^a | P(p) \rangle = a_P \quad \text{Anomalous matrix element}$$

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P = any quarkonium with quantum numbers 0^- i.e. η_b , η_c , η , η' , π^0

2-body pseudoscalar quarkonium decays

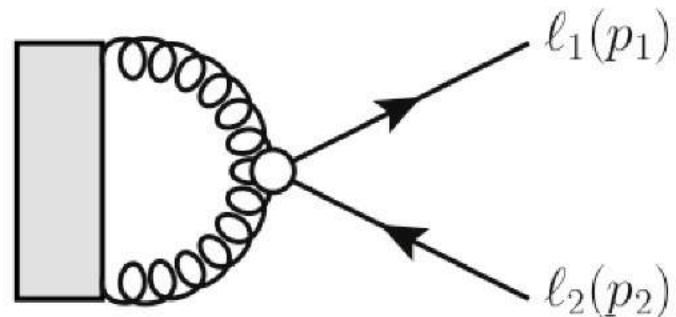
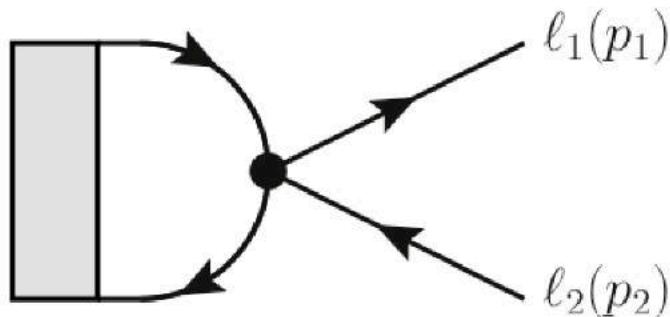
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Anomalous matrix element

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Amplitude

$$\mathcal{A}(P \rightarrow \ell_1 \bar{\ell}_2) = \bar{u}(p_1, s_1) [E_P^{\ell_1 \ell_2} + i F_P^{\ell_1 \ell_2} \gamma_5] v(p_2, s_2)$$

2-body pseudoscalar quarkonium decays

Branching ratio

$$\mathcal{B}(P \rightarrow \ell_1 \bar{\ell}_2) = \frac{m_P}{8\pi\Gamma_P} (1 - y^2)^2 \left[|E_P^{\ell_1 \ell_2}|^2 + |F_P^{\ell_1 \ell_2}|^2 \right]$$

Dimensionless Constants

$$E_P^{\ell_1 \ell_2} = y \frac{m_P}{4\Lambda^2} \left[-i f_P \left[2 \left(C_{AL}^{q\ell_1 \ell_2} + C_{AR}^{q\ell_1 \ell_2} \right) - m_P^2 G_F \left(C_{PL}^{q\ell_1 \ell_2} + C_{PR}^{q\ell_1 \ell_2} \right) \right] + 9 G_F a_P \left(C_{\tilde{G}L}^{\ell_1 \ell_2} + C_{\tilde{G}R}^{\ell_1 \ell_2} \right) \right]$$

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m_P = pseudoscalar meson mass

2-body pseudoscalar quarkonium decays

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m_P = pseudoscalar meson mass

$y = m_2/m_P$

2-body pseudoscalar quarkonium decays

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m_P = pseudoscalar meson mass

$y = m_2/m_P$

Γ_s = total decay rate

2-body pseudoscalar quarkonium decays

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$$F_P^{\ell_1 \ell_2} = -y \frac{m_P}{4\Lambda^2} \left[f_P \left[2 \left(C_{AL}^{q\ell_1 \ell_2} - C_{AR}^{q\ell_1 \ell_2} \right) - m_P^2 G_F \left(C_{PL}^{q\ell_1 \ell_2} - C_{PR}^{q\ell_1 \ell_2} \right) \right] + 9 i G_F a_P \left(C_{\tilde{G}L}^{\ell_1 \ell_2} - C_{\tilde{G}R}^{\ell_1 \ell_2} \right) \right]$$

m_P = pseudoscalar meson mass Axial operator dependence

$y = m_2/m_P$

Γ_S = total decay rate

2-body pseudoscalar quarkonium decays

Branching ratio

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m_P = pseudoscalar meson mass

Axial operator dependence

$y = m_2/m_P$

Pseudoscalar operator dependence

Γ_S = total decay rate

2-body pseudoscalar quarkonium decays

Branching ratio

$$\mathcal{B}(P \rightarrow \ell_1 \bar{\ell}_2) = \frac{m_P}{8\pi\Gamma_P} (1 - y^2)^2 \left[|E_P^{\ell_1 \ell_2}|^2 + |F_P^{\ell_1 \ell_2}|^2 \right]$$

Dimensionless Constants

$$E_P^{\ell_1 \ell_2} = y \frac{m_P}{4\Lambda^2} \left[-i f_P \left[2 \left(C_{AL}^{q\ell_1 \ell_2} + C_{AR}^{q\ell_1 \ell_2} \right) - m_P^2 G_F \left(C_{PL}^{q\ell_1 \ell_2} + C_{PR}^{q\ell_1 \ell_2} \right) \right] + 9 G_F a_P \left(C_{\tilde{G}L}^{\ell_1 \ell_2} + C_{\tilde{G}R}^{\ell_1 \ell_2} \right) \right]$$
$$F_P^{\ell_1 \ell_2} = -y \frac{m_P}{4\Lambda^2} \left[f_P \left[2 \left(C_{AL}^{q\ell_1 \ell_2} - C_{AR}^{q\ell_1 \ell_2} \right) - m_P^2 G_F \left(C_{PL}^{q\ell_1 \ell_2} - C_{PR}^{q\ell_1 \ell_2} \right) \right] + 9 i G_F a_P \left(C_{\tilde{G}L}^{\ell_1 \ell_2} - C_{\tilde{G}R}^{\ell_1 \ell_2} \right) \right]$$

m_P = pseudoscalar meson mass

Axial operator dependence

$y = m_2/m_P$

Pseudoscalar operator dependence

Γ_s = total decay rate

Gluonic operator dependence

2-body pseudoscalar quarkonium decays

Decay constants used to constrain Wilson coefficients

State	η_b	η_c	$\eta, u(d)$	η, s	$\eta', u(d)$	η', s	π
$f_P^q, \text{ MeV}$	667 ± 6	387 ± 7	108 ± 3	-111 ± 6	89 ± 3	136 ± 6	130.41 ± 0.20

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- $a_\eta = -0.022 \pm 0.002 \text{ GeV}^3$
- $a_{\eta'} = -0.057 \pm 0.002 \text{ GeV}^3$

2-body pseudoscalar quarkonium decays

Experimental upper limits of BR for 2-body pseudoscalar decays from the PDG.

$\ell_1 \ell_2$	$e\mu$
$\mathcal{B}(\eta \rightarrow \ell_1 \ell_2)$	6×10^{-6}
$\mathcal{B}(\eta' \rightarrow \ell_1 \ell_2)$	4.7×10^{-4}
$\mathcal{B}(\pi^0 \rightarrow \ell_1 \ell_2)$	3.6×10^{-10}

2-body pseudoscalar quarkonium decays

Constraints on Wilson coefficients of 4-fermion operators.

	Leptons			Initial state			
Wilson coefficient	$\ell_1 \ell_2$	η_b	η_c	$\eta(u/d)$	$\eta(s)$	$\eta'(u/d)$	$\eta'(s)$
$ C_{AL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	—	—	n/a	n/a	n/a	n/a
	$e\tau$	—	—	n/a	n/a	n/a	n/a
	$e\mu$	—	—	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$ C_{AR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	—	—	n/a	n/a	n/a	n/a
	$e\tau$	—	—	n/a	n/a	n/a	n/a
	$e\mu$	—	—	3×10^{-3}	2×10^{-3}	2.1×10^{-1}	1.9×10^{-1}
$ C_{PL}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	—	—	n/a	n/a	n/a	n/a
	$e\tau$	—	—	n/a	n/a	n/a	n/a
	$e\mu$	—	—	2×10^3	1×10^3	3.9×10^4	3.6×10^4
$ C_{PR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	—	—	n/a	n/a	n/a	n/a
	$e\tau$	—	—	n/a	n/a	n/a	n/a
	$e\mu$	—	—	2×10^3	1×10^3	3.9×10^4	3.6×10^4

2-body pseudoscalar quarkonium decays

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$ C_{PR}^{q\ell_1\ell_2}/\Lambda^2 $	$\mu\tau$	—	—	n/a	n/a	n/a	n/a
	$e\tau$	—	—	n/a	n/a	n/a	n/a
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2-body pseudoscalar quarkonium decays

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- Not a breakdown of the EFT!
- Existing data doesn't allow for strong constraints

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2-body pseudoscalar quarkonium decays

Constraints on Wilson coefficients of gluonic operators.

Gluonic Wilson coefficient (GeV^{-2})	Leptons	$\ell_1 \ell_2$	η_b	η_c	Initial state	η	η'
$ C_{GL}^{\ell_1 \ell_2} / \Lambda^2 $	$e\mu$		—	—		2×10^2	5.0×10^3
$ C_{GR}^{\ell_1 \ell_2} / \Lambda^2 $	$e\mu$		—	—		2×10^2	5.0×10^3

2-body pseudoscalar quarkonium decays

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2-body scalar quarkonium decays

$$\langle 0 | \bar{q}q | S(p) \rangle = -im_S f_S$$

Decay constant

$$\langle 0 | \frac{\alpha_s}{4\pi} G^{a\mu\nu} G^a_{\mu\nu} | S(p) \rangle = a_S$$

Anomalous matrix element

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S = any quarkonium with quantum numbers 0^{++} i.e. X_{b0} , X_{c0}

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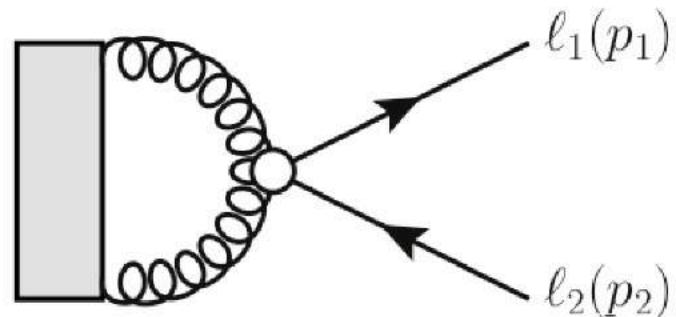
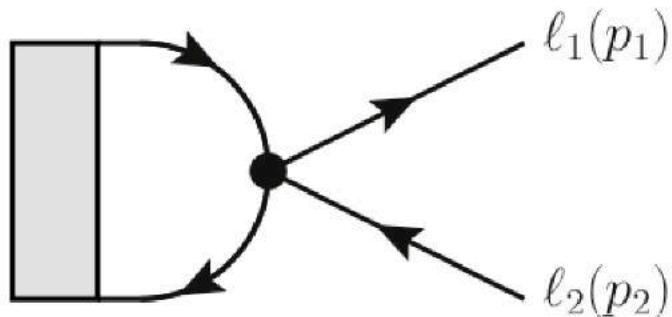
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Amplitude

$$\mathcal{A}(S \rightarrow \ell_1 \bar{\ell}_2) = \bar{u}(p_1, s_1) [E_S^{\ell_1 \ell_2} + i F_S^{\ell_1 \ell_2} \gamma_5] v(p_2, s_2)$$

2-body scalar quarkonium decays

Branching ratio

$$\mathcal{B}(S \rightarrow \ell_1 \bar{\ell}_2) = \frac{m_S}{8\pi\Gamma_S} (1 - y^2)^2 \left[|E_S^{\ell_1 \ell_2}|^2 + |F_S^{\ell_1 \ell_2}|^2 \right]$$

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2-body scalar quarkonium decays

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Scalar operator dependence

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Gluonic operator dependence

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2-body scalar quarkonium decays

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- States χ_{b0} and χ_{c0} produced via
 - gluon-gluon fusion at LHC
 - B decays at flavor factories
 - Radiative decays of $\Upsilon(2S)$, $\Upsilon(3S)$, $\psi(2S)$,
 $\psi(3770)$

3-body vector quarkonium decays

Resonant transitions

- $V \rightarrow \gamma$ ($S \rightarrow l_1 l_2$) may be used to study scalar 2-body decays

3-body vector quarkonium decays

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- If soft γ can be tagged at B factories then:

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3-body vector quarkonium decays

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- Quite useful:

$$\mathcal{B}(\psi(2S) \rightarrow \gamma \chi_{c0}(1P)) = 9.99 \pm 0.27\% ,$$

$$\mathcal{B}(\psi(3770) \rightarrow \gamma \chi_{c0}(1P)) = 0.73 \pm 0.09\%$$

3-body vector quarkonium decays

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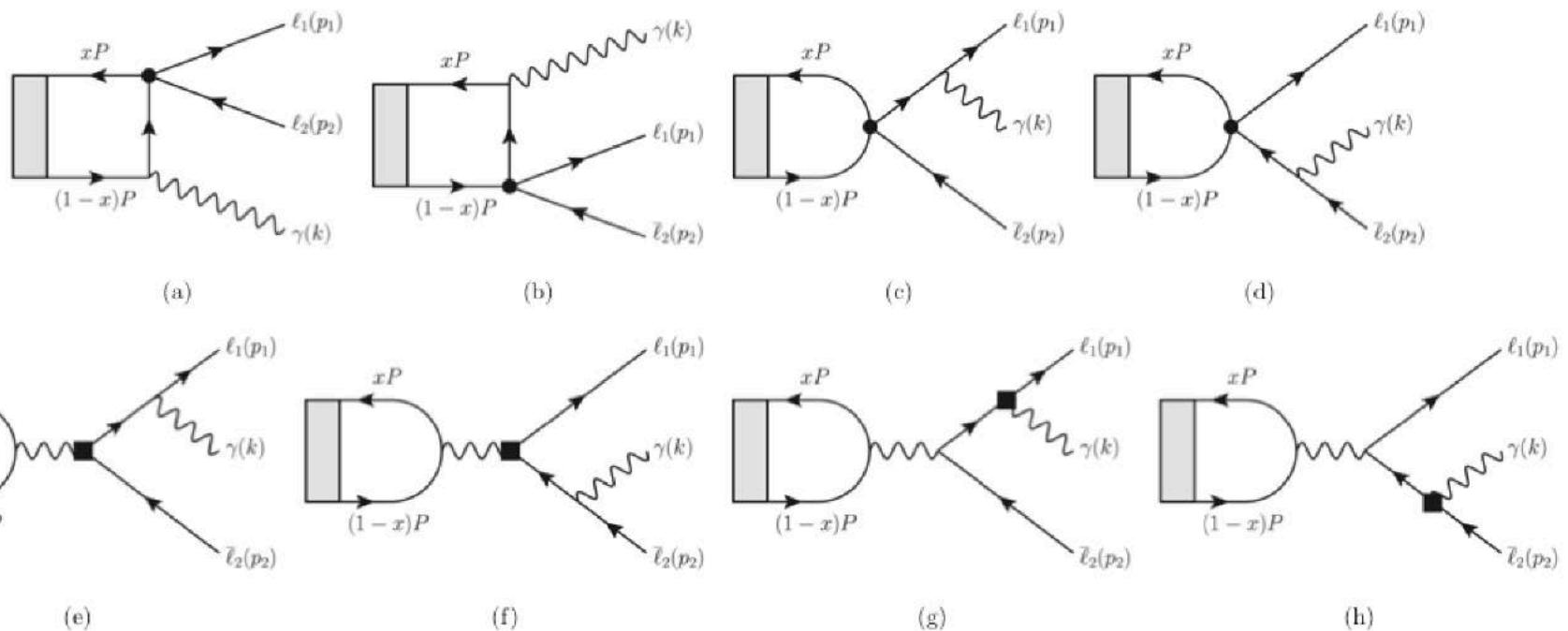
$$\mathcal{B}(\Upsilon(2S) \rightarrow \gamma \chi_{b0}(1P)) = 3.8 \pm 0.4\% ,$$

$$\mathcal{B}(\Upsilon(3S) \rightarrow \gamma \chi_{b0}(1P)) = 0.27 \pm 0.04\% ,$$

$$\mathcal{B}(\Upsilon(3S) \rightarrow \gamma \chi_{b0}(2P)) = 5.9 \pm 0.6\% .$$

3-body vector quarkonium decays

Nonresonant transitions



Diagrams for $V \rightarrow \gamma l_1 l_2$

3-body vector quarkonium decays

Nonresonant transitions

$$\Psi_V = \frac{I_c}{\sqrt{6}} \Phi_V(x) (m_V \gamma^\alpha + i p^\beta \sigma^{\alpha\beta}) \epsilon^\alpha(p) \quad \text{wave function}$$

3-body vector quarkonium decays

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wave function

$$\Phi_V(x) = \frac{f_V}{2\sqrt{6}} \delta(x - 1/2)$$

distribution amplitude

3-body vector quarkonium decays

Nonresonant transitions

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distribution amplitude

$$\langle 0 | \bar{q} \Gamma^\mu q | V \rangle = \int_0^1 \text{Tr}[\Gamma^\mu \Psi_V] dx$$

nonlocal matrix element

3-body vector quarkonium decays

Nonresonant transitions

Differential decay rates

$$\frac{d\Gamma_{V \rightarrow \gamma \ell_1 \bar{\ell}_2}^A}{dm_{12}^2} = \frac{1}{9} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2}{\Lambda^4} (C_{AL}^2 + C_{AR}^2) \frac{(m_V^2 - m_{12}^2)(2m_V^2 y^2 + m_{12}^2)(m_V^2 y^2 - m_{12}^2)^2}{m_V m_{12}^6}$$

$$\frac{d\Gamma_{V \rightarrow \gamma \ell_1 \bar{\ell}_2}^S}{dm_{12}^2} = \frac{1}{24} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V}{\Lambda^4} (C_{SL}^2 + C_{SR}^2) \frac{y^2 (m_V^2 - m_{12}^2)(m_V^2 y^2 - m_{12}^2)^2}{m_{12}^2}$$

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3-body vector quarkonium decays

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$$m_{12}^2 = (P-k)^2$$

3-body vector quarkonium decays

Nonresonant transitions

Total decay rates

$$\Gamma_A(V \rightarrow \gamma \ell_1 \bar{\ell}_2) = \frac{1}{18} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 m_V^3}{\Lambda^4} (C_{AL}^2 + C_{AR}^2) f(y^2)$$

$$\Gamma_S(V \rightarrow \gamma \ell_1 \bar{\ell}_2) = \frac{1}{144} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V^7}{\Lambda^4} (C_{SL}^2 + C_{SR}^2) y^2 f(y^2)$$

$$\Gamma_P(V \rightarrow \gamma \ell_1 \bar{\ell}_2) = \frac{1}{144} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V^7}{\Lambda^4} (C_{PL}^2 + C_{PR}^2) y^2 f(y^2)$$

$$f(y^2) = 1 - 6y^2 - 12y^4 \log(y) + 3y^4 + 2y^6$$

3-body vector quarkonium decays

Nonresonant transitions

Total decay rates

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$$\Gamma_S(V \rightarrow \gamma \ell_1 \bar{\ell}_2) = \frac{1}{144} \frac{\alpha Q_q^2}{(4\pi)^2} \frac{f_V^2 G_F^2 m_V^7}{\Lambda^4} (C_{SL}^2 + C_{SR}^2) y^2 f(y^2)$$

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3-body vector quarkonium decays

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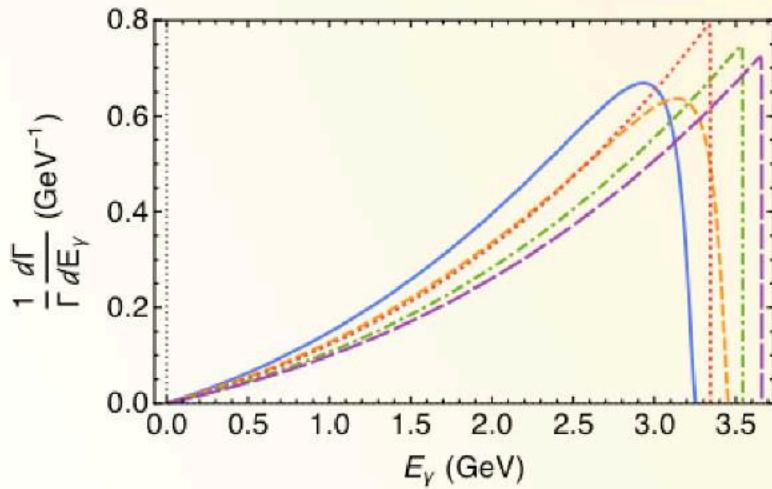
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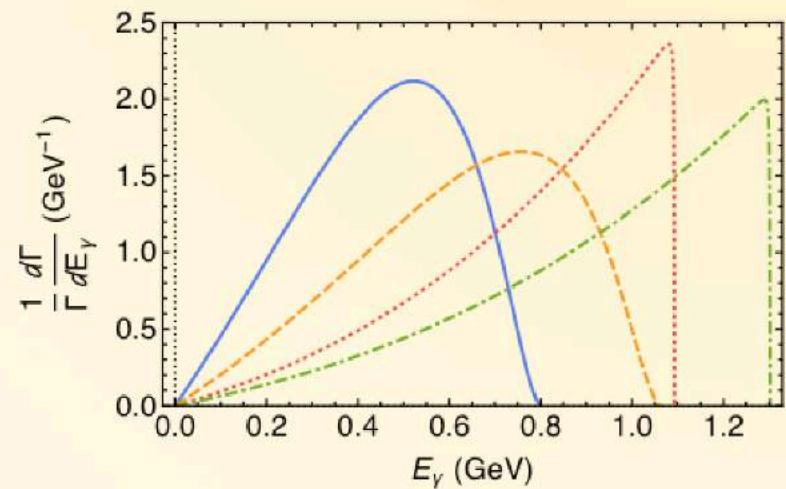
3-body vector quarkonium decays

Nonresonant transitions

Axial operator differential decay rates



(a)



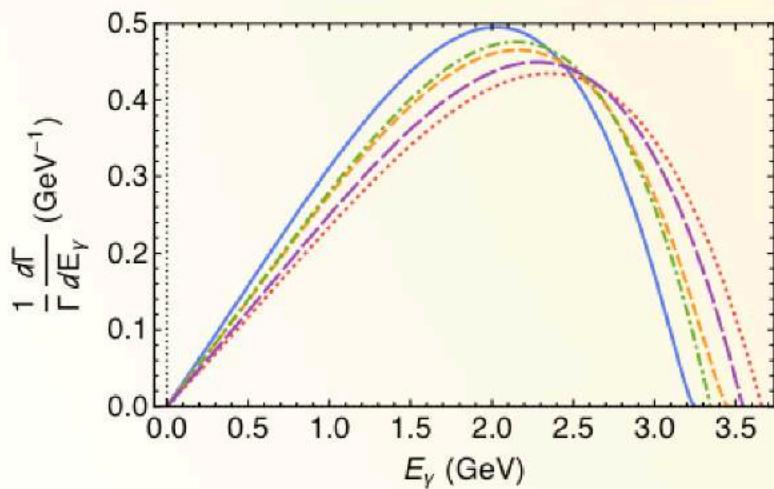
(b)

- (a) $Y(1S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (solid blue), $Y(2S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (short-dashed gold), $Y(3S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (dotted red), $Y(1S) \rightarrow \gamma e \mu$ (dot-dashed green), $Y(2S) \rightarrow \gamma e \mu$ and $Y(3S) \rightarrow \gamma e \mu$ (long-dashed purple); (b) $J/\psi \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (solid blue), $\psi(2S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (short-dashed gold), $J/\psi \rightarrow \gamma e \mu$ (dotted red), $\psi(2S) \rightarrow \gamma e \mu$ (dot-dashed green)

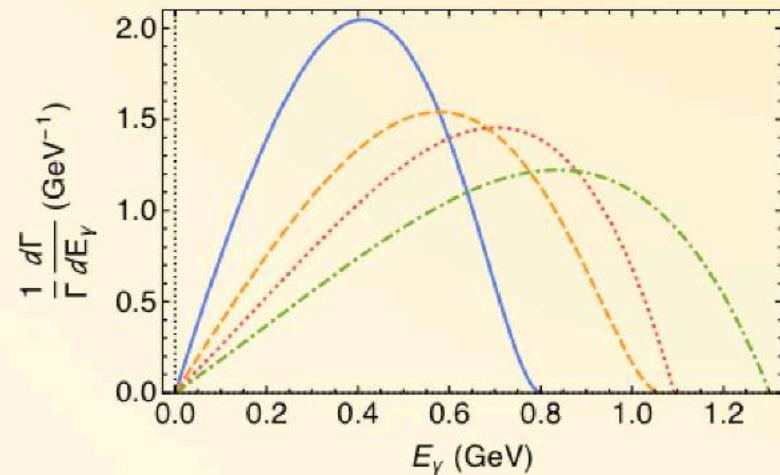
3-body vector quarkonium decays

Nonresonant transitions

Scalar/Pseudoscalar operator differential decay rates



(a)



(b)

- (a) $Y(1S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (solid blue), $Y(2S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (short-dashed gold), $Y(3S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (dotted red), $Y(1S) \rightarrow \gamma e \mu$ (dot-dashed green), $Y(2S) \rightarrow \gamma e \mu$ and $Y(3S) \rightarrow \gamma e \mu$ (long-dashed purple); (b) $J/\psi \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (solid blue), $\psi(2S) \rightarrow \gamma \mu \tau$ or $\gamma e \tau$ (short-dashed gold), $J/\psi \rightarrow \gamma e \mu$ (dotted red), $\psi(2S) \rightarrow \gamma e \mu$ (dot-dashed green)

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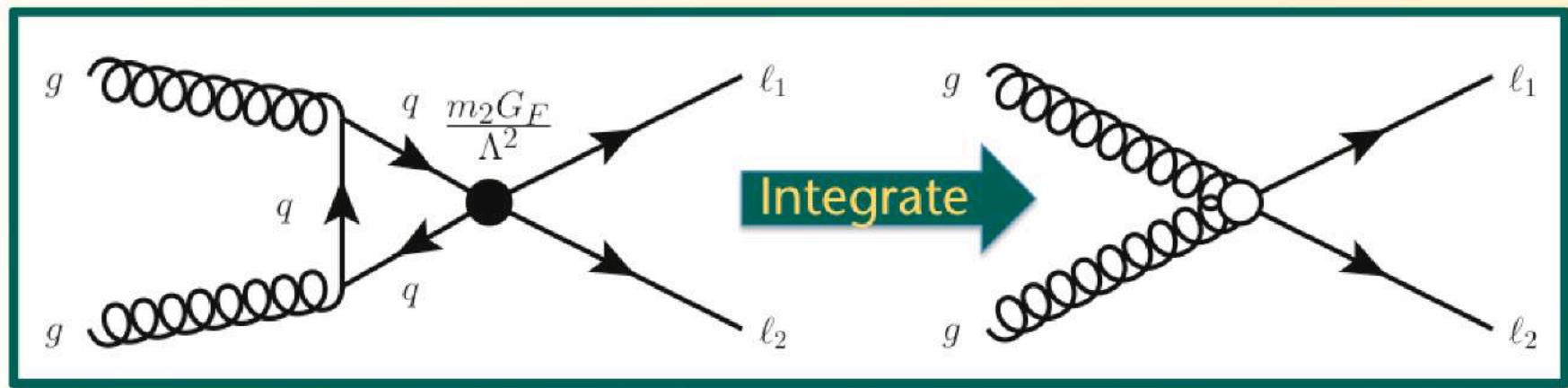
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- Any NP model with flavor violation at high scales can be cast in terms of the L_{eff} at low energies.
- Two-body decays allow for operator selection and reduces single operator dominance reliance.
- RLFV decays can provide complimentary operator access.

Why is $L_G \sim 1/\Lambda^2$ and not $1/\Lambda^3$?

- Naively it should be $1/\Lambda^3$

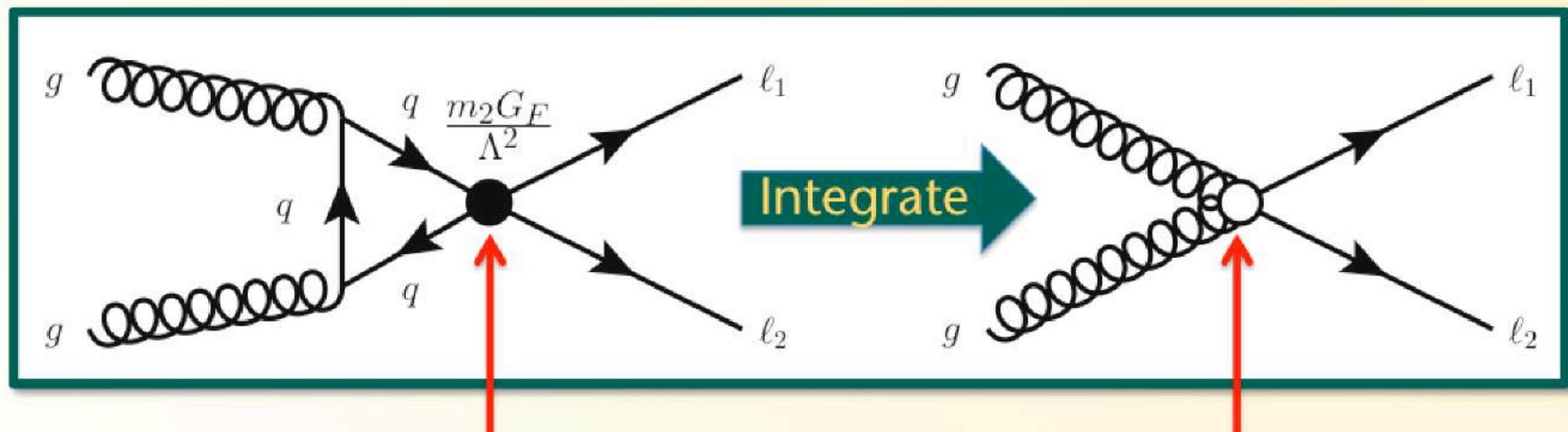
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4-fermion operators $\sim m_2 G_F / \Lambda^2$

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