3rd HiggsTools Annual Meeting (Torino)

16 May 2017

Chapter 3:
Higgs + Jet at 1-loop and the Heavy Top-Quark Limit

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Contribution to HiggsTools Book

MAX-PLANCK-GESELLSCHAFT

higgstools

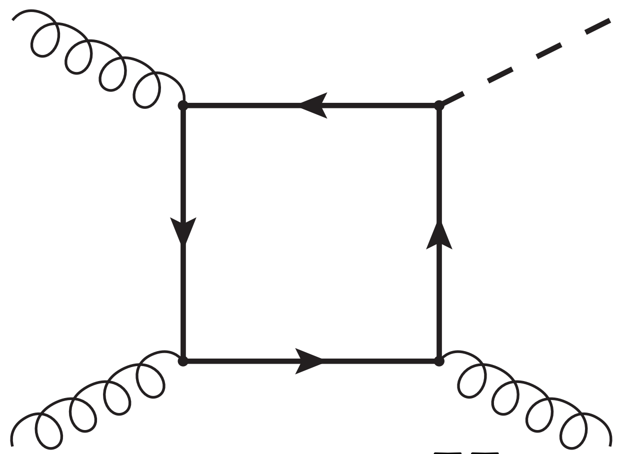
Introduction

- Overview of $pp \to Hj$ at 1-loop in the SM
- 1-loop calculation of gluon channel using projectors & IBPs

  (a.k.a how to crack a nut with a sledgehammer)
- Heavy top-quark limit at 1-loop
  1. Expanding the 1-loop result
  2. Using the Higgs Effective Field Theory (HEFT)

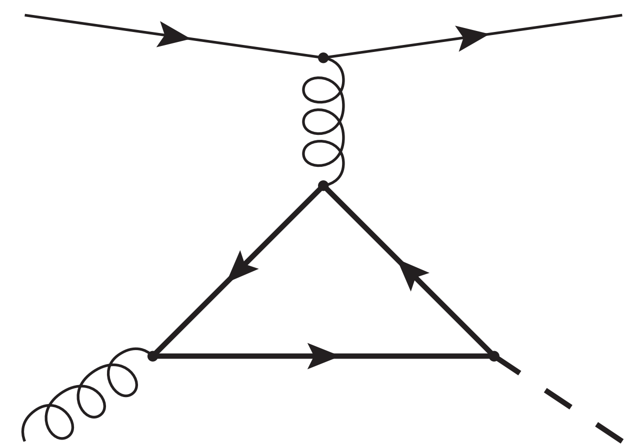
SM Partonic Channels (1-loop)

Gluon



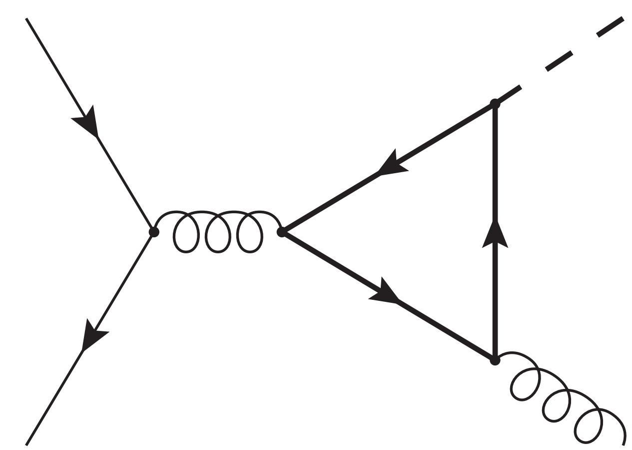
$$gg \to gH$$

Quark



$$qg \to qH, \quad gq \to qH$$

$$\bar{q}g \to \bar{q}H, \quad g\bar{q} \to qH$$



$$q\bar{q} \to gH, \quad \bar{q}q \to gH$$

- Higgs coupling vanishes for massless quark
- Only 2 independent matrix elements
- At LO only one quark flavour runs in the loop of a given diagram

SM Partonic Channels (1-loop)

SM LO (1-loop) Cross Section:

Baur, Glover 90

Channel	Cross Section (fb)	Fraction
gg	2890.0	71.77%
gq	435.26	10.81%
qg	435.36	10.81%
$g\bar{q}$	122.04	3.03%
$\bar{q}g$	121.74	3.02%
$q\bar{q}$	11.127	0.28%
$\bar{q}q$	11.119	0.28%
Total	4026.60	

NNPDF21_lo_as_0130_100

$\sqrt{s} = 8$ TeV

$m_H = 125$ GeV

$m_T = 172$ GeV

$\mu_R = \mu_F = 125$ GeV

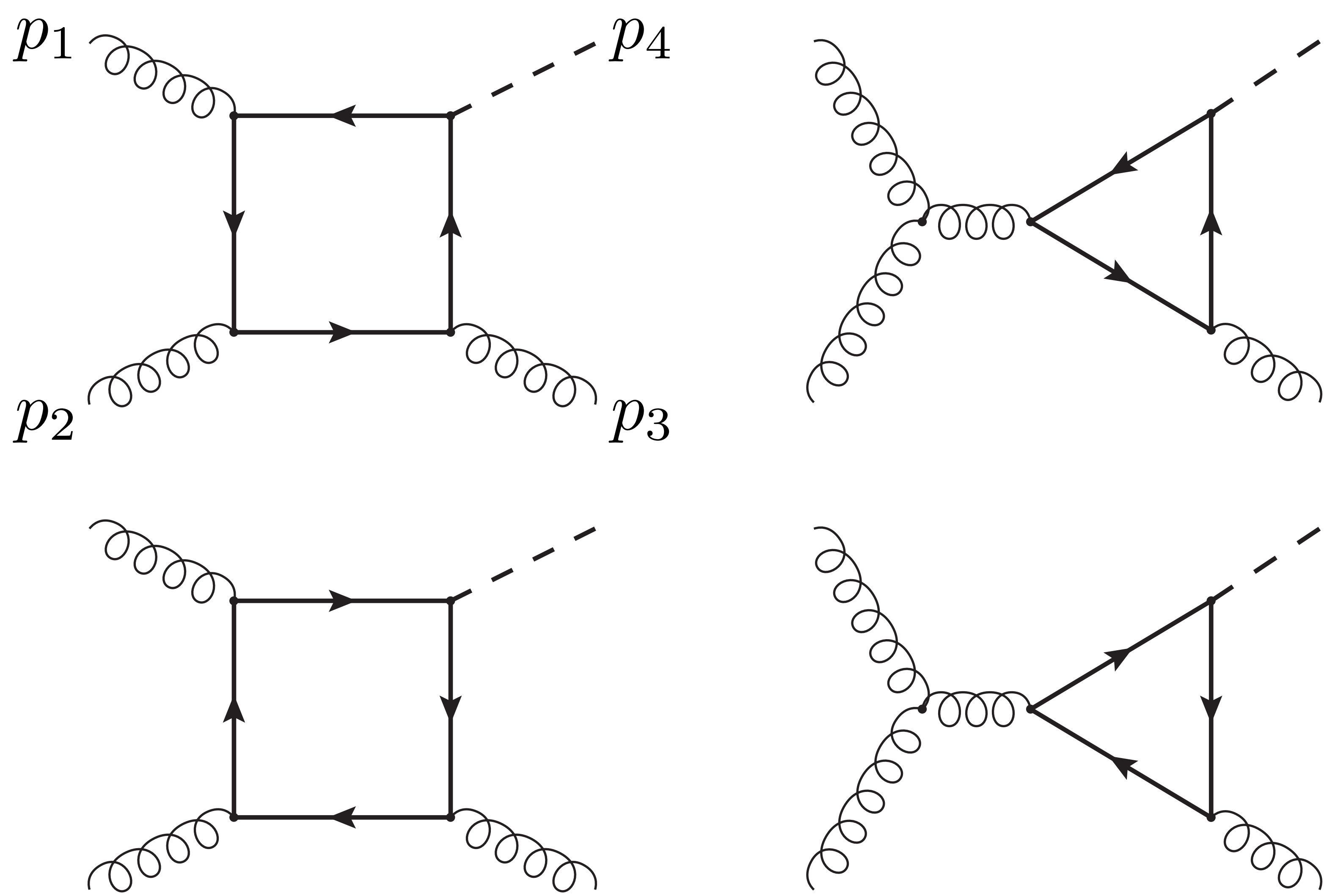
$p_T^j > 30$ GeV

MCFM 8.0

Gluon channel dominant due to high gluon luminosity (PDF)

Gluon-Quark channels still contribute significantly (22%)

Gluon Channel: Feynman Diagams



+ 2 (flip fermion line)

+ 4 (cyclic permutation $p_1 \to p_2, p_2 \to p_3, p_3 \to p_1$)

+ 4 (cyclic permutation $p_1 \to p_3, p_2 \to p_1, p_3 \to p_2$)

Total: 12 diagrams

Tensor Decomposition

Write $g(p_1) + g(p_2) + g(p_3) \to H(p_4)$ amplitude as $M = \varepsilon_\mu^{h1}\varepsilon_\nu^{h2}\varepsilon_\tau^{h3} M^{\mu\nu\tau}$

Impose transversality of the pol. vectors and the Ward identity:

$$
\begin{aligned}
M^{\mu\nu\tau}_{\text{physical}} = {} & F_{212}(s g^{\mu\nu} - 2p_2^\mu p_1^\nu)(u p_1^\tau - t p_2^\tau)/(2t) \\
& + F_{332}(u g^{\nu\tau} - 2p_3^\nu p_2^\tau)(t p_2^\mu - s p_3^\mu)/(2s) \\
& + F_{311}(t g^{\tau\mu} - 2p_1^\tau p_3^\mu)(s p_3^\nu - u p_1^\nu)/(2u) \\
& + F_{312}\Big(g^{\mu\nu}(u p_1^\tau - t p_2^\tau) + g^{\nu\tau}(t p_2^\mu - s p_3^\mu) + g^{\tau\mu}(s p_3^\nu - u p_1^\nu) \\
& \quad + 2p_3^\mu p_1^\nu p_2^\tau - 2p_2^\mu p_3^\nu p_1^\tau\Big)/2
\end{aligned}
$$

Our job: Compute the Form Factors $F_{212}, F_{332}, F_{311}, F_{312}$

Here: $s = (p_1 + p_2)^2,\ t = (p_1 + p_3)^2,\ u = (p_2 + p_3)^2$

this convention differs from appendix!

Thanks: Frellesvig, Glover

Tensor Decomposition

Can construct projectors with the property:

$$Q^{212}_{\mu\nu\tau} M^{\mu\nu\tau} = F_{212}$$
$$Q^{332}_{\mu\nu\tau} M^{\mu\nu\tau} = F_{332}$$
$$Q^{311}_{\mu\nu\tau} M^{\mu\nu\tau} = F_{311}$$
$$Q^{312}_{\mu\nu\tau} M^{\mu\nu\tau} = F_{312}$$

(Or alternatively: $\sum_{h_1,h_2,h_3} Q_i^{\mu\nu\tau} (\varepsilon_\mu^{h_1})^* (\varepsilon_\nu^{h_2})^* (\varepsilon_\tau^{h_3})^* \varepsilon_{\hat\mu}^{h_1} \varepsilon_{\hat\nu}^{h_2} \varepsilon_{\hat\tau}^{h_3} M^{\hat\mu\hat\nu\hat\tau} = F_i$)

For example:

depends on external momenta, d

$$Q_{312}^{\mu\nu\tau} = \Big(ut\big(sug^{\mu\tau}p_1^\nu + sg^{\mu\nu}(tp_2^\tau - up_1^\tau) + p_2^\mu((d-2)p_1^\nu(tp_2^\tau - up_1^\tau) - stg^{\nu\tau})\big)$$
$$+ s\big((d-2)tp_2^\tau(sp_3^\mu - tp_2^\mu) - sutg^{\mu\tau} + up_1^\tau(dtp_2^\mu - (d-2)sp_3^\mu)\big)p_3^\nu$$
$$+ su\big(stg^{\nu\tau} + p_1^\nu((d-2)up_1^\tau - dtp_2^\tau)\big)p_3^\mu \Big) / ((d-3)s^2u^2t^2)$$

Thanks: Frellesvig, Glover

Integral Families

Can rewrite tensor integrals/scalar products as inverse propagators

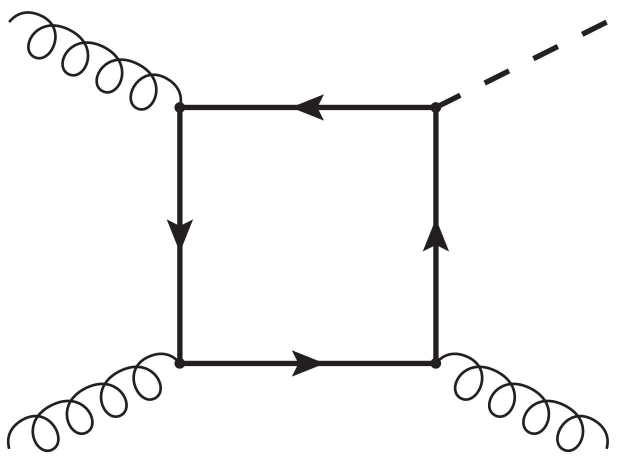
#scalar products

$$S = \frac{l(l+1)}{2} + lm \qquad \begin{array}{r} l = 1 \\ m = 3 \end{array} \quad \begin{array}{l} \textbf{\#loops} \\ \textbf{\#l.i. external momenta} \end{array} \qquad \Longrightarrow \quad S = 4$$

Introduce 1 integral family with 4 propagators:

$$I_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} = \int \mathrm{d}^d k_1 \frac{1}{D_1^{\alpha_1} D_2^{\alpha_2} D_3^{\alpha_3} D_4^{\alpha_4}} \qquad \alpha_i \in \mathbb{Z}$$

$$\begin{aligned} D_1 &= k_1^2 - m_T^2, \\ D_2 &= (k_1 - p1)^2 - m_T^2, \\ D_3 &= (k_1 - p1 - p2)^2 - m_T^2, \\ D_4 &= (k_1 - p1 - p2 - p3)^2 - m_T^2. \end{aligned}$$



All integrals in the amplitude can be expressed in terms of $I_{\alpha_1,\alpha_2,\alpha_3,\alpha_4}$

Integral Families (2-loop)

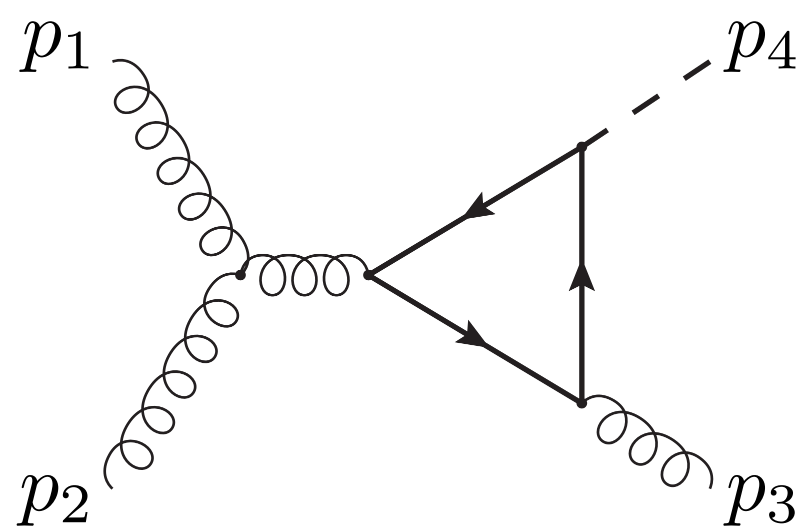
1-loop is a bit too simple to see generally what is going on
look briefly at 2-loop (but we will not calculate anything):

PL1	PL2	NPL
k_1^2	$k_1^2 - m_T^2$	$k_1^2 - m_T^2$
$(k_1 - p_1)^2$	$(k_1 - p_1)^2 - m_T^2$	$(k_1 + p_1)^2 - m_T^2$
$(k_1 - p_1 - p_2)^2$	$(k_1 - p_1 - p_2)^2 - m_T^2$	$(k_1 - p_2 - p_3)^2 - m_T^2$
$(k_1 - p_1 - p_2 - p_3)^2$	$(k_1 - p_1 - p_2 - p_3)^2 - m_T^2$	$k_2^2 - m_T^2$
$k_2^2 - m_T^2$	$k_2^2 - m_T^2$	$(k_2 + p_1)^2 - m_T^2$
$(k_2 - p_1)^2 - m_T^2$	$(k_2 - p_1)^2 - m_T^2$	$(k_2 - p_3)^2 - m_T^2$
$(k_2 - p_1 - p_2)^2 - m_T^2$	$(k_2 - p_1 - p_2)^2 - m_T^2$	$(k_1 - k_2)^2$
$(k_2 - p_1 - p_2 - p_3)^2 - m_T^2$	$(k_2 - p_1 - p_2 - p_3)^2 - m_T^2$	$(k_1 - k_2 - p_2)^2$
$(k_1 - k_2)^2 - m_T^2$	$(k_1 - k_2)^2$	$(k_1 - k_2 - p_2 - p_3)^2$

Melnikov, Tancredi, Wever 16

At 2-loop $S = 9$, diagrams have ≤ 7 propagators need "auxiliary propagators" also 1 integral family is not enough, need at least 3
Other than this everything proceeds exactly as at 1-loop

Computing a Diagram



51 terms

$$F_{312} \supset f^{abc} C_\epsilon \frac{e g_{ht} g_s^3}{(d-3)} \left[(d-4) \frac{2 m_T}{s^2 t} I^{(132)}_{1,-1,0,1} + d \frac{4 m_T}{s^2 t} I^{(132)}_{1,1,-1,0} + \dots \right]$$

$$g_{ht} = -\frac{1}{2 s_w} \frac{m_T}{m_W}$$

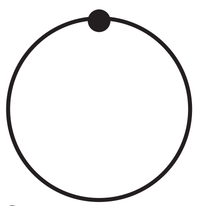
inverse propagators from numerator scalar products involving loop momenta

Notation: $I^{(132)}_{\dots}$ means permute $p_1 \to p_3,\ p_3 \to p_2,\ p_2 \to p_1$ in propagators of integral family

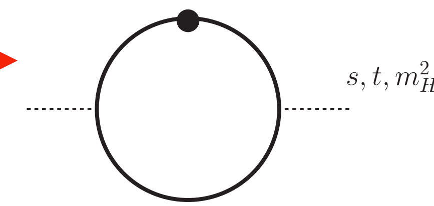
Integration By Parts (IBP) identities allow us to reduce the number of integrals to a minimal set (we use **REDUZE**) von Manteuffel, Studerus 12

Integrals

Change basis of integrals to select a set of "pre-canonical" master integrals:



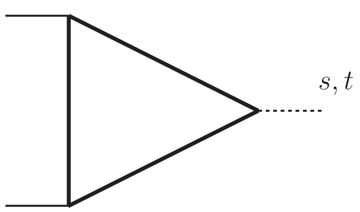
dots on tadpoles and bubbles



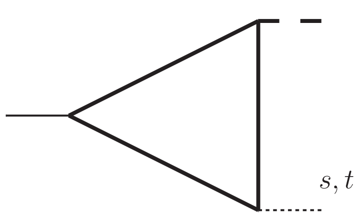
$$f_1(m_T^2) = I_{2,0,0,0}$$

$$f_2(s, m_T^2) = I_{2,0,1,0}, \;\; f_2(t, m_T^2) = I^{(132)}_{2,0,1,0}$$

$$f_3(m_H^2, m_T^2) = I_{2,0,0,1}$$



no dots for triangles, boxes

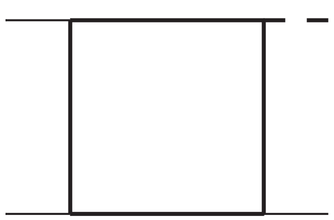


$$f_4(s, m_T^2) = I_{1,1,1,0}$$

$$f_4(t, m_T^2) = I^{(132)}_{1,1,1,0}$$

$$f_5(s, m_H^2, m_T^2) = I^{(13)}_{1,1,0,1}$$

$$f_5(t, m_H^2, m_T^2) = I^{(12)}_{1,1,0,1}$$



$$f_6(s, t, m_H^2, m_T^2) = I^{(12)}_{1,1,1,1}$$

Integrals

Form a vector of the "pre-canonical" master integrals $\vec{f}$

Write differential equation: $\mathrm{d}\vec{f} = B\vec{f}$

Change basis: $\vec{f} = T\vec{g}$

epsilon factorised

Obtain canonical form: $\mathrm{d}\vec{g} = \epsilon \, \mathrm{d}A \, \vec{g}$ $\quad \mathrm{d}A = \sum_{i=1}^{n} M_i \mathrm{d}\log \eta_i$

Our alphabet (for the box integral) contains square roots which are not simultaneously rationalised by changing to the "Landau variables":

$$x = \frac{-(1-y)^2}{y}$$

Solved by Hjalte using techniques presented in the appendix

Amplitude

$$F_{312}(s,t,u) = f^{abc} C_\epsilon \frac{2m_T e g_{ht} g_s^3}{(d-3)(d-2)} \Bigg[\left(\frac{8(d-4)}{st+tu} \right) A(m_H^2)$$

$$+ \frac{4\left((d-2)t^2 + 2dtu + (d-2)u^2\right)}{tu(t+u)^2} B(s, m_H^2, m_T^2)$$

$$+ \frac{(d-2)s\left(t^2+u^2\right)}{t^2u^2} C(s,t,u,m_T^2)$$

$$+ \frac{4(d-3)\left((d-2)(t+u) - 8m_T^2\right)}{s(t+u)} f_5(s, m_H^2, m_T^2)$$

$$+ \frac{(d-2)\left(-(d-2)su + (d-3)t^2 - 4m_T^2 t\right)}{t^2} f_6(s,t,m_H^2,m_T^2) \Bigg]$$

$$+ (s \to t, t \to u, u \to s) + (s \to u, t \to s, u \to t),$$

$$F_{311}(t,u,s) = F_{332}(u,s,t) = F_{212}(s,t,u) = \ldots$$

although result is finite, receives contribution from ϵ/ϵ "rational term"

full expression in book

Abbreviations:

$$A(m_H^2, m_T^2) = f_1(m_T^2) + (4m_T^2 - m_H^2) f_3(m_H^2, m_T^2),$$

$$B(x, m_H^2, m_T^2) = (4m_T^2 - x) f_2(x, m_T^2) - (4m_T^2 - m_H^2) f_3(m_H^2, m_T^2),$$

$$C(s,t,u,m_T^2) = (d-2) f_4(s, m_T^2) - (d-2)\frac{u+t}{s} f_5(s, m_H^2, m_T^2).$$

Squared Amplitude

For unpolarised cross sections can now square the amplitude and average over the incoming gluon polarisations and colours:

colour ↓ **polarisations** ↓

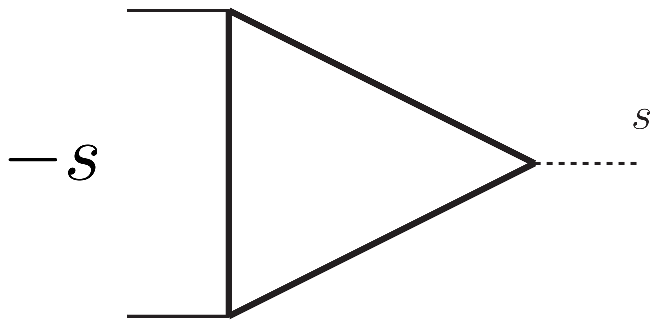
$$\overline{|M|^2} = \frac{1}{N_A^2}\frac{1}{(d-2)^2}|M|^2,$$

$$|M|^2 = \sum_{h_1,h_2,h_3} (\varepsilon_\mu^{h_1})^* \varepsilon_{\hat\mu}^{h_1} (\varepsilon_\nu^{h_2})^* \varepsilon_{\hat\nu}^{h_2} (\varepsilon_\tau^{h_3})^* \varepsilon_{\hat\tau}^{h_3} (M^{\mu\nu\tau})^* M^{\hat\mu\hat\nu\hat\tau}$$

$$= \frac{1}{4}\Big[\frac{(d-2)|F_{212}|^2 s^3 u}{t} + F_{212}F_{332}^* s u^2 + F_{212}F_{311}^* s^2 t + (d-2)F_{212}F_{312}^* s^2 u$$
$$+ F_{332}F_{212}^* s u^2 + \frac{(d-2)|F_{332}|^2 t u^3}{s} + F_{332}F_{311}^* t^2 u + (d-2)F_{332}F_{312}^* t u^2$$
$$+ F_{311}F_{212}^* s^2 t + F_{311}F_{332}^* t^2 u + \frac{(d-2)|F_{311}|^2 s t^3}{u} + (d-2)F_{311}F_{312}^* s t^2$$
$$+ (d-2)F_{312}F_{212}^* s^2 u + (d-2)F_{312}F_{332}^* t u^2 + (d-2)F_{312}F_{311}^* s t^2 + (3d-8)|F_{312}|^2 stu\Big]$$

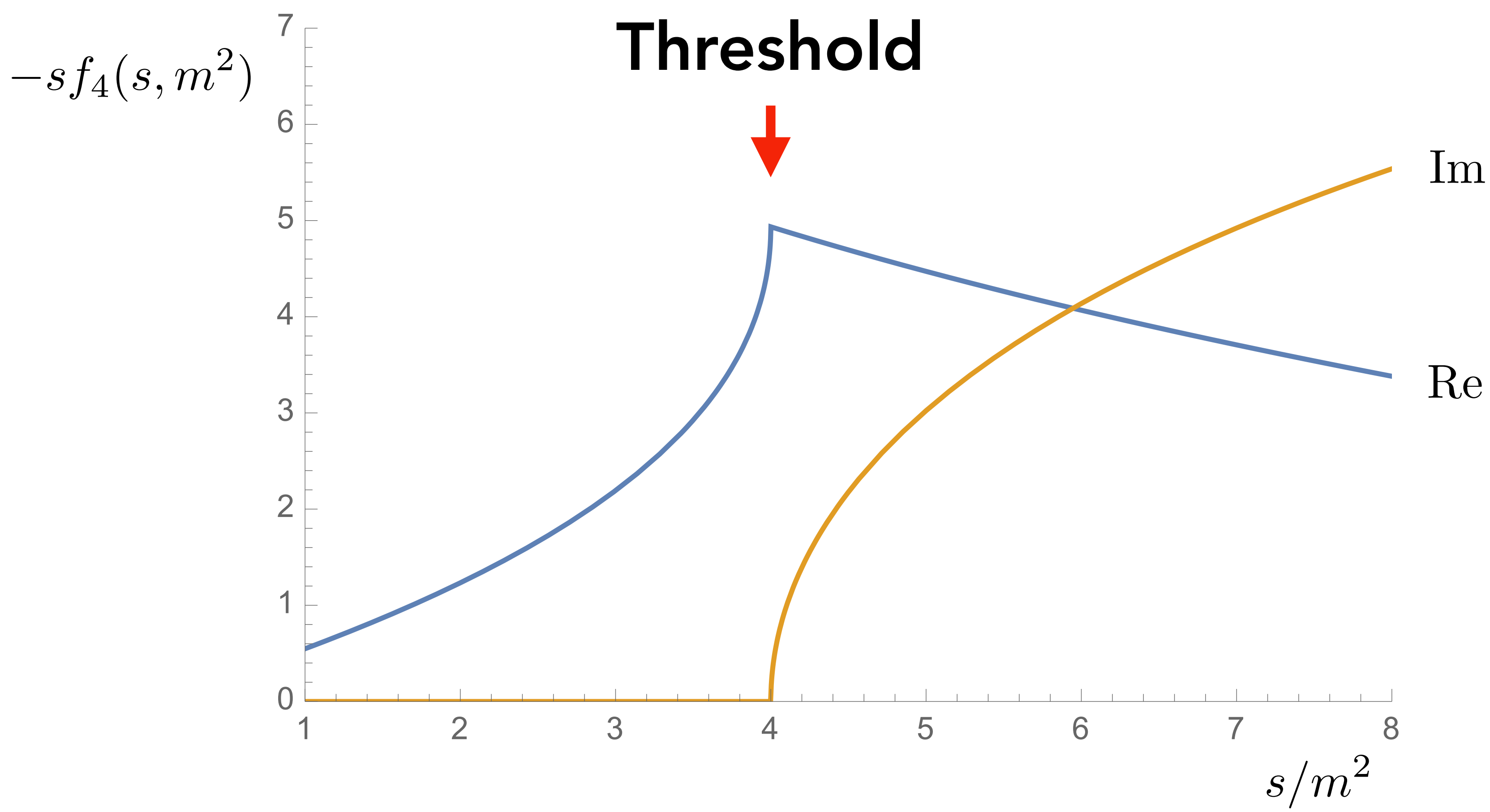
Here we used: $\sum_{pols} (\varepsilon_1^\mu(p_1))^* \, \varepsilon_1^\nu(p_1) = -g^{\mu\nu} + \frac{p_1^\mu n_1^\nu + p_1^\nu n_1^\mu}{p_1 \cdot n_1}$

Integral Expansion (Part 1)

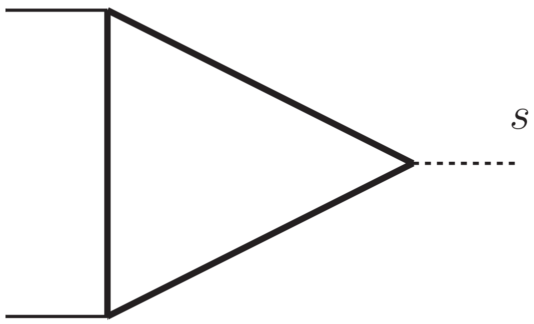


$$= -\frac{1}{2}\ln^2\left(-\frac{1-\beta}{1+\beta}\right) \qquad \beta = \sqrt{1-\frac{4m^2}{s}}$$

Threshold: for $s > 4m^2$ propagators can be on-shell, integral has a branch point and develops an imaginary part



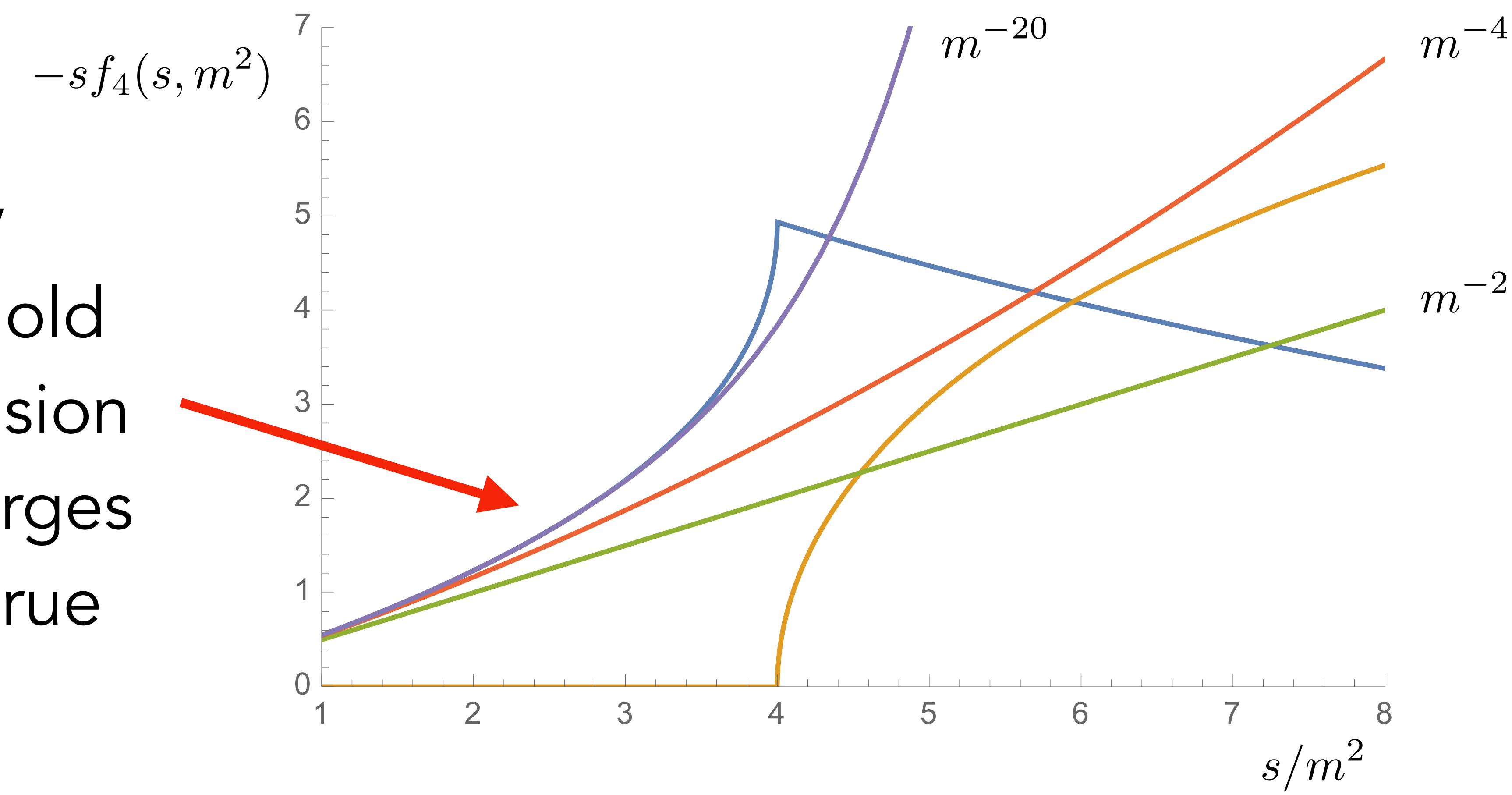
Integral Expansion (Part 1)

$-s$ 

$$= -\frac{1}{2}\ln^2\left(-\frac{1-\beta}{1+\beta}\right) = \frac{s}{2m^2} + \frac{s^2}{24m^4} + \frac{s^3}{180m^6} + \ldots$$

Above threshold the expansion does not capture the true behaviour, near to threshold higher terms in the expansion are needed

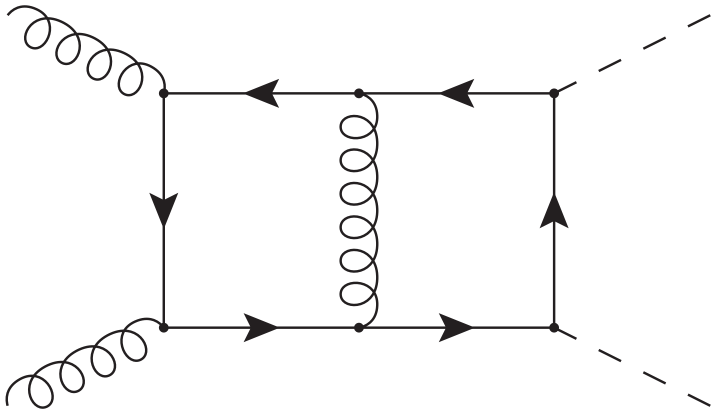
Below threshold expansion converges onto true result

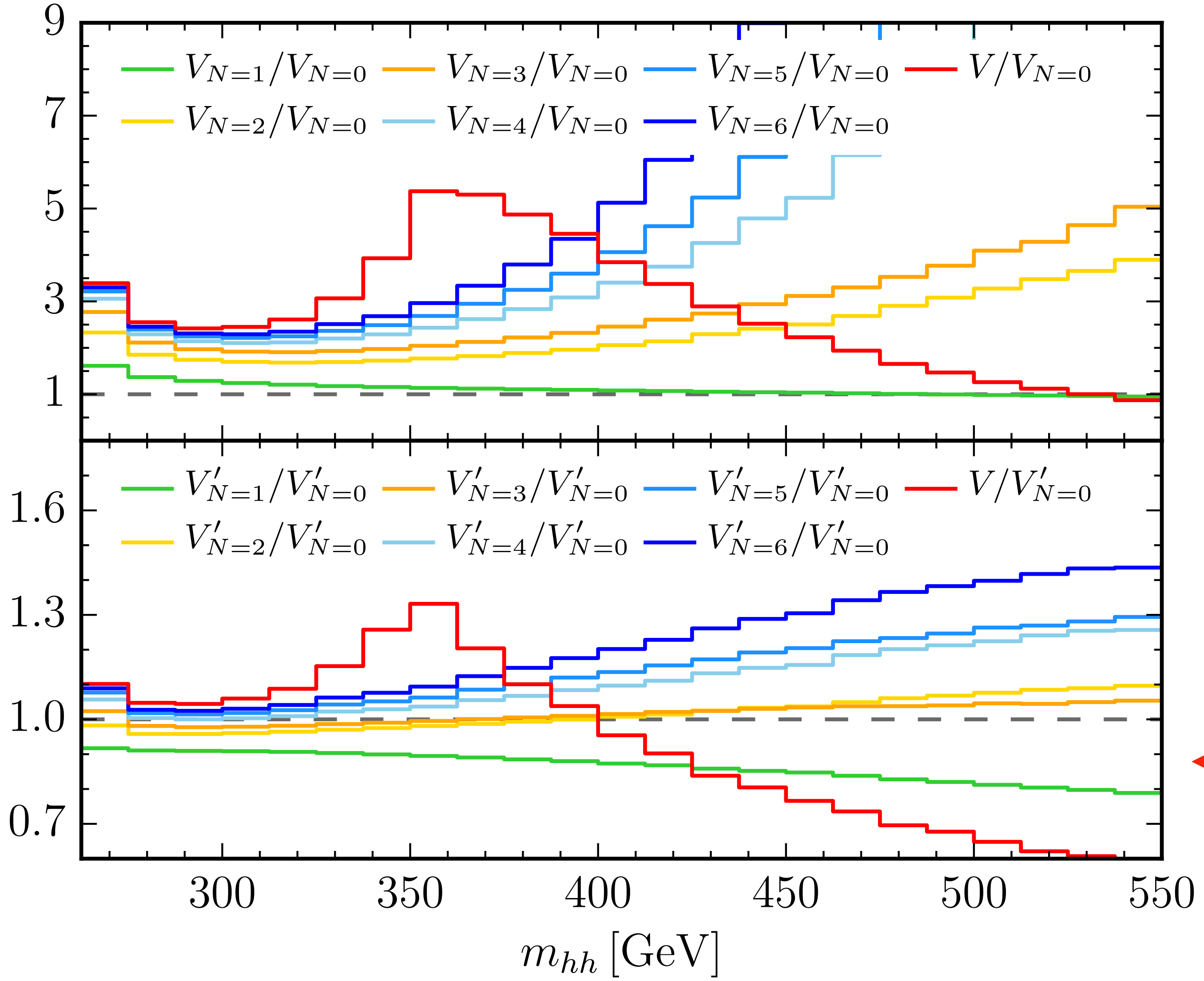


gg → HH (2-loop)

Can compare just virtual ME to expansion:

$$d\hat{\sigma}_N = \sum_{\rho=0}^{N} d\hat{\sigma}^{(\rho)} \left(\frac{\Lambda}{m_t}\right)^{2\rho} \qquad \Lambda \in \left\{\sqrt{\hat{s}}, \sqrt{\hat{t}}, \sqrt{\hat{u}}, m_h\right\}$$





$$V_N = \left(d\hat{\sigma}_N^V + d\hat{\sigma}_N^{LO} \otimes \mathbf{I}\right)$$

$$V'_N = V_N \frac{d\hat{\sigma}^{LO}}{d\hat{\sigma}_N^{LO}}$$

Rescaled better but does not describe full above threshold

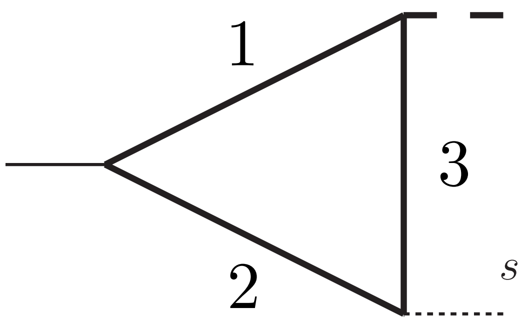
S. Borowka, N. Greiner, G. Heinrich, SJ, M. Kerner, J. Schlenk, T. Zirke 16

$V_{N\geq4}$ thanks to J. Hoff

Grigo, Hoff, Steinhauser 15

Integral Expansion (Part 2)

Suppose instead that we did not first compute all integrals, we can also expand integrals before integration



$$= (-1)^3\Gamma(3-d/2)\int_0^\infty \left(\prod_{i=1}^{3} \mathrm{d}x_i\right) \delta(1-\sum_{i=1}^{3} x_i)\frac{\mathcal{U}^{3-d}}{\mathcal{F}^{3-d/2}}.$$

$$\mathcal{U} = x_1 + x_2 + x_3,$$
$$\mathcal{F} = -sx_2x_3 - m_H^2 x_1 x_3 + m_T^2(x_1+x_2+x_3)^2.$$

Substitute: $m_T^2 = \bar{m}_T^2/\rho, s = \bar{s}, m_H^2 = \bar{m}_H^2$ series expand about $\rho = 0$

Integral Expansion (Part 2)

$$f_5(s, m_H^2) = (-1)^3\Gamma(3-d/2)\int_0^\infty \left(\prod_{i=1}^3 \mathrm{d}x_i\right)\delta(1-\sum_{i=1}^3 x_i)\rho^{3-d/2}$$
$$\left[\frac{(x_1+x_2+x_3)^{3-d}}{(\bar{m}_T^2(x_1+x_2+x_3)^2)^{3-d/2}}\right.$$
$$\left. -\rho(3-d/2)(-\bar{s}x_2x_3 - \bar{m}_H^2 x_1 x_3)\frac{(x_1+x_2+x_3)^{3-d}}{(\bar{m}_T^2(x_1+x_2+x_3)^2)^{4-d/2}} + \mathcal{O}(\rho^2)\right].$$

Note that after series expansion the integrals do not depend on the invariants or masses, they are simple "tadpole" type integrals:

$$\int_0^\infty \left(\prod_{i=1}^3 \mathrm{d}x_i\right)\delta(1-\sum_{i=1}^3 x_i)(x_1+x_2+x_3)^{-3} = \frac{1}{2}$$
$$\int_0^\infty \left(\prod_{i=1}^3 \mathrm{d}x_i\right)\delta(1-\sum_{i=1}^3 x_i)x_2x_3(x_1+x_2+x_3)^{-4} = \frac{1}{24}$$

At higher orders the tools **`q2e/exp+ matad`** help to automate the large mass expansion of the amplitude Harlander, Seidensticker, Steinhauser 97,99; Steinhauser 00

Amplitude Expansion

Expand each integral coefficient and each integral, obtain very simple result for each form factor:

$$F_{212} = N\frac{1}{u}, \quad F_{332} = N\frac{1}{t}, \quad F_{311} = N\frac{1}{s}, \qquad N = f^{abc} C_\epsilon \frac{8}{3} \frac{e g_{ht} g_s^3}{m_T}$$

$$F_{312} = N\frac{(st + tu + us)}{stu} \qquad\qquad = f^{abc} C_\epsilon \frac{8}{3} \frac{g_s^3}{v}$$

Compute mod-squared amplitude (no colour, spin averaging):

$$|M|^2 = \frac{4 C_A N_A}{9} \frac{\alpha_s^3}{\pi v^2} \frac{(m_H^8 + s^4 + t^4 + u^4)(1 - 2\epsilon) + \epsilon/2(m_H^4 + s^2 + t^2 + u^2)^2}{stu}$$

This result is remarkably simple!

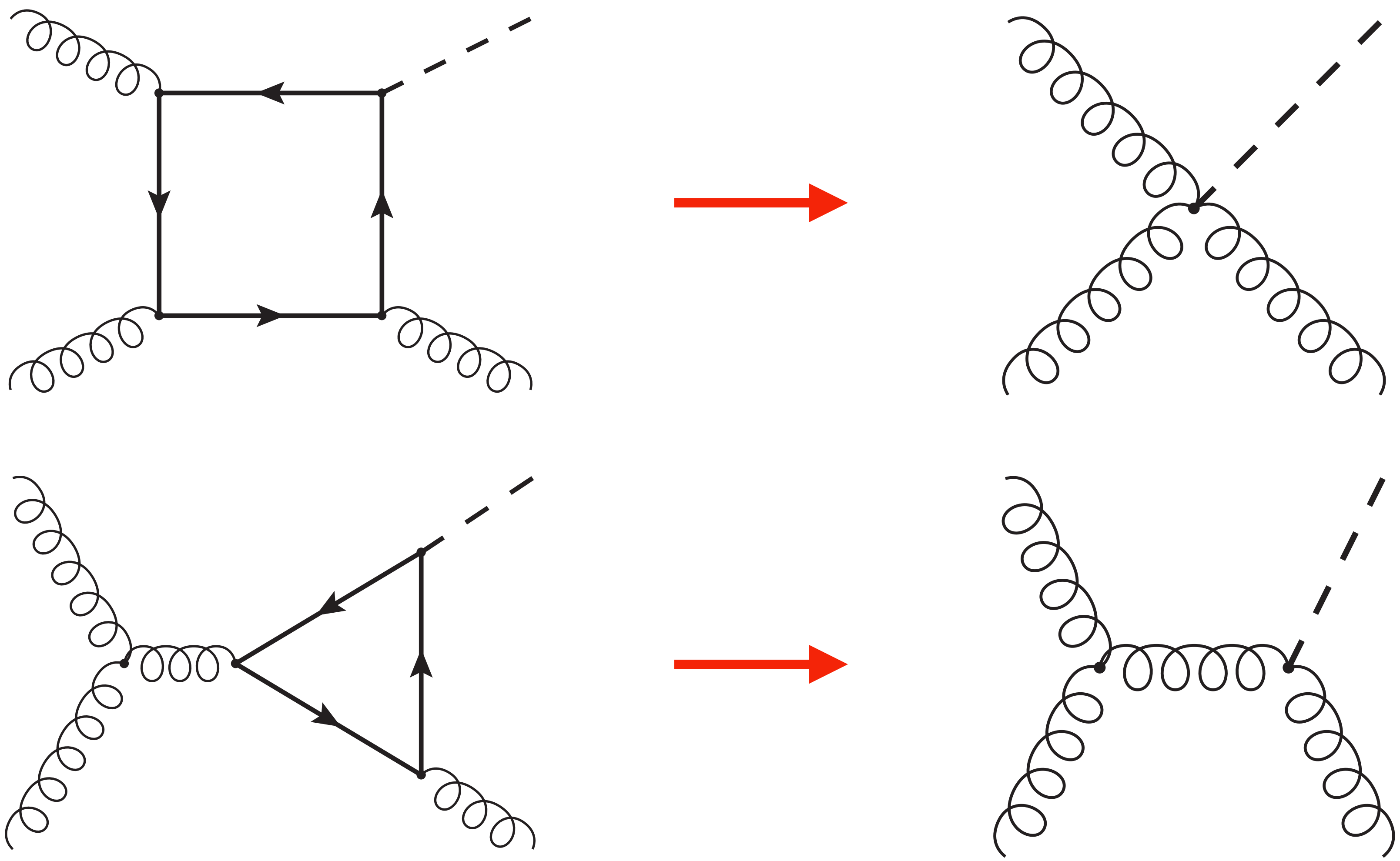
Independent of the top quark mass

Can we obtain it in an easier way?

Higgs Effective Field Theory

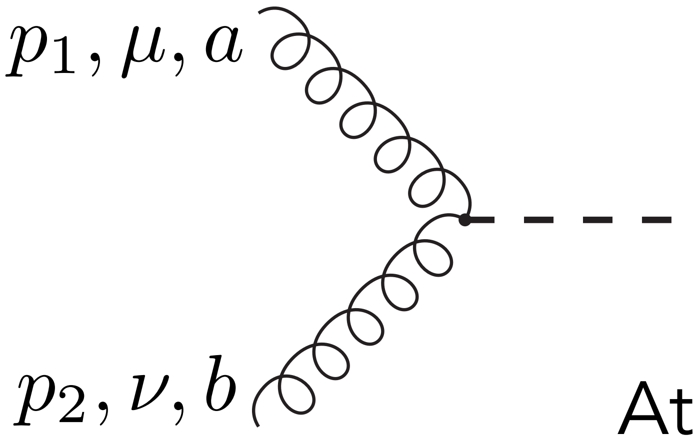
Introduce a coupling between gluons and Higgs in Lagrangian, this corresponds to taking the large quark mass limit ($m_T \to \infty$)
Induces effective tree level interaction:

$$\mathcal{L}_{eff} \supset -\frac{1}{4} AHG^A_{\mu\nu} G^{A\ \mu\nu}$$



HEFT Feynman Rules

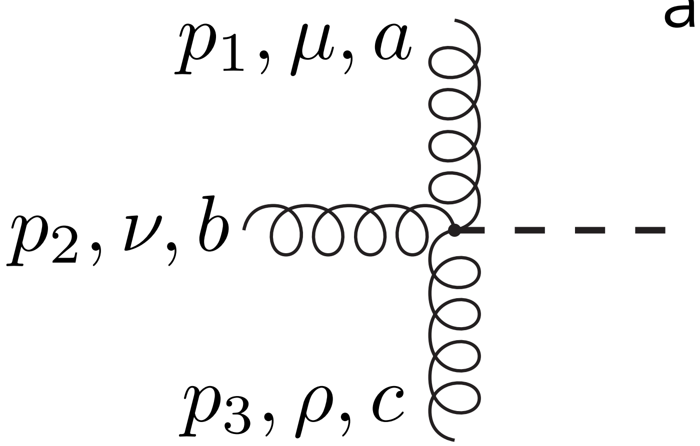
$$\mathcal{L}_{eff} \supset -\frac{1}{4} A H G^A_{\mu\nu} G^{A\ \mu\nu}$$



$$= iA\delta^{ab} H^{\mu\nu}(p_1, p_2)$$

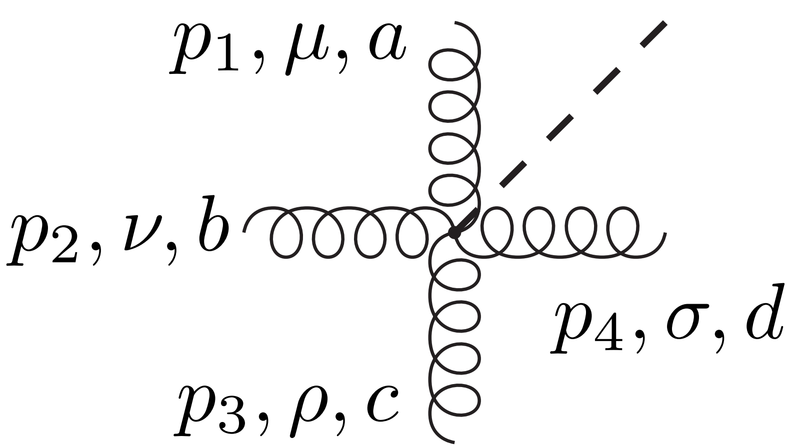
$$H^{\mu\nu}(p_1, p_2) = - g^{\mu\nu} p_1 \cdot p_2 + p_2^{\mu} p_1^{\nu}$$

At this point just some arbitrary parameter



$$= -A g_s f^{abc} V^{\mu\nu\rho}(p_1, p_2, p_3)$$

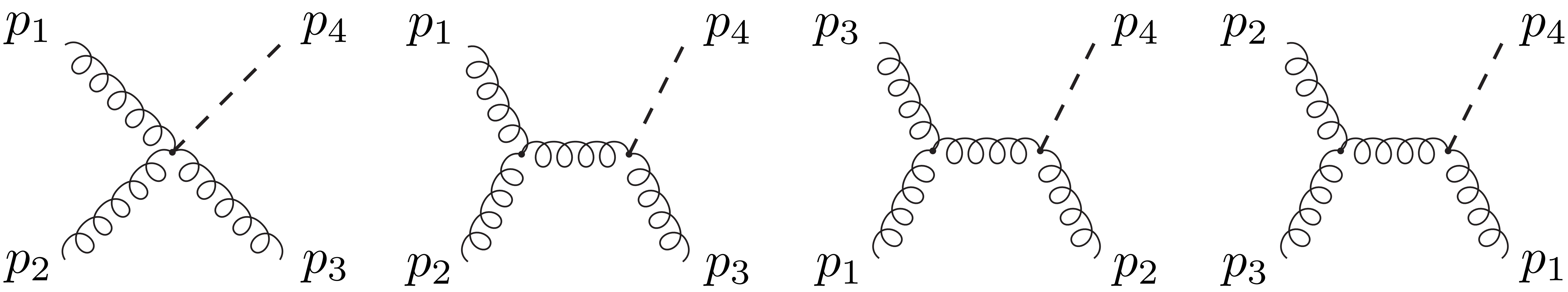
$$V^{\mu\nu\rho}(p_1, p_2, p_3) = g^{\mu\nu}(p_1 - p_2)^{\rho} + g^{\nu\rho}(p_2 - p_3)^{\mu} + g^{\rho\mu}(p_3 - p_1)^{\nu}$$



$$= -iAg_s^2 X^{abcd}_{\mu\nu\rho\sigma}$$

$$X^{abcd}_{\mu\nu\rho\sigma} = \ldots$$

HEFT Feynman Diagrams



Total: 4 diagrams

In the HEFT the computation becomes a tree-level calculation:
Can apply projectors and compute individual form factors
Easier to just square the tree level amplitude

we do both to check the projectors

$$|M|^2 = C_A N_A 4\pi\alpha_s A^2 \frac{(m_H^8 + s^4 + t^4 + u^4)(1-2\epsilon) + \epsilon/2(m_H^4 + s^2 + t^2 + u^2)^2}{stu}$$

HEFT Matching

By comparing our expanded 1-loop result to the HEFT result we can determine the coefficient A:

$$|M|^2 = \frac{4C_A N_A}{9} \frac{\alpha_s^3}{\pi v^2} \frac{(m_H^8 + s^4 + t^4 + u^4)(1-2\epsilon) + \epsilon/2(m_H^4 + s^2 + t^2 + u^2)^2}{stu}$$

$$A = \frac{\alpha_s}{3\pi v} + \mathcal{O}(\alpha_s^2)$$

Note: the coefficient also has an expansion in the coupling

$$|M|^2 = C_A N_A 4\pi\alpha_s A^2 \frac{(m_H^8 + s^4 + t^4 + u^4)(1-2\epsilon) + \epsilon/2(m_H^4 + s^2 + t^2 + u^2)^2}{stu}$$

The coefficient is a parameter of the HEFT and can be obtained by matching any quantity that depends on it

In the literature it has been obtained to higher orders from the decoupling relations of the strong coupling constant

Chetyrkin, Kniehl, Steinhauser 97

Conclusion

Higgs + Jet at LO in the Standard Model

- Summarised partonic channels and their relative importance
- Presented details of calculation of $gg \to gH$ at LO (1-loop)

Large mass expansion

- Expansion of individual integrals & breakdown near threshold
- Expansion of the amplitude
- HEFT approach

Work to be added to book (?)

- Details of the calculation of 1-loop integrals via DE using methods presented in the appendix

Thank you for listening!