

Renormalization of the THDM and NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4\text{fermions}$

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(in collaboration with L.Altenkamp and H.Rzehak; see arXiv:1704.02645 + paper in preparation)

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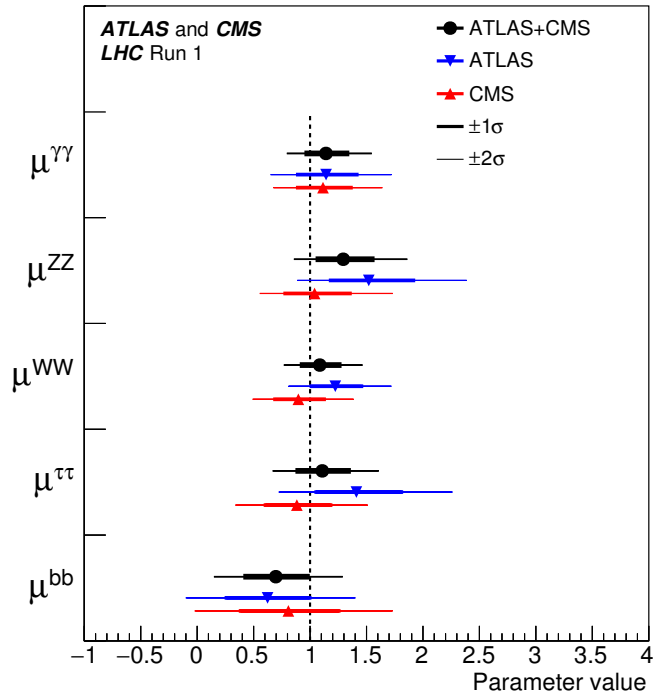


Introduction



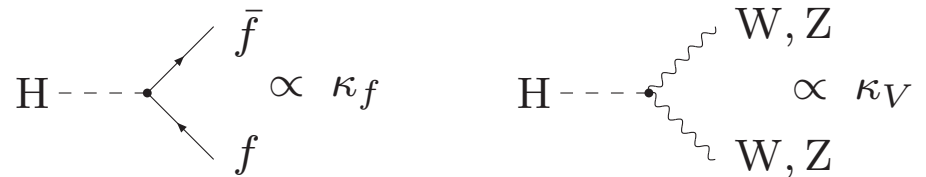
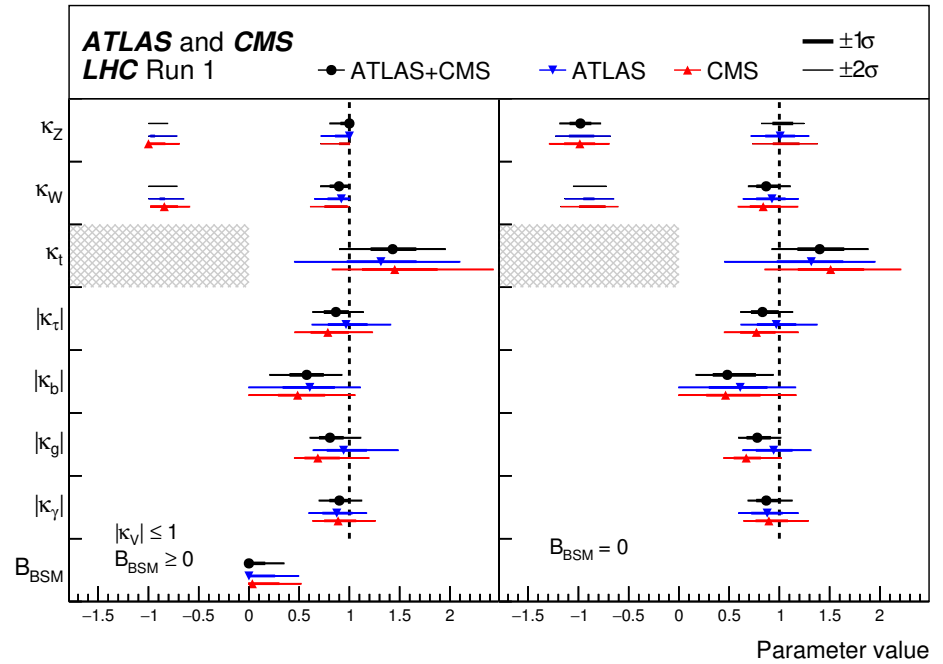
Some central LHC results from profiling the Higgs boson

Decay signal strength:



$$\mu = \frac{\Gamma_{\text{exp}}}{\Gamma_{\text{SM}}}$$

Fit of coupling modifiers:



Compatibility with Standard Model

Reveal BSM effects with higher precision ?

⇒ Precision calculations in BSM models necessary

→ THDM considered in this talk

Renormalization of the THDM



THDM Lagrangian and Higgs fields

Lagrangian: restriction to CP-conserving case!

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2) - V(\Phi_1, \Phi_2),$$
$$D_\mu = \partial_\mu - ig_2 I_W^a W_\mu^a + ig_1 \frac{Y_W}{2} B_\mu$$

Higgs potential:

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1)$$
$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1) (\Phi_2^\dagger \Phi_2)$$
$$+ \lambda_4 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]$$

Two complex scalar SU(2) doublets: $v_{1,2} = \text{vevs}$

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (\eta_1 + i\chi_1 + v_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (\eta_2 + i\chi_2 + v_2) \end{pmatrix}, \quad Y_W(\Phi_{1,2}) = 1$$

Transition to the “mass basis”:

$$\text{CP-even neutral fields: } \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\text{CP-odd neutral fields: } \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G_0 \\ A_0 \end{pmatrix}, \quad \tan \beta = \frac{v_2}{v_1}$$

$$\text{charged fields: } \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}$$

Higgs potential after diagonalization:

$$V = \underbrace{-t_h h - t_H H}_{\text{tadpoles} \rightarrow 0} + \frac{1}{2} M_h^2 h^2 + \frac{1}{2} M_H^2 H^2 + \frac{1}{2} M_{A_0}^2 A_0^2 + M_{H^\pm}^2 H^+ H^- + \dots$$

Transformation of input parameters:

$$\text{original set: } \{ \lambda_1, \dots, \lambda_5, m_{11}^2, m_{22}^2, m_{12}^2, v_1, v_2, g_1, g_2 \}$$

↓

$$\text{mass basis: } \{ \underbrace{M_H, M_h, M_{A_0}, M_{H^\pm}, M_W, M_Z, e}_{\text{renormalized on-shell}}, \underbrace{\lambda_5, \alpha, \beta}_{\overline{\text{MS}}}, \underbrace{t_H, t_h}_{2 \text{ ren. variants}} \}$$

Yukawa couplings:

Avoid FCNC at tree level!

↪ Couple each fermion flavour only to one Φ_n (\mathbb{Z}_2 symmetry)

$$\mathcal{L}_{\text{Yukawa}} = -\bar{L}'^L Y^l l'^R \Phi_{n_1} - \bar{Q}'^L Y^u u'^R \tilde{\Phi}_{n_2} - \bar{Q}'^L Y^d d'^R \Phi_{n_3} + h.c.$$

THDM type	u_i	d_i	e_i	\mathbb{Z}_2 symmetry
Type I	Φ_2	Φ_2	Φ_2	$\Phi_1 \rightarrow -\Phi_1$
Type II	Φ_2	Φ_1	Φ_1	$(\Phi_1, d_i, e_i) \rightarrow -(\Phi_1, d_i, e_i)$
Lepton-specific	Φ_2	Φ_2	Φ_1	$(\Phi_1, e_i) \rightarrow -(\Phi_1, e_i)$
Flipped	Φ_2	Φ_1	Φ_2	$(\Phi_1, d_i) \rightarrow -(\Phi_1, d_i)$

Yukawa couplings modified by THDM factors ξ_{H,h,A_0}^f :

	Type I	Type II	Lepton-specific	Flipped
ξ_H^l	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$	$\cos \alpha / \cos \beta$	$\sin \alpha / \sin \beta$
ξ_H^u	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$	$\sin \alpha / \sin \beta$
ξ_H^d	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$	$\sin \alpha / \sin \beta$	$\cos \alpha / \cos \beta$
ξ_h^l	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$	$-\sin \alpha / \cos \beta$	$\cos \alpha / \sin \beta$
ξ_h^u	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$	$\cos \alpha / \sin \beta$
ξ_h^d	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$	$\cos \alpha / \sin \beta$	$-\sin \alpha / \cos \beta$
$\xi_{A_0}^l$	$\cot \beta$	$-\tan \beta$	$-\tan \beta$	$\cot \beta$
$\xi_{A_0}^u$	$\cot \beta$	$\cot \beta$	$\cot \beta$	$\cot \beta$
$\xi_{A_0}^d$	$\cot \beta$	$-\tan \beta$	$\cot \beta$	$-\tan \beta$

Renormalization (see also Santos/Barroso '97; Kanemura et al. '04; Lopez-Val/Sola '09; Degrande '14)

↪ follow on-shell renormalization as far as possible/reasonable
related work by Krause et al. '16; Denner et al. '16

On-shell renormalization:

- all particle masses: $M_W, M_Z, M_h, M_H, \dots$

- matrix-valued renormalization for all fields:

$$\begin{pmatrix} H_0 \\ h_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_H & \frac{1}{2}\delta Z_{Hh} \\ \frac{1}{2}\delta Z_{hH} & 1 + \frac{1}{2}\delta Z_h \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}, \quad \text{etc.}$$

↪ no mixing of external (on-shell) states

- elmg. coupling α_{em} in the Thomson limit

\overline{MS} renormalization:

- mixing angles α, β

↪ e.g. determined by Higgs mixing self-energies

- Higgs self-coupling λ_5

↪ e.g. determined by HA_0A_0 vertex correction

⇒ Renormalization-scale-dependent parameters $\alpha(\mu_r), \beta(\mu_r), \lambda_5(\mu_r)$

Tadpole renormalization:

Note: No physical effect (just bookkeeping)
if all parameters are fixed by “physical renormalization conditions”!

But: $\overline{\text{MS}}$ parameters in general depend on tadpole renormalization!

Two commonly used variants:

a) **Vanishing renormalized tadpoles** t_S : $t_{S,0} = t_S + \delta t_S = 0 + \delta t_S$

- (explicit tadpole loops Γ^S) + $\delta t_S = 0 \Rightarrow$ explicit tadpoles can be ignored
- (implicit) tadpole contributions δt_S in counterterms
- **drawback:** $t_{S,0} = \delta t_S$ enters relation between bare basic input parameters
 \hookrightarrow potentially gauge-dependent terms $\propto \delta t_S$ enter relations
between renormalized parameters and predicted observables

b) **Vanishing bare tadpoles** $t_{S,0}$: $t_{S,0} = 0$ **Fleischer/Jegerlehner '80; Actis et al. '06**

- explicit tadpole loops Γ^S have to be included everywhere,
technical variant: remove Γ^S from 2-point functions by shift $v_S \rightarrow v_S + \Delta v_s$
- **advantage:** no gauge-dep. δt_S terms in relations between bare parameters
 \hookrightarrow relation between ren. parameters and observables gauge independent

Different schemes employed in NLO calculation for $h \rightarrow 4f$:

- $\overline{\text{MS}}(\alpha)$: see also by Krause et al. '16; Denner et al. '16
 - ◇ input: β, λ_5, α
 - ◇ tadpole treatment a): $t_S = 0$
 - ◇ gauge dependent: results tied to 't Hooft–Feynman gauge
- $\text{FJ}(\alpha)$: see also by Krause et al. '16; Denner et al. '16
 - ◇ input: β, λ_5, α
 - ◇ FJ tadpole treatment b): $t_{S,0} = 0$
 - ◇ gauge independent
- $\overline{\text{MS}}(\lambda_3)$:
 - ◇ as $\overline{\text{MS}}(\alpha)$, but α replaced by coupling λ_3 as input
 - ◇ gauge independent only in R_ξ gauges at NLO
- $\text{FJ}(\lambda_3)$:
 - ◇ as $\text{FJ}(\alpha)$, but α replaced by coupling λ_3 as input
 - ◇ gauge independent

↪ Study renormalization scheme and renormalization scale dependence of results

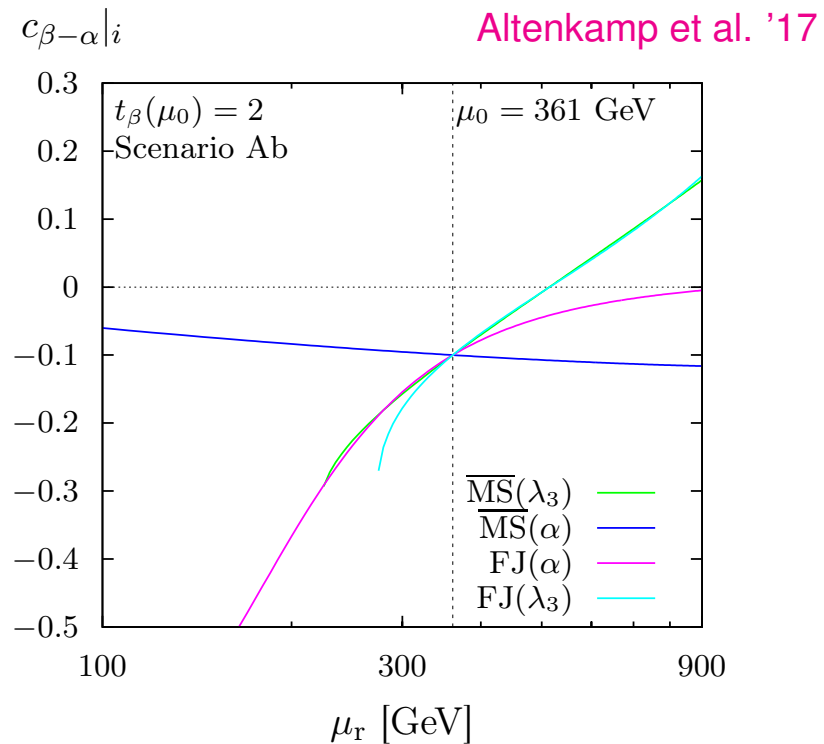
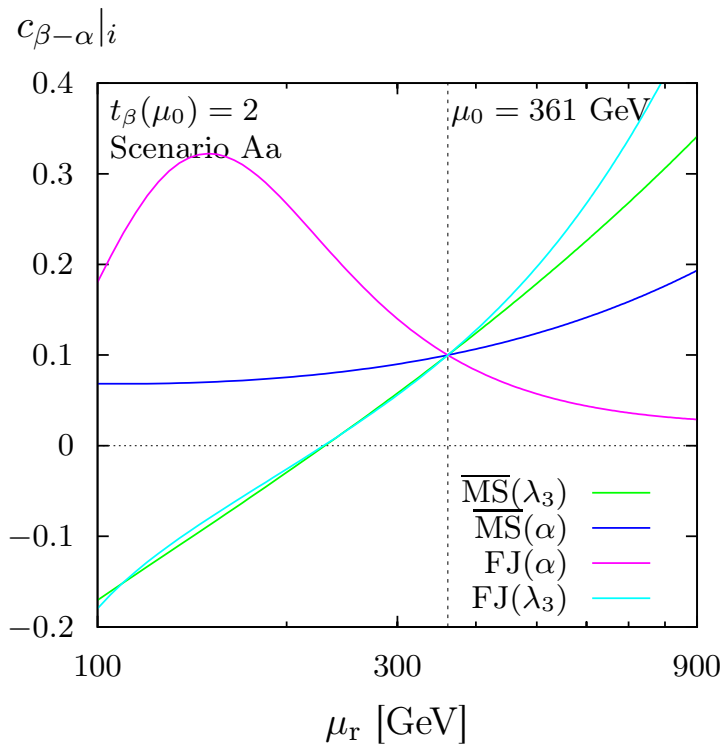
Running of \overline{MS} parameters: (numerical solution of ren. group eqs.)

Example: $c_{\beta-\alpha}$ in a THDM low-mass scenario of Type I

Scenario A: $M_h = 125 \text{ GeV}$, $c_{\beta-\alpha} = +0.1$ (Aa) or $c_{\beta-\alpha} = -0.1$ (Ab)

$M_H = 300 \text{ GeV}$, $M_{A_0} = M_{H^+} = 460 \text{ GeV}$, $\lambda_5 = -1.9$, $\tan \beta = 2$

default scale: $\mu_0 = \frac{1}{5}(M_h + M_H + M_{A_0} + 2M_{H^+}) = 361 \text{ GeV}$



Strong dependence of running on renormalization scheme

Conversion between renormalization schemes:

Note: Values of ren. parameters of a model scenario depend on the ren. scheme!

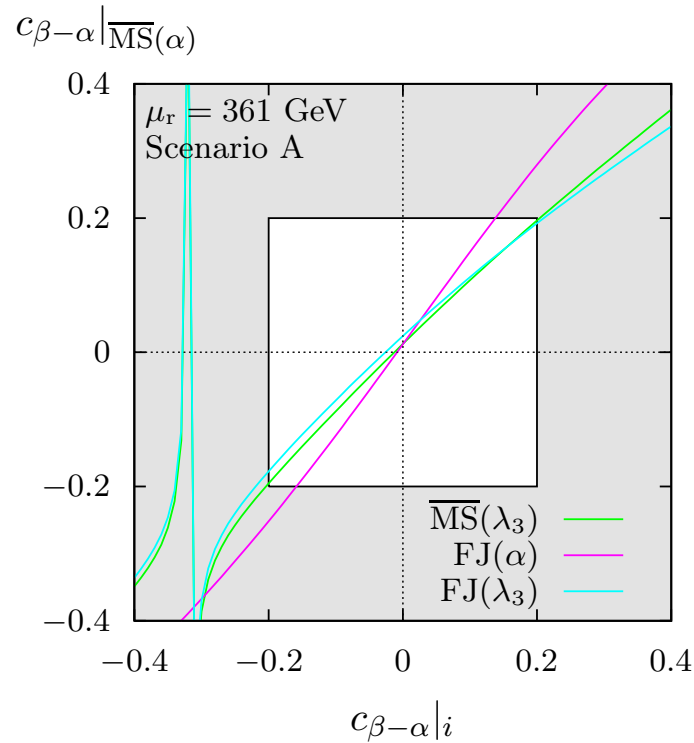
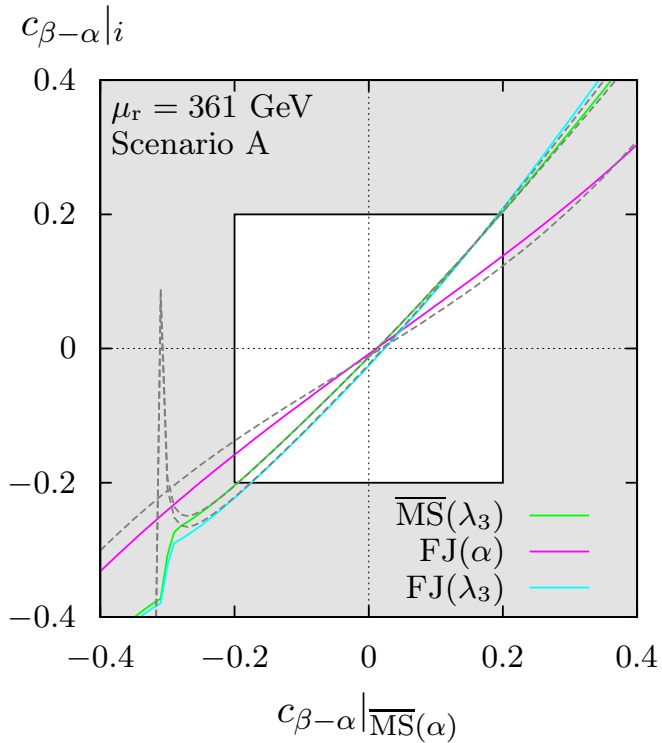
Conversion between schemes (1) and (2) via equality of bare parameters:

$$p_0 = p^{(1)} + \delta p^{(1)}(p^{(1)}) = p^{(2)} + \delta p^{(2)}(p^{(2)})$$

$$\Rightarrow p^{(2)} = p^{(1)} + \delta p^{(1)}(p^{(1)}) - \delta p^{(2)}(p^{(2)}) \stackrel{\text{NLO}}{=} p^{(1)} + \delta p^{(1)}(p^{(1)}) - \delta p^{(2)}(p^{(1)}) + \dots$$

Example: $c_{\beta-\alpha}$ in low-mass scenario A

Altenkamp et al. '17



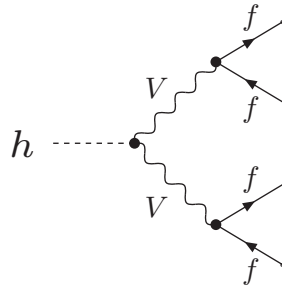
Sizeable
conversion
effects!

NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4\text{fermions}$



Survey of Feynman diagrams for NLO corrections to $h \rightarrow WW/ZZ \rightarrow 4f$

Lowest order:

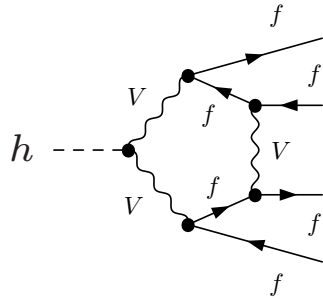


$$= \sin(\beta - \alpha) \mathcal{M}_{\text{SM,LO}}$$

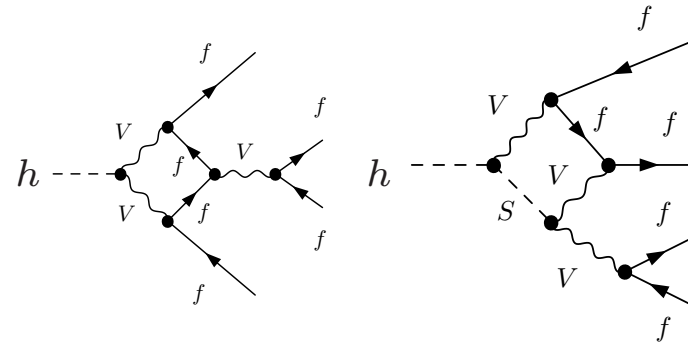
Typical one-loop diagrams:

diagrams = $\mathcal{O}(200-400)$

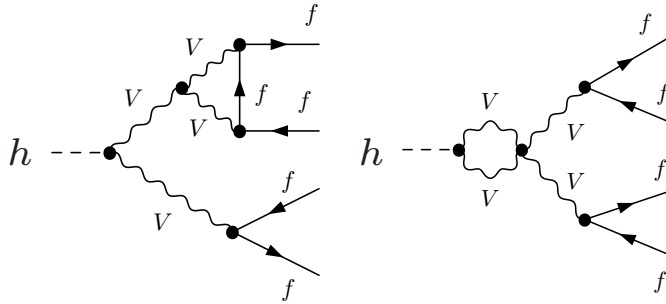
pentagons



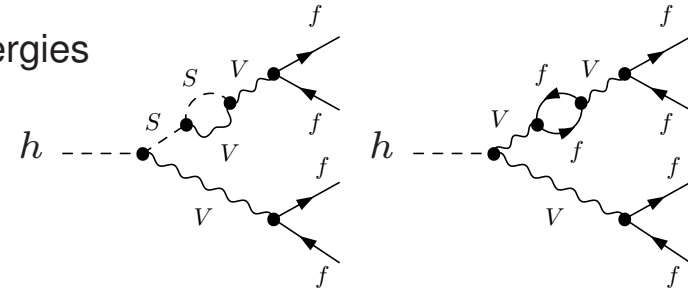
boxes



vertices



self-energies

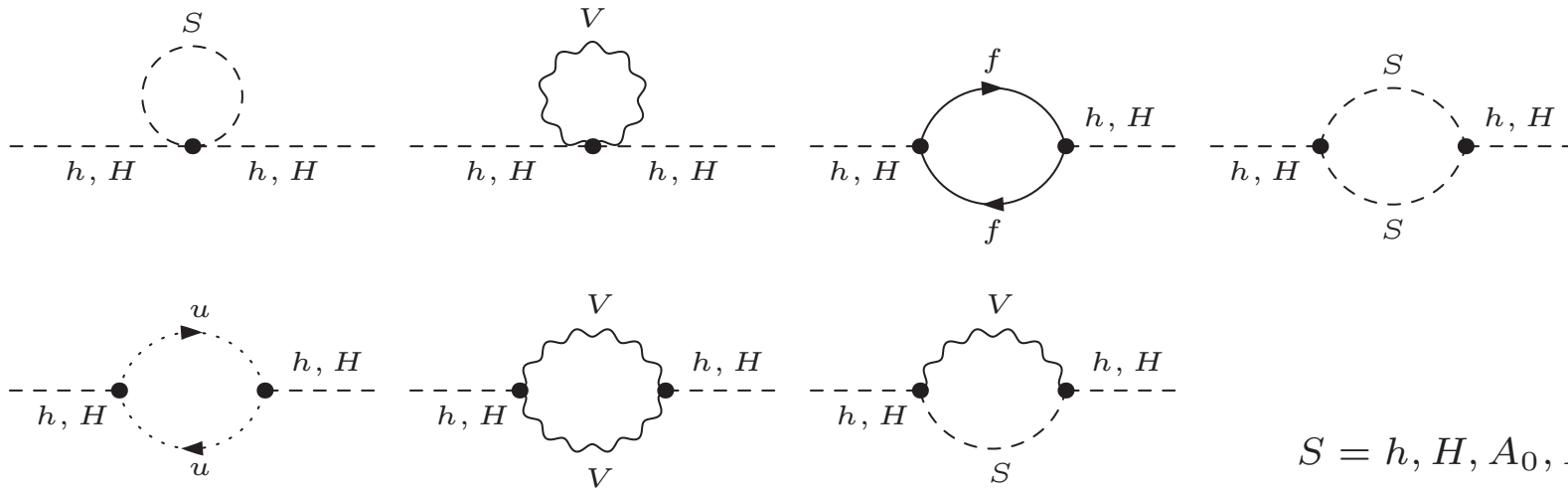


+ counterterms

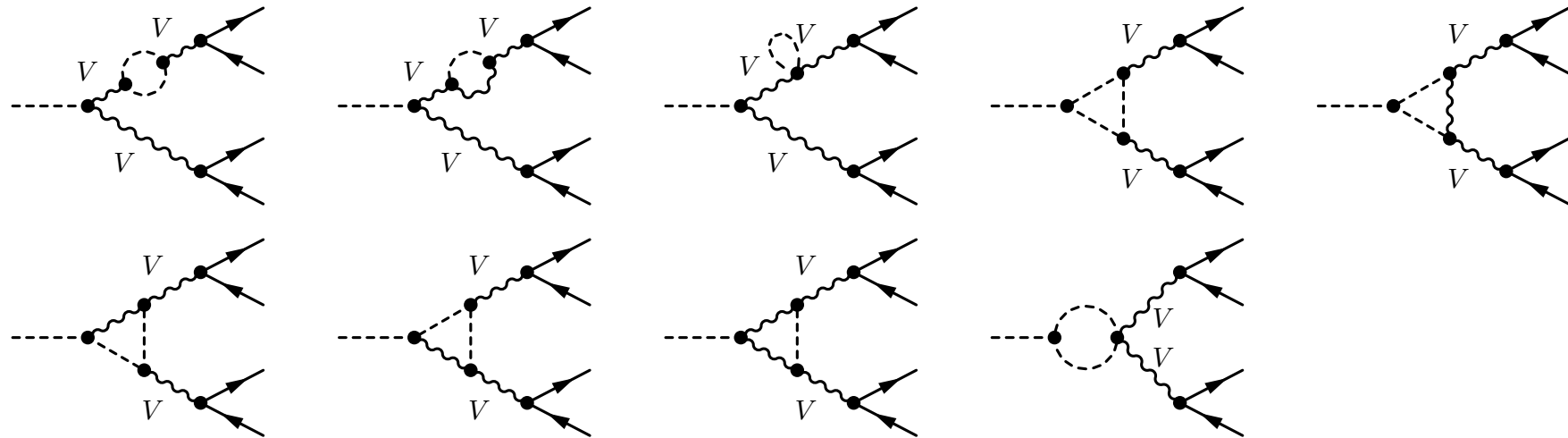
+ tree graphs with real gluon or photons

Generic diagrams for hh, hH, HH self-energies

↪ external wave-function renormalization + hH mixing

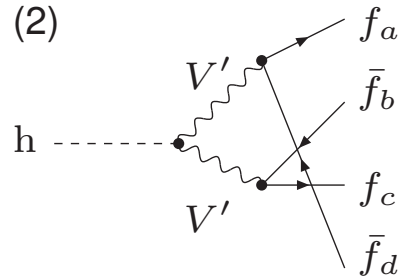
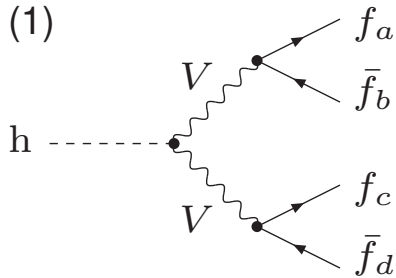


Generic diagrams with internal heavy Higgs bosons H, A₀, H[±]



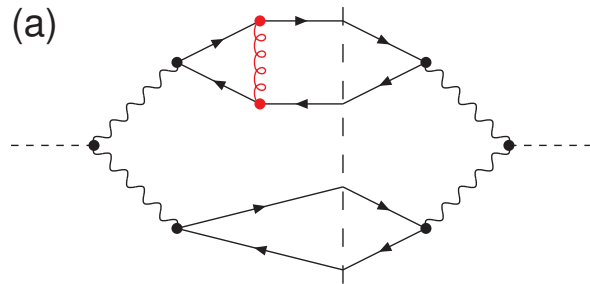
Classification of QCD corrections

Possible Born diagrams:

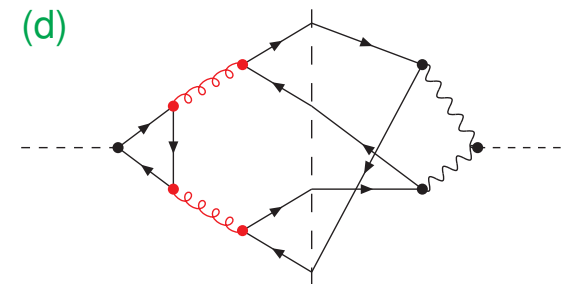
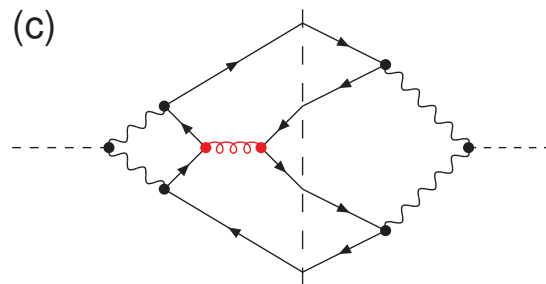
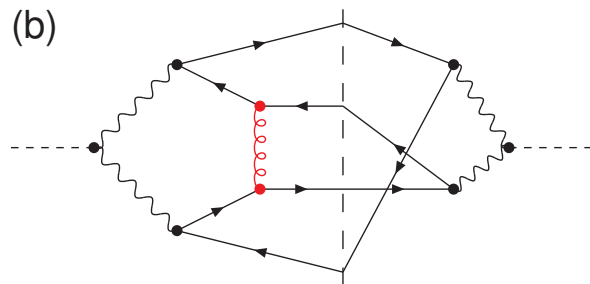


diagrams (2) only for $f\bar{f}f\bar{f}$ and $f\bar{f}f'\bar{f}'$ channels
 (f' = weak-isospin partner of f)

Classification of QCD corrections into four categories: (typical diagrams shown)



(d) only QCD correction without universal scaling $\propto s_{\beta-\alpha}$ from \mathcal{M}_{SM}



(b,c,d) = corrections to interferences (only for $q\bar{q}q\bar{q}$ and $q\bar{q}q'\bar{q}'$ channels)

Details of the NLO calculation

Virtual corrections

- model file generation with **FEYNRULES**
- diagram generation with **FEYNARTS**
- amplitude reduction with inhouse Mathematica routines or **FORMCALC**
- W/Z resonances treated in the *complex-mass scheme* **Denner, S.D., Roth, Wieders '05**
- loop integrals evaluated with **COLLIER**

Real corrections and Monte Carlo integration

- all amplitudes from SM calculation via rescaling with factor $s_{\beta-\alpha}$
- IR singularities treated with dipole subtraction **Catani, Seymour '96; S.D. '99; S.D. et al. '08**
- multi-channel Monte Carlo integration within **PROPHECY4F**

Two independent calculations of all ingredients

- model
- diagra
- amplit
- W/Z
- loop in

- all am
- IR sing
- multi-c

Collier is hosted by Hepforge, IPPP Durham



A Complex One-Loop Library with Extended Regularizations

Authors

Ansgar Denner *Universität Würzburg, Germany*
 Stefan Dittmaier *Universität Freiburg, Germany*
 Lars Hofer *Universitat de Barcelona, Spain*

Released in April 2016!

Features of the library

COLLIER is a fortran library for the numerical evaluation of one-loop scalar and tensor integrals appearing in perturbative relativistic quantum field theory with the following features:

- ✧ scalar and tensor integrals for high particle multiplicities
- ✧ dimensional regularization for ultraviolet divergences
- ✧ dimensional regularization for soft infrared divergences (mass regularization for abelian soft divergences is supported as well)
- ✧ dimensional regularization or mass regularization for collinear mass singularities
- ✧ complex internal masses (for unstable particles) fully supported (external momenta and virtualities are expected to be real)
- ✧ numerically dangerous regions (small Gram or other kinematical determinants) cured by dedicated expansions
- ✧ two independent implementations of all basic building blocks allow for internal cross-checks
- ✧ cache system to speed up calculations

If you use Collier for a publication, please cite all the references listed [here!](#)

LC

oth, Wieders '05

'99; S.D. et al. '08

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Two independent calculations of all ingredients

Prophecy4f

A Monte Carlo generator for a
Proper description of the
Higgs decay into **4 fermions**

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Axel Bredenstein
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Prophecy4f is a Monte Carlo integrator for Higgs decays $H \rightarrow WW/ZZ \rightarrow 4$ fermions

It includes:

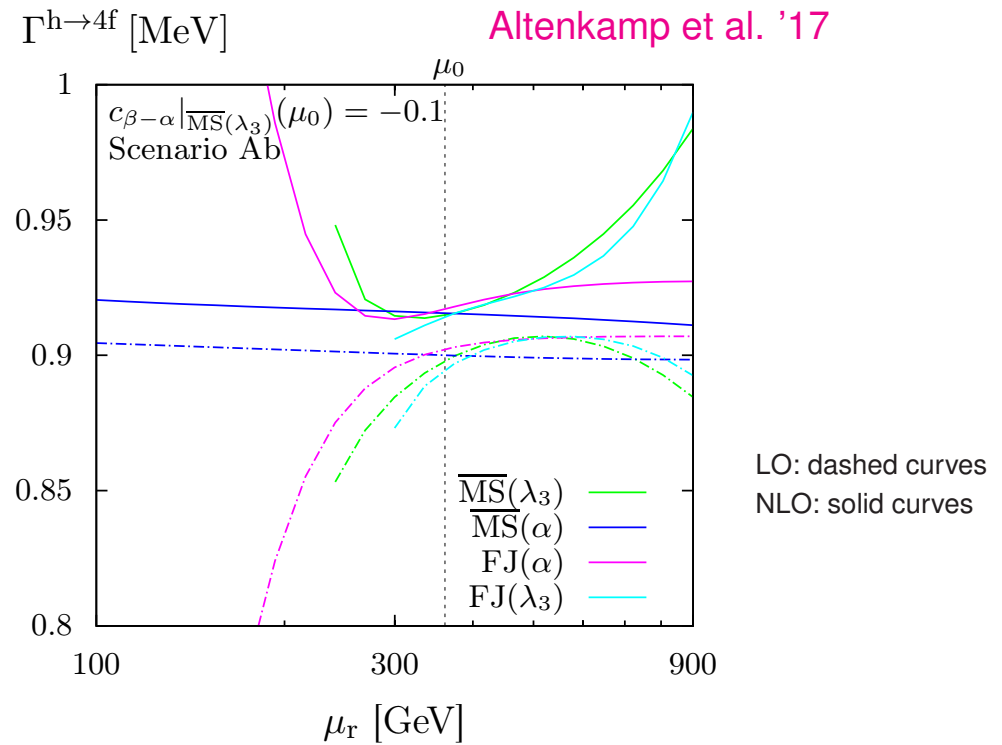
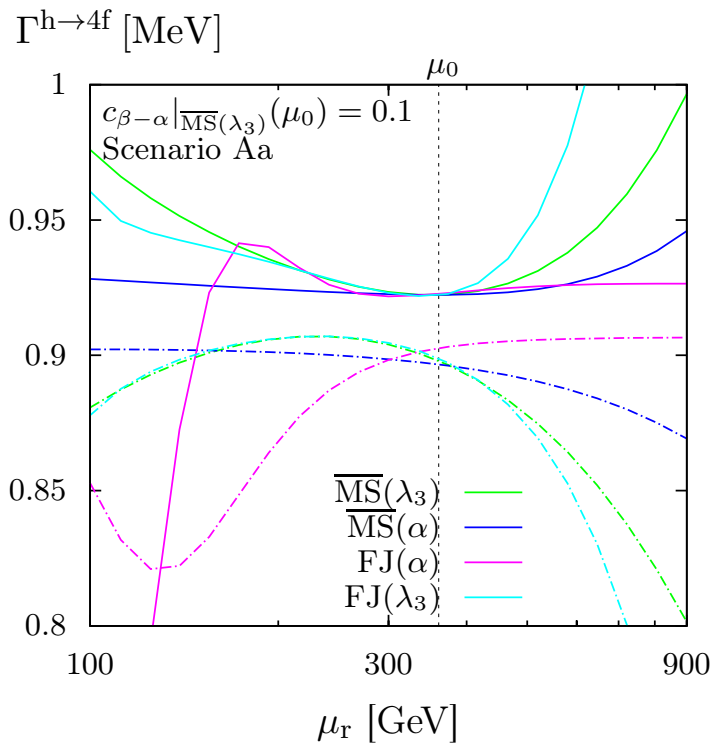
- all four-fermion final states
- NLO QCD and electroweak corrections
- all interferences at LO and NLO
- effects beyond NLO from heavy-Higgs effects
- alternatively an Improved Born Approximation (IBA) with leading effects of the corrections
- production of unweighted events for leptonic final states
- optional inclusion of a 4th fermion generation (w/ or w/o leading two-loop improvements)

↔ **New PROPHECY4F version available on request** (on hepforge soon)

Numerical results



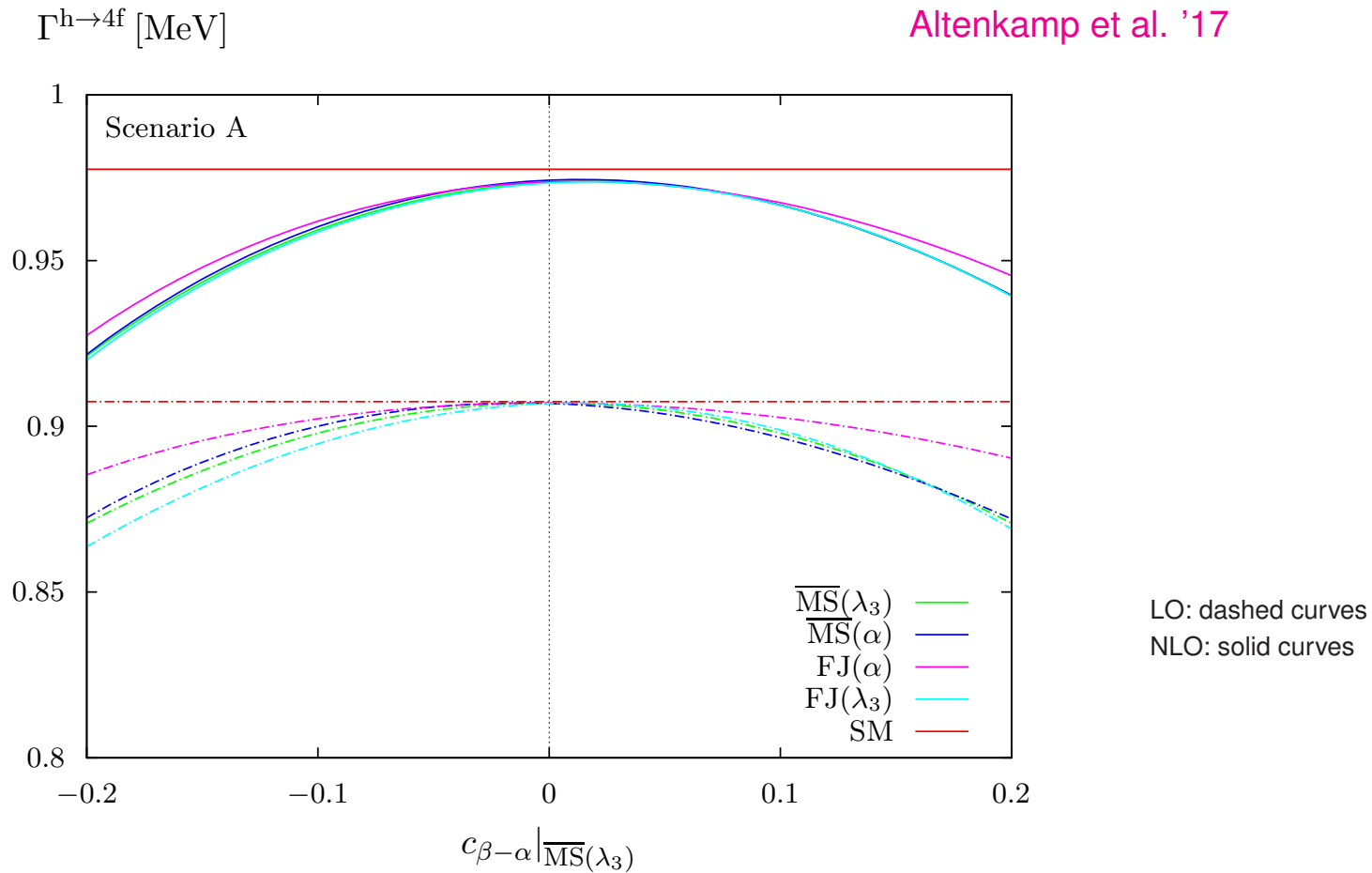
Scale dependence of the $h \rightarrow 4f$ width in scenario A:



- Ren. scale dependence: reduction from LO \rightarrow NLO in all schemes
Note: scale $\mu_r = M_h$ inappropriate
- Ren. scheme dependence: reduction from LO \rightarrow NLO
Note: consistent parameter conversion mandatory!

$c_{\beta-\alpha}$ dependence of $h \rightarrow 4f$ width in scenario A:

Altenkamp et al. '17

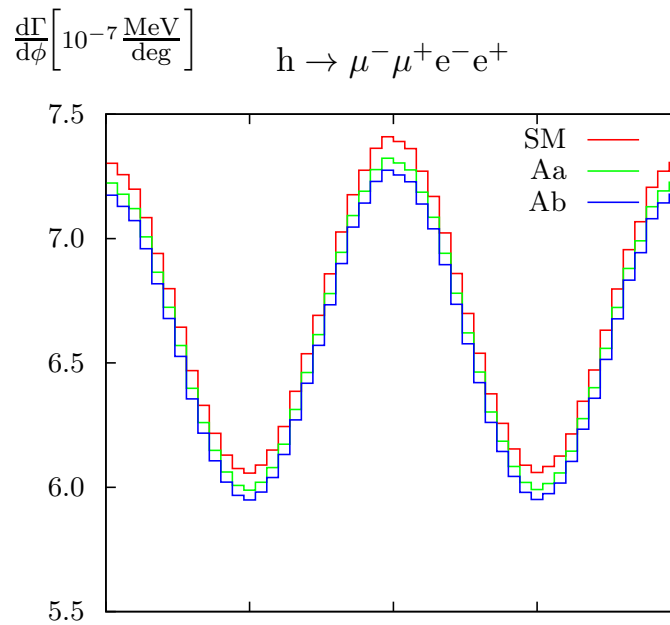
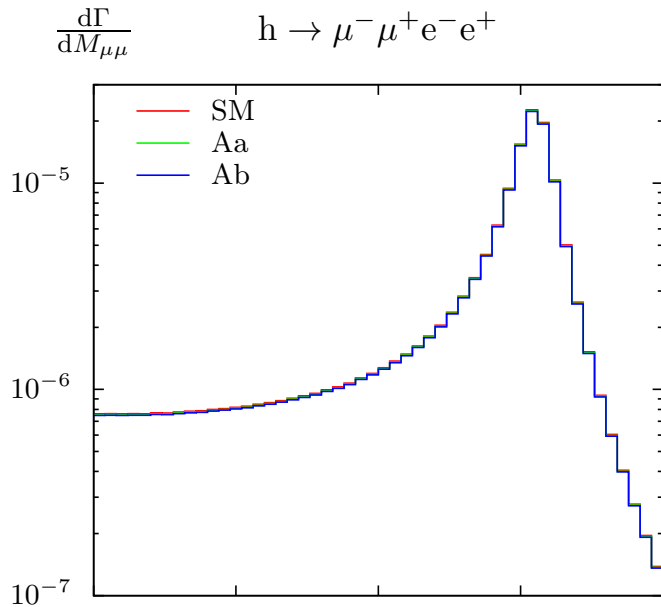


- $\overline{MS}(\lambda_3)$ scheme used $\Rightarrow \Gamma_{\text{THDM,LO}}^{h \rightarrow 4f} |_{\overline{MS}(\lambda_3)} = s_{\beta-\alpha}^2 \Gamma_{\text{SM,LO}}^{h \rightarrow 4f}$
- relative difference to SM: $\Delta_{\text{SM}} \lesssim 2\% (6\%)$ for $|c_{\beta-\alpha}| < 0.1 (0.2)$

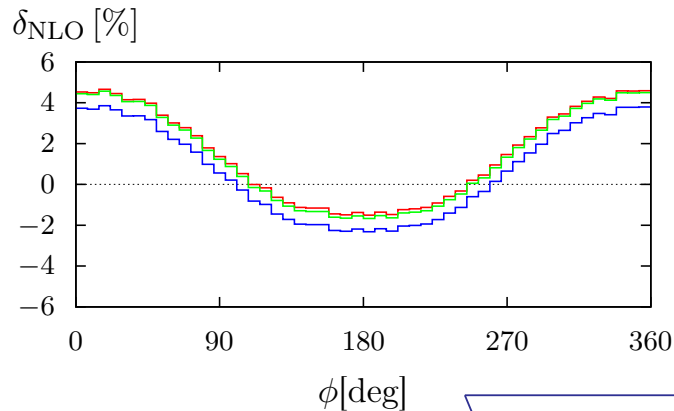
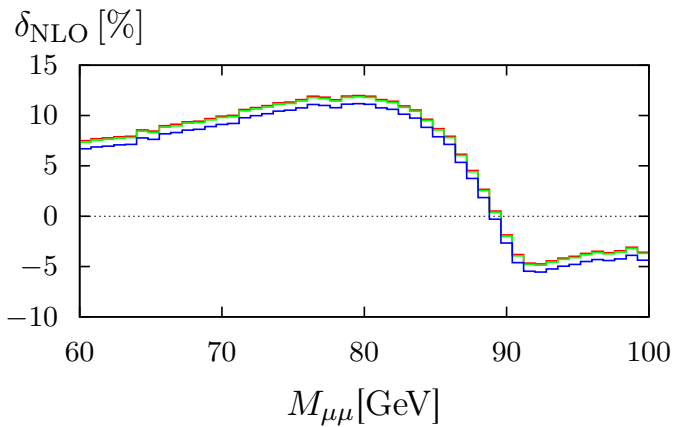
Final state	$\Gamma_{\text{NLO}}^{h \rightarrow 4f}$ [MeV]	δ_{EW} [%]	δ_{QCD} [%]	$\Delta_{\text{SM}}^{\text{NLO}}$ [%]	$\Delta_{\text{SM}}^{\text{LO}}$ [%]
inclusive $h \rightarrow 4f$	0.967297	2.71	4.96	-1.05	-1.00
ZZ	0.106126	0.34	4.88	-1.13	-1.00
WW	0.866304	3.00	5.01	-1.04	-1.00
WW/ZZ int.	-0.005134	1.28	11.99	-0.51	-1.00
$\nu_e e^+ \mu^- \bar{\nu}_\mu$	0.010201	3.03	0.00	-1.04	-1.00
$\nu_e e^+ u \bar{d}$	0.031719	3.02	3.76	-1.04	-1.00
$u \bar{d} s \bar{c}$	0.098465	2.97	7.52	-1.04	-1.00
$\nu_e e^+ e^- \bar{\nu}_e$	0.010197	3.12	0.00	-1.04	-1.00
$u \bar{d} d \bar{u}$	0.100473	2.85	7.35	-1.06	-1.00
$\nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$	0.000949	3.01	0.00	-1.14	-1.00
$e^- e^+ \mu^- \mu^+$	0.000239	1.30	0.00	-1.13	-1.00
$\nu_e \bar{\nu}_e \mu^- \mu^+$	0.000477	2.45	0.00	-1.13	-1.00
$\nu_e \bar{\nu}_e \nu_e \bar{\nu}_e$	0.000569	2.90	0.00	-1.14	-1.00
$e^- e^+ e^- e^+$	0.000132	1.12	0.00	-1.12	-1.00
$\nu_e \bar{\nu}_e u \bar{u}$	0.001679	0.60	3.76	-1.12	-1.00
$\nu_e \bar{\nu}_e d \bar{d}$	0.002177	1.69	3.76	-1.12	-1.00
$e^- e^+ u \bar{u}$	0.000845	0.11	3.76	-1.12	-1.00
$e^- e^+ d \bar{d}$	0.001088	0.47	3.76	-1.12	-1.00
$u \bar{u} c \bar{c}$	0.002971	-1.80	7.51	-1.11	-1.00
$d \bar{d} d \bar{d}$	0.002556	-0.38	4.38	-1.21	-1.00
$d \bar{d} s \bar{s}$	0.004956	-0.36	7.51	-1.12	-1.00
$u \bar{u} s \bar{s}$	0.003852	-0.66	7.51	-1.11	-1.00
$u \bar{u} u \bar{u}$	0.001506	-1.92	4.06	-1.24	-1.00

NLO corrections to leptonic distributions in scenario A

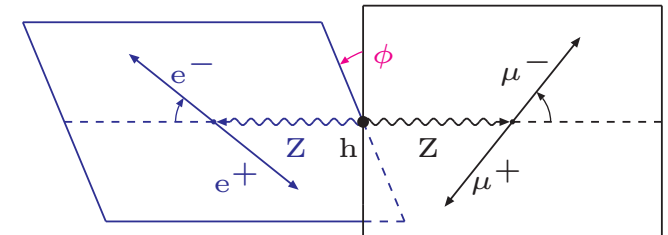
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$\overline{\text{MS}}(\lambda_3)$

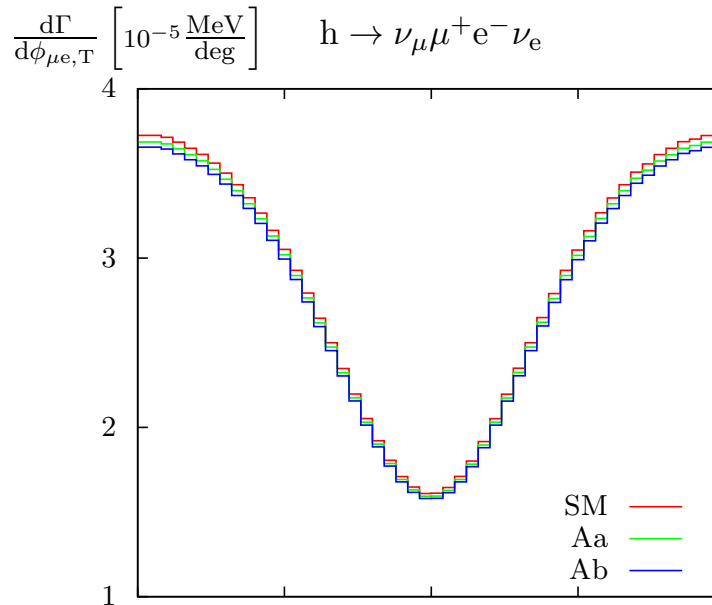
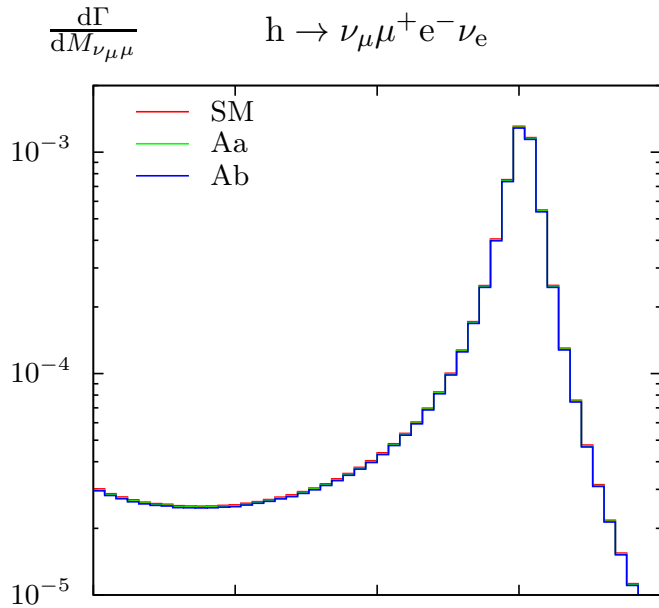


correction $\delta_{\text{THDM}} \approx \delta_{\text{SM}} + \text{const.}$
mainly due to external hH mixing

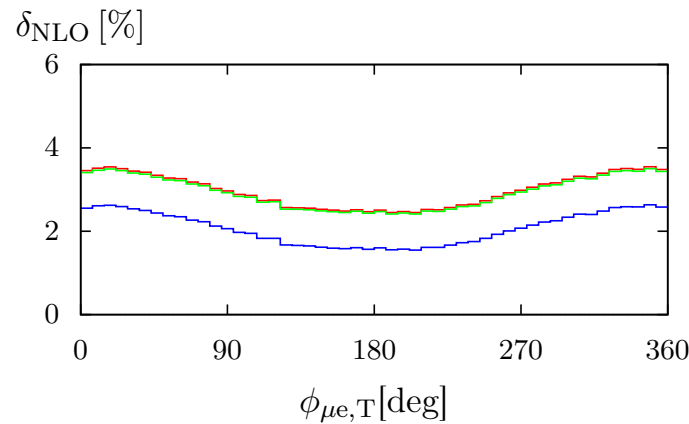
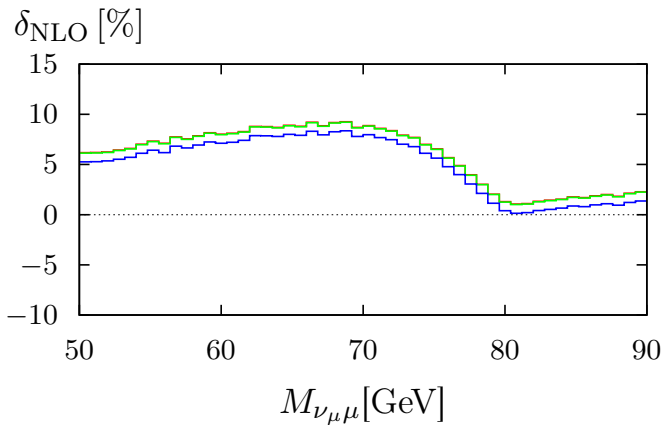


NLO corrections to leptonic distributions in scenario A

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correction $\delta_{\text{THDM}} \approx \delta_{\text{SM}} + \text{const.}$
mainly due to external hH mixing

$\phi_{\Gamma, \mu e} = \angle(\mu, e)$ in a fixed plane \approx (plane \perp beams)

Conclusions



NLO corrections in the THDM

- in principle straightforward with techniques used in the SM
 - **renormalization involves some issues**
 - ◇ choice of input parameters, which ones in $\overline{\text{MS}}$?
 - ◇ gauge dependences, perturbative stability, etc.
- ↪ several schemes proposed and applied in recent literature

$h \rightarrow WW/ZZ \rightarrow 4f$ at NLO in the THDM

- **results presented for a low-mass scenario** ($M_{H,A_0,H^+} \sim 300\text{--}460 \text{ GeV}$)
 - ◇ $|\text{THDM} - \text{SM}| \lesssim 5\%$ for viable THDM parameters $c_{\beta-\alpha}$
 - ◇ significant **reduction in ren. scale and scheme dependence** for LO \rightarrow NLO
 - ◇ no further distortion of distributions in SM \rightarrow THDM at NLO
 - ◇ no sensitivity of $h \rightarrow 4f$ to the type of THDM
- **results for large M_{H,A_0,H^+} in preparation**
 - ◇ results generically similar
 - ◇ but: pathologies for scenarios near exp. exclusion and theoretical bounds

Outlook: similar studies for SM singlet extensions

↪ work in progress within HiggsTools with Michele Boggia (ESR in Freiburg)

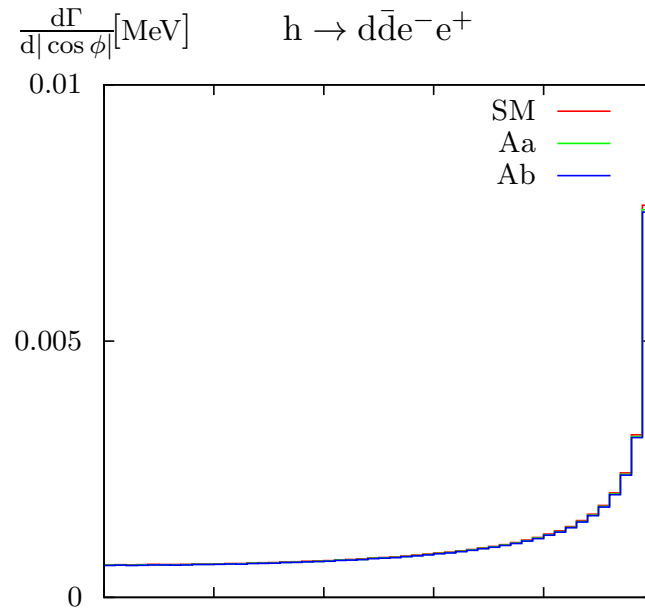
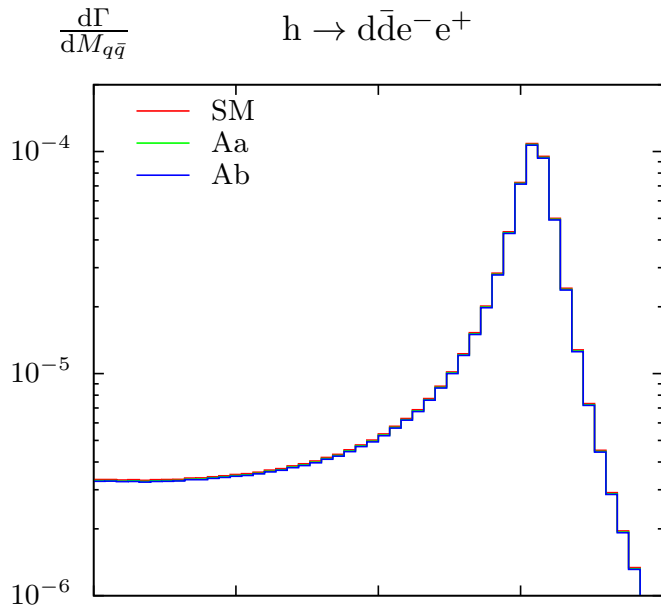
Backup slides



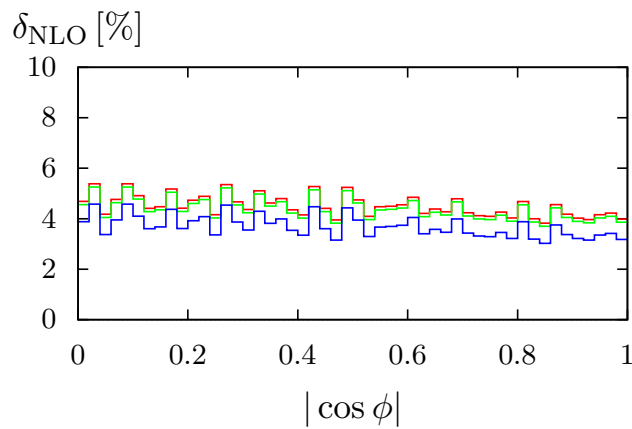
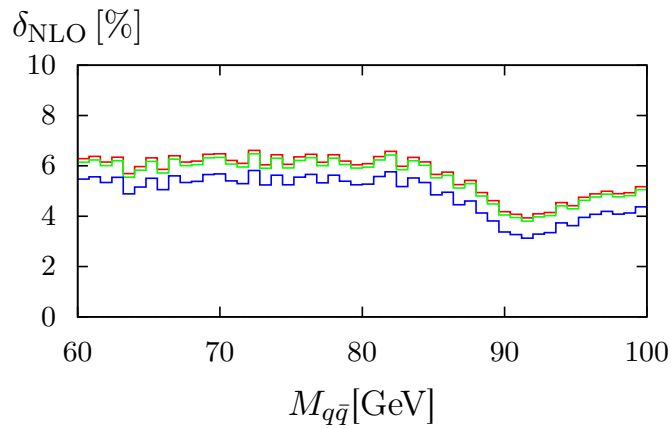
Final state	$\Gamma_{\text{NLO}}^{h \rightarrow 4f}$ [MeV]	δ_{EW} [%]	δ_{QCD} [%]	$\Delta_{\text{SM}}^{\text{NLO}}$ [%]	$\Delta_{\text{SM}}^{\text{LO}}$ [%]
inclusive $h \rightarrow 4f$	0.959800	1.87	4.97	-1.82	-1.00
ZZ	0.105464	-0.34	4.90	-1.75	-1.00
WW	0.859376	2.14	5.01	-1.83	-1.00
WW/ZZ int.	-0.005040	0.51	10.70	-2.32	-1.00
$\nu_e e^+ \mu^- \bar{\nu}_\mu$	0.010116	2.17	0.00	-1.87	-1.00
$\nu_e e^+ u \bar{d}$	0.031463	2.16	3.76	-1.84	-1.00
$u \bar{d} s \bar{c}$	0.097695	2.11	7.52	-1.81	-1.00
$\nu_e e^+ e^- \bar{\nu}_e$	0.010112	2.27	0.00	-1.87	-1.00
$u \bar{d} d \bar{u}$	0.099720	1.99	7.38	-1.80	-1.00
$\nu_e \bar{\nu}_e \nu_\mu \bar{\nu}_\mu$	0.000943	2.34	0.00	-1.78	-1.00
$e^- e^+ \mu^- \mu^+$	0.000237	0.62	0.00	-1.79	-1.00
$\nu_e \bar{\nu}_e \mu^- \mu^+$	0.000474	1.78	0.00	-1.78	-1.00
$\nu_e \bar{\nu}_e \nu_e \bar{\nu}_e$	0.000565	2.23	0.00	-1.79	-1.00
$e^- e^+ e^- e^+$	0.000131	0.45	0.00	-1.78	-1.00
$\nu_e \bar{\nu}_e u \bar{u}$	0.001668	-0.08	3.76	-1.76	-1.00
$\nu_e \bar{\nu}_e d \bar{d}$	0.002163	1.02	3.76	-1.76	-1.00
$e^- e^+ u \bar{u}$	0.000840	-0.57	3.76	-1.77	-1.00
$e^- e^+ d \bar{d}$	0.001081	-0.21	3.76	-1.76	-1.00
$u \bar{u} c \bar{c}$	0.002952	-2.48	7.51	-1.75	-1.00
$d \bar{d} d \bar{d}$	0.002545	-1.06	4.57	-1.67	-1.00
$d \bar{d} s \bar{s}$	0.004925	-1.04	7.51	-1.74	-1.00
$u \bar{u} s \bar{s}$	0.003828	-1.35	7.51	-1.74	-1.00
$u \bar{u} u \bar{u}$	0.001500	-2.60	4.31	-1.65	-1.00

NLO corrections to semileptonic distributions in scenario A

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$\overline{\text{MS}}(\lambda_3)$



NLO corrections to semileptonic distributions in scenario A

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$\overline{\text{MS}}(\lambda_3)$

