COMBINED RESUMMATION AND THE HIGGS p_T SPECTRUM

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UNIVERSITÀ DI MILANO & INFN

3RD ANNUAL HIGGSTOOLS MEETING

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SUMMARY

- THE p_T DISTRIBUTION & RESUMMATION
  - why is it interesting?
  - information from resummation: kinematic regions
- HIGH-ENERGY RESUMMATION OF p_t DISTRIBUTIONS
  - the structure of resummed results
  - mass dependence from resummation
- COMBINED RESUMMATION
  - high energy vs. soft in combination with p_T
  - consistent transverse momentum resummation

THE HIGGS p_t DISTRIBUTION & COMBINED RESUMMATION

RESOLVING HIGGS COUPLINGS

THE DEGENERACY

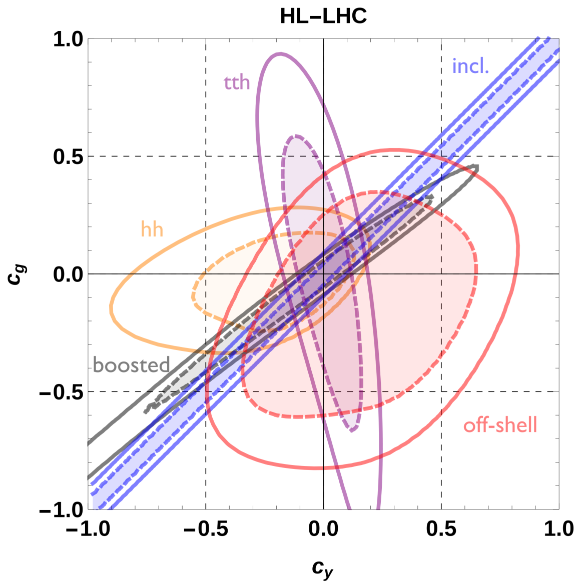
- NEW STATES MOST LIKELY REVEALED THROUGH HIGGS-TOP COUPLINGS
- DEGENERACY IN HIGGS COUPLINGS TO DIMENSION 6 OPERATORS:

$$\mathcal{L} = -c_t \frac{m_t}{v} \bar{t}th + \frac{g_s^2}{48\pi^2} c_g \frac{h}{v} G_{\mu\nu} G^{\mu\nu}$$

- TOTAL CROSS-SECTIONS ONLY DEPENDS ON $c_g + c_t$

LIFTING THE DEGENERACY

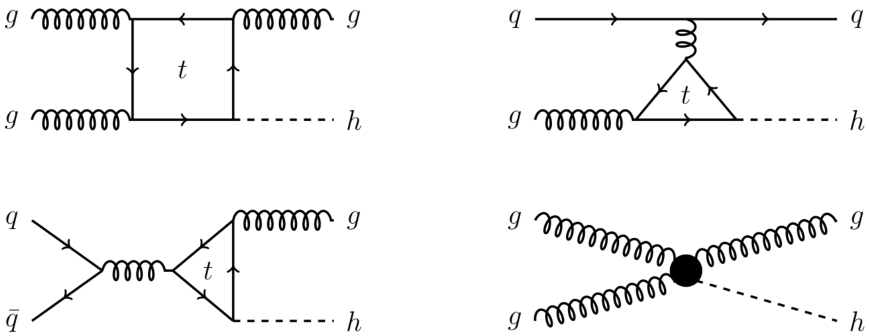
EXCLUSION REGIONS AT HL-LHC



(Azatov, Paul, Grojean, Salvioni, 2016)

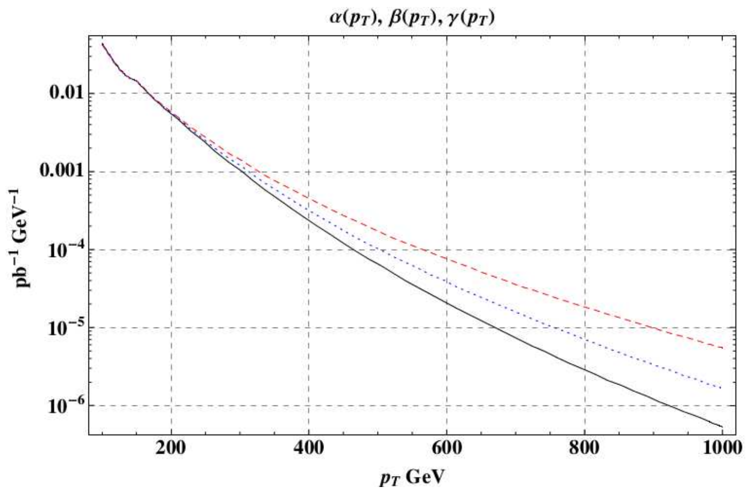
- DOUBLE HIGGS
- OFF-SHELL
- PRODUCTION IN ASSOCIATION WITH TOP
- BOOSTED

BOOSTED HIGGS



(Grojean, Salvioni, Schlaffer, Weiler, 2013)

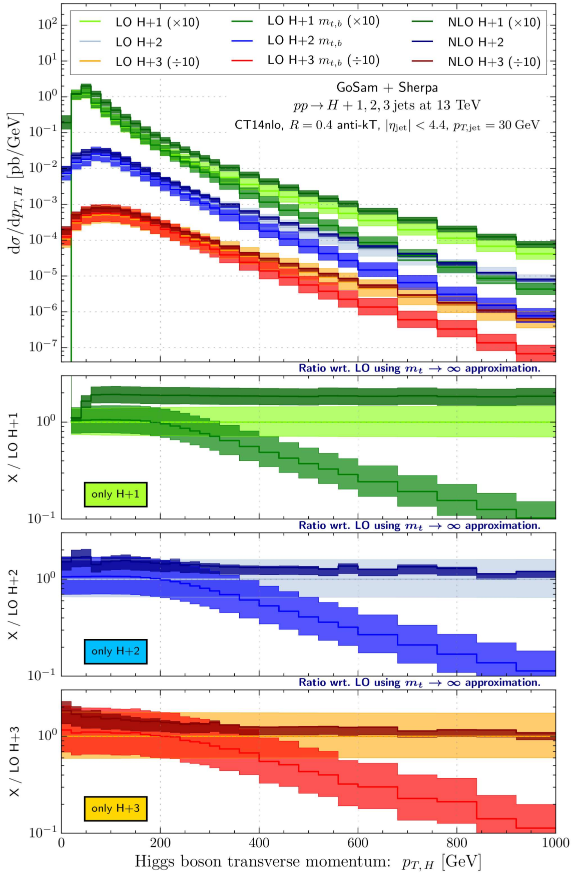
- HIGGS+ JET CROSS SECTION: EFFECTIVE c_g INDUCED BY CONTACT TERM
- CROSS-SECTION $\frac{d\sigma}{dp_T} = \alpha(p_T)c_t^2 + 2\gamma(p_t)c_t c_g + \beta(p_T)c_g^2$
- CAN DISENTAGLE CONTACT TERM BY SLOWER DROP WITH p_T



(Azatov, Paul, 2013)

THE HIGGS TRANSVERSE-MOMENTUM SPECTRUM
QUARK MASS SENSITIVITY

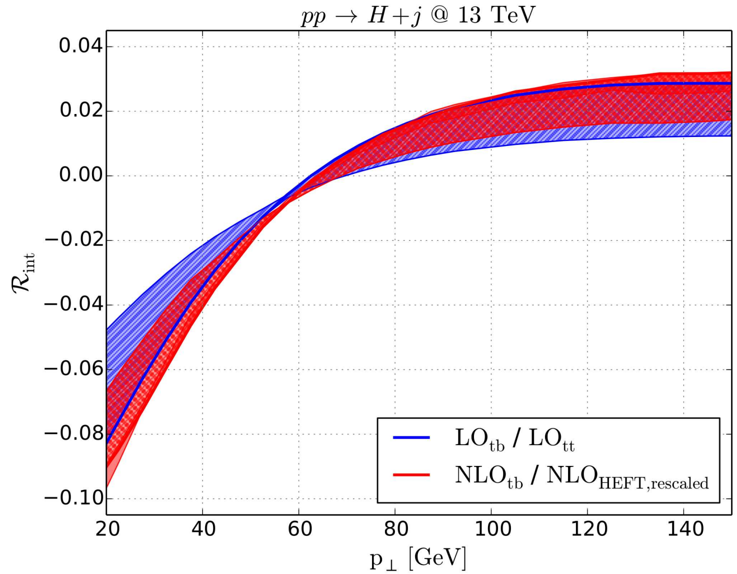
H+N JETS: EFT VS. EXACT m_t DEP.



(Greiner, Höche, Luisoni, Schönherr, Winter, 2016)

EFT BREAKS DOWN
AT LARGE $p_T \sim 200$ GeV

TOP-BOTTOM INTERFERENCE



(Lindert, Melnikov, Tancredi, Wever, 2017)

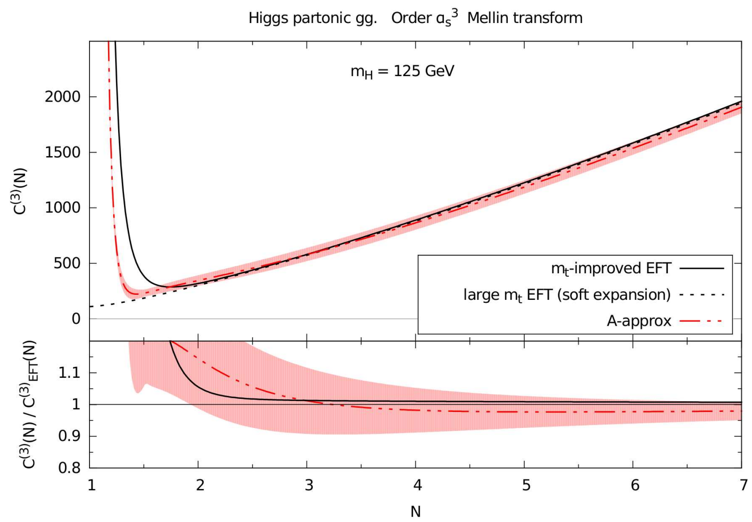
SIZABLE B-MASS
EFFECTS AT LOW p_T

APPROXIMATION FROM RESUMMATION

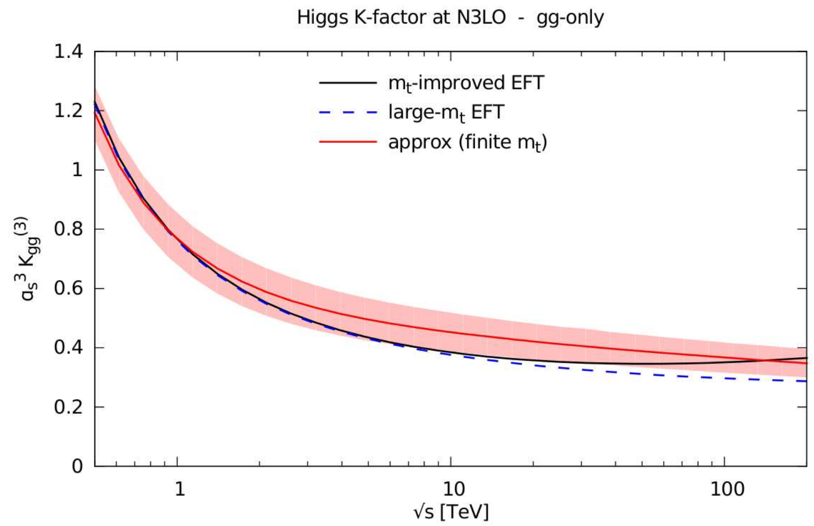
- TOTAL PARTONIC CROSS-SECTION $\Rightarrow$ DISTRIBUTION IN $\tau = \frac{m_h^2}{s}$
- MELLIN TRANSFORM W.R. TO $\tau \Rightarrow$ ANALYTIC FUNCTION OF COMPLEX MELLIN N
- SOFT RESUMMATION $\rightarrow N \rightarrow \infty$ (LOG) SINGULARITY
- HIGH-ENERGY RESUMMATION $\rightarrow$ RIGHTMOST N POLES
- TOP MASS FULLY INCLUDED $\Rightarrow$ CAN IMPROVE EFT RESULTS

(Marzani, Ball, Del Duca, sf, Vicini, 2008; Ball, Bonvini, sf, Marzani, Ridolfi, 2013)

N^3LO COEFFICIENT FUNCTION (MELLIN SPACE)



N^3LO K-FACTOR



(Bonvini, Marzani, Muselli, Rottoli 2016)

CAN WE DO IT AT A MORE DIFFERENTIAL LEVEL?

TRANSVERSE MOMENTUM: KINEMATICS

- INCLUSIVE KINEMATICS: $Q^2 = M_H^2$, $\tau = \frac{M^2}{s}$, $\hat{s} = x_1 x_2 s$;

  AT FIXED p_T, $\hat{s} \geq \hat{s}_{\min} = \left(\sqrt{M^2 + p_{\rm T}^2} + p_{\rm T}\right)^2$: $\Rightarrow$ INTEGRAL OVER x_1, x_2 NOT A CONVOLUTION

  $\tau \leq \tau_{\max} = \frac{\sqrt{M^2+p_T^2} - \sqrt{p_t^2}}{M^2}$

- DIFFERENTIAL KINEMATICS: $Q^2 = \left(\sqrt{M^2 + p_{\rm T}^2} + p_{\rm T}\right)^2$, $\tau' = \frac{Q^2}{s}$; $\xi_p = \frac{p_{\rm T}^2}{M^2}$ CONVOLUTION!

$$\frac{d\sigma}{d\xi_p}\left(\tau', \xi_p, M^2\right) = \tau' \sum_{ij} \int_{\tau'}^1 \frac{dx}{x} \mathcal{L}_{ij}\left(\frac{\tau'}{x}, \mu_{\rm F}^2\right) \frac{1}{x} \frac{d\hat{\sigma}_{ij}}{d\xi_p}\left(x, \xi_p, \alpha_s\left(\mu_{\rm R}^2\right), \mu_{\rm F}^2\right)$$

- MELLIN SPACE FACTORIZATION:

$$\frac{d\sigma}{d\xi_p}\left(N, \xi_p, M^2\right) = \sum_{ij} \mathcal{L}_{ij}\left(N+1, \mu_{\rm F}^2\right) \frac{d\hat{\sigma}_{ij}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_{\rm R}^2\right), \mu_{\rm F}^2\right)$$

- TOTAL VS DIFFERENTIAL: MOMENTUM SPACE (HADRONIC):

$$\sigma(\tau) = \int_0^{\frac{(1-\tau)^2}{4\tau}} d\xi \, \frac{d\sigma}{d\xi}(\tau, \xi)$$

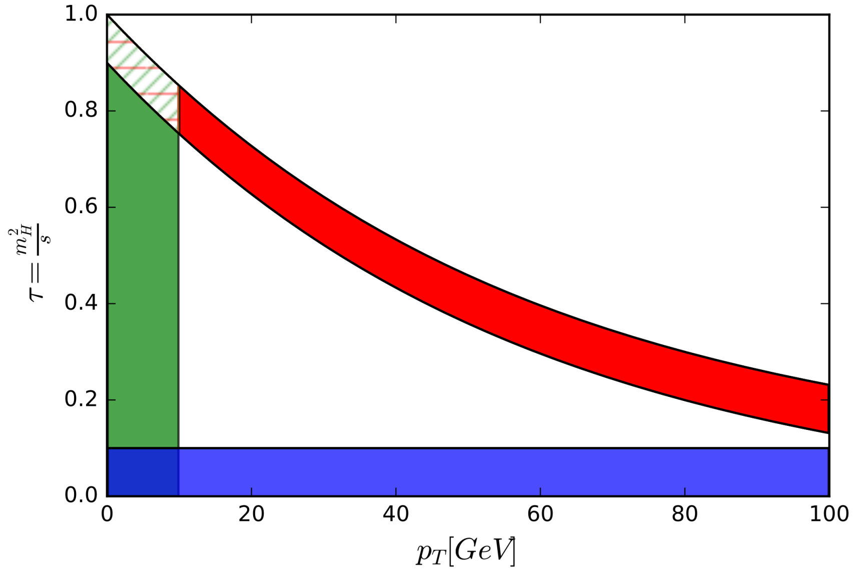
- TOTAL VS DIFFERENTIAL: MELLIN SPACE:

$$\hat{\sigma}_{ij}(N) = \int_0^\infty d\xi_p \left(\sqrt{1+\xi_p} - \sqrt{\xi_p}\right)^{2N} \frac{d\hat{\sigma}_{ij}}{d\xi_p}(N, \xi_p)$$

  EXTRA FACTOR IN MELLIN TRANSFORM

KINEMATIC REGIONS

INCLUSIVE KINEMATICS



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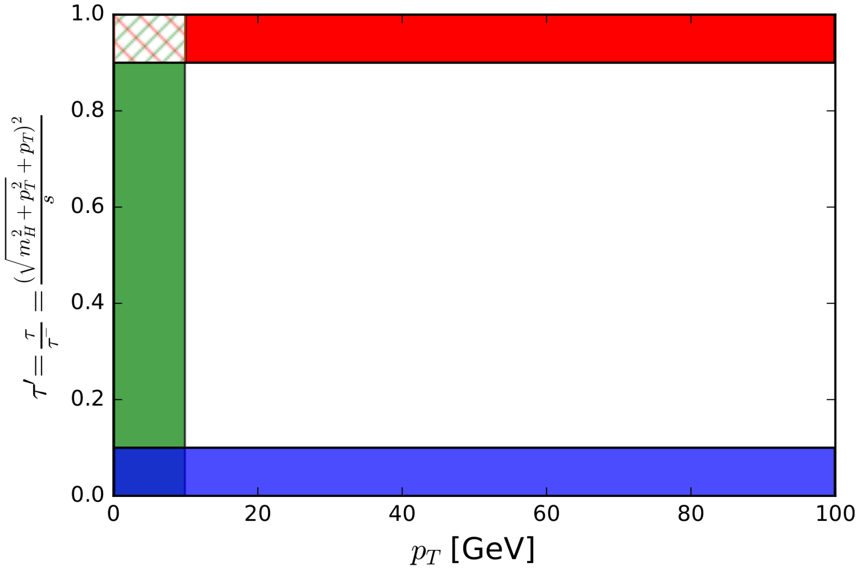
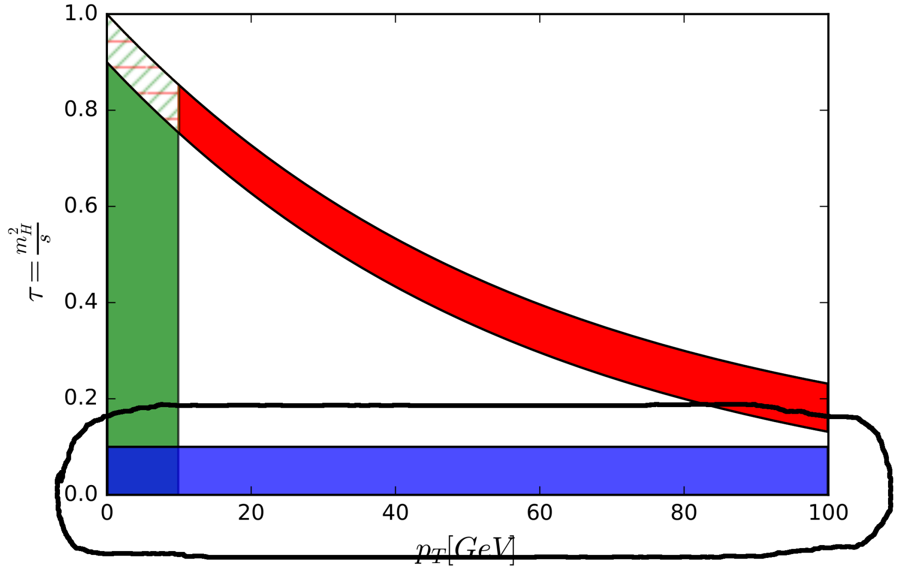


- HIGH ENERGY: $\tau \sim \tau' << 1$
- SOFT: $\tau \sim \tau_{\max} = \frac{\sqrt{M^2 + p_T^2} - \sqrt{p_t^2}}{M^2}$, $\tau' \sim 1$
- SMALL p_T: $p_t \lesssim m$
- JOINT HIGH ENERGY AND SMALL p_T
- JOINT SOFT AND SMALL p_T

KINEMATIC REGIONS

INCLUSIVE KINEMATICS



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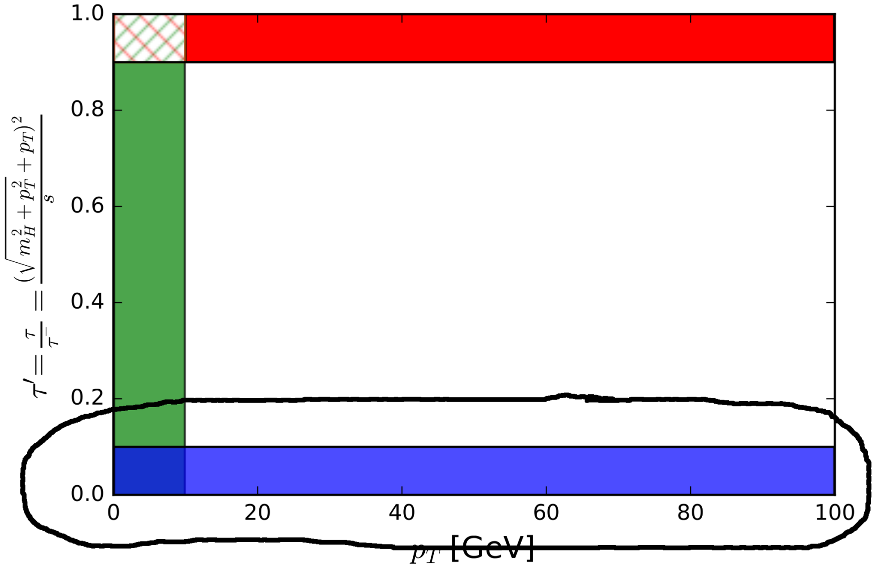

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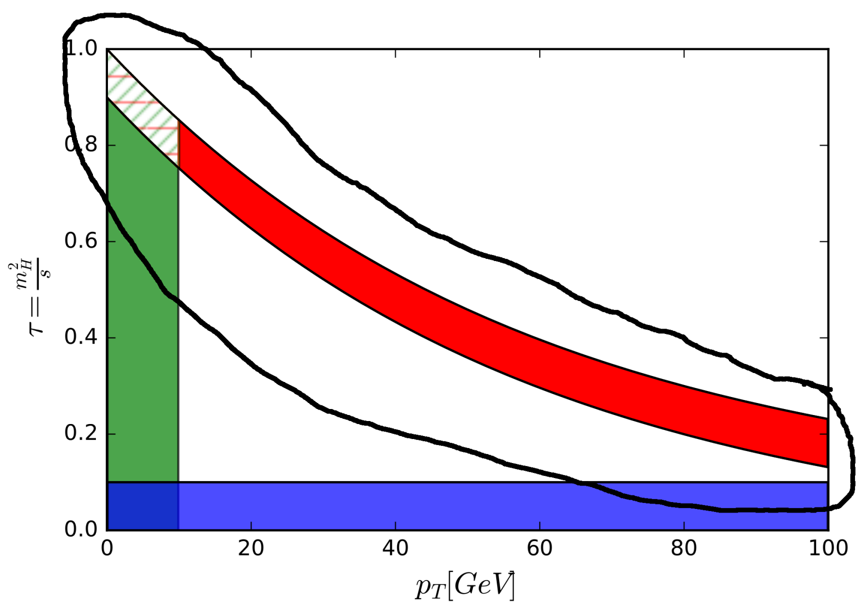

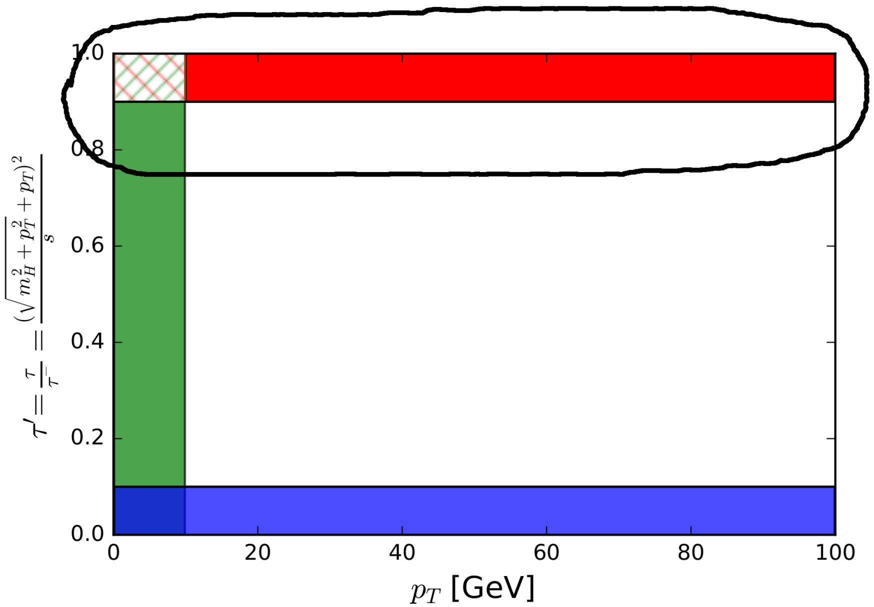

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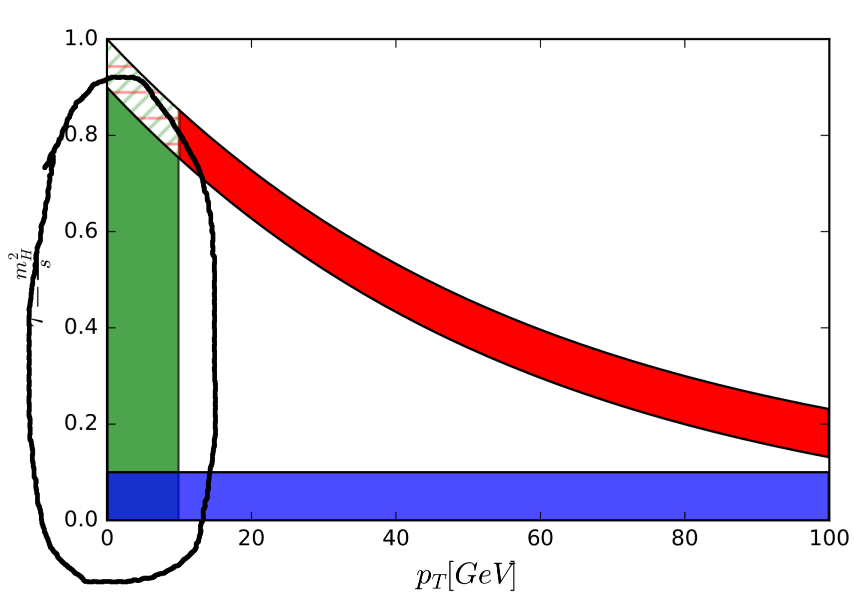

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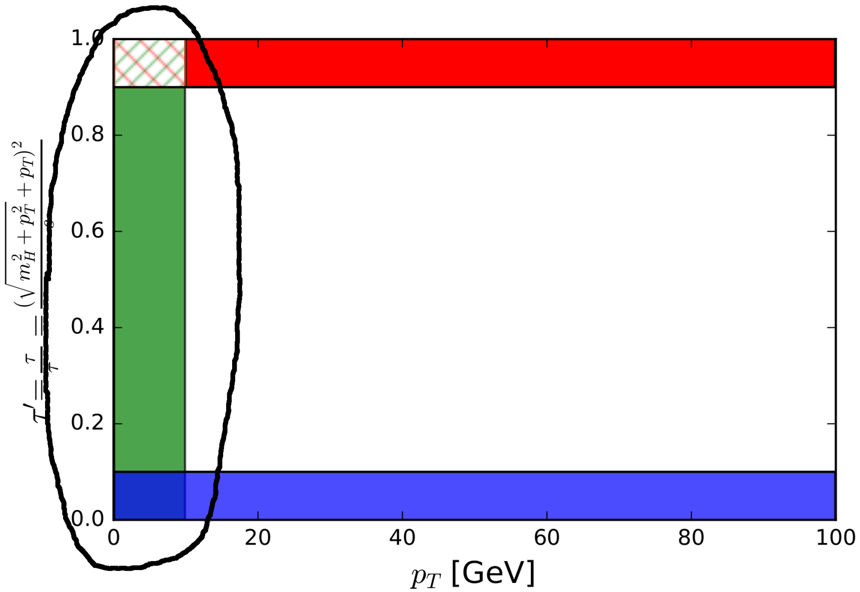

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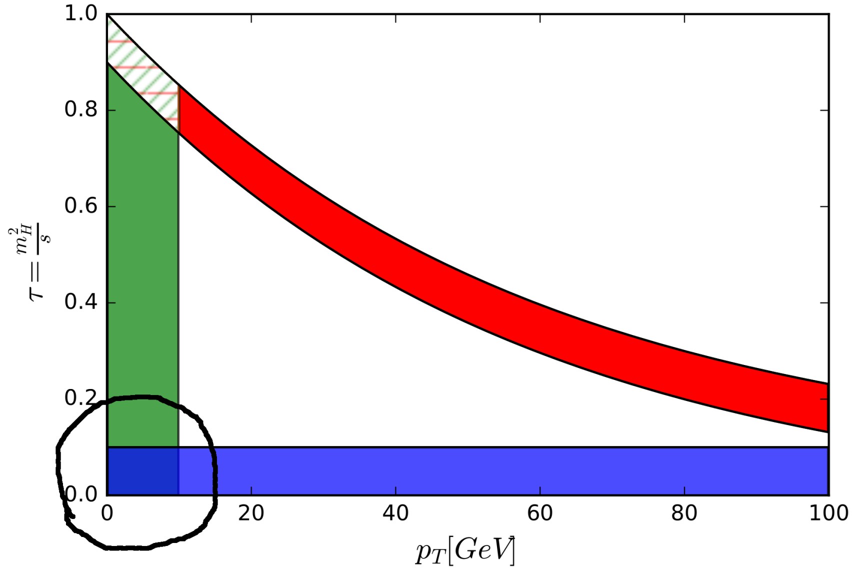

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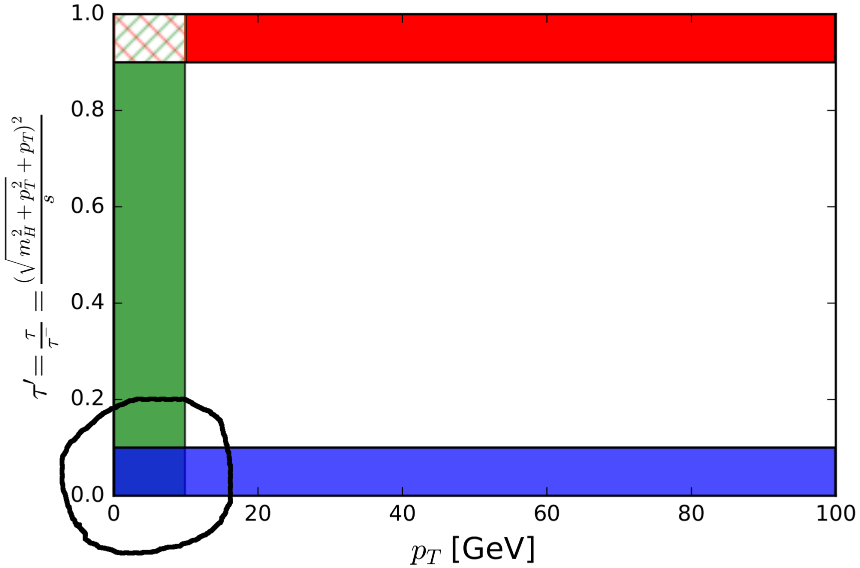

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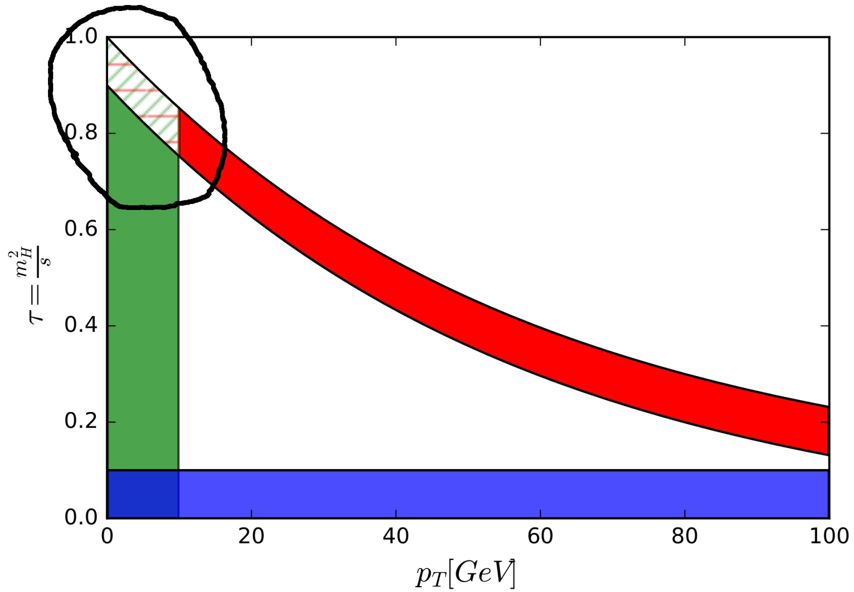

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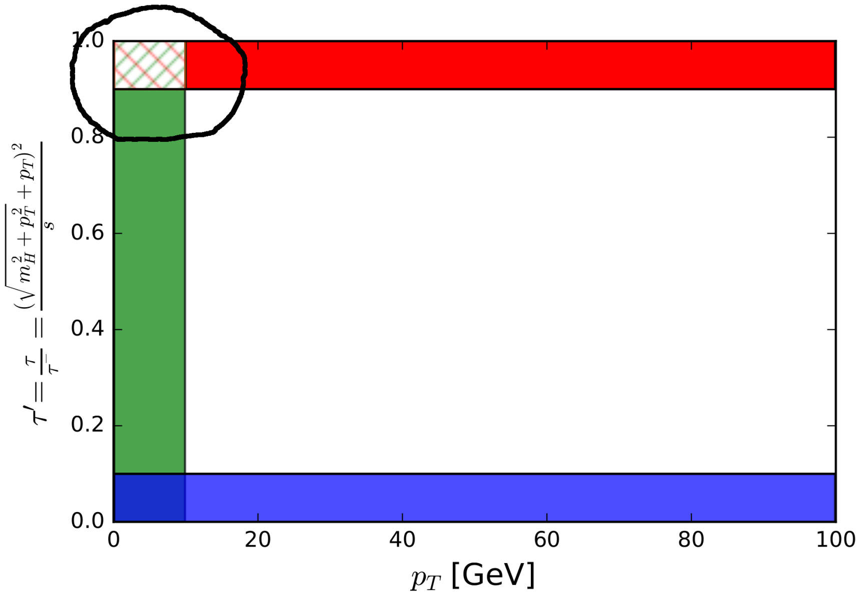

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TASKS

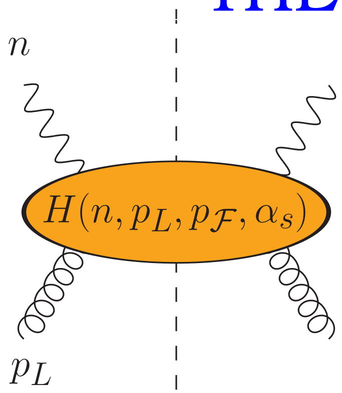
- PERFORM HIGH-ENERGY RESUMMATION OF TRANSVERSE MOMENTUM DISTRIBUTIONS
- COMBINE HIGH-ENERGY AND SMALL p_T RESUMMATION
- PERFORM SMALL p_T RESUMMATION CONSISTENTLY WITH SOFT RESUMMATION $\rightarrow$ INTEGRAL OF RESUMMED p_T DISTRIBUTION EQUAL TO SOFT-RESUMMED TOTAL CROSS SECTION (see Laenen, Sterman, Vogelsang, 2001)
- COMBINE CONSISTENT SMALL p_T RESUMMATION WITH SOFT RESUMMATION AT THE DIFFERENTIAL LEVEL (see Lustermans, Waalewijn, Zeune, 2016)

HIGH ENERGY RESUMMATION OF p_T DISTRIBUTIONS

HIGH ENERGY RESUMMATION: A LONG STORY

- ANOMALOUS DIMENSIONS: SMALL x TERMS IN DGLAP RESUMMED TO ALL ORDERS AT THE LEADING AND SUBLEADING LEVEL (BFKL 1975-1976, Fadin-Lipatov 1998)
- GENERAL RESUMMATION THEORY: TO LLx, FOR PHOTO-, ELECTRO- AND HADROPRODUCTION (Catani, Ciafaloni, Hautmann, 1991)
- ELECTROPRODUCTION: SMALL x CORRECTIONS TO DEEP-INELASTIC COEFFICIENT FUNCTION KNOWN AT THE LEADING NONTRIVIAL LEVEL FOR INCLUSIVE DIS (Catani, Hautmann, 1994) AND HQ PRODUCTION (Catani, Ciafaloni, Hautmann, 1991)
- DGLAP RESUMMATION: TWO ALTERNATIVE APPROACHES (1998-2004)
  - SMALL x RESUMMATION OF DGLAP (Altarelli, Ball, sf)
  - INCLUSION OF FIXED-ORDER DGLAP INTO BFKL(Ciafaloni, Colferai, Salam)
- INCLUSIVE HADRONIC CROSS-SECTIONS: SMALL x CORRECTIONS TO INCLUSIVE HARD CROSS-SECTIONS KNOWN AT THE LEADING NONTRIVIAL LEVEL FOR HQ PRODUCTION (Ball, K.Ellis, 2001); GG→HIGGS EFT (Hautmann, 2002) & FULL HQ MASS DEPENDENCE (Marzani, Ball, Del Duca, sf, Vicini, 2008); DRELL-YAN (Marzani, Ball, 2009); ISOLATED PHOTON (Diana, 10)
- RAPIDITY DISTRIBUTIONS: GENERAL THEORY, & HIGGS IN GLUON FUSION (FULL HQ MASS DEPENDENCE) RESUMMED AT LLx (Caola, sf, Marzani, 2011);
- TRANSVERSE MOMENTUM SPECTRUM DISTRIBUTIONS: GENERAL THEORY,& HIGGS IN GLUON FUSION (FULL HQ MASS DEPENDENCE) RESUMMED AT LLx (sf, Muselli, 2015-2016);

THE OFF-SHELL CROSS-SECTION



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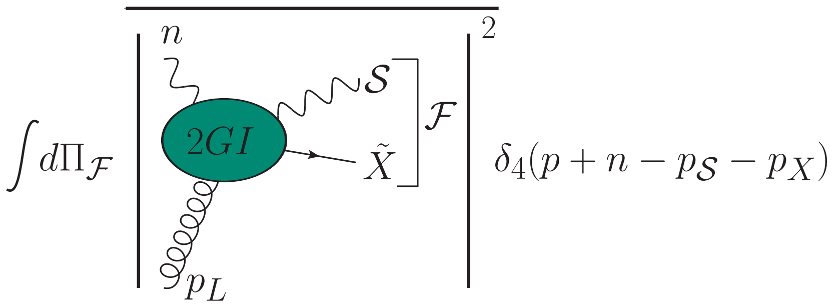


RESUMMATION $\Leftrightarrow$ OFF-SHELL CROSS-SECTION WITH $M = \gamma$ (DUALITY)

(Catani, Ciafaloni, Hautmann , 1991)

$$\sigma_{\rm res}\,(N, \alpha_s) = h\,\left(N, \gamma\left(\tfrac{\alpha_s}{N}\right), \gamma\left(\tfrac{\alpha_s}{N}\right), \alpha_s\right)$$

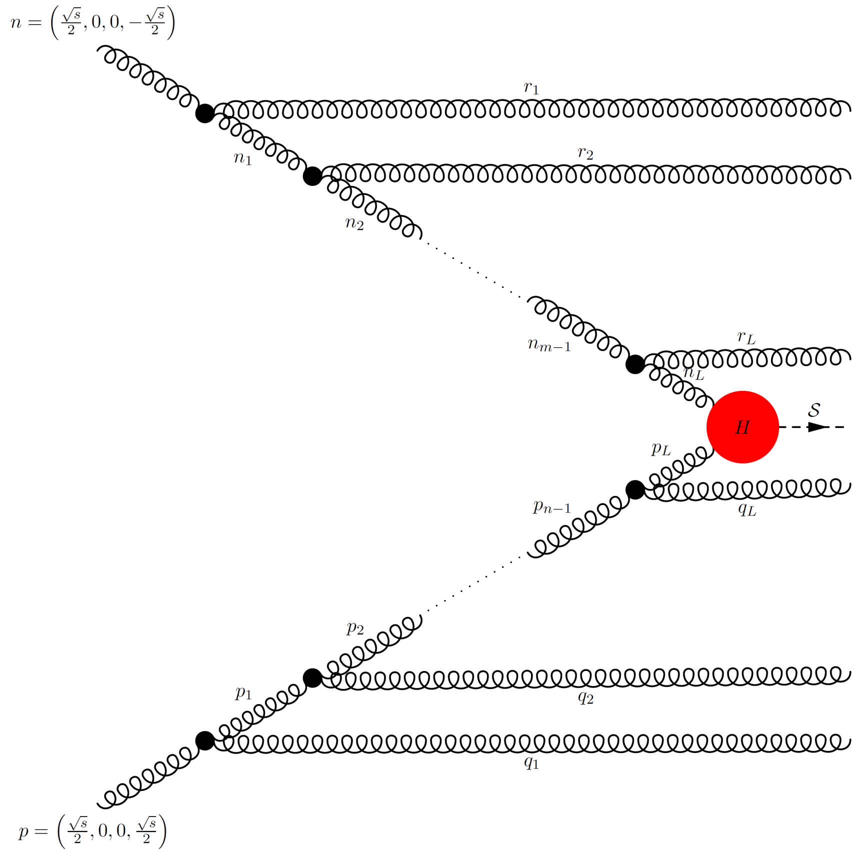
$$h\,(N, M_1, M_2, \alpha_s) = M_1 M_2\, R\,(M_1)\, R\,(M_2) \int_0^\infty d\xi\, \xi^{M_1 - 1} \int_0^\infty d\bar{\xi}\, \bar{\xi}^{M_2 - 1}\, C\,\left(N, \xi, \bar{\xi}, \alpha_s\right)$$

- THE ITERATED KERNEL (ANOMALOUS DIMENSION) EXPONENTIATES
- THE CONVOLUTIONS LOOK LIKE k_t-SPACE MELLIN-TRANSFORMS ($\xi = \frac{k_t^2}{Q^2}$; k_t^2 GLUON OFF-SHELLNESS)

THE TRANSVERSE MOMENTUM DISTRIBUTION

(sf, Muselli, 2015)

$$
\begin{aligned}
p_1 &= z_1\, p - \mathbf{k}_1 \\
q_1 &= (1 - z_1)\, p + \mathbf{k}_1 \\
p_2 &= z_2 z_1\, p - \mathbf{k}_2 \\
q_2 &= (1 - z_1 z_2)\, z_1\, p + \mathbf{k}_2 - \mathbf{k}_1 \\
&\cdots\cdots\cdots \\
p_L &= z p - \mathbf{k} \\
q_L &= (1 - z)\, p + \mathbf{k} - \mathbf{k_{n-1}} \\
\\
n_1 &= \bar{z}_1\, p - \bar{\mathbf{k}}_1 \\
r_1 &= (1 - \bar{z}_1)\, p + \bar{\mathbf{k}}_1 \\
n_2 &= \bar{z}_2 \bar{z}_1\, p - \bar{\mathbf{k}}_2 \\
r_2 &= (1 - \bar{z}_1 \bar{z}_2)\, \bar{z}_1\, p + \bar{\mathbf{k}}_2 - \bar{\mathbf{k}}_1 \\
&\cdots\cdots\cdots \\
n_L &= \bar{z} n - \bar{\mathbf{k}} \\
r_L &= (1 - \bar{z})\, n + \bar{\mathbf{k}} - \bar{\mathbf{k}}_{\mathbf{m-1}}\, .
\end{aligned}
\tag{1}
$$



- LADDER EXPANSION HOLDS ALSO FOR TRANSVERSE MOM. DISTN
- TRANSVERSE MOMENTUM INTEGRATIONS ARE INDEPENDENT OF EACH OTHER
- p_T-DEPENDENCE FINAL STATE DUE TO p_T-DEPENDENCE OF INCOMING OFF-SHELL GLUONS

RESUMMED RESULTS: QUALITATIVE BEHAVIOUR

(sf, Muselli, 2015)

TOTAL CROSS-SECTION

TRANSVERSE MOMENTUM DISTRIBUTION

$$\sigma \underset{x\to 0}{\sim} \sigma_{\rm LO} \times \begin{cases} \delta(1-x) + \sum_{k=1}^{\infty} c_k \alpha_s^k \ln^{2k-1}\frac{1}{x}, & \text{pointl.} \\ \delta(1-x) + \sum_{k=1}^{\infty} d_k \alpha_s^k \ln^{k-1}\frac{1}{x}, & \text{resolved} \end{cases} ; \quad \frac{d\sigma}{d\xi_p} \underset{x\to 0}{\sim} \frac{\sigma_{LO}}{\xi_p} \times \begin{cases} \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1}\frac{1}{x} \sum_{n=0}^{k-1} c_{kn} \ln^n \xi_p, \\ \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1}\frac{1}{x} d_k(\xi_p) \end{cases}$$

$$\xi_p = \frac{p_T^2}{Q^2}$$

- INCLUSIVE XSECT: DOUBLE ENERGY LOGS (POINTLIKE) VS SINGLE LOGS (RESOLVED)
- TRANSVERSE MOMENTUM DISTRIBUTION:
  - SINGLE ENERGY LOGS (ALWAYS) WHOSE COEFFICIENTS ARE
  - POINTLIKE:
    SERIES IN $\ln p_t$ WITH CONSTANT COEFFICIENTS (HARD PART p_t-INDEP.)
  - RESOLVED:
    COEFFICIENTS ARE $d_k(\xi_p)$ FUNCTIONS $\Rightarrow$ VANISH AS $\frac{1}{p_t}$ AT LARGE p_t;
    SMALL p_t: POINTLIKE LIMIT RECOVERED

RESUMMED RESULTS: HIGGS IN POINTLIKE LIMIT

(sf, Muselli, 2015)

IMPACT FACTOR

$$h_{p_{\rm T}} = R(M_1)\,R(M_2)\,\sigma_{\rm LO}\,\frac{\xi_p^{M_1+M_2-1}}{(1+\xi_p)^N}\left[\frac{\Gamma(1+M_1)\Gamma(1+M_2)\Gamma(2-M_1-M_2)}{\Gamma(2-M_1)\Gamma(2-M_2)\Gamma(M_1+M_2)}\left(1+\frac{2M_1M_2}{1-M_1-M_2}\right)\right]$$

SUBSTITUTING $M_1 = \gamma_s\left(\frac{\alpha_s}{N}\right)$ $M_2 = \gamma_s\left(\frac{\alpha_s}{N}\right)$ & EXPANDING

p_t DISTRIBUTION

RECALL $\frac{1}{N^k} \leftrightarrow \ln^{k-1}\frac{1}{x}$

$$\frac{d\sigma}{d\xi_p}(N,\alpha_s) = \sigma_{LO}\sum_{k=1}^{\infty} C_k(\xi_p)\,\alpha_s^k\,\frac{\ln^{k-1}x}{k-1!}$$

$$C_1(\xi_p) = \frac{2C_{\rm A}}{\pi}\,\frac{1}{\xi_p}$$

$$C_2(\xi_p) = \frac{4C_{\rm A}^2}{\pi^2}\,\frac{\ln\xi_p}{\xi_p}$$

$$C_3(\xi_p) = \frac{2C_{\rm A}^3}{\pi^3}\,\frac{1+2\ln^2\xi_p}{\xi_p}$$

$$C_4(\xi_p) = \frac{4C_{\rm A}^4}{\pi^4}\,\frac{3+3\ln\xi_p+2\ln^3\xi_p+17\zeta_3}{3\xi_p}$$

- AGREES WITH EXPECTED BEHAVIOUR
- LO AND NLO RESULTS CHECKED AGAINST EXACT EXPRESSIONS
- NNLO RESULT CHECKED AGAINST NNLL TRANSVERSE MOMENTUM RESUMMATION

RESUMMED RESULTS: HIGGS IN RESOLVED CASE

(Caola, sf, Marzani, Muselli, Vita, 2016)

IMPACT FACTOR

$$h_{p_{\mathrm{T}}} = \sigma_0\left(y_i\right) R\left(M_1\right) R\left(M_2\right) \frac{\xi_p^{M_1+M_2-1}}{(1+\xi_p)^N} \left[c_0\left(\xi_p, y_i\right)\left(M_1+M_2\right)+\sum_{j\geq k>0} c_{j,k}\left(\xi_p, y_i\right)\left(M_1^k M_2^j + M_1^j M_2^k\right)\right]$$

SUBSTITUTING $M_1 = \gamma_s\left(\frac{\alpha_s}{N}\right)$ $M_2 = \gamma_s\left(\frac{\alpha_s}{N}\right)$ & EXPANDING

p_t DISTRIBUTION

$$\frac{d\sigma}{d\xi_p}\left(x, \xi_p, y_t, y_b\right) = \sigma_0\left(y_t, y_b\right) \sum_{k=1}^{\infty} C_k\left(\xi_p, y_t, y_b\right) \alpha_s^k (-1)^{k+1} \frac{\ln^{k-1} x}{(k-1)!}$$

$$C_1\left(\xi_p, y_t, y_b\right) = \frac{2C_{\mathrm{A}}}{\pi} \frac{c_0\left(\xi_p, y_t, y_b\right)}{\xi_p}$$

$$C_2\left(\xi_p, y_t, y_b\right) = \frac{2C_{\mathrm{A}}^2}{\pi^2} \frac{2c_0\left(\xi_p, y_t, y_b\right)\ln\xi_p + c_{1,1}\left(\xi_p, y_t, y_b\right)}{\xi_p}$$

$$C_3\left(\xi_p, y_t, y_b\right) = \frac{2C_{\mathrm{A}}^3}{\pi^3} \frac{2c_0\left(\xi_p, y_t, y_b\right)\ln^2\xi_p + 2c_{1,1}\left(\xi_p, y_t, y_b\right)\ln\xi_p + c_{2,1}\left(\xi_p, y_t, y_b\right)}{\xi_p}$$

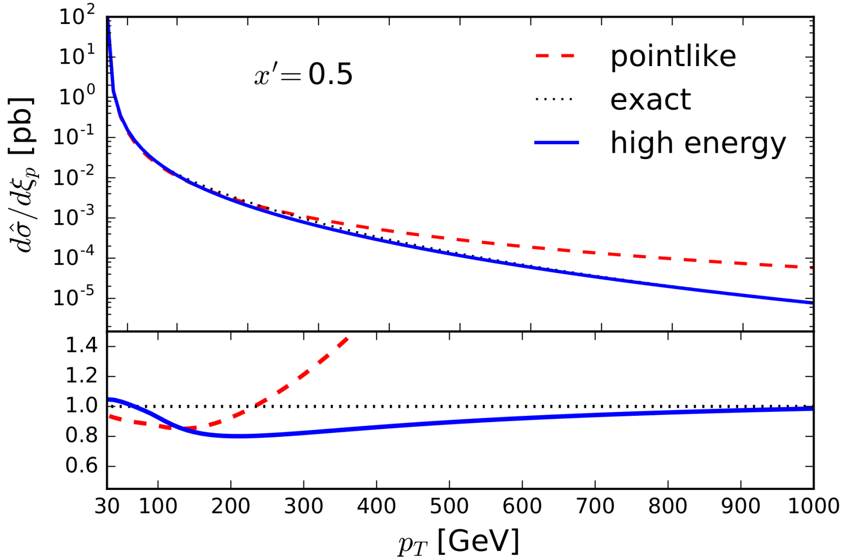
$\Rightarrow$ LEADING POWER OF $\ln\xi_p$ PROPORTIONAL TO LO c_0

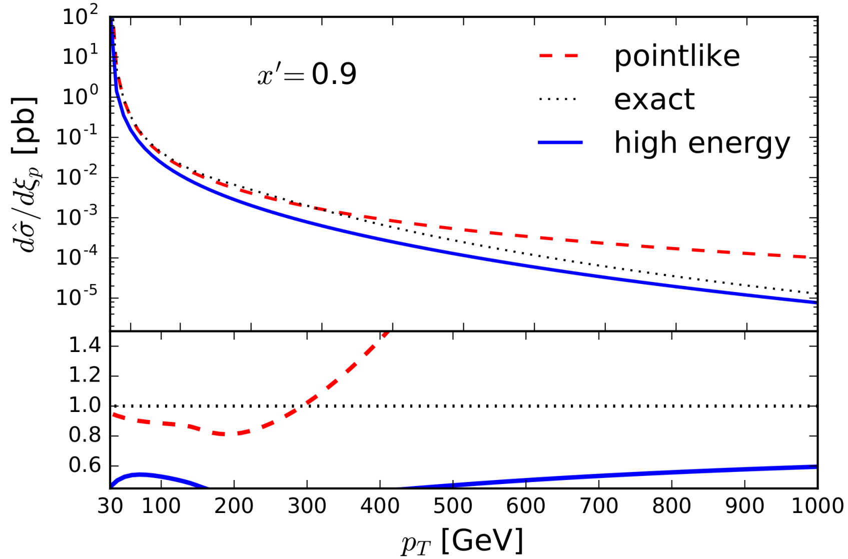
THE HIGH-ENERGY APPROXIMATION
PARTON LEVEL AT LEADING ORDER

(Caola, sf, Marzani, Muselli, Vita, 2016)

- CAN COMPARE TO EXACT RESULT WITH MASS DEP. (Baur, Glover, 1990)
  & IN POINTLIKE LIMIT (Ellis, Hinchliffe, Soldate, van der Bij 1988)
- LO HE RESULT DOES NOT DEPEND ON x: $\frac{d\sigma^{\mathrm{LLx-LO}}}{d\xi_p} = \sigma_0\left(y_b, y_t\right) c_0\left(\xi_p, y_t, y_b\right) \frac{2C_{\mathrm{A}}\alpha_s}{\pi}\frac{1}{\xi_p}$

PARTON-LEVEL TRANSVERSE MOMENTUM DISTRIBUTION





- POINTLIKE LIMIT FAILS BADLY FOR $p_t \gtrsim m_t$: POINTLIKE $\underset{p_t\to\infty}{\sim} \frac{1}{p_t}$, MASSIVE $\underset{p_t\to\infty}{\sim} \frac{1}{p_t^2}$
- BOTTOM MASS CORRECTION (DUE TO INTERFERENCE) SMALL BUT VISIBLE FOR $m_b \lesssim p_t \lesssim m_t$
- HE APPROXIMATION EXCELLENT! FOR $x \lesssim 0.5$
- EVEN WHEN $x >> 0.5$, HE APPROXIMATION DOES NOT DETERIORATE AT LARGE p_t

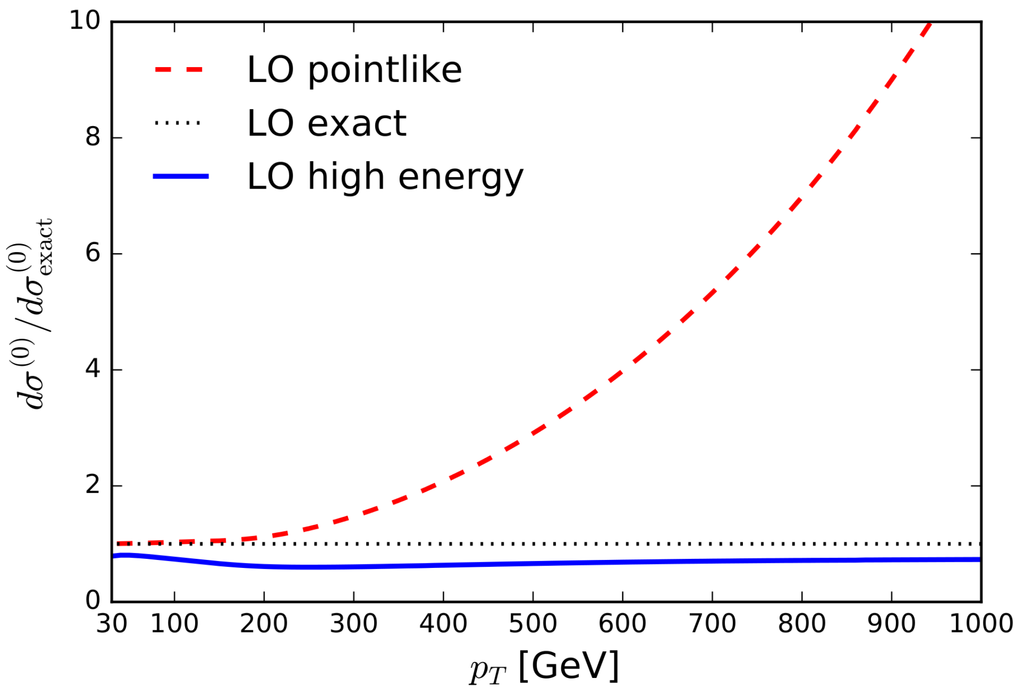
THE HIGH ENERGY APPROXIMATION
HADRON LEVEL AT LO

(Caola, sf, Marzani, Muselli, Vita, 2016)

THE FACTORIZED RESULT: $\frac{d\sigma_{ij}}{d\xi_p}(\tau, \xi_p, y_t, \alpha_s) = \tau' \int_{\tau'}^{1} \frac{dx'}{x'} \mathcal{L}_{ij}\left(\frac{\tau'}{x'}\right) \left[\frac{1}{x'} \frac{d\hat{\sigma}}{d\xi_p}(x', \xi_p, y_t, \alpha_s)\right]$

SCALE: $Q^2 = \left(\sqrt{m_H^2 + p_T^2} + \sqrt{p_T^2}\right)^2$; $0 \le \tau',\ x' \le 1$; LOW p_t: $Q^2 \approx m_h^2$; HIGH p_t: $Q^2 \approx 4p_t^2$

p_t SPECTRUM AT LHC 13: RATIO TO EXACT RESULT (Baur, Glover, 1990)



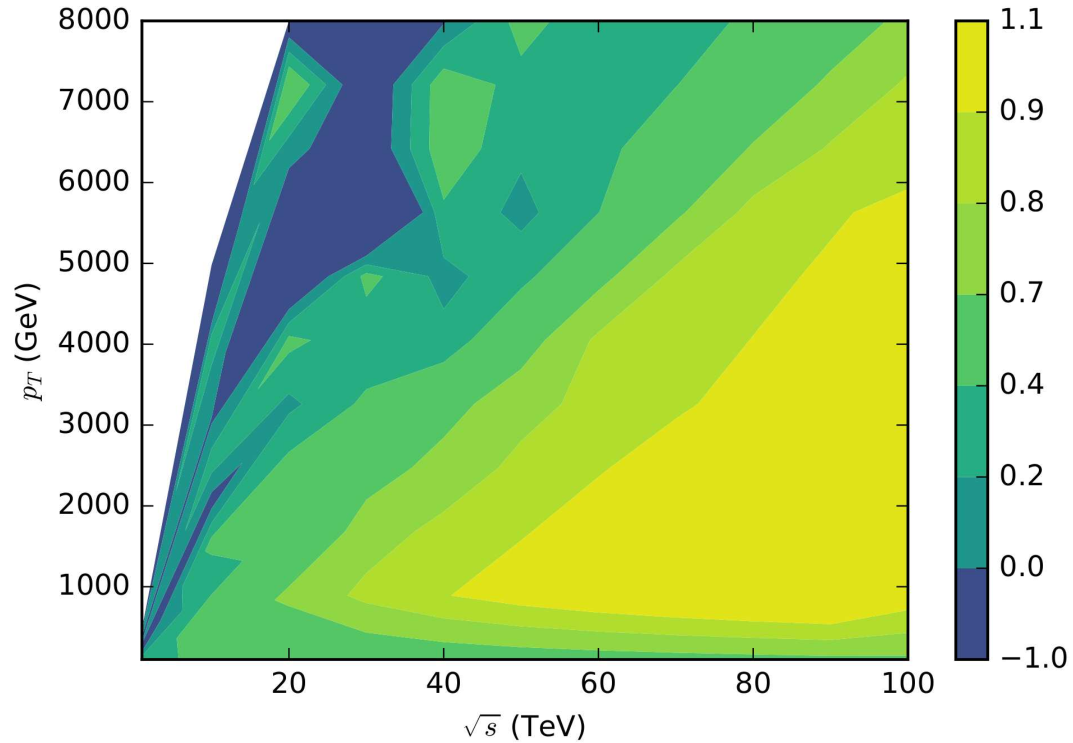
HIGH ENERGY IS MORE ACCURATE THAN POINTLIKE FOR $p_t \gtrsim m_t$

THE HIGH ENERGY APPROXIMATION
HADRON LEVEL AT NLO

(Caola, sf, Marzani, Muselli, Vita, 2016)

BEYOND LO, ONLY EFT RESULT KNOWN ⇒ HIGH-ENERGY VS. EXACT IN POINTLIKE LIMIT

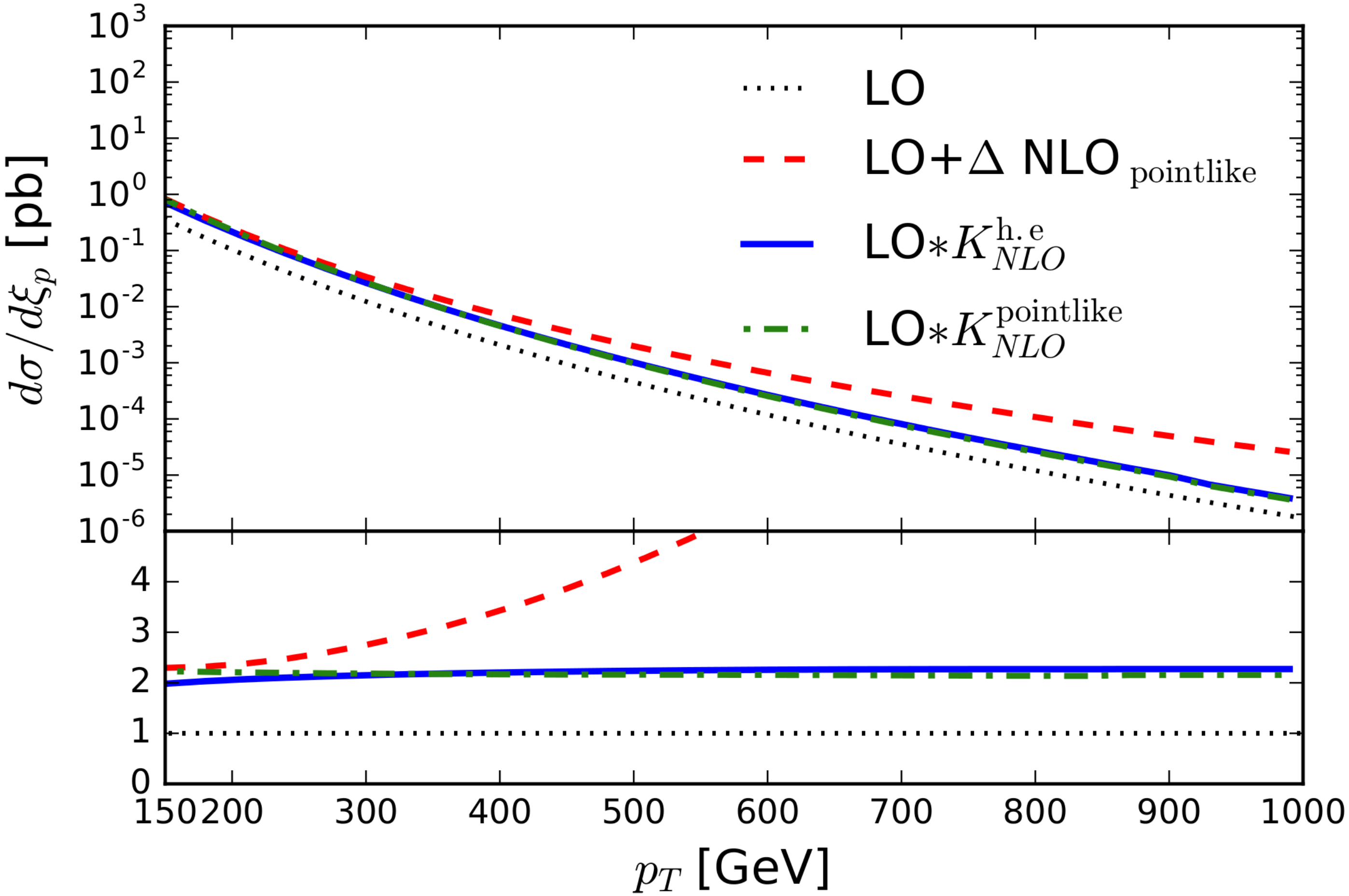
$$d\sigma^{(1)}/d\sigma^{(1)}_{\rm point}$$



HIGH ENERGY APPROX. GOOD FOR $2m_t \lesssim p_t \lesssim p_t^{\rm max}/5$

THE NLO SPECTRUM (LHC 13)

(Caola, sf, Marzani, Muselli, Vita, 2016)



- BEST APPROX AT LARGE p_t : EXACT LO$\times$ HE K-FACTOR
- UNCERTAINTY DRIVEN BY HE APPROX (ABOUT 30% AT LHC 13)
- POINTLIKE NLO FAILS FOR $p_t \gtrsim m_t$

SMALL p_T RESUMMATION: FROM LOW TO HIGH ENERGY

THE STRUCTURE OF TRANSVERSE MOMENTUM RESUMMATION

$$\frac{d\hat{\sigma}^{\rm tr}_{ij}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(M^2\right),M^2\right)=\sigma_0\int_0^\infty db\,\frac{b}{2}\,J_0\left(bp_{\rm T}\right)H_{ij}\left(N,\alpha_s\left(M^2\right)\right)S(M,N,b)$$

$$=\sigma_0\int_0^\infty db\,\frac{b}{2}\,J_0\left(bp_{\rm T}\right)H_{ij}\left(N,\alpha_s\left(M^2\right)\right)\bar{S}(M,b)\Gamma_i[\alpha_s,b^2,M^2,N]\Gamma_j[\alpha_s,b^2,M^2,N]$$

- ij PARTONIC SUBCHANNEL, FOURIER W.R. TO b, J_0 BESSEL
- RESUMMATION $\Rightarrow$ SUDAKOV EXPONENT

$$S(M,b)=\exp\left[-\int_{\frac{b_0^2}{b^2}}^{M^2}\frac{dq^2}{q^2}\left[A^{p_{\rm T}}\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2}+B^{p_{\rm T}}\left(\alpha_s\left(q^2\right),N\right)\right]\right]$$

NOTE N DEP IN $B^{p_{\rm T}}$ (BY DEFINITION)

$$\bar{S}(M,b)=\exp\left[-\int_{\frac{b_0^2}{b^2}}^{M^2}\frac{dq^2}{q^2}\left[\bar{A}^{p_{\rm T}}\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2}+\bar{B}^{p_{\rm T}}\left(\alpha_s\left(q^2\right)\right)\right]\right]$$

$\Gamma_i[\alpha_s,b^2,Q^2,N]\Rightarrow$ PDF EVOLUTION FROM b^2 TO M^2

- HARD FUNCTION

$$H_{ij}(\alpha_s)=[C_i(N,\mathbf{b})C_i(N,\mathbf{b})+G_i(N,\mathbf{b})G_j(N,\mathbf{b})]$$

C, G UNIVERSAL (DEP. ON PARTON)

(Catani, Grazzini et al., 2001-2012)

COMBINED TRANSVERSE – HIGH-ENERGY

TRANSVERSE MOMENTUM

$$\frac{d\hat{\sigma}^{\rm tr}_{ij}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(M^2\right),M^2\right)=\sigma_0\int_0^\infty db\,\frac{b}{2}\,J_0\left(bp_{\rm T}\right)H_{ij}\left(N,\alpha_s\left(M^2\right)\right)\bar{S}(M,b)\Gamma_i[\alpha_s,b^2,M^2,N]\Gamma_j[\alpha_s,b^2,M^2,N]$$

- RESUMMATION $\Rightarrow$ SUDAKOV EXPONENT

$$\bar{S}(M,b)=\exp\left[-\int_{\frac{b_0^2}{b^2}}^{M^2}\frac{dq^2}{q^2}\left[\bar{A}^{p_{\rm T}}\left(\alpha_s\left(q^2\right)\right)\ln\frac{M^2}{q^2}+\bar{B}^{p_{\rm T}}\left(\alpha_s\left(q^2\right)\right)\right]\right]$$

$\Gamma_i[\alpha_s,b^2,Q^2,N]\Rightarrow$ PDF EVOLUTION FROM b^2 TO M^2

- HARD FUNCTION

$$H_{ij}(\alpha_s)=[C_i(N,\mathbf{b})C_i(N,\mathbf{b})+G_i(N,\mathbf{b})G_j(N,\mathbf{b})]$$

C, G UNIVERSAL (DEP. ON PARTON) (Catani, Grazzini et al., 2001-2012)

HIGH ENERGY

$$h_{p_{\rm T}}=\sigma_0 e^{-(M_1+M_2)\ln\frac{b^2m_h^2}{4}}R\left(M_1\right)R\left(M_2\right)\times\left[\frac{\Gamma\left(1+M_1\right)}{\Gamma\left(1-M_1\right)}\frac{\Gamma\left(1+M_2\right)}{\Gamma\left(1-M_2\right)}+M_1\frac{\Gamma\left(1+M_1\right)}{\Gamma\left(2-M_1\right)}M_2\frac{\Gamma\left(1+M_2\right)}{\Gamma\left(2-M_2\right)}\right]$$

- HIGH ENERGY $\Rightarrow M=\gamma$ AT LLX; BECOMES POINTLIKE AS $p_T\to 0$
- UNIVERSAL FACTORIZED STRUCTURE OF H_{ij} REPRODUCED $\Rightarrow$ SMALL p_T LIMIT OF HIGH-ENERGY HAS UNIVERSAL STRUCTURE (sf, Muselli, 2016)
- SUDAKOV IS N-INDEPENDENT $\Rightarrow$ RESUMMATION SUBLEADING

COMBINED

$C_i=C_i^{p_T}+C_i^{HE}-d.c.$; $G_i=G_i^{p_T}+G_i^{HE}-d.c.$, DITTO FOR G (Marzani, 2016)

THE STRUCTURE OF SOFT RESUMMATION

THE CASE OF p_t DISTRIBUTIONS

$$\frac{d\hat{\sigma}^{\rm th}_{ij}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(Q^2\right),Q^2\right)=\sigma_0\,C_0\left(N,\xi_p\right)\,g_{0\;ij}\left(\xi_p\right)\exp\left[G\left(N\right)\right]\exp\left[S\left(N,p_{\rm T}\right)\right]$$

(de Florian, Kulesza, Vogelsang, 2005)

- SUDAKOV:

$$G\left(N\right)=\Delta_i\left(N\right)+\Delta_j\left(N\right)+J_k\left(N\right)$$

$$\Delta_i\left(N\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2}^{Q^2(1-z)^2}\frac{dq^2}{q^2}A_i^{\rm th}\left(\alpha_s\left(q^2\right)\right)$$

$$J_k\left(N\right)=\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\int_{Q^2(1-z)^2}^{Q^2(1-z)}\frac{dq^2}{q^2}A_k^{\rm th}\left(\alpha_s\left(q^2\right)\right)+B_k^{\rm th}\left(\alpha_s\left(Q^2\left(1-z\right)\right)\right)$$

SIMILAR TO PROMPT PHOTON, A^{th} $\Rightarrow$ SINGULAR PART OF ANOMALOUS DIMENSION

- SOFT:

$$S\left(N,\xi_p\right)=-\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}A_k^{\rm th}\left(\alpha_s\left(Q^2\left(1-z\right)^2\right)\right)\ln\frac{\left(\sqrt{1+\xi_p}+\sqrt{\xi_p}\right)^2}{\xi_p}$$

NEW FOR p_T $\Rightarrow$ INTERFERENCE BETWEEN JACOBIAN IN LO & SOFT RADIATION

SOFT VS. TRANSVERSE MOMENTUM RESUMMATION

TWO PROBLEMS:

- SMALL p_T INCLUDES RADIATION WHICH DOES NOT CONTRIBUTE TO SOFT RESUMMATION (COLLINEAR BUT NOT SOFT)
- THE SOFT AND SMALL p_T LIMIT DO NOT COMMUTE

PHASE SPACE IN THE SOFT LIMIT

MULTIPLE SOFT RADIATION

(sf, Ridolfi, Muselli, 2017)

$$d\Phi_{n+1}(p_1, p_2; q, k_1, \ldots, k_n) = x \frac{\pi^{3-2\epsilon}}{\Gamma(1-\epsilon)} d\xi_p \int db^2 \; (bp_{\mathrm{T}})^{-\epsilon} \, b^{n\epsilon} J_{-\epsilon}(bp_{\mathrm{T}}) \, \delta(x - z_1 \ldots z_n)$$

$$\times J_{-\epsilon}\left(bk_{\mathrm{T}_1}\right) \frac{M^{-\epsilon} (\xi_1)^{-\frac{\epsilon}{2}} \, dz_1 d\xi_1}{4(2\pi)^{2-\epsilon} \sqrt{(1-z_1)^2 - 4z_1\xi_1}} \ldots J_{-\epsilon}\left(bk_{\mathrm{T_n}}\right) \frac{M^{-\epsilon} (\xi_n)^{-\frac{\epsilon}{2}} \, dz_n d\xi_n}{4(2\pi)^{2-\epsilon} \sqrt{(1-z_n)^2 - 4z_n\xi_n}}$$

- IN THE $\xi \to 0$ LIMIT, $\frac{1}{\sqrt{(1-z)^2-4\xi}} \sim \frac{1}{1-x} \Rightarrow$ SOFT LOGS
- BUT

$$\int_0^1 dz \, z^{N-1} \int_0^{\frac{(1-z)^2}{4}} d\xi \, J_0\left(bM\sqrt{\xi}\right) \frac{1}{\sqrt{(1-z)^2 - 4\xi}} = \frac{2}{b^2 M^2} \left(1 - \frac{4N^2}{b^2 M^2} + \frac{16N^4}{b^4 M^4} + \ldots\right)$$

NEGLECTED TERMS ARE $O(N/b) \Rightarrow$ INCORRECT LARGE N LIMIT

- MUST TAKE $b \to \infty$ LIMIT AT FIXED N/b

SIMPLE: COMBINING RESUMMATIONS

(sf, Ridolfi, Muselli, 2017)

- FACTORIZATION OF PHASE SPACE IN THE SOFT LIMIT AT FIXED p_T INCOMPATIBLE WITH FACTORIZATION IN SMALL p_T LIMIT $\Leftrightarrow$ DIFFERENT KINEMATIC CONFIGURATIONS

- ASSUME WE CAN DETERMINE CONSISTENT p_T RESUMMED $\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}$
  $\Rightarrow$ INTEGRAL OVER $\xi_p = \frac{p_T}{M}$ IS THE SOFT-RESUMMED TOTAL σ

COMBINED THRESHOLD AND TRANSVERSE MOMENTUM RESUMMATION

$$\frac{d\hat{\sigma}_{ij}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_R^2\right), \mu_F^2\right) = \left(1 - T\left(N, \xi_p\right)\right) \frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_R^2\right), \mu_F^2\right) + T\left(N, \xi_p\right) \frac{d\hat{\sigma}_{ij}^{\mathrm{th}}}{d\xi_p}\left(N, \xi_p, \alpha_s\left(\mu_R^2\right), \mu_F^2\right)$$

$$\lim_{N\to\infty} T\left(N, \xi_p\right) = 1 \qquad \lim_{\xi_p\to 0} T\left(N, \xi_p\right) = 0$$

- AS $p_T \to 0$ MATCHING FUNCTION T KILLS $\xi_p \to 0$ SINGULARITIES $\Rightarrow$ ONLY $\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}'}}{d\xi_p}$ CONTRIBUTES TO TOTAL CROSS-SECTION IN SOFT (LARGE N) LIMIT

- FOR FINITE p_TM AS $N \to \infty$ MATCHING FUNCTION T KILLS LARGE N SINGULARITIES $\Rightarrow$ ONLY $\frac{d\hat{\sigma}_{ij}^{\mathrm{th}}}{d\xi_p}$ CONTRIBUTES TO SOFT LIMIT

HARDER: SOFT CONSISTENCY

(sf, Ridolfi, Muselli, 2017)

- PERFORM TRANSVERSE MOMENTUM RESUMMATION WITH MODIFIED PHASE SPACE
- GENERIC LOGS BECOME

$$G_{k,1}(N,b) = \int_0^\infty d\xi \left(\sqrt{1+\xi} - \sqrt{\xi}\right)^{2N} J_0\left(bM\sqrt{\xi}\right) \int_0^1 dz\, z^{N-1}$$

$$\left(\left[\frac{\ln^k \xi}{\xi}\right]_+^{p_T} \left[\frac{1}{\sqrt{(1-z)\left(1-(\sqrt{1+\xi}-\sqrt{\xi})^4 z\right)}}\right]_+^z + \delta(1-z)\frac{1}{2(\sqrt{1+\xi}-\sqrt{\xi})^2}\left(\frac{\ln(1+\xi)\ln^k \xi}{\xi} - \left[\frac{\ln^{k+1}\xi}{\xi}\right]_+^{p_T}\right)\right)$$

$$G_{k,2}(N,b) = \int_0^\infty d\xi \left(\sqrt{1+\xi} - \sqrt{\xi}\right)^{2N} J_0\left(bM\sqrt{\xi}\right) \left[\frac{\ln^k \xi}{\xi}\right]_+^{p_T}$$

COMPARE TO $\left[\frac{\ln^k \xi}{\xi}\right]_+$, $\left[\frac{1}{1-x}\right]_+$

- COMPUTE BY
  - DETERMINING A GENERATING FUNCTION FOR BOTH CLASSES OF LOGS
  - EVALUATING THE GENERATING FUNCTION IN THE $b \to \infty$ LIMIT AT FIXED $\frac{N}{b}$.

$$G_{k,1}(N,b) = \frac{(-1)^k}{2}\left[-\frac{1}{k+2}\ln^{k+2}\chi + \frac{\ln N^2}{k+1}\ln^{k+1}\chi + \ln^k N^2\, \mathrm{Li}_2\left(\frac{N^2}{\chi}\right) + \mathcal{O}\left(\ln^j N^2 \ln^{k-1-j}\chi\right)\right]$$

$$G_{k,2}(N,b) = -\frac{(-1)^k}{k+1}\ln^{k+1}\chi + \mathcal{O}\left(\ln^{k-1}\chi\right)$$

ARGUMENT OF LOGS: $\chi^2 = N^2 + b^2M^2$

CONSISTENT TRANSVERSE-SOFT RESUMMATION

HIGGS IN GLUON FUSION AT NNLO

(sf, Ridolfi, Muselli, 2017)

$$\frac{d\hat{\sigma}_{ij}^{\mathrm{tr}\prime}}{d\xi_p}\left(N,\xi_p,\alpha_s\left(M^2\right),M^2\right)=\sigma_0\int_0^\infty db\,\frac{b}{2}\,J_0\left(bM\sqrt{\xi_p}\right)\left(\sqrt{1+\xi_p}-\sqrt{\xi_p}\right)^{-2N}\bar{\mathcal{H}}_{ij}\left(N,\alpha_s\left(M^2\right)\right)$$

$$\times\exp\left[\ln\chi\, g_1\left(\lambda_N,\lambda_\chi\right)+g_2\left(\lambda_N,\lambda_\chi\right)+\alpha\, g_3\left(\lambda_N,\lambda_\chi\right)\right]$$

$$\lambda_N=\alpha_s\left(M^2\right)\beta_0\ln\bar{N}^2;\quad \lambda_\chi=\alpha_s\left(M^2\right)\beta_0\ln\chi;\qquad \bar{N}=N\,e^{\gamma_E},\quad \chi=\bar{N}^2+b^2M^2e^{2\gamma_E}/4$$

$$g_1\left(\lambda_\chi,\lambda_N\right)=\frac{A_g^{p_\mathrm{T},(1)}}{\beta_0}\left(\frac{\lambda_\chi+\ln\left(1-\lambda_\chi\right)}{\lambda_\chi}\right)-\frac{A_g^{p_\mathrm{T},(1)}}{\beta_0}\ln\left(1-\lambda_\chi\right)\frac{\lambda_N}{\lambda_\chi}$$

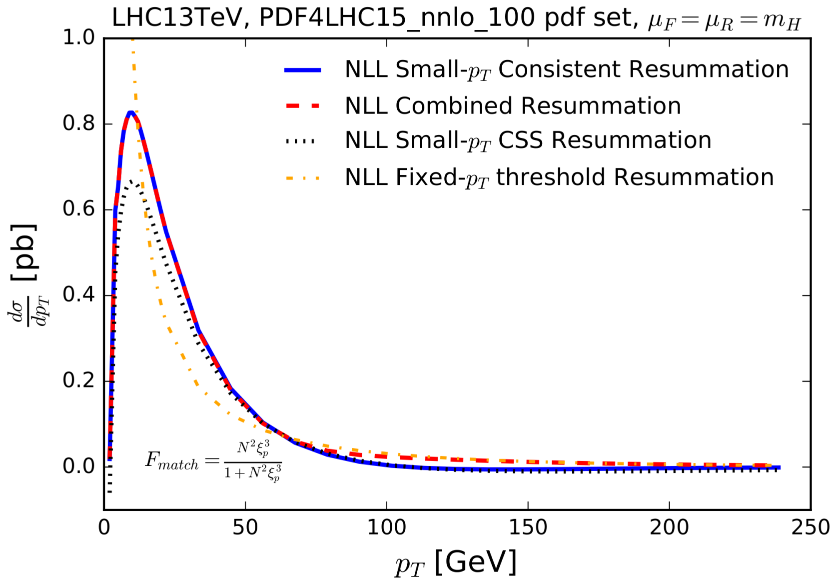
- $\ln\chi$ INTERPOLATES BETWEEN $\ln N^2$ AS $b\to 0$ (INCLUSIVE CROSS SECTION) & $\ln b$ AS $b\to\infty$ ($p_T\to 0$)
  PREVIOUS ATTEMPTS INTRODUCED PHENOMENOLOGICALLY INTERPOLATIONS $\chi=N+bM$(Kulesza, Sterman, Vogelsang, 2004)

- COEFFICIENTS OF THRESHOLD & TRANSVERSE MOMENTUM RESUMMATION RELATED TO EACH OTHER $\Rightarrow A^{th}\neq A^{p_T}$ BEYOND NLO "COLLINEAR ANOMALY"
  A^{th} COEFF OF $\ln N$ IN ANOMALOUS DIMENSION $\Leftrightarrow A^{p_T}$ COEFF OF $\ln N\ln b$ IN TRANSVERSE MOMENTUM SPECTRUM

PHENOMENOLOGY

(PRELIMINARY) sf, Muselli, Ridolfi

NLL



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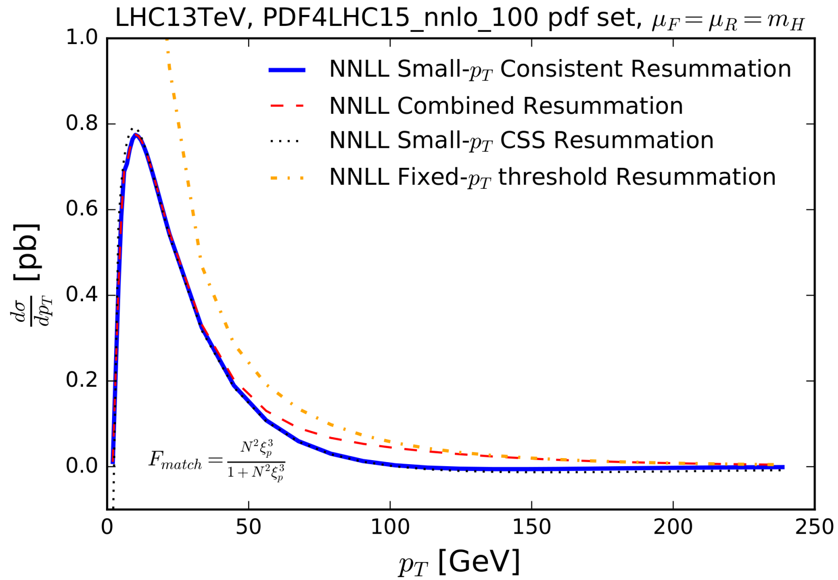

- FIXED-p_t THRESHOLD AND SMALL p_T RESUMMATION WIDELY DIFFERENT
- CONSISTENT SMALL p_T RESUMMATION AGREES WITH STANDARD CSS AT SMALL p_T, ANTICIPATES THE HIGHER-ORDER PEAK
- COMBINED REPRODUCES CONSISTENT IN THE PEAK REGION AND BELOW, & FIXED p_T THRESHOLD AT LARGER p_T

OUTLOOK

- FULLY DIFFERENTIAL HIGH-ENERGY RESUMMATION
  - INCLUDING p_t & RAPIDITY DEPENDENCE
- DOUBLE AND TRIPLE RESUMMATION
  - INCLUDING HIGH-ENERGY, p_t & THRESHOLD RESUMMATIONS
- $\Rightarrow$ MATCHED APPROXIMATIONS
  - MATCH $1/m_t$ TO HIGH-ENERGY (AKIN TO Harlander, Mantler, Marzani, Ozeren 2010 FOR INCLUSIVE CROSS-SECTION)
  - MATCH HIGH-ENERGY TO THRESHOLD (AKIN TO Ball, Bonvini, sf, Marzani, Ridolfi 2013 FOR INCLUSIVE CROSS-SECTION)
- OTHER PROCESSES?
  - DRELL-YAN
  - JETS???