Recent results from NNLOJET

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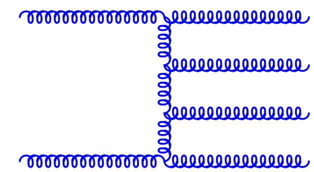
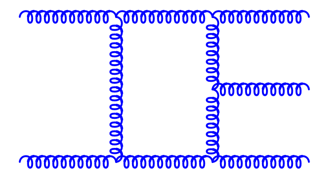
Anatomy of a Higher Order calculation

e.g. pp to JJ at NNLO



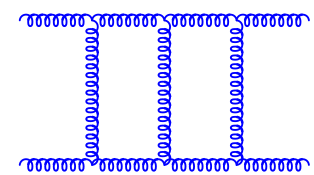
✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$

- implicit poles from double unresolved emission



✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$

- explicit infrared poles from loop integral
- implicit poles from soft/collinear emission



✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$

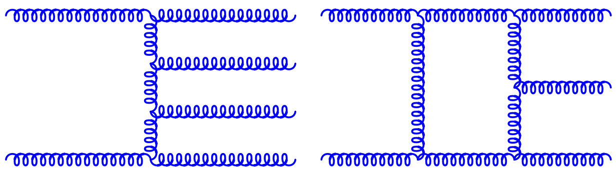
- explicit infrared poles from loop integral

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

Anatomy of a Higher Order calculation

e.g. pp to JJ at NNLO

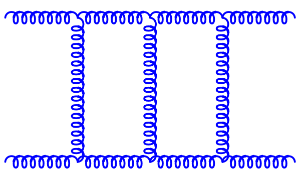
✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet



Can exploit NLO automation

...but needs to be evaluated in regions of phase space where extra jet is not resolved

✚ Two loop amplitudes - very limited set known



...currently far from automation

✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

...currently not automated

NNLOJET

X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, NG, A. Huss, M. Jaquier, T. Morgan, J. Niehues, J. Pires

Implementing NNLO corrections including decays for

- ✓ $pp \to H, W, Z$
- ✓ $pp \to H + J$ 1408.5325, 1607.08817
- ✓ $pp \to Z + J$ 1507.02850, 1605.04295, 1610.01843
- ✓ $pp \to JJ$ 1301.7310, 1310.3993, 1611.01460, 1704.00923
- ✓ $ep \to JJ + (J)$ 1606.03991, 1703.05977
- ✓ $\ldots$

using Antenna subtraction

$$|M(1,\ldots,i,j,k,\ldots,n)|^2 \to X(i,j,k)|M(1,\ldots,I,K,\ldots,n)|^2$$

✓ all singularities associated with j soft or collinear with i or k are concentrated in antenna X

✓ I and K are resolved partons

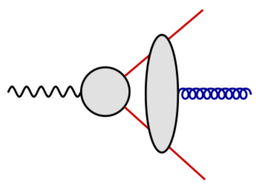
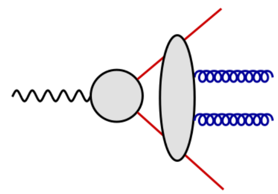
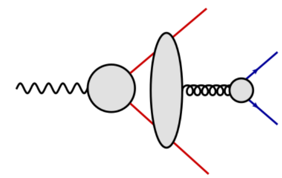
Antenna subtraction at NNLO

✓ Antenna subtraction exploits the fact that matrix elements already possess the intricate overlapping divergences

$$X_3^0(i,j,k) \sim \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2}, \qquad X_4^0(i,j,k,l) \sim \frac{|\mathcal{M}_4^0(i,j,k,l)|^2}{|\mathcal{M}_2^0(I,L)|^2}$$

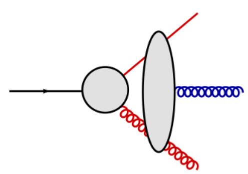
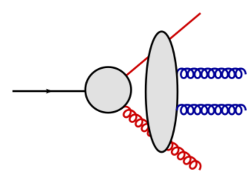
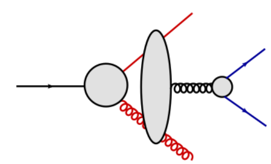
Quark-antiquark:

$\gamma^* \to q\bar{q} + \cdots$

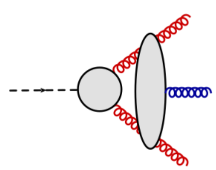
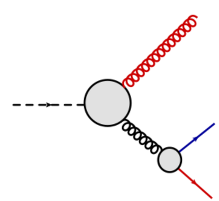
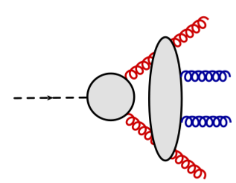
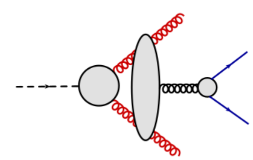
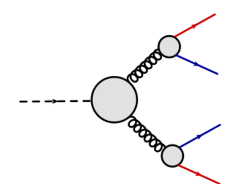
Quark-gluon:

$\tilde{\chi}^0 \to \tilde{g}g + \cdots$

Gluon-gluon:

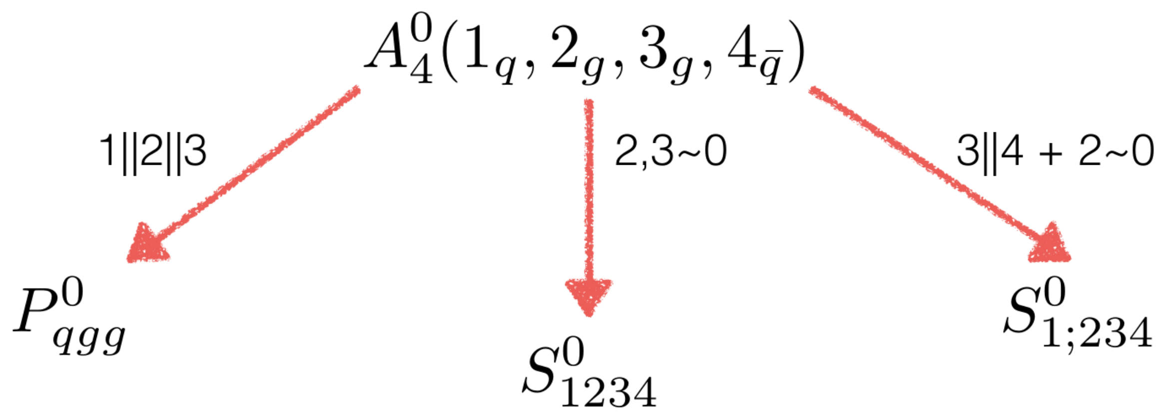
$H \to gg + \cdots$

✓ plus mappings $i+j+k \to I+J$, $i+j+k+l \to I+L$

Antenna subtraction at NNLO

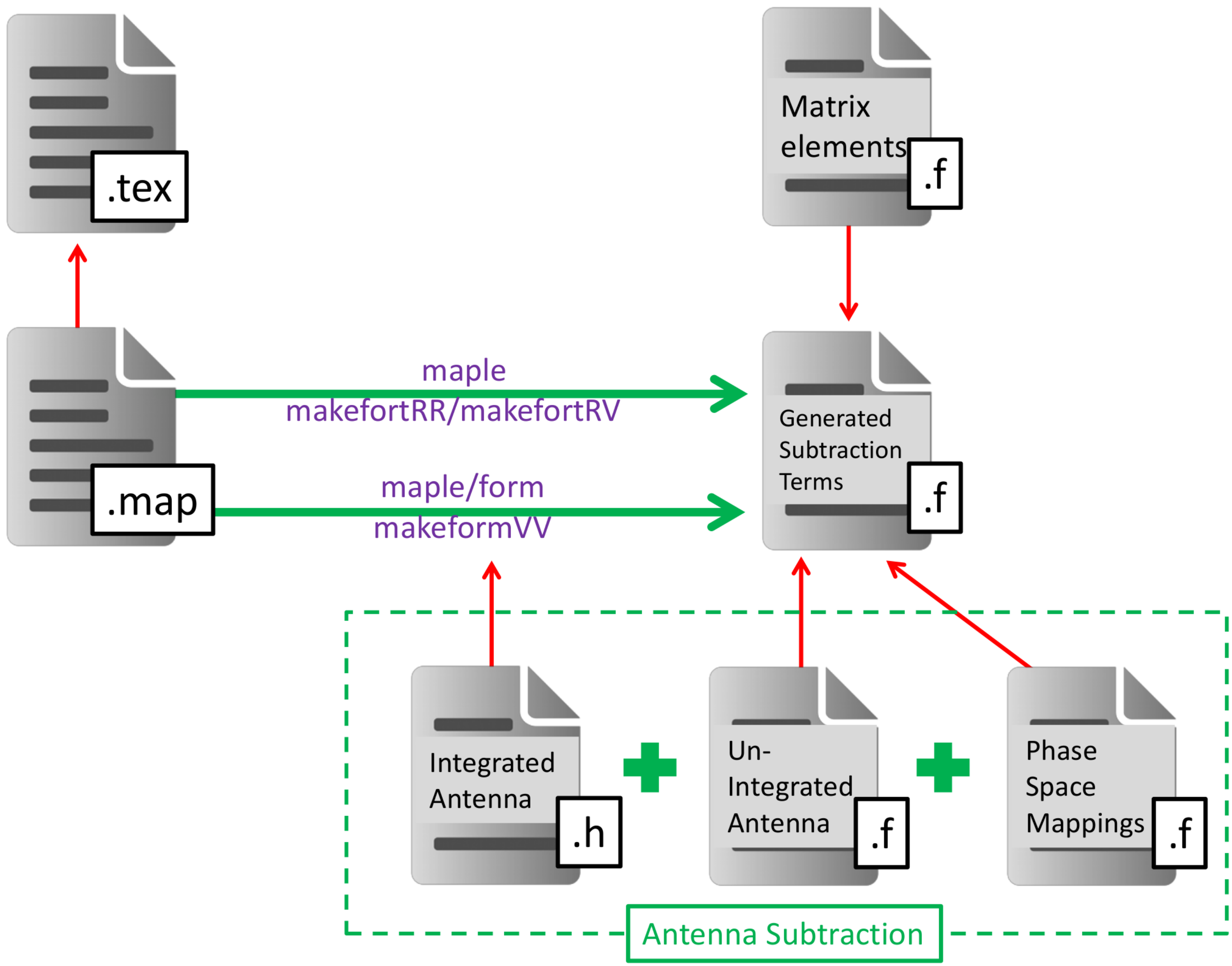
✓ Antenna mimics all singularities of QCD



✓ Phase space map smoothly interpolates momenta for reduced matrix element between limits

$$
\begin{aligned}
\widetilde{(123)} &= x p_1 + r_1 p_2 + r_2 p_3 + z p_4 \\
\widetilde{(234)} &= (1-x) p_1 + (1-r_1) p_2 + (1-r_2) p_3 + (1-z) p_4
\end{aligned}
$$

Automatically generating the code (1)



Maple script: RR example


```
+F40a(i,j,k,l)*A4g0(1,2,[i,j,k],[j,k,l])
-f30FF(i,j,k)*f30FF([i,j],[j,k],l)
*A4g0(1,2,[[i,j],[j,k]],[[j,k],l])
...
```

$$+\,F_4^{0,a}(i,j,k,l)\,A_4^0(1,2,(\widetilde{ijk}),(\widetilde{jkl}))$$

$$-\,f_3^0(i,j,k)\,f_3^0((\widetilde{ij}),(\widetilde{jk}),l)\,A_4^0(1,2,[(\widetilde{ij}),(\widetilde{jk})],(\widetilde{(\widetilde{jk})l}))$$

...

- ✓ X_4^0, X_3^0 (and X_3^1 in RV) are unintegrated antennae
- ✓ `[i,j,k]` or $(\widetilde{ijk})$ are mapped momenta

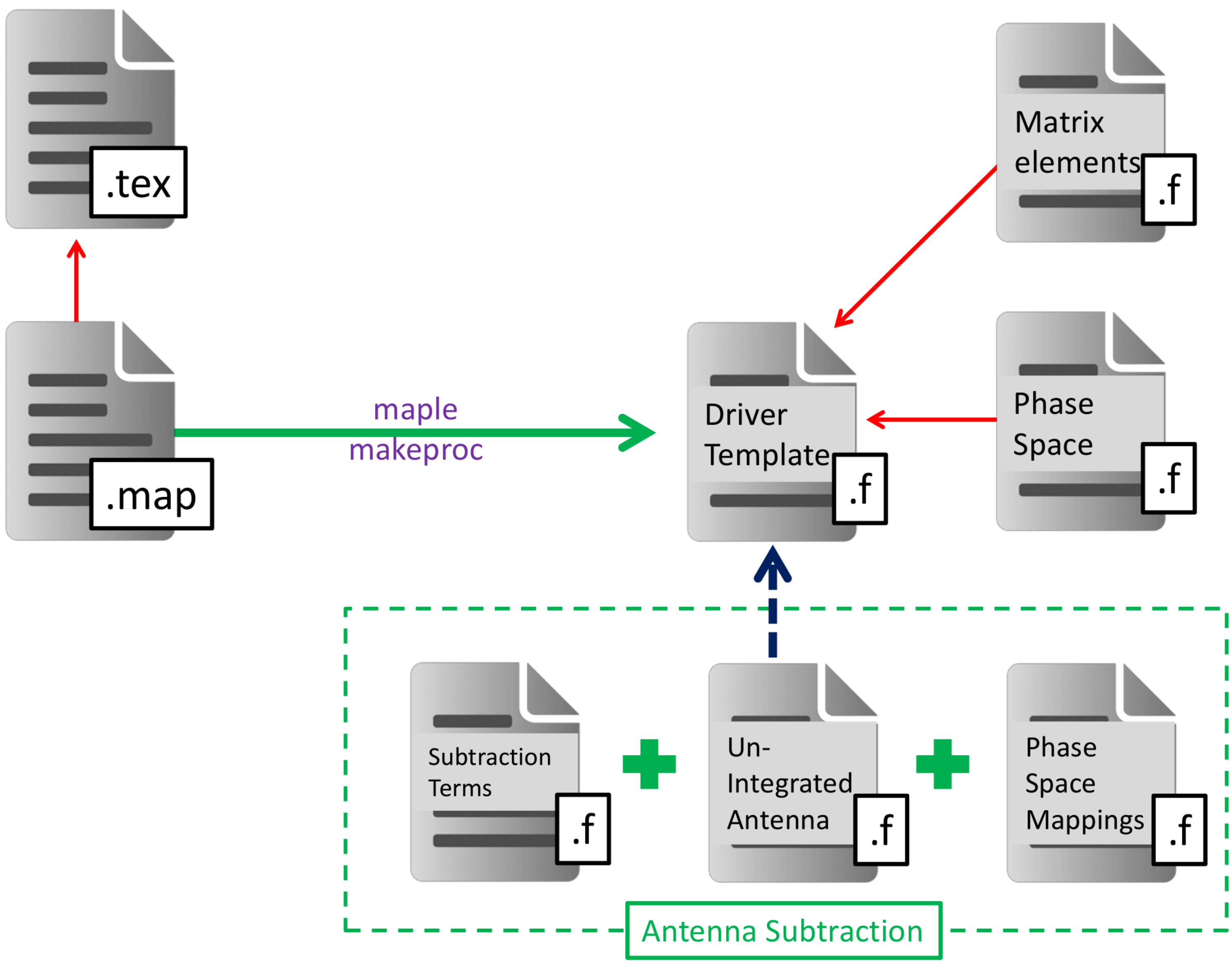
Maple script: VV example


```
-(+1/2*calgF40FI(2,3)
+1/2*calgF31FI(2,3)
+b0/e*1/2*QQ(s23)*calgF30FI(2,3)
-b0/e*1/2*calgF30FI(2,3)
-1/2*calgF30FI(2,3)*1/2*calgF30FI(2,3)
-1/2*gamma2gg(z2)
+b0/e*1/2*gamma1gg(z2)
)*A4g0(1,2,3,4)
...
```

$$
\begin{aligned}
+\Big[\quad & - \frac{1}{2}\, \mathcal{F}^{0}_{4,g}(s_{23}) \\
& - \frac{1}{2}\, \mathcal{F}^{1}_{3,g}(s_{23}) \\
& - \frac{b_0}{2\epsilon} \left(\frac{s_{23}}{\mu_R^2} \right)^{-\epsilon} \mathcal{F}^{0}_{3,g}(s_{23}) \\
& + \frac{b_0}{2\epsilon}\, \mathcal{F}^{0}_{3,g}(s_{23}) \\
& + \frac{1}{4}\, \mathcal{F}^{0}_{3,g}(s_{23}) \otimes \mathcal{F}^{0}_{3,g}(s_{23}) \\
& + \frac{1}{2}\, \Gamma^{(2)}_{gg}(z_2)
\end{aligned}
$$

✓ $\mathcal{X}_4^0$, $\mathcal{X}_3^0$ and $\mathcal{X}_3^1$ are integrated antennae

Automatically generating the code (2)



Maple script to produce driver template


```
R:=[
[A5g0,[g,g,g,g,g],1],
[B3g0,[qb,g,g,g,q],1/nc],
...
]:
```

$$d\sigma_{gg}^{R} = \mathcal{N}_{LO}\left(\frac{\alpha_s N}{2\pi}\right)\Bigg[$$

$$+2\,\frac{1}{3!}\left(\sum_{12} \texttt{A5g0}(1,2,3,4,5) - \color{red}{\texttt{ggA5g0SNLO}}\color{black}(1,2,3,4,5)\right)$$

$$+\frac{N_F}{N}\left(\sum_{6} \texttt{B3g0}(3,1,2,4,5) - \color{red}{\texttt{ggB3g0SNLO}}\color{black}(3,1,2,4,5)\right)$$

$$\ldots\Bigg]$$

Checks

✓ Analytic pole cancellations for RV, VV

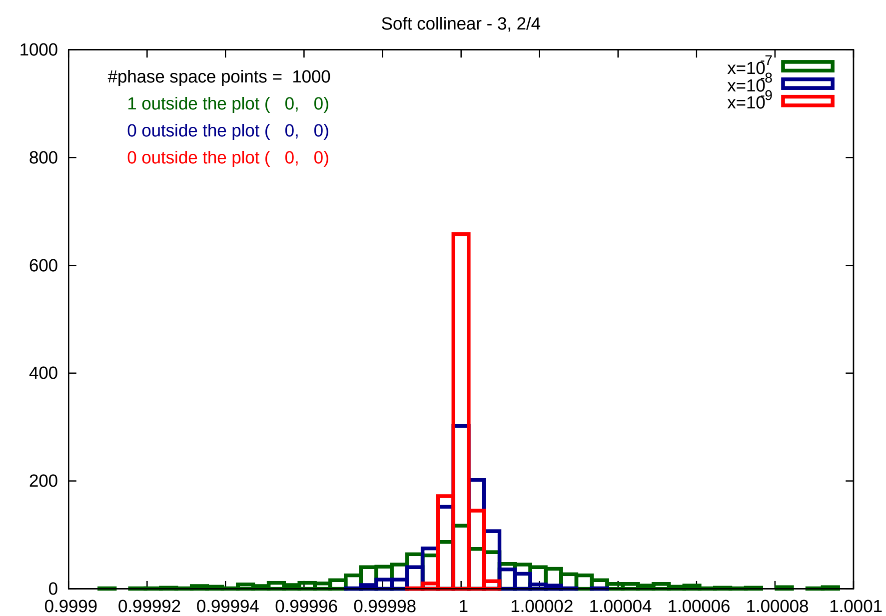
$$\mathrm{Poles}\left(d\sigma^{RV} - d\sigma^{T}\right) = 0$$
$$\mathrm{Poles}\left(d\sigma^{VV} - d\sigma^{U}\right) = 0$$

```
09:26:35  ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
    #-

   poles = 0;

  6.58 sec out of 6.64 sec
```

✓ Unresolved limits for RR, RV

$$d\sigma^{S} \longrightarrow d\sigma^{RR}$$
$$d\sigma^{T} \longrightarrow d\sigma^{RV}$$

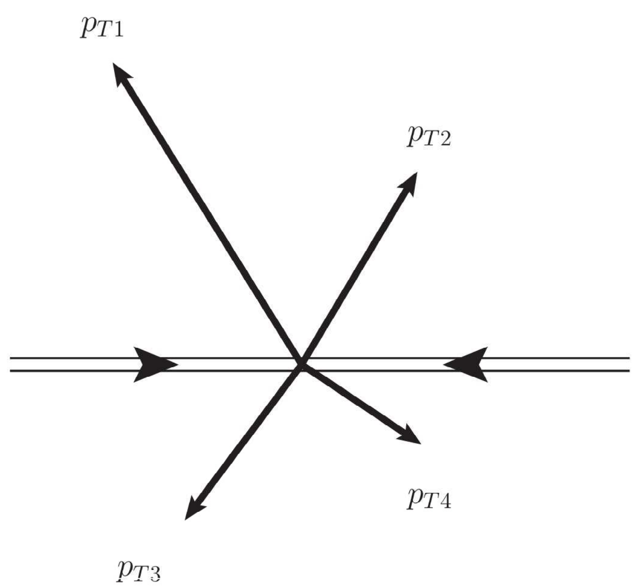
$q\bar{q} \to Z + g_3\ g_4\ g_5$ (g_3 soft & $g_4 \parallel \bar{q}$)



Single Jet Inclusive Distribution

Currie, NG, Pires (16)

✓ Classic jet observable



✓ Every jet in the event enters in the distribution

✓ Expect sensitivity to PDFs

✓ ... and to α_s

✓ All sub-processes included
– gg, gq, $q\bar{q}$, qq etc

✓ in leading colour approximation
i.e. all $\alpha_s^2 N^2$, $\alpha_s^2 N N_F$, $\alpha_s^2 N_F^2$
contributions relative to Born

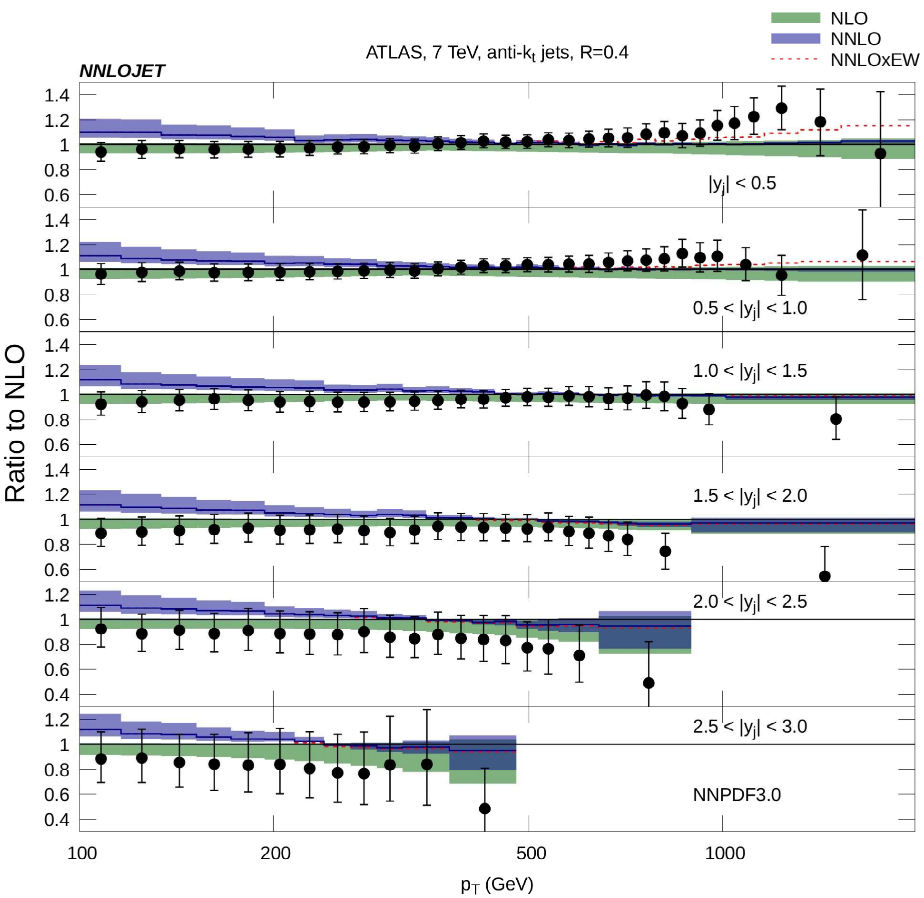
✘ missing corrections
O(1), N_F/N, $1/N^2$, N_F/N^3, $1/N^4$

✓ expect to be less than 10% of the NNLO correction (as at NLO)

Single Jet Inclusive Distribution – R=0.4

Currie, NG, Pires (16)

- ✓ ATLAS 7 TeV data, 4.5 fb^{-1}
  JHEP02(2015)153
  JHEP09(2015)141 (Erratum)
- ✓ anti-k_T algorithm with $R = 0.4$
- ✓ six rapidity slices,
  0 – 0.5, 0.5 – 1.0, 1.0 – 1.5, 1.5 – 2.0,
  2.0 – 2.5, 2.5 – 3.0
- ✓ NNPDF3.0_NNLO PDFs
- ✓ negligible NP corrections



Single Jet Inclusive Distribution – R=0.4

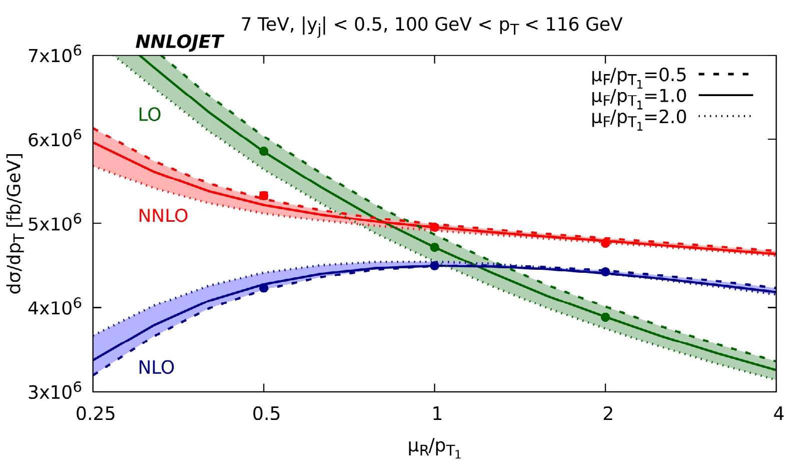
Currie, NG, Pires (16)

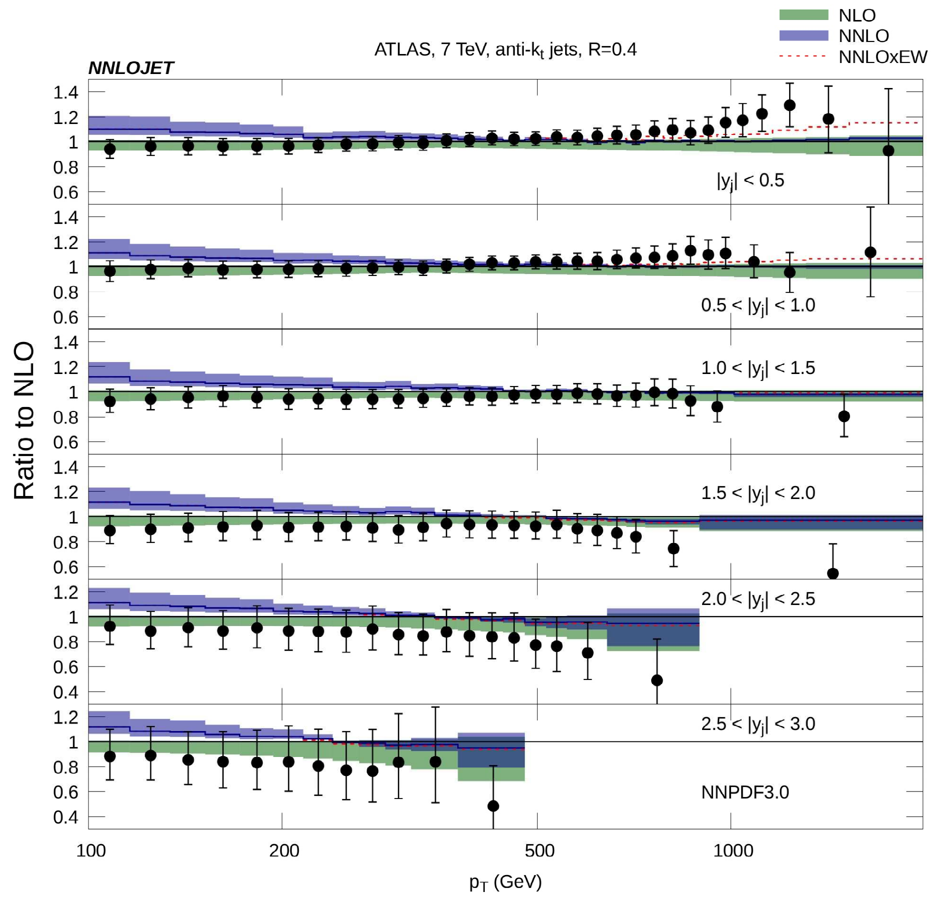
✓ NLO describes the data pretty well

✓ NLO has relatively small scale dependence

– because the central scale choice lies close to the turning point in the scale variation plot

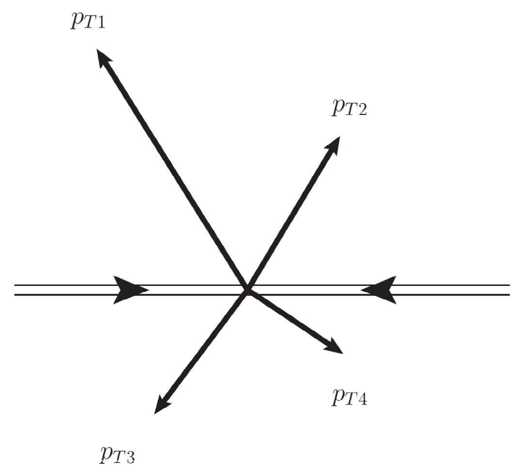
✓ NNLO effects around 10% at low p_T and small at high p_T



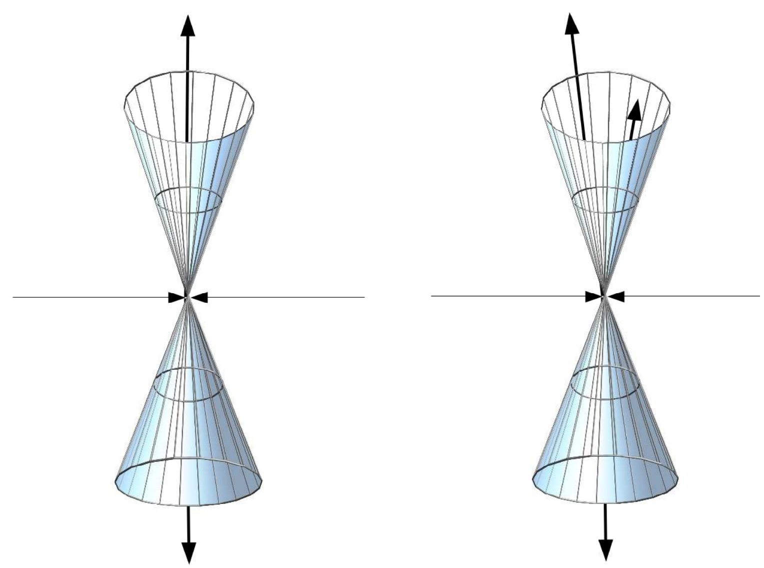


Scale Choice

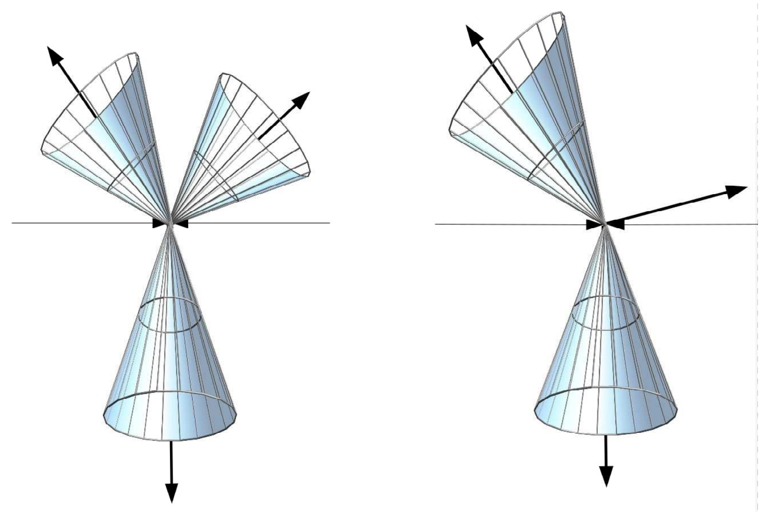
- ✓ no fixed hard scale for jet production
- ✓ two widely used scale choices
  - leading jet p_T (p_{T1})
  - individual jet p_T (p_T)



- ✓ different scale changes PDF and α_s
- ✓ no difference for back-to-back jet configurations (only arises at higher orders)

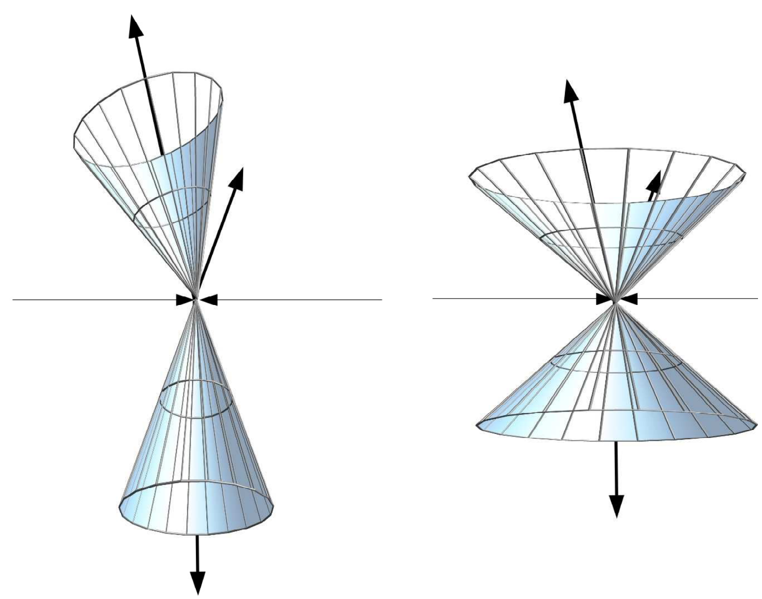


Scale Choice



At NLO, $p_T \neq p_{T1}$ for

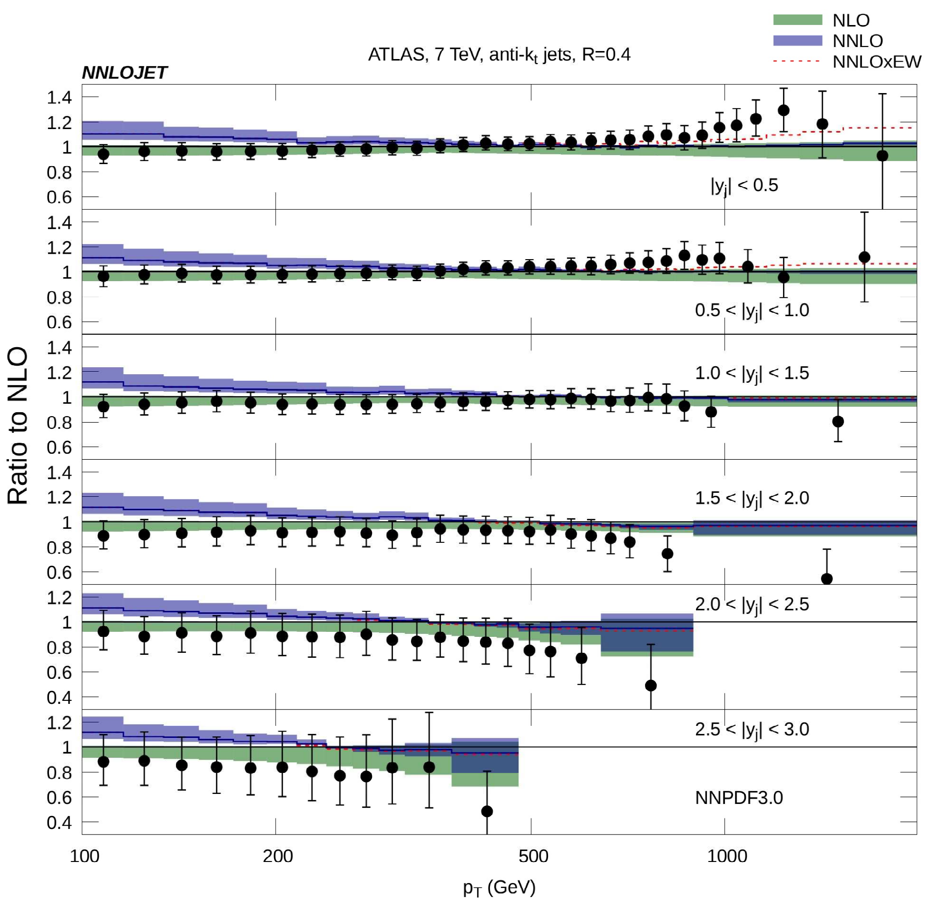
- ✓ 3-jet rate (small effect)
- ✓ 2-jet rate (3rd parton falls outside jet)



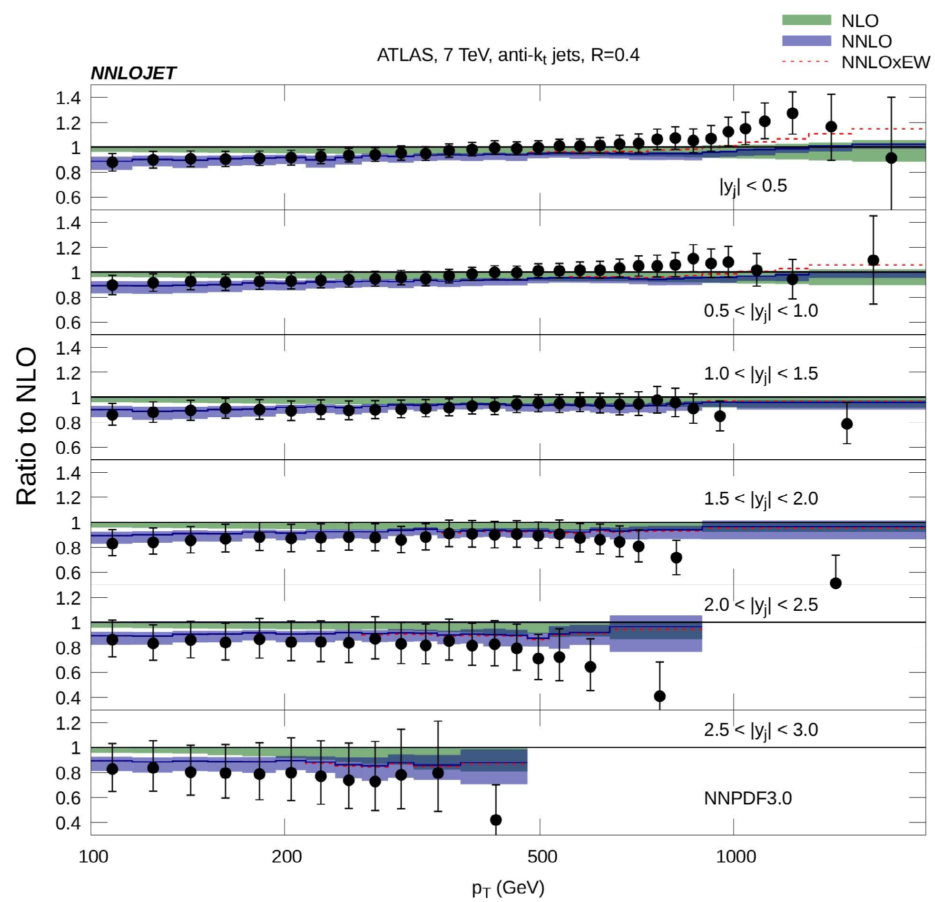
Changing R has an effect on the cross section, but also on the scale choice:

- ✓ introduces spurious R-dependence in scale choice
- ✓ p_{T1} scale has no R-dependence at NLO, unlike p_T
- ✓ at NNLO p_{T1} scale depends on R in some four-parton configurations

Single Jet Inclusive Distribution – R=0.4



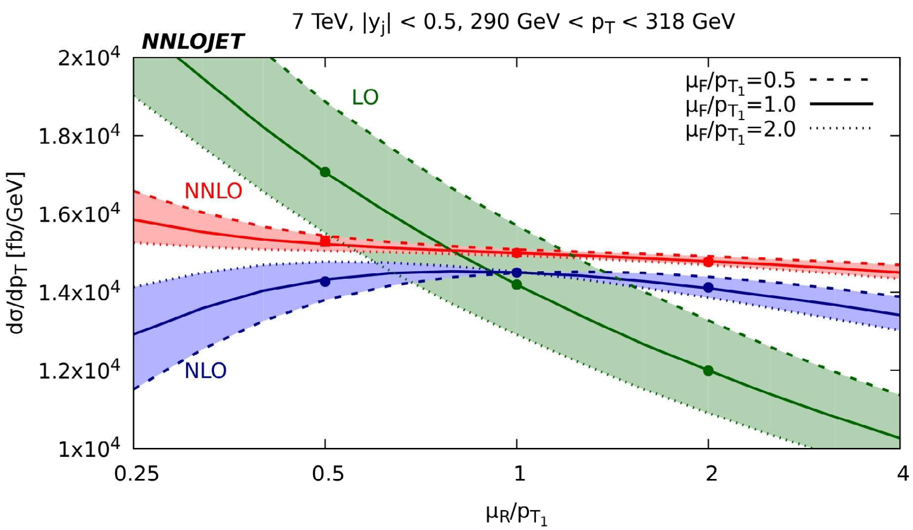
$$\mu_R = \mu_F = p_{T1}$$



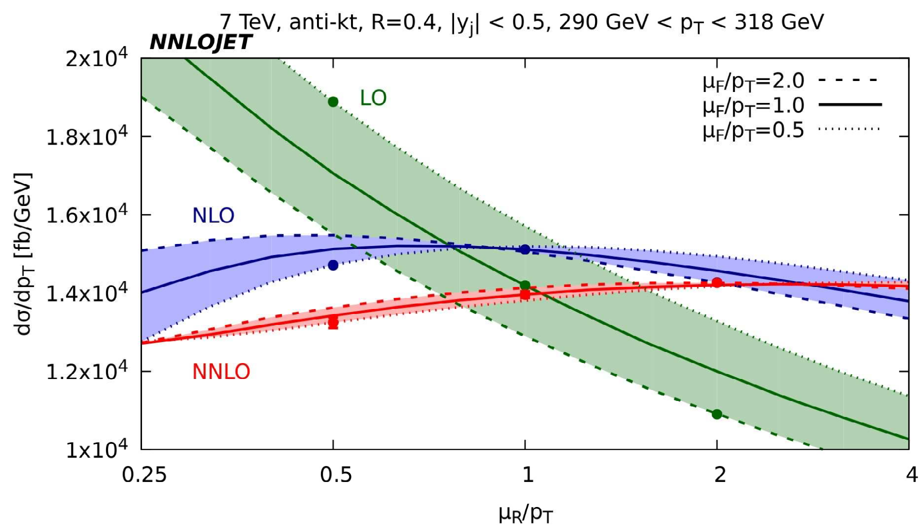
$$\mu_R = \mu_F = p_T$$

- ✘ Quite different behaviour!
- ✓ NLO with $\mu = p_{T1}$ describes $R = 0.4$ data quite well
- ✓ NNLO with $\mu = p_T$ describes $R = 0.4$ data quite well

Single Jet Inclusive Distribution – R=0.4



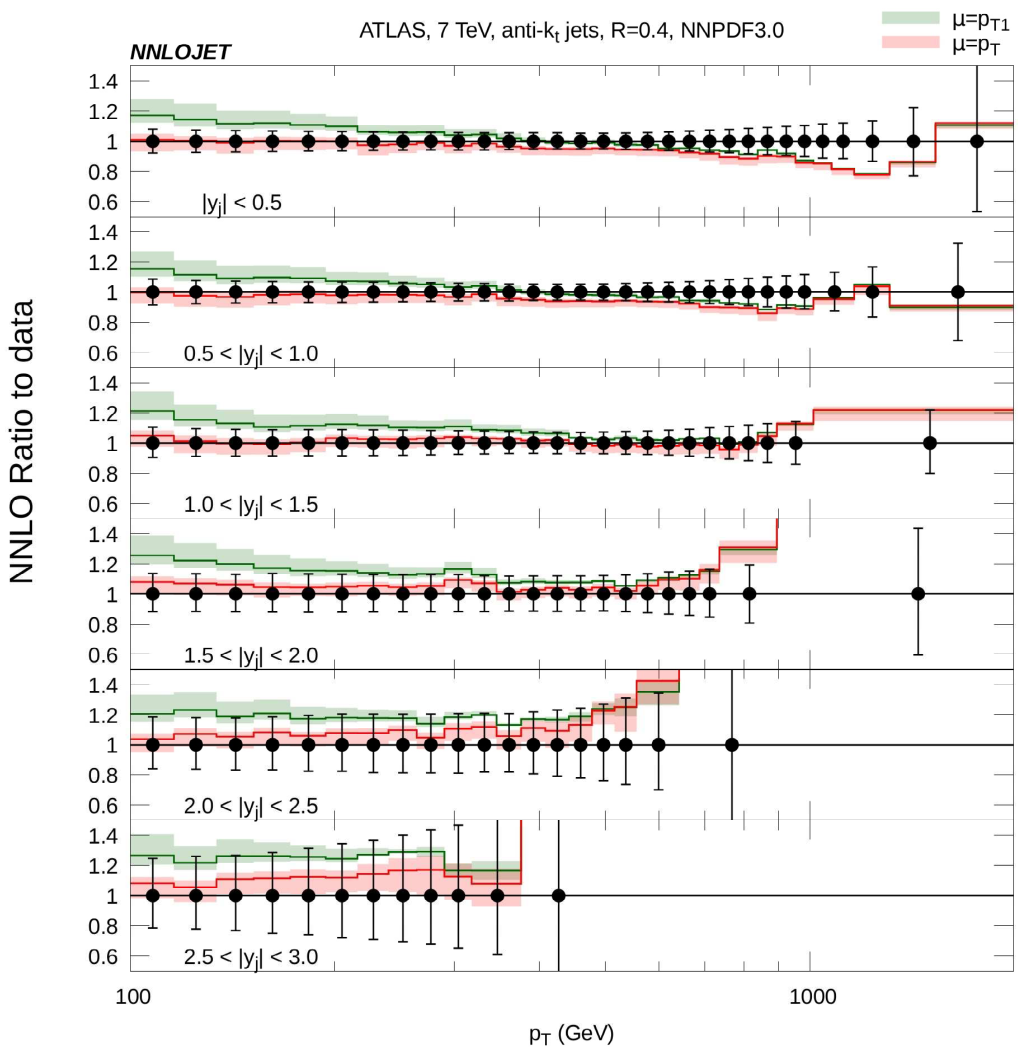
$$\mu_R = \mu_F = p_{T1}$$



$$\mu_R = \mu_F = p_T$$

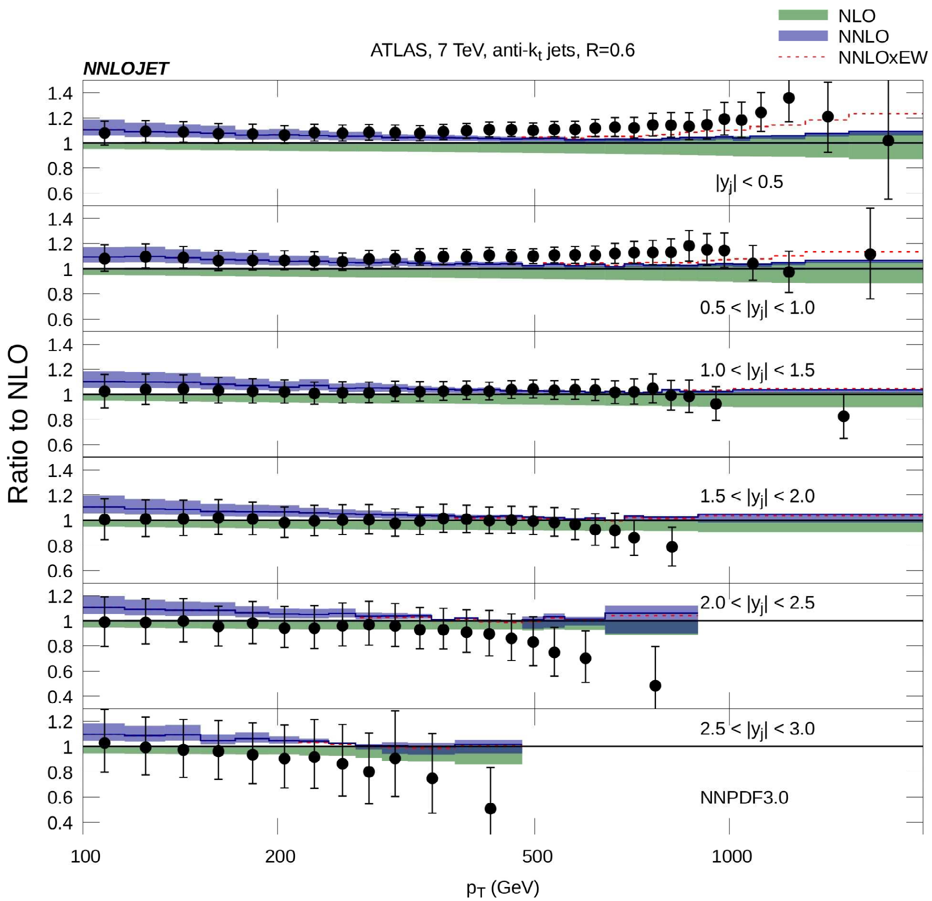
- ✘ Quite different behaviour!
- ➠ scale uncertainty much smaller than difference between scale choices
- ➠ explore alternative scale choices

Single Jet Inclusive Distribution – R=0.4

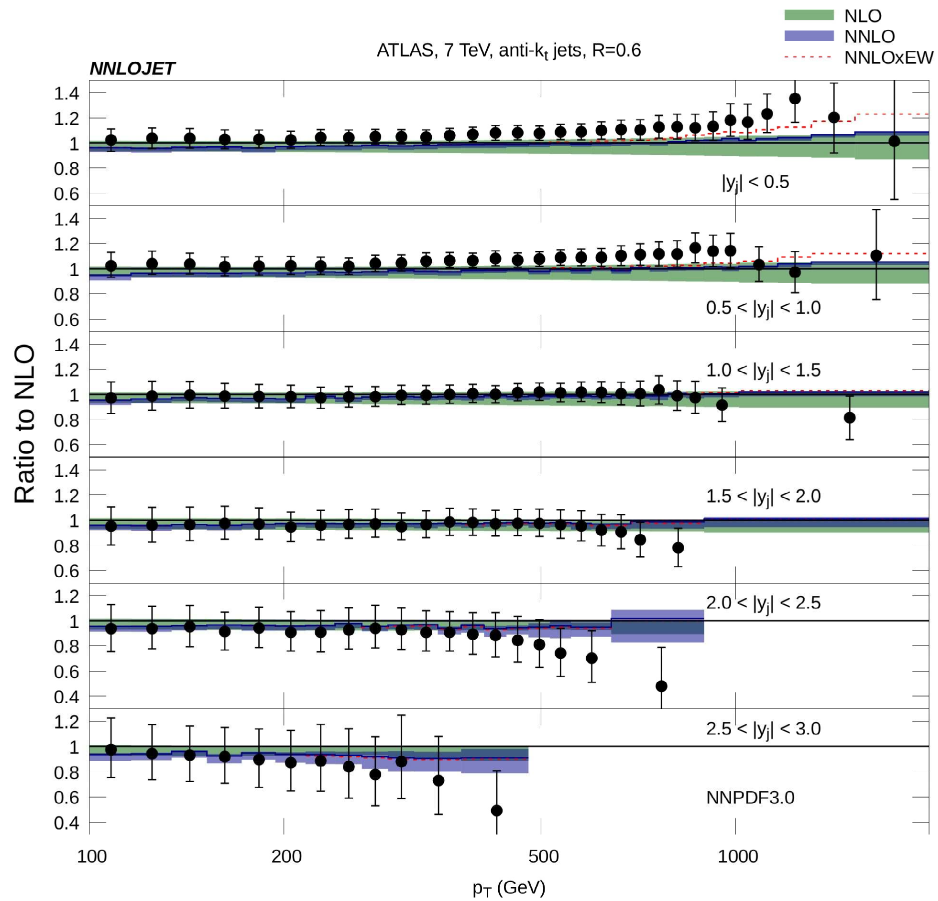


✘ Scale uncertainty is smaller than the uncertainty in choosing p_T or p_{T1}

Single Jet Inclusive Distribution – R=0.6



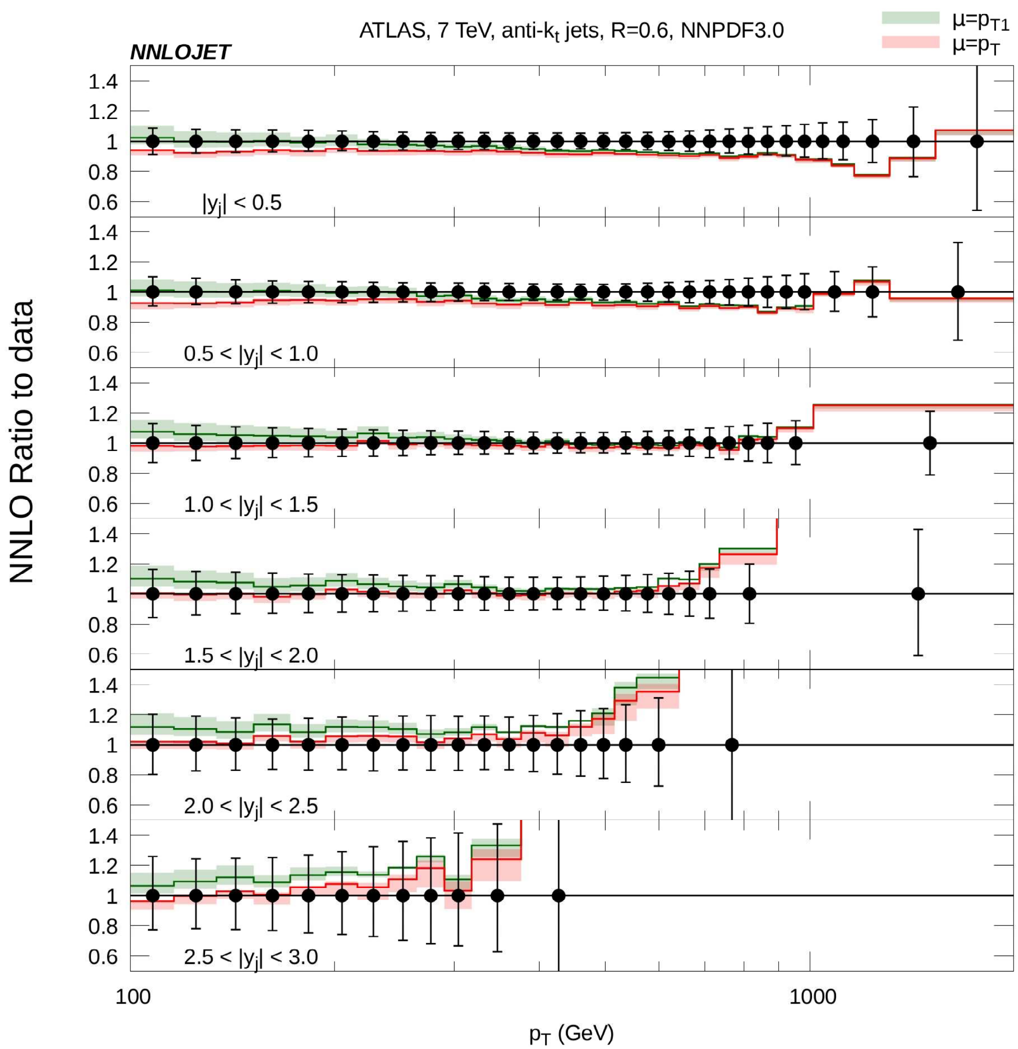
$$\mu_R = \mu_F = p_{T1}$$



$$\mu_R = \mu_F = p_T$$

- ✘ Quite different behaviour!
- ✓ NLO with $\mu = p_T$ describes $R = 0.6$ data quite well
- ✓ NNLO with $\mu = p_{T1}$ describes $R = 0.6$ data quite well

Single Jet Inclusive Distribution – R=0.6



- ✘ Scale uncertainty is smaller than the uncertainty in choosing p_T or p_{T1}

CPU cost

✓ Standalone production run with fixed $\sqrt{s}$, fixed R, fixed PDF, three scale variation for $\mu = p_{T1}$ and $\mu = p_T$ (Warmup $\sim$ 1-2%)

Job Type	No. Jobs	Runtime/Job (hr)	Total Runtime
LO	200	0.5	100
NLO-V	500	1.5	750
NLO-R	500	2	1000
NNLO-VV	600	20	12000
NNLO-RV	2500	50	125000
NNLO-RRa	3500	50	175000
NNLO-RRb	2000	20	40000
			353850

✓ because LO is independent of R and $p_T = p_{T1}$ to obtain different cone sizes/different scales can do a (much cheaper) NLO 3-jet calculation

$$\frac{d\sigma^{NNLO}(R_2)}{dp_T} = \frac{d\sigma^{NNLO}(R_1)}{dp_T} + \left(\frac{d\sigma^{R}(R_2)}{dp_T} - \frac{d\sigma^{R}(R_1)}{dp_T}\right)$$
$$+ \left(\frac{d\sigma^{RV}(R_2)}{dp_T} - \frac{d\sigma^{RV}(R_1)}{dp_T}\right) + \left(\frac{d\sigma^{RR}(R_2)}{dp_T} - \frac{d\sigma^{RR}(R_1)}{dp_T}\right)$$

Summary

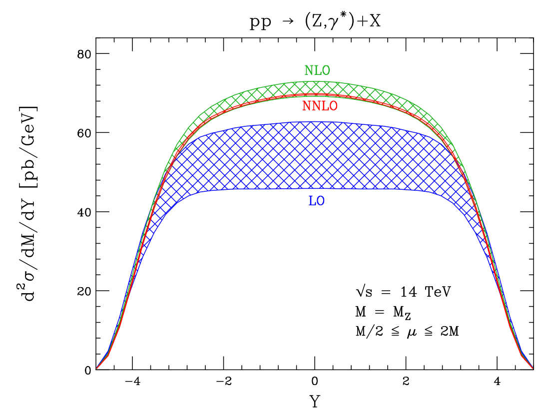
- ✚ NNLOJET is able to make a range of fully differential parton level NNLO predictions that can be compared with LHC fiducial cross sections
- — code is partially automated and typically requires significant CPU resource
- — need validation with different IR subtraction schemes
- ✚ results show anticipated features of NNLO calculations - reduction of scale uncertainty, stabilisation of perturbative series, etc
- — serious study of choice of scales and pdf uncertainties needed and in progress
- ✓ Single jet inclusive distribution
  - ✚ Reduction of the scale uncertainty but . . .
  - — difference between common scale choices p_T and p_{T1} larger than scale uncertainty
  - ✚ NP effects important at large R, low p_T (~ 30% for $R = 0.7$, $p_T \sim 50$ GeV)
  - ✚ EW effects important at large p_T (~ 5% for $p_T \sim 1000$ GeV)

Work in progress:

- ✓ Including other processes, such as dijets, other Higgs decays, etc
- ✓ Studying potential of data to constrain PDF sets and interface to `APPLgrid`, `fastNLO`

What to expect from NNLO (1)

✓ Reduced renormalisation scale dependence

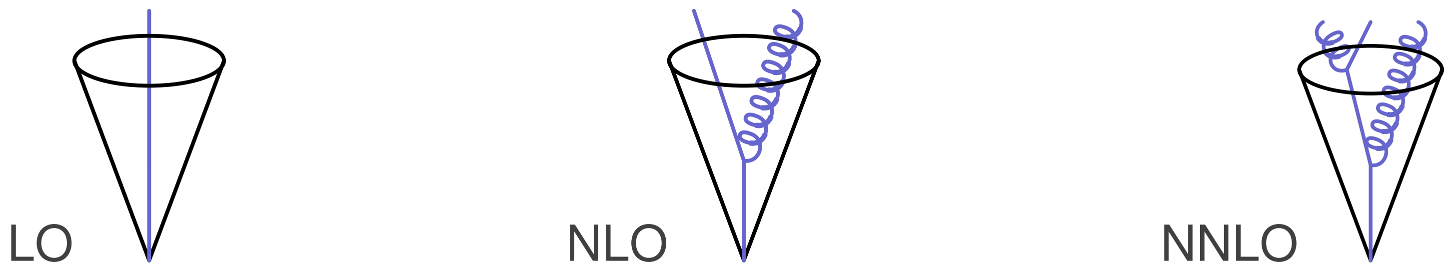


✓ Better able to judge convergence of perturbation series

✓ Fiducial (parton level) cross sections. Fully differential, so that experimental cuts can be applied directly

✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower

➠ better matching of jet algorithm between theory and experiment

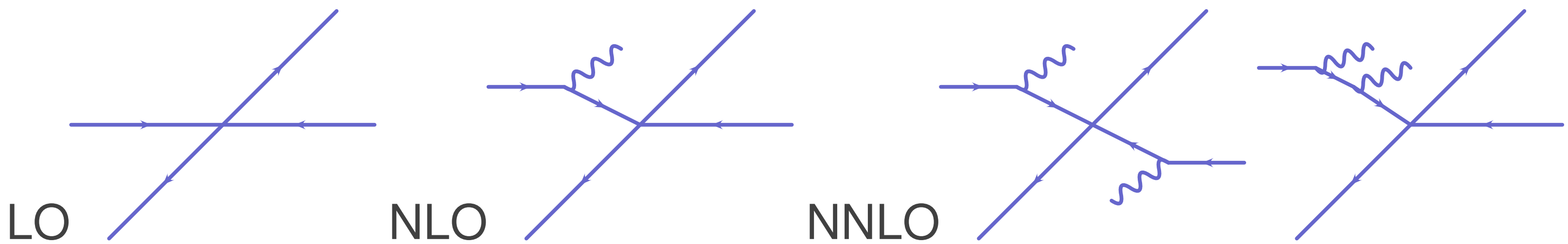


What to expect from NNLO (2)

- ✓ All channels present at NNLO

LO	NLO	NNLO
gg	gg, qg	gg, qg, qq
$q\bar{q}$	$q\bar{q}$, qg	$q\bar{q}$, qg, gg

- ✓ Better description of transverse momentum of final state due to double radiation off initial state



  - ✓ At LO, final state has no transverse momentum
  - ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
  - ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state