Jet axes and jet substructure

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Outline

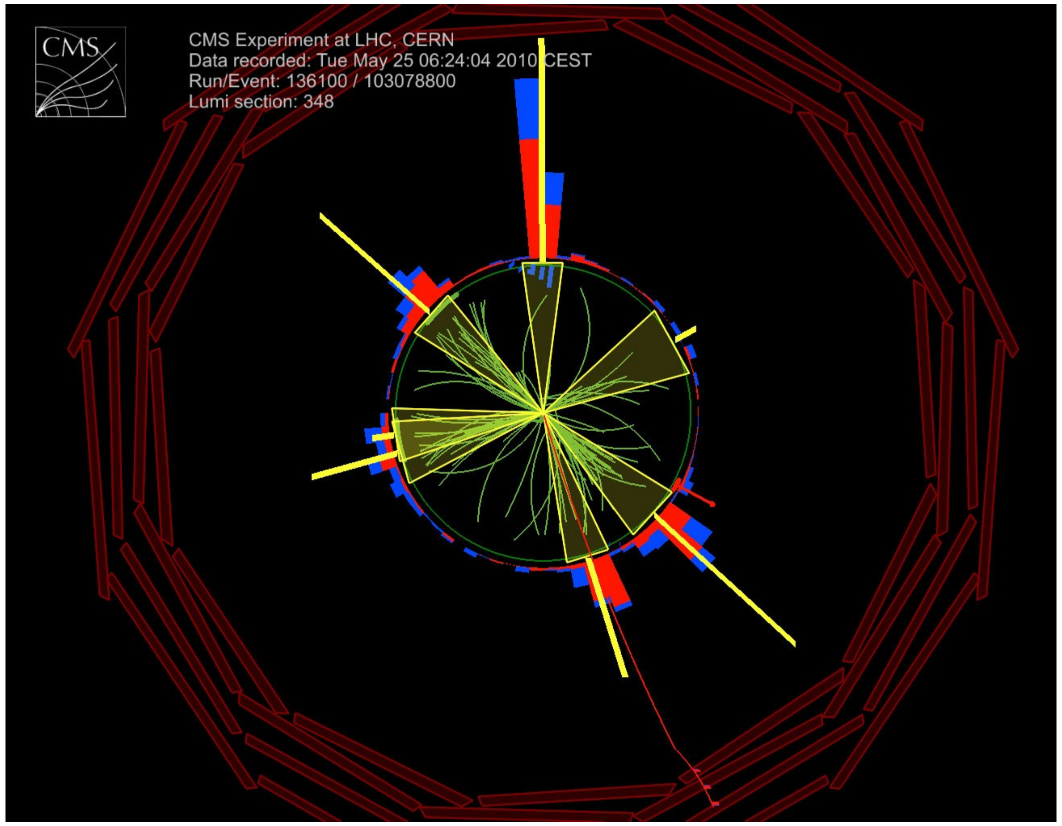
1. Introduction
2. Inclusive subjets
3. Winner-take-all jet shape
4. Standard jet shape
5. Conclusions

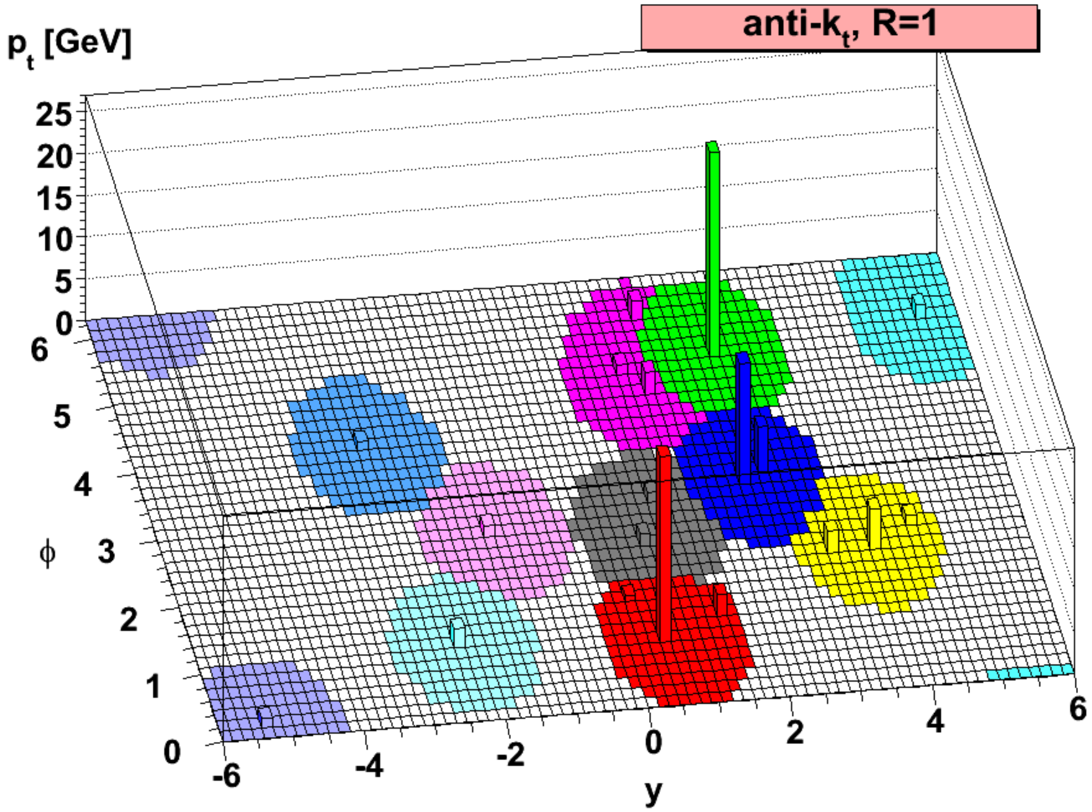
Based on work with Z. Kang, F. Ringer
and D. Neill, A. Papaefstathiou, L. Zoppi

1. Introduction

What is a jet?

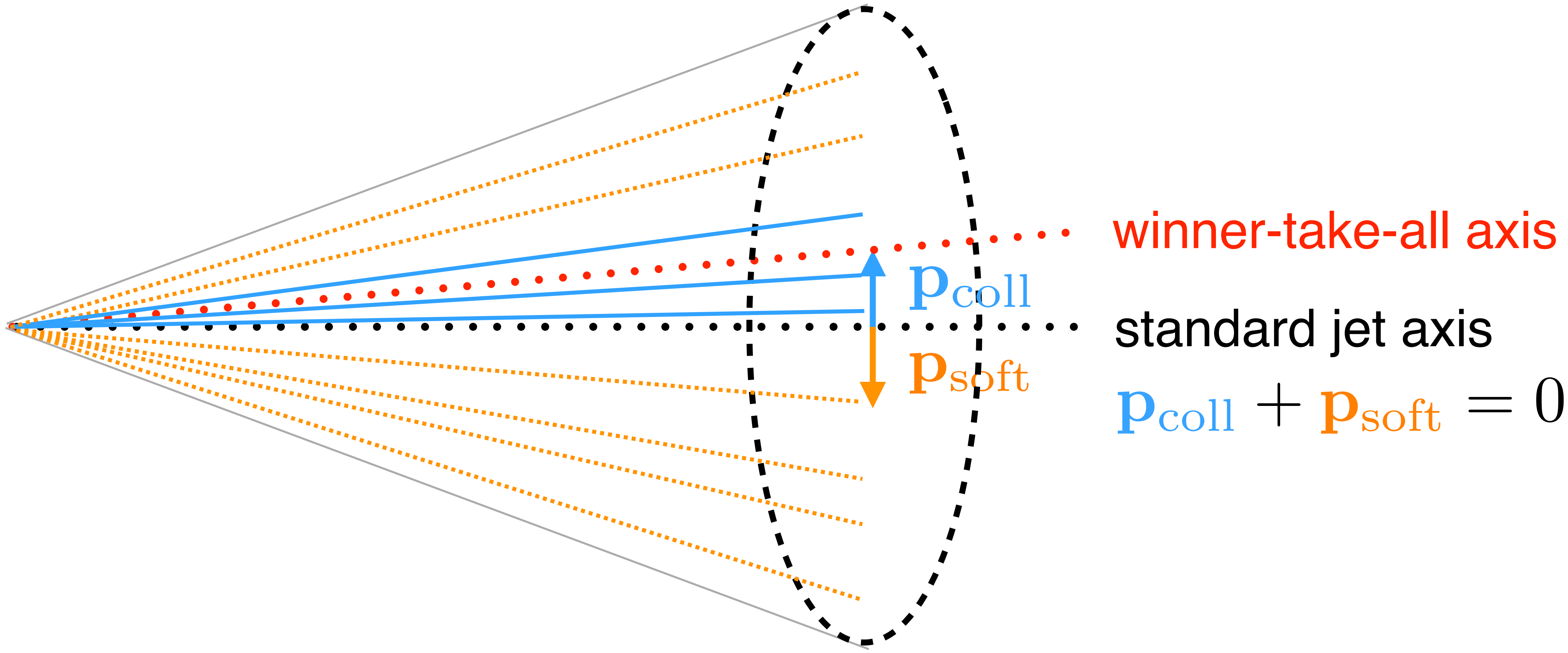
- Energetic quark or gluon radiates and hadronizes → jet





- 30 years to settle on jet definition: anti-k_T [Cacciari, Salam, Soyez]
- Today I want to discuss the jet axis

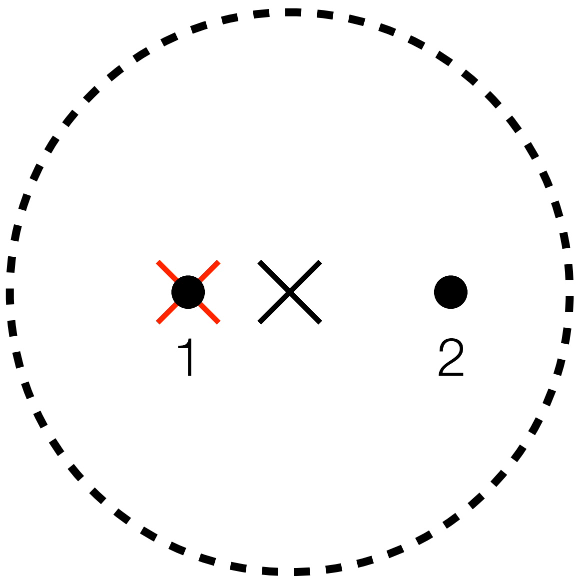
Recoil of jet axis



- Standard jet axis along total momentum of jet
  → Recoils against soft radiation inside jet
- Winner-take-all axis is insensitive to soft radiation [Larkoski, Neill, Thaler]

Winner-take-all axis

- Standard clustering algorithm:
  - Determine distance between all particles
  - Merge nearest particles $p_1, p_2 \to p = p_1 + p_2$
  - Repeat until distance exceeds jet radius *R*



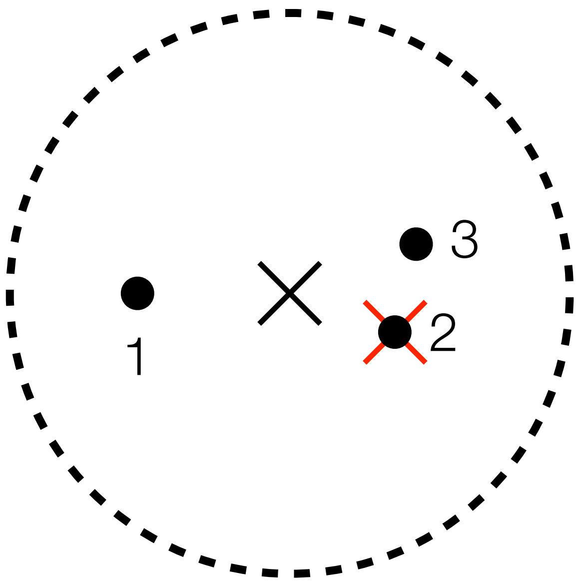
$E_1 > E_2$

- Winner-take-all modifies merging

$$E = E_1 + E_2$$

$$\hat{n} = \begin{cases} \hat{n}_1 & \text{if } E_1 > E_2 \\ \hat{n}_2 & \text{if } E_2 > E_1 \end{cases}$$

[Bertolini, Chan, Thaler]



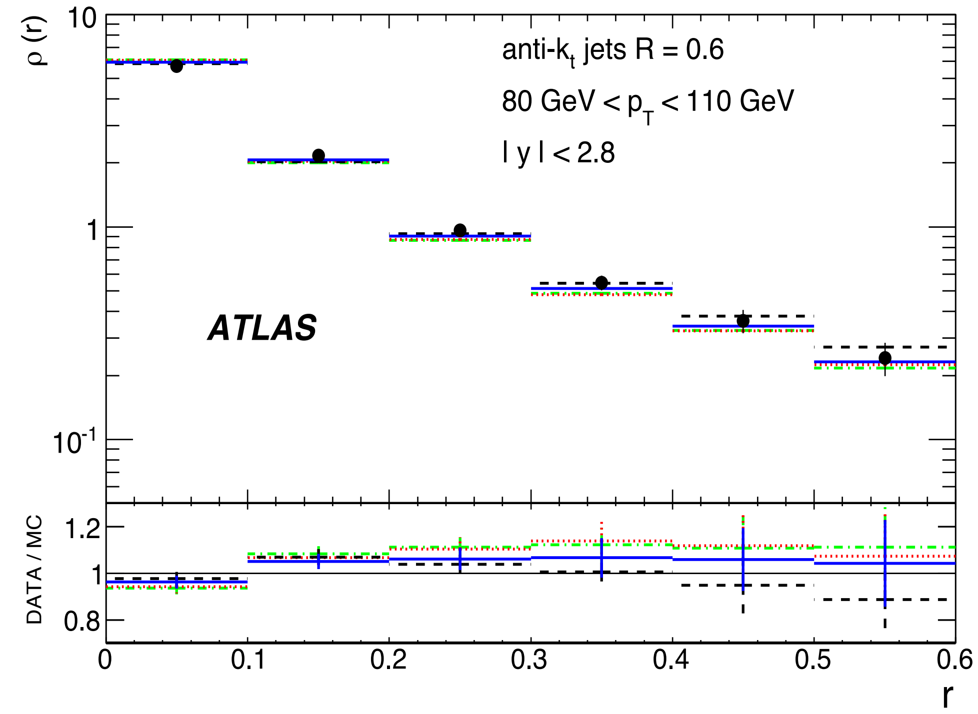
$E_2 + E_3 > E_1$

$E_1 > E_2 > E_3$

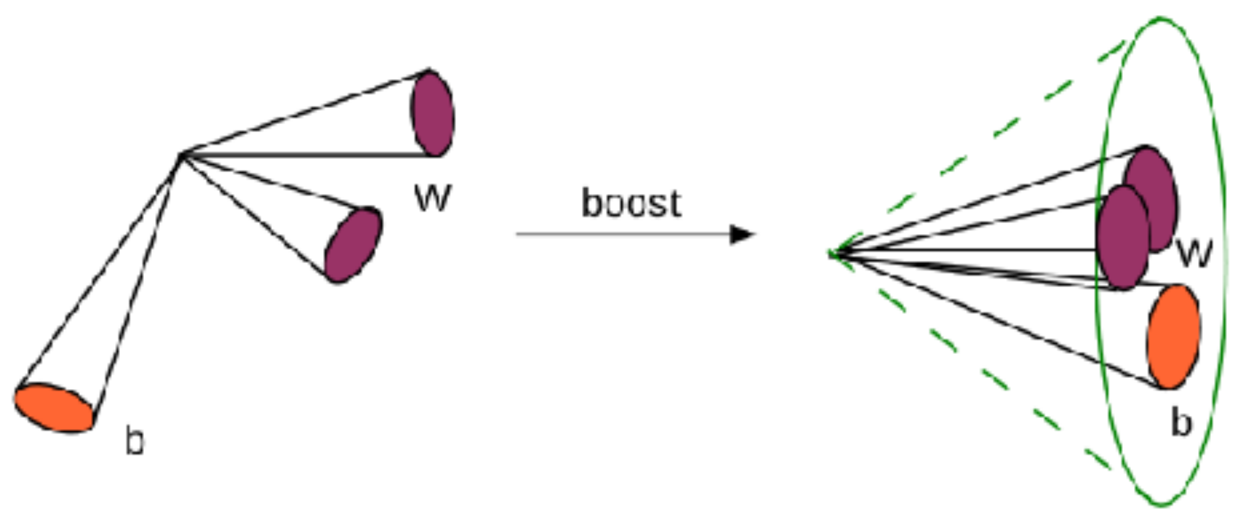
- Axis not recoiled by soft radiation

Jet shape and subjets

- Jet shape:
  - Average energy fraction in cone of radius $r < R$
  - Potentially sensitive to soft radiation through recoil
  - Measured by ATLAS, CMS, …



- Prototype for other observables: tagging boosted *t*, Higgs, …



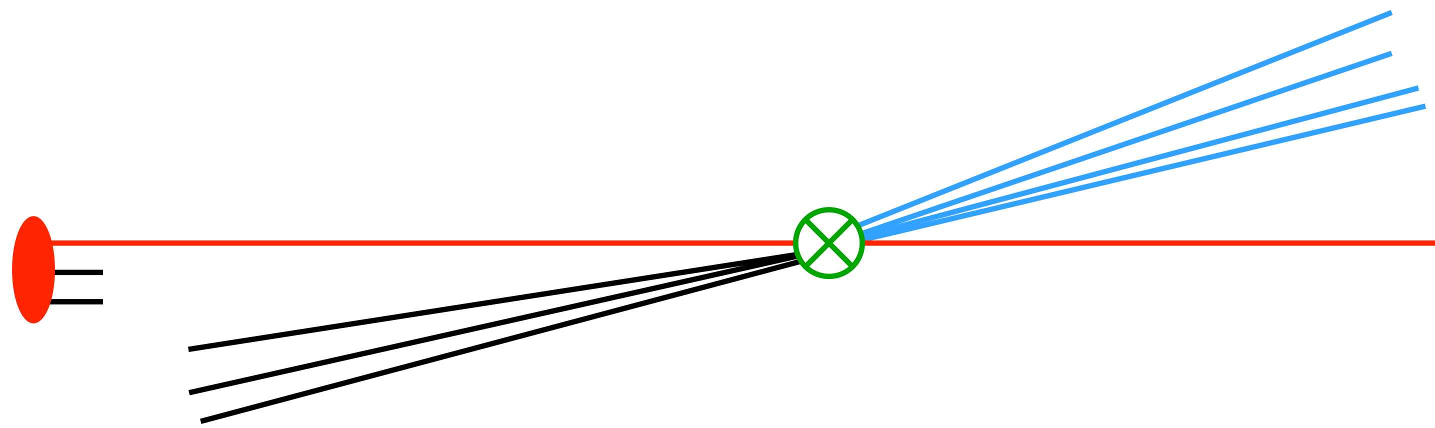
3 jets

3 subjets

2. Inclusive subjets

Warming up with inclusive subjets

- Energy fraction z_r of inclusive subjets in inclusive jet cross section



$$\frac{d\sigma}{d\eta\, dp_T\, dz_r} = \sum_{a,b,c} f_a(x_a,\mu) \otimes f_b(x_b,\mu) \otimes \mathcal{H}_{ab}^{c}(x_a,x_b,\eta,p_T/z,\mu) \otimes \mathcal{G}_c^{\text{jet}}(z_r,r;z,p_T R,\mu)$$

- Collinear factorization of parton distributions, hard collision and jet, assuming R<<1

Subjet function and ln *R* resummation

- Subjet function $\mathcal{G}_i^{\text{jet}}$ at NLO [Kang, Ringer, WW]

$$\mathcal{G}_q^{\text{jet}}(z_r, r;, z, p_T R, \mu) = \delta(1-z)\delta(1-z_r) + \frac{\alpha_s}{\pi}\bigg\{\delta(1-z_r)\ln\Big(\frac{\mu}{p_T R}\Big)\big[P_{qq}(z) + P_{gq}(z)\big]$$
$$-\,\delta(1-z)\ln\Big(\frac{r}{R}\Big)\big[P_{qq}(z_r) + P_{qq}(1-z_r)\big] + C_F\delta(1-z_r)\bigg[\delta(1-z)\Big(\frac{13}{4}-\frac{\pi^2}{3}\Big)$$
$$-\,(1+z^2)\Big(\frac{\ln(1-z)}{1-z}\Big)_+ - \ln(1-z)\,\frac{1+(1-z)^2}{z} - \frac{1}{2}\bigg]\bigg\}$$

  - DGLAP evolution for fraction *z* of parton *i* going into jet

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathcal{G}_i^{\text{jet}}(z_r, r; z, p_T R, \mu) = \sum_j \int_z^1 \frac{\mathrm{d}z'}{z'}\,\frac{\alpha_s}{\pi}P_{ji}\Big(\frac{z}{z'},\mu\Big)\,\mathcal{G}_j^{\text{jet}}(z_r, r; z', p_T R, \mu)$$

  - ln *R* resummed by evolving from jet scale $p_T R$ to hard scale p_T
    [Dasgupta, Dreyer, Salam, Soyez; Kang, Ringer, Vitev; Dai, Kim, Leibovich]

Subjet function and ln R resummation

- Subjet function $\mathcal{G}_i^{\text{jet}}$ at NLO [Kang, Ringer, WW]

$$\mathcal{G}_q^{\text{jet}}(z_r, r; , z, p_T R, \mu) = \delta(1-z)\delta(1-z_r) + \frac{\alpha_s}{\pi}\Bigg\{\delta(1-z_r)\ln\left(\frac{\mu}{p_T R}\right)[P_{qq}(z) + P_{gq}(z)] - \delta(1-z)\ln\left(\frac{r}{R}\right)[P_{qq}(z_r) + P_{qq}(1-z_r)] + C_F\delta(1-z_r)\left[\delta(1-z)\left(\frac{13}{4} - \frac{\pi^2}{3}\right) - (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \ln(1-z)\frac{1+(1-z)^2}{z} - \frac{1}{2}\right]\Bigg\}$$

  - DGLAP evolution for fraction z of parton i going into jet

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\mathcal{G}_i^{\text{jet}}(z_r, r; z, p_T R, \mu) = \sum_j \int_z^1 \frac{\mathrm{d}z'}{z'}\frac{\alpha_s}{\pi}P_{ji}\left(\frac{z}{z'}, \mu\right)\mathcal{G}_j^{\text{jet}}(z_r, r; z', p_T R, \mu)$$

  - ln R resummed by evolving from jet scale $p_T R$ to hard scale p_T
[Dasgupta, Dreyer, Salam, Soyez; Kang, Ringer, Vitev; Dai, Kim, Leibovich]
  - z_r dependence given by splitting functions
  - For $r \ll R$, logarithms of r/R require resummation

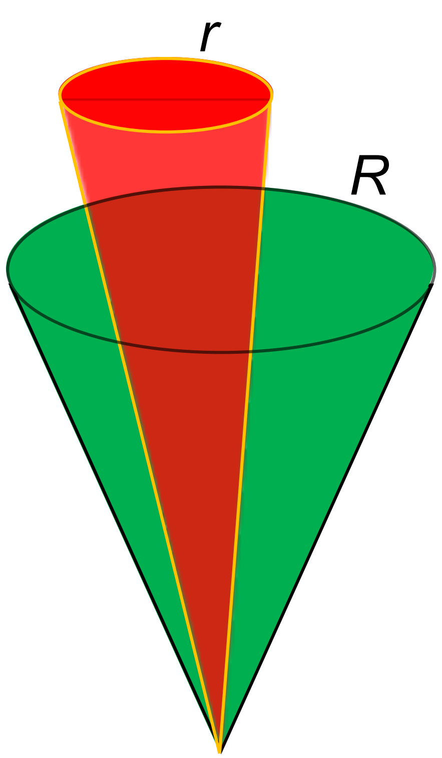
Resummation of ln (*r*/*R*)

- Collinear factorization for *r* << *R*

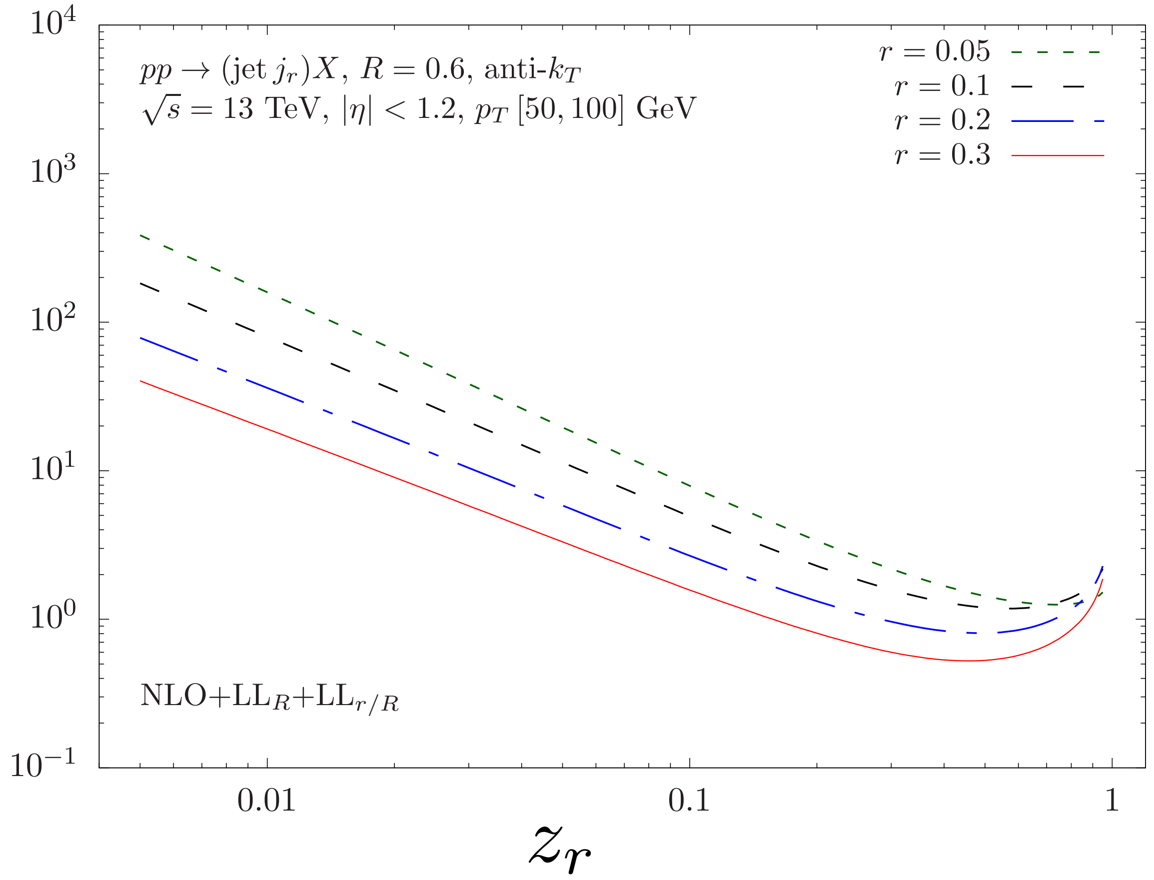
$$\mathcal{G}_i^{\text{jet}}(z_r, r; , z, p_T R, \mu) = \sum_j \int_{z_r}^1 \frac{\mathrm{d}z_r'}{z_r'} \mathcal{J}_{ij}(z_r'; z, p_T R, \mu)\, J_j\left(\frac{z_r}{z_r'}, p_T r, \mu\right) \left[1 + \mathcal{O}\left(\frac{r^2}{R^2}\right)\right]$$

[Dai, Kim, Leibovich; Kang, Ringer, WW]

- Jet function J_j for subjet is same as for inclusive jet production
- ln (*r*/*R*) resummed by DGLAP from subjet scale $p_T r$ to jet scale $p_T R$
- Matching coefficients $\mathcal{J}_{ij}$ same whether subjets or hadrons in jet, so limit *r* → 0 continuous



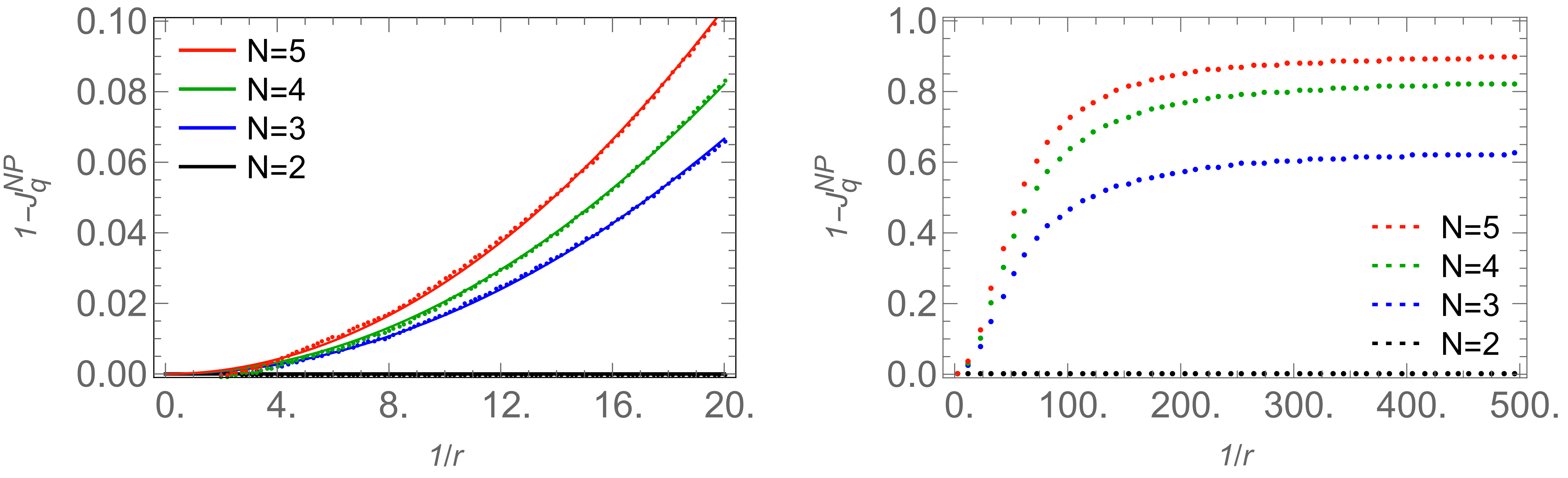
Numerics for $pp \to$ jet + X



- Spectrum of inclusive subjets is splitting function at $\mathcal{O}(\alpha_s)$, with deviations due to ln *r*/*R* resummation

- Not monotonically decreasing with z_r, unlike hadron spectrum
  Must be restored for $r \to 0$

Fragmentation limit $r \to 0$



- Parametrize nonperturbative effects [Kang, Ringer, WW]

$$J_i(z_r, Er, \mu) = \int_{z_r}^{1} \frac{\mathrm{d}z_r'}{z_r'} \, J_i^{\mathrm{pert}}(z_r', Er, \mu) \, J_i^{\mathrm{NP}}\left(\frac{z_r}{z_r'}, Er\right)$$

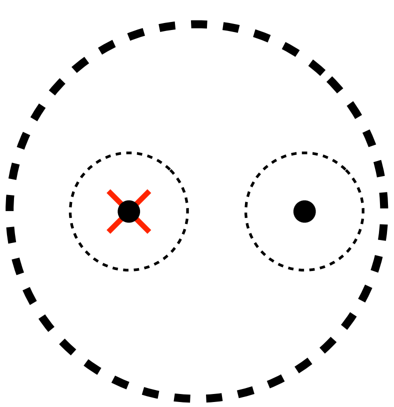
  - Extract Mellin moments of J_q^{NP} from Pythia e^+e^- with E = 250 GeV
  - Leading nonperturbative corrections $\sim \Lambda_{\mathrm{QCD}}^2/(Er)^2$
  - J_q^{NP} asymptotes to fragmentation function

3. Winner-take-all jet shape

Central subjet function

- Central subjet function $\tilde{\mathcal{G}}_i^{\rm jet}$ gives energy fraction z_r of subjet centered on winner-take-all axis
    - Factorization of jet and resummation of ln R same as before
    - Jet shape is average z_r
- At NLO, two subjets with energy fractions z_r and 1 - z_r WTA axis along most energetic one → restrict to $z_r > 1/2$

$$\tilde{\mathcal{G}}_q^{\rm jet}(z_r, r;, z, p_T R, \mu) = \delta(1-z)\delta(1-z_r) + \frac{\alpha_s}{\pi}\Big\{\delta(1-z_r)\ln\left(\frac{\mu}{p_T R}\right)\left[P_{qq}(z)+P_{gq}(z)\right] - \delta(1-z)\ln\left(\frac{r}{R}\right)\theta\left(z_r-\frac{1}{2}\right)\left[P_{qq}(z_r)+P_{qq}(1-z_r)\right] + C_F\delta(1-z_r)\Big[\delta(1-z)\left(\frac{13}{4}-\frac{\pi^2}{3}\right) - (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - \ln(1-z)\,\frac{1+(1-z)^2}{z} - \frac{1}{2}\Big]\Big\}$$



Resummation of ln (*r*/*R*)

- Similar factorization for *r* << *R*

$$\tilde{\mathcal{G}}_q^{\text{jet}}(z_r, r; , z, p_T R, \mu) = \sum_j \int_{z_r}^1 \frac{\mathrm{d}z_r'}{z_r'} \tilde{\mathcal{J}}_{ij}(z_r'; z, p_T R, \mu)\, \tilde{J}_j\left(\frac{z_r}{z_r'}, p_T r, \mu\right)\left[1 + \mathcal{O}\left(\frac{r^2}{R^2}\right)\right]$$

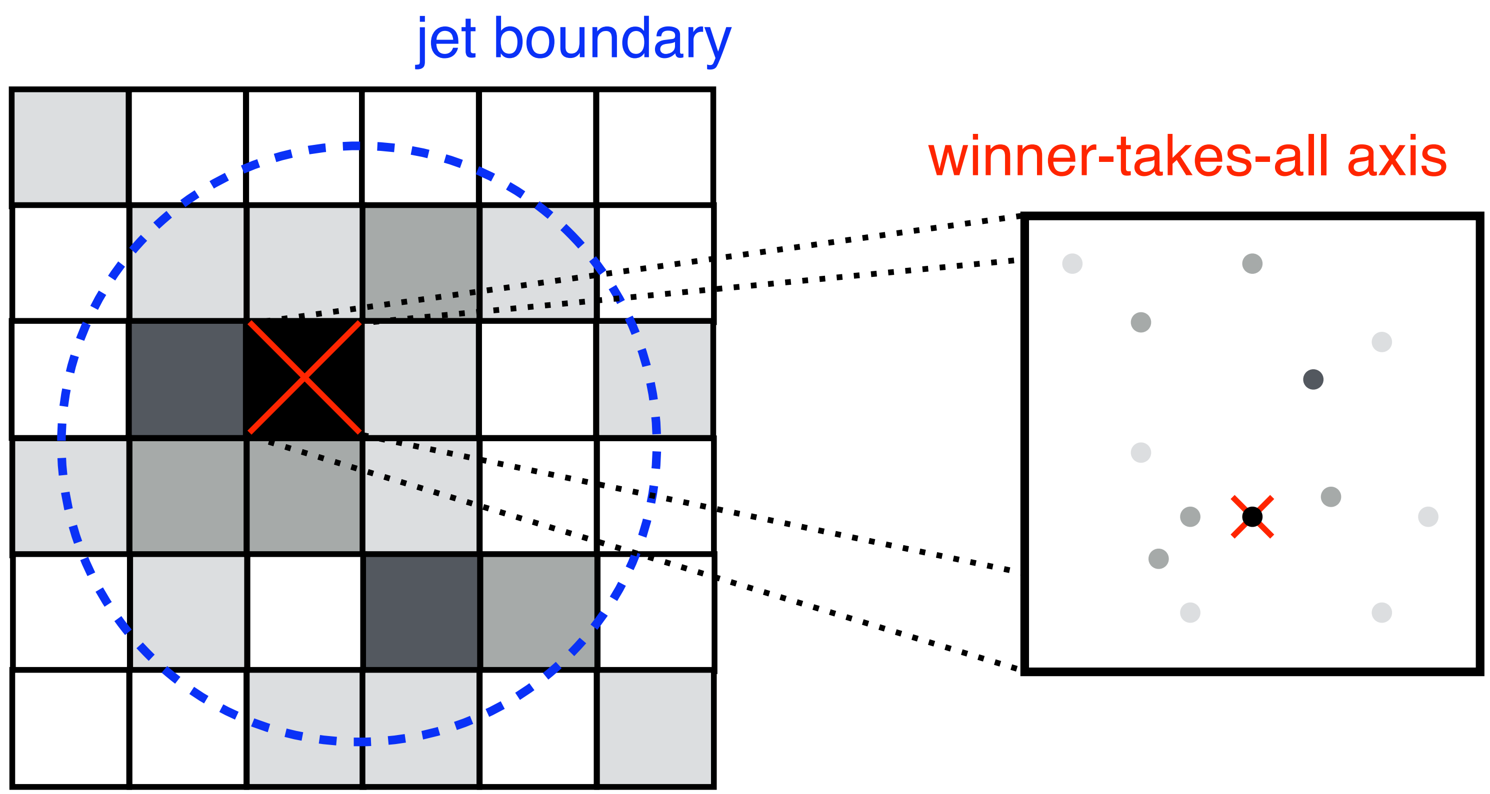
[Neill, Scimemi, WW; Kang, Ringer, WW]

  - ln (*r*/*R*) resummed by evolving from subjet scale $p_T r$ to jet scale $p_T R$
  - Jet function $\tilde{J}_j$ has modified DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} \tilde{J}_i(z_r, p_T r, \mu) = \sum_j \int_{z_r}^1 \frac{\mathrm{d}z_r'}{z_r'} \frac{\alpha_s}{\pi}\, \theta\left(z_r - \frac{1}{2}\right) P_{ji}(z_r, \mu)\, \tilde{J}_j\left(\frac{z_r}{z_r'}, p_T r, \mu\right)$$

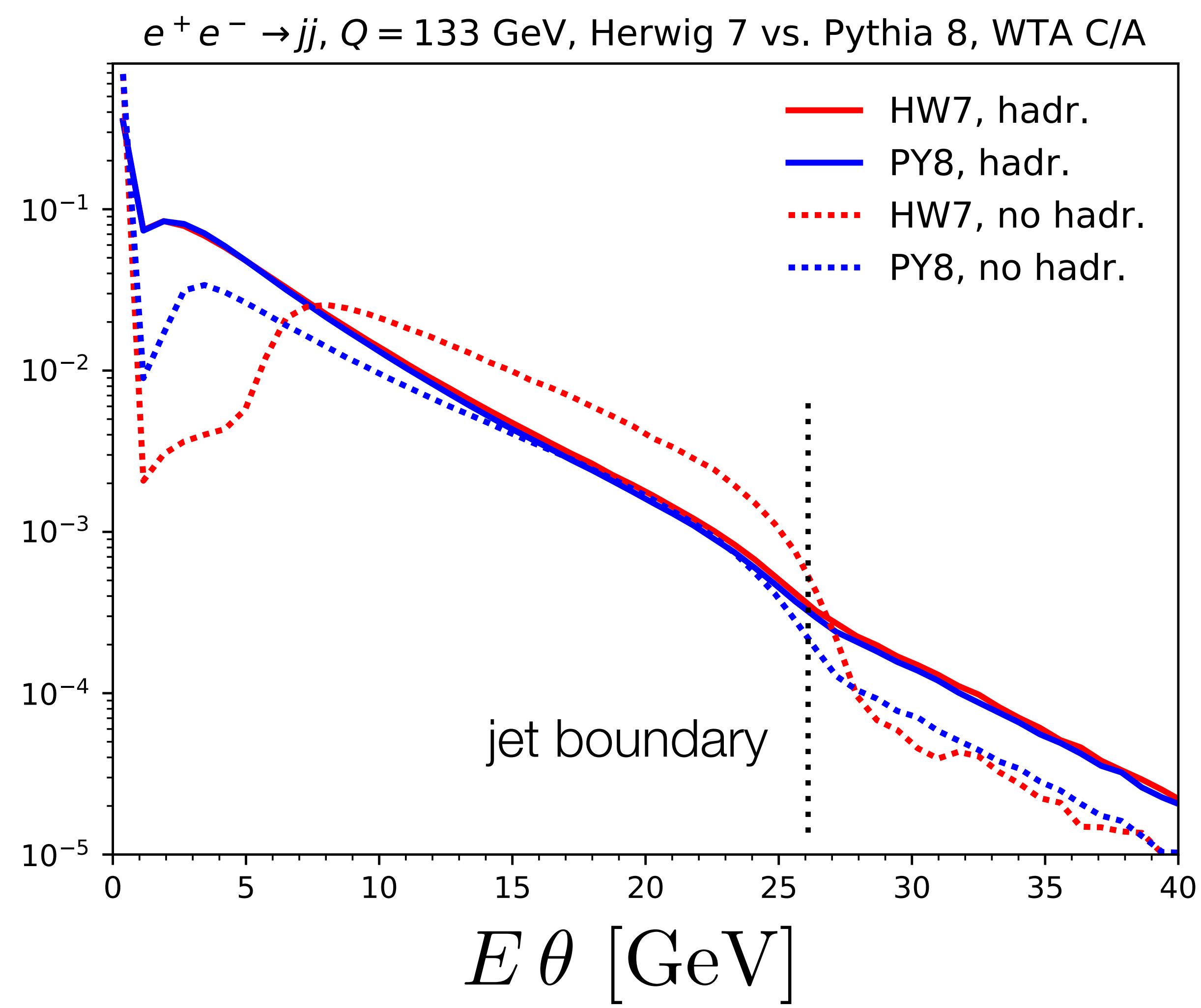
  - Matching coefficients $\tilde{\mathcal{J}}_{ij}$ not the same as for inclusive subjets

Factorization in pictures



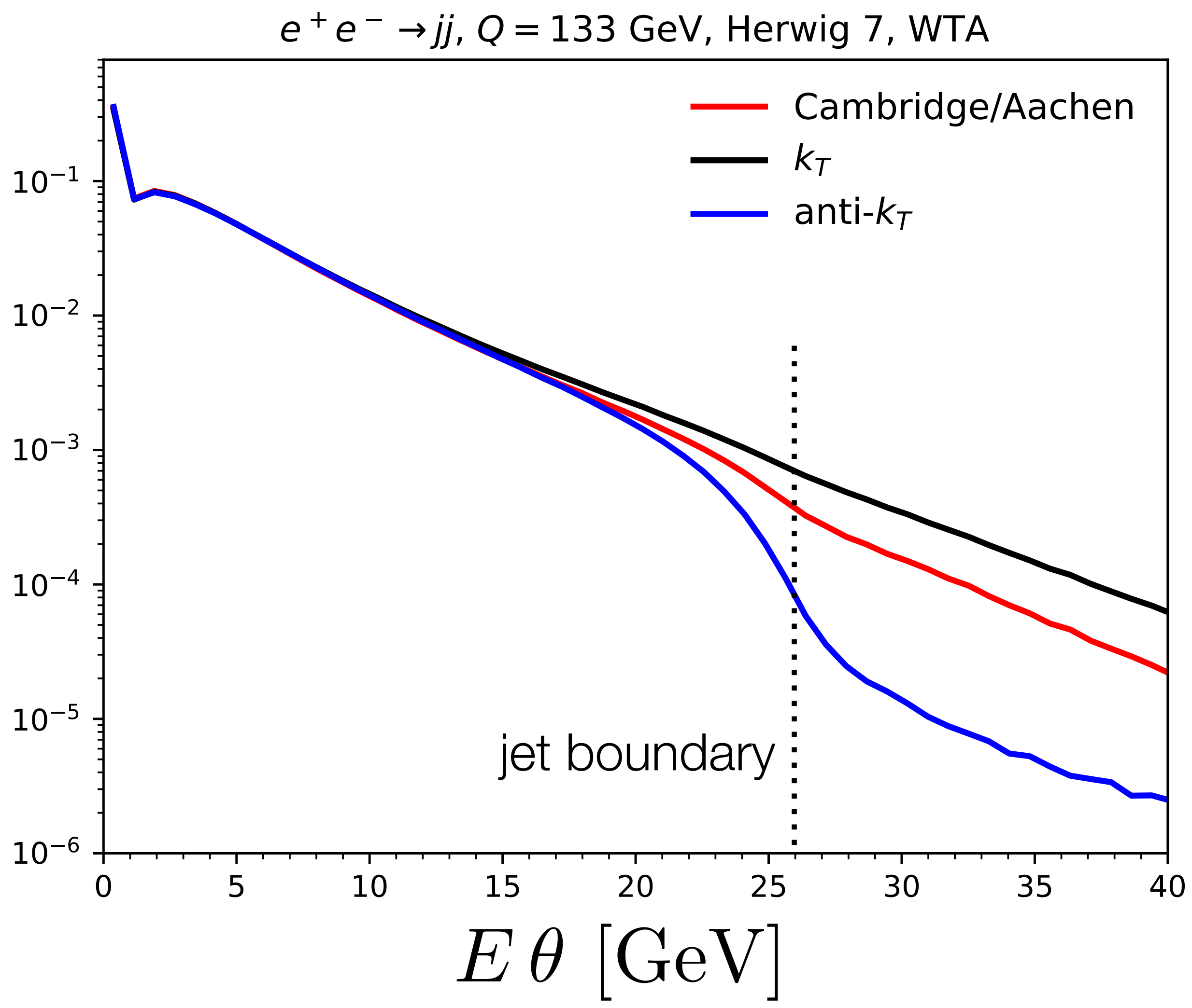
- Measurement factorizes:
  - $\tilde{\mathcal{J}}_{ij}$ identifies "pixel" with axis, sensitive to jet boundary
  - $\tilde{J}_j$ determines axis in pixel and z_r of subjet
- Amplitude factorizes as winner-take-all axis on energetic particle

Jet shape in e^+e^- with winner-take-all



- Winner-take-all axis along particle → peak at $\theta = 0$
- Pythia and Herwig agree after hadronization
- Herwig's hadronization changes energy flow considerably

Comparing jet algorithms



- Jet algorithm dependence only enters near jet boundary
  → Agrees with factorization of $\tilde{\mathcal{J}}_{ij}$ and $\tilde{J}_j$
- Anti-k_T has hardest and k_T has softest boundary

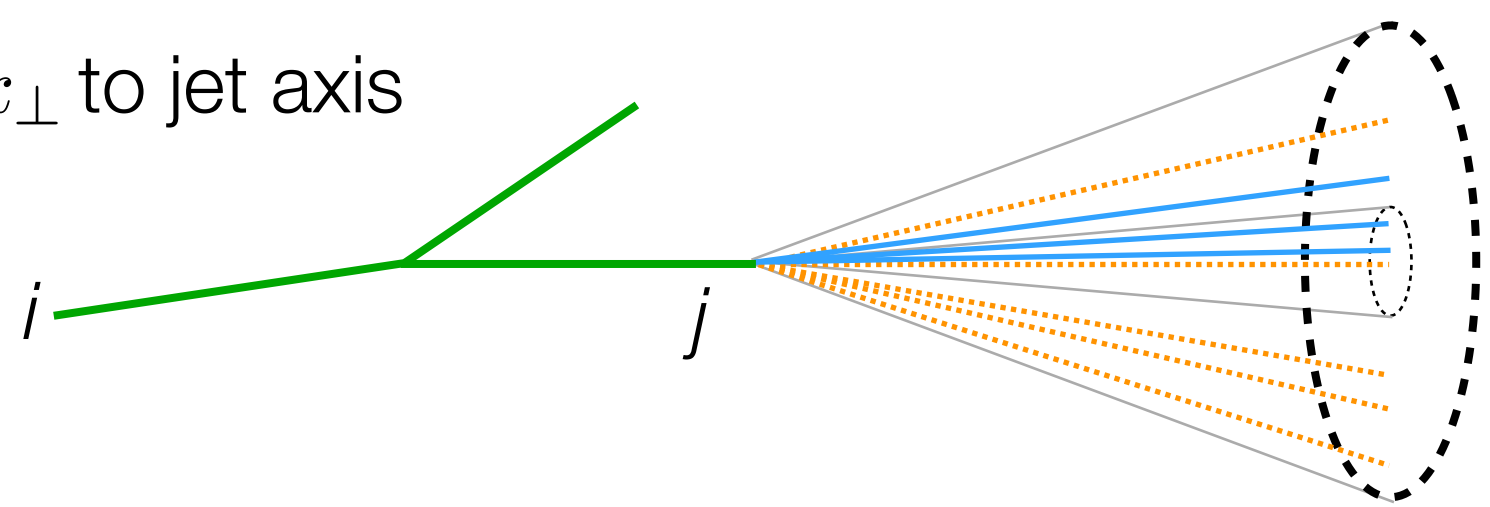
4. Standard jet shape

Factorization for $r << R$

- Central subjet function $\hat{\mathcal{G}}_i^{\rm jet}$ describes energy fraction z_r of subjet centered on standard jet axis
  - NLO calculation contains $\ln^2 (r/R)$ → soft sensitivity
- Factorization for $r << R$:

$$\hat{\mathcal{G}}_i(z_r, r; , z, p_T R, \mu) = H_{ij}(z, \omega_R R, \mu) \int d^2 k_\perp \, C_j(z_r, \omega_r r, k_\perp, \mu, \nu) \, S_j(k_\perp, R, \mu, \nu)$$

  - Hard: splittings of parton *i* outside jet, determines *z*
  - Collinear: splittings inside jet from parton *j* at angles r, determines z_r
  - Soft: gives recoil $k_\perp$ to jet axis



Resummation on ln (*r*/*R*)

- ln (*r*/*R*) resummed by evaluating ingredients at natural scales

$$\mu_H \sim p_T R\,, \quad \mu_C \sim \mu_S \sim p_T\, r\,, \quad \nu_C \sim p_T\,, \quad \nu_S \sim p_T\,\frac{r}{R}$$

and evolving to common μ and ν using

$$\gamma_q^C + \gamma_q^S = \frac{\alpha_s C_F}{\pi}\Big(2\ln\frac{\mu}{p_T R} + \frac{3}{2}\Big)$$

$$\gamma_q^\nu = \frac{\alpha_s C_F}{\pi}\,\frac{1}{\mu^2}\,\frac{1}{(k_\perp^2/\mu^2)}_+$$

  - Recoil overlooked in earlier calculations, enters at NLL
  [Seymour; Li, Li, Yuan; Chien, Vitev]
  - All-orders resummation is hinderen by non-global logarithms, as only soft radiation inside the jet is constrained

Conclusions

- Standard jet axis is sensitive to recoil, winner-take-all axis is not
- Resummation of ln R independent of jet substructure
- Resummation of ln (r/R) depends on subjet measurement:
    - Inclusive subjets: DGLAP resums ln (r/R)
    - Winner-take-all: **modified** DGLAP resums ln (r/R)
    - Standard jet axis: Sudakov resummation of $\ln^2 (r/R)$, suffers from nonglobal logarithms

Thank you!