Status of hadronic light-by-light scattering and the muon (g-2)

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Work done in collaboration with
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Outline

- Short Introduction
- Hadronic light-by-light scattering contribution to the muon g-2:
  - its role in the 3σ deviation
  - its framework and reference numbers
  - its new trends
- Outlook

The anomalous magnetic moment of the muon

- gyromagnetic ratio: g

$$\vec{\mu} = g \frac{e}{2m} \cdot \vec{S}$$

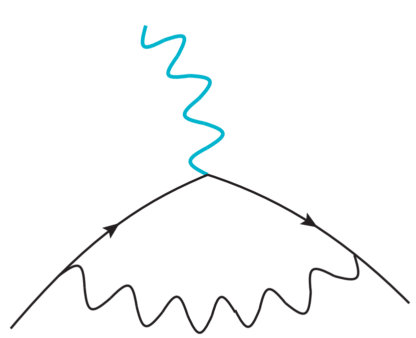
spin $\frac{1}{2}$ $\rightarrow$ Dirac theory: $g = 2$

QFT (Rad. Corr): $g \neq 2$

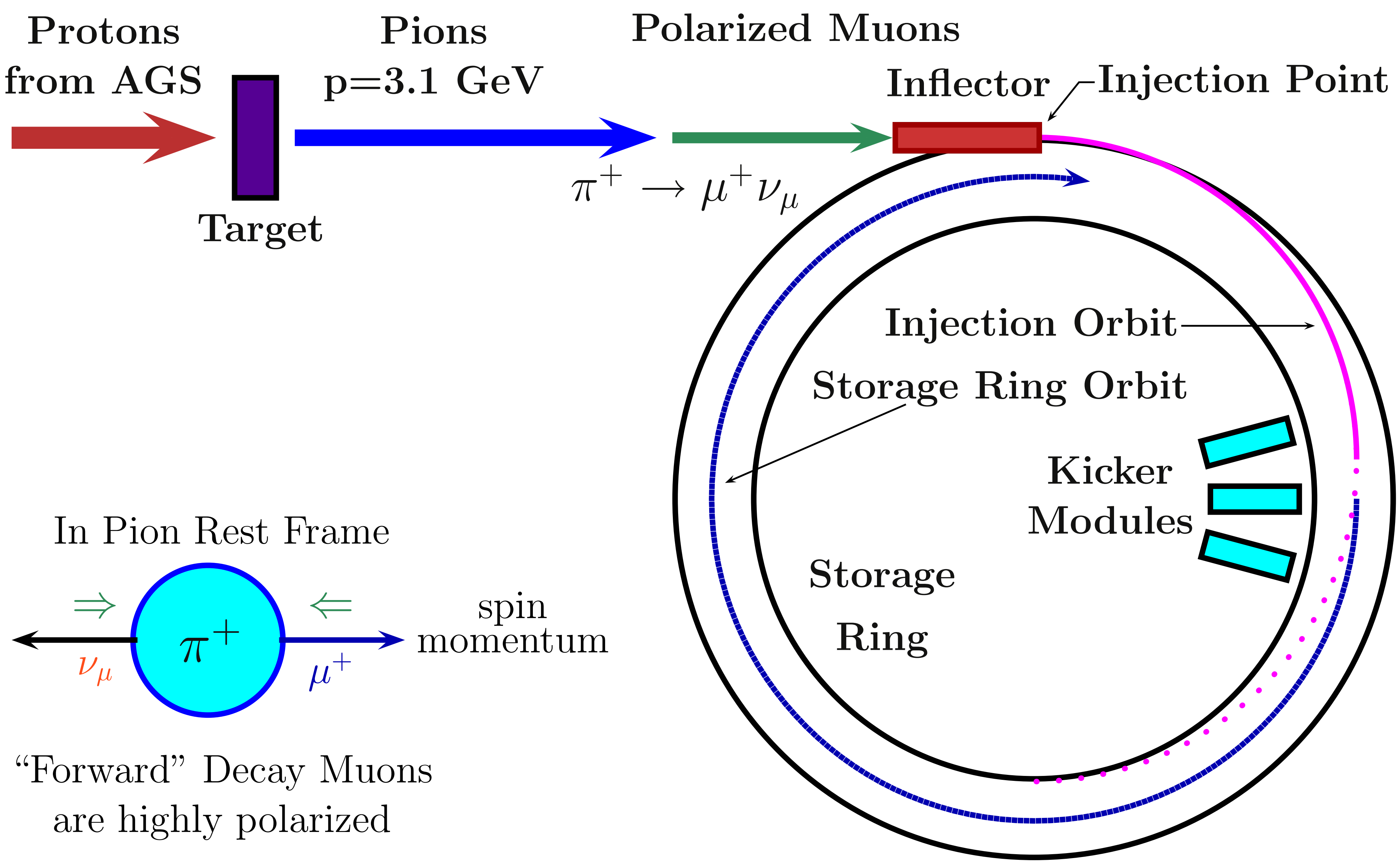
- Deviation from the Dirac value $g = 2$ is:

$$a_\mu = \frac{g_\mu - 2}{2}$$

$$a_\mu^{\mathrm{QED,LO}} = \alpha / 2\pi \sim 1.16 \times 10^{-3}$$



The anomalous magnetic moment of the muon: the experiment

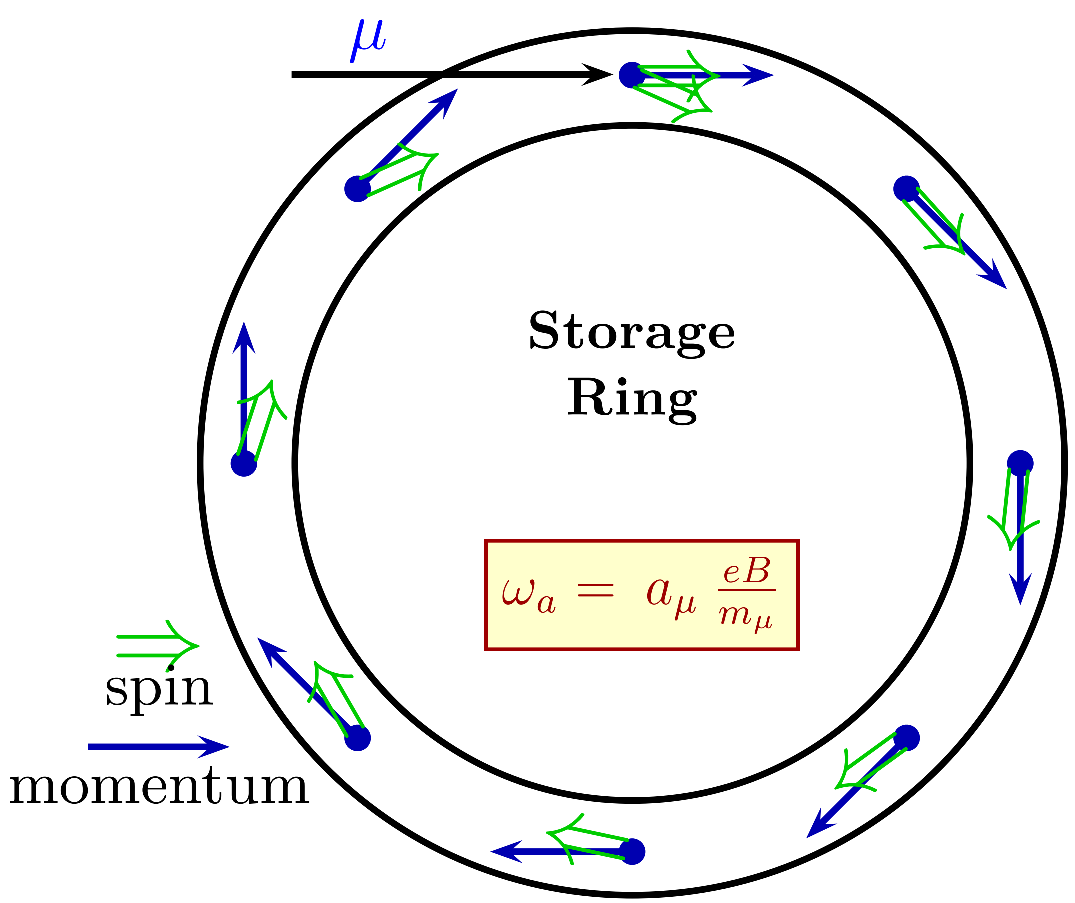


The anomalous magnetic moment of the muon: the experiment

$$\omega_c = -\frac{qB}{m\gamma}$$ cyclotron precession

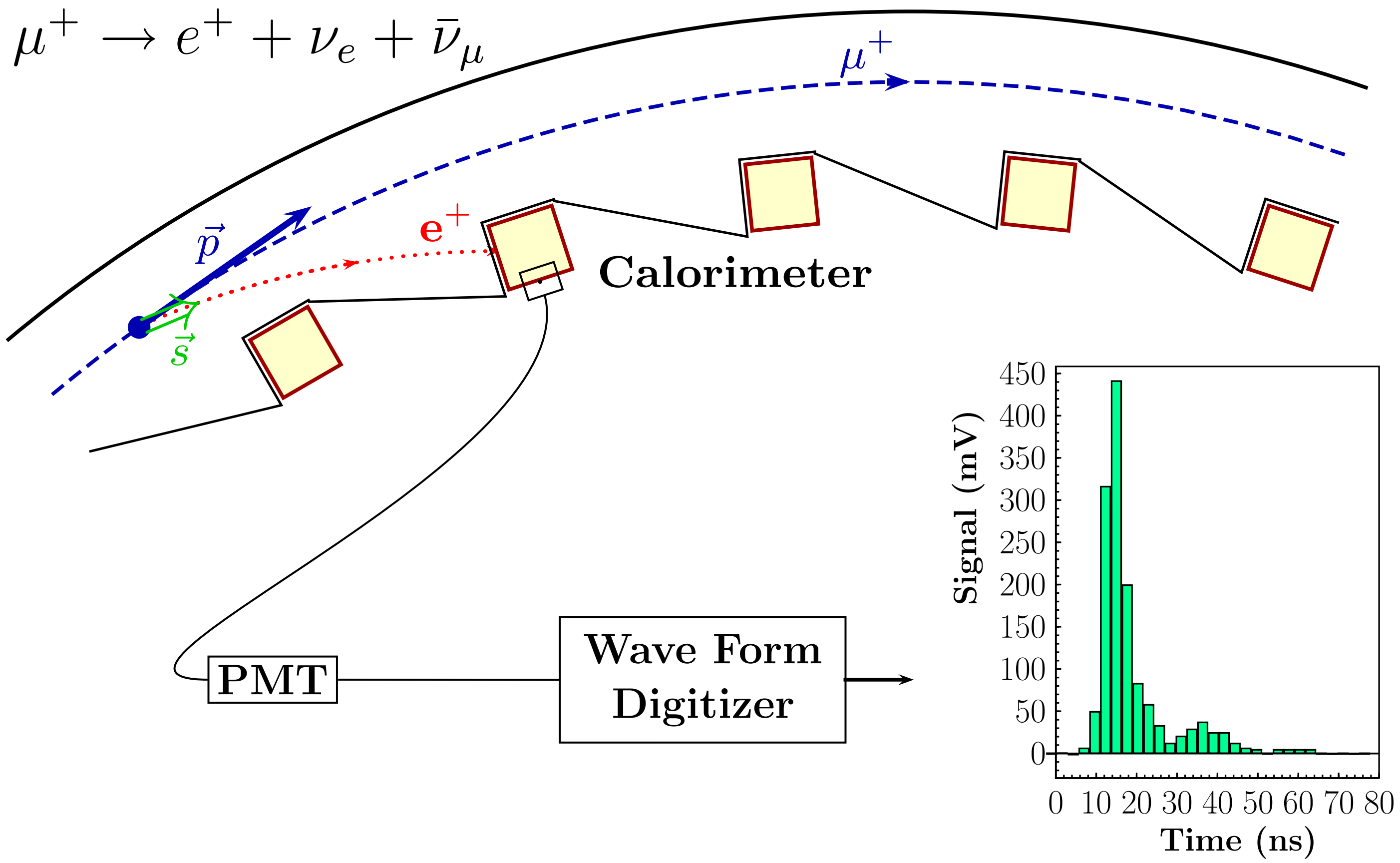
$$\omega_s = -\frac{gqB}{2m} - (1-\gamma)\frac{qB}{m\gamma}$$

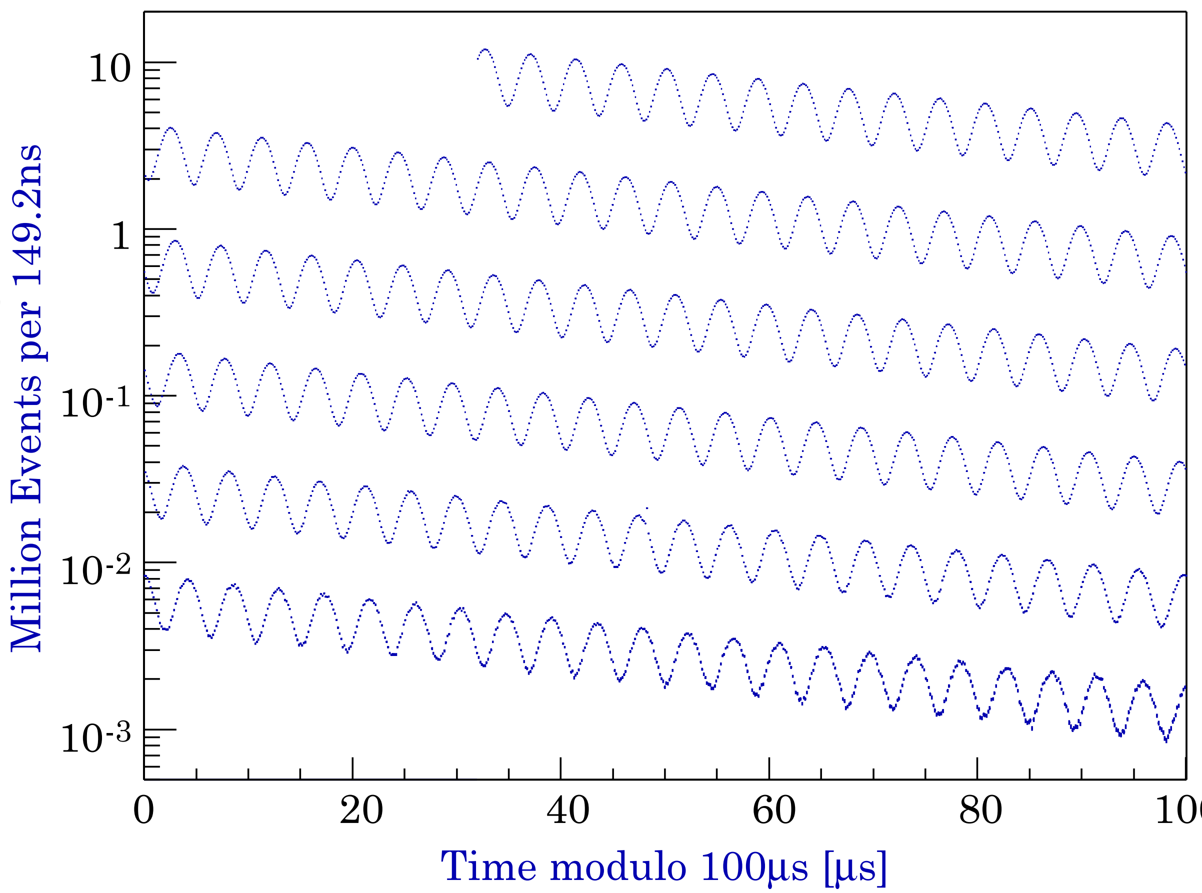
spin precession (Larmor)



$$\omega_a = \omega_s - \omega_c = -\left(\frac{g-2}{2}\right)\frac{qB}{m} = -a_\mu \frac{qB}{m}$$

The anomalous magnetic moment of the muon: the experiment





The anomalous magnetic moment of the muon

- The E821 experiment at BNL

Bennet et al, PRD73,072003 (2006)

$$a_{\mu+}^{\exp} = 11\,659\,204(6)(5) \times 10^{-10}$$ [2000]

$$a_{\mu-}^{\exp} = 11\,659\,215(8)(3) \times 10^{-10}$$ [2001]

- Assuming CPT invariance

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$$a_{\mu}^{\exp} = 11\,659\,209.1\,\underbrace{(5.4)(3.3)}_{(6.3)} \times 10^{-10}$$

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Forthcoming exp: FNAL & J-PARC $\sim 1.6 \times 10^{-10}$

The anomalous magnetic moment of the muon

Anomalous magnetic moment a_μ (anomaly):

$$g_\mu = 2\left(1 + a_\mu = \frac{\alpha}{2\pi} + \cdots\right)$$

$$a_\mu^{th} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{had}$$

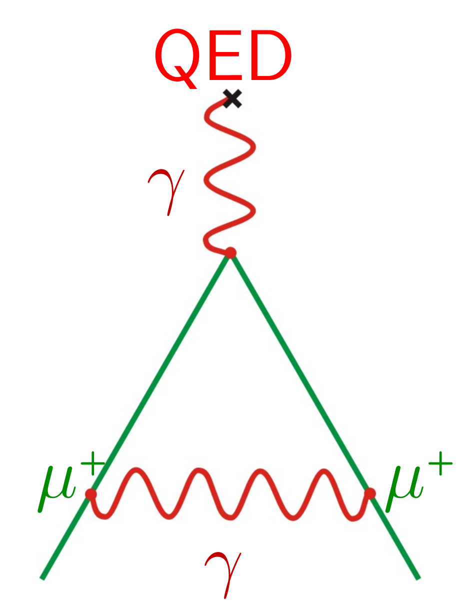
Contribution	Result in power of $\frac{\alpha}{\pi}$
$a_\mu^{(2)}$	$0.5\left(\frac{\alpha}{\pi}\right)$
$a_\mu^{(4)}$	$0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2$
$a_\mu^{(6)}$	$24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3$
$a_\mu^{(8)}$	$130.879\,6(63)\left(\frac{\alpha}{\pi}\right)^4$
$a_\mu^{(10)}$	$753.29(1.04)\left(\frac{\alpha}{\pi}\right)^5$
a_μ^{QED}	$11\,658\,471.885(4) \times 10^{-10}$

Schwinger 1948

Petermann and Sommerfield 1958

Laporta and Remiddi 1996

Kinoshita *et al* 2012



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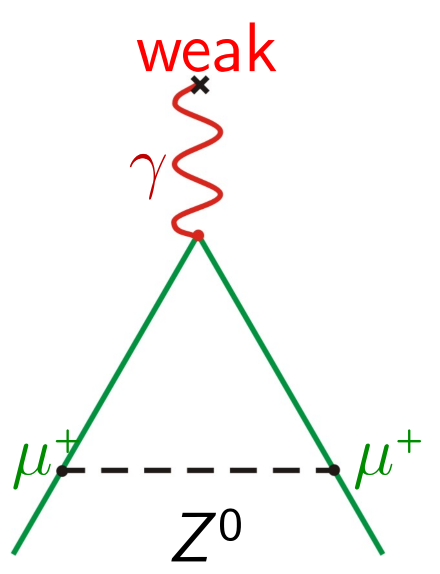
Contribution	Result in 10^{-10} units	
QED(leptons)	11658471.885 ± 0.004	Kinoshita *et al* 2012, Remiddi
HVP(leading order)	690.8 ± 4.7	Davier *et al* 2011
HVP(NLO)	-9.93 ± 0.07	Hagiwara *et al* 2009
HVP(NNLO)	1.22 ± 0.01	Kurz *et al* 2014
HLBL (+NLO)*	11.7 ± 4.0	Jegerlehner, Nyffeler 2009
→ EW (2 loop)	15.4 ± 0.1	Czarnecki 2003, Gnendinger 2013
Total	11659179.1 ± 6.2	

* NLO: Colangelo et al 2014



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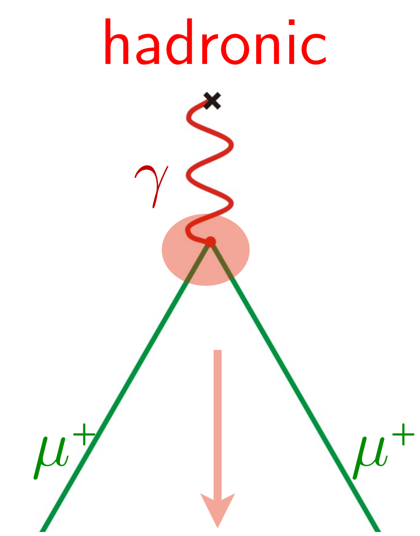

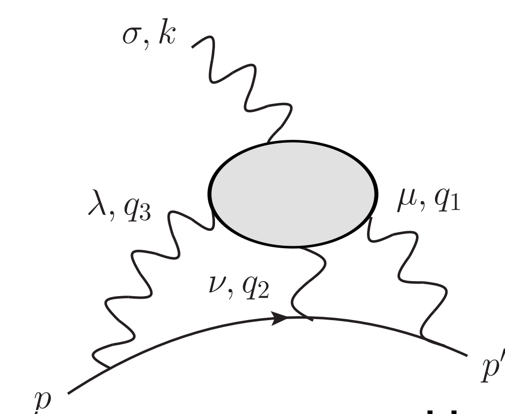

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$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 28.0(8.8) \times 10^{-10} \Rightarrow 3.2\,\sigma$$

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Forthcoming exp:
FNAL
JPAC

$\sim 1.6 \times 10^{-10}$

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Hints to NP

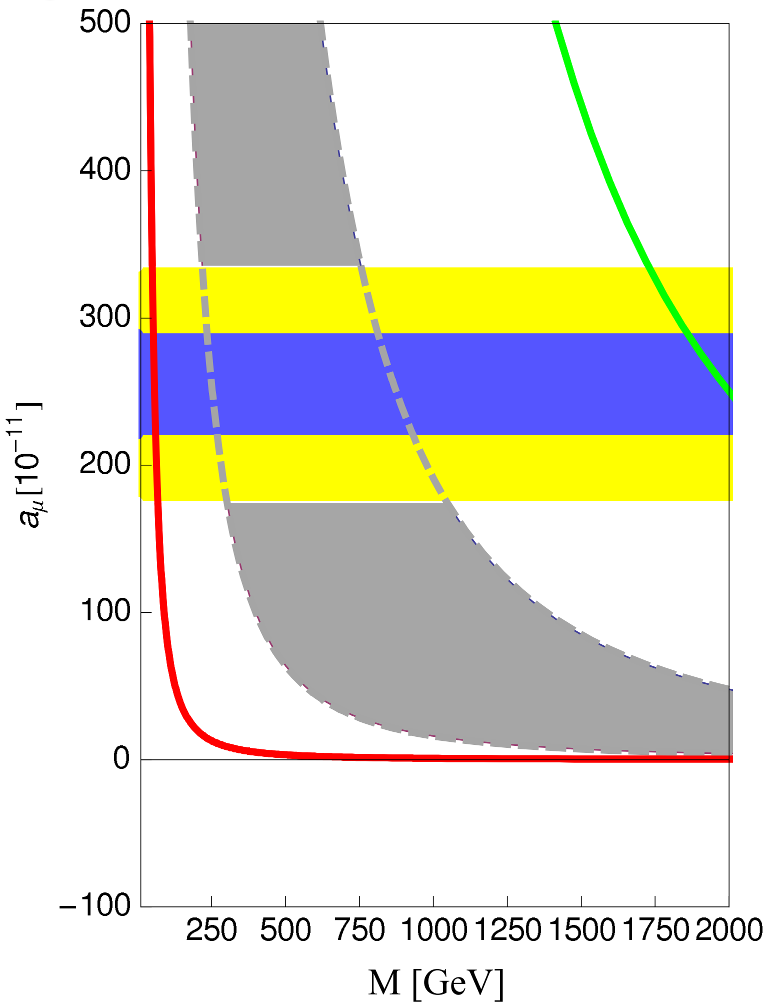
(talk from Stöckinger)

Very different contributions to a_μ

generally: $$C = \frac{\delta m_\mu(\text{N.P.})}{m_\mu}, \quad \delta a_\mu(\text{N.P.}) = \mathcal{O}(C)\left(\frac{m_\mu}{M}\right)^2$$

classify new physics: C **very** model-dependent

Very useful constraints on new physics



$\mathcal{O}(1)$	radiative muon mass generation ... [Czarnecki,Marciano '01] [Crivellin, Girrbach, Nierste '11][Dobrescu, Fox '10]
$\mathcal{O}(\frac{\alpha}{4\pi}\ldots)$	supersymmetry (tan β), unparticles [Cheung, Keung, Yuan '07] extra dim. (ADD/RS) (n_c)... [Davioudasl, Hewett, Rizzo '00] [Graesser,'00][Park et al '01][Kim et al '01]
$\mathcal{O}(\frac{\alpha}{4\pi})$	Z', W', UED, Littlest Higgs (LHT)...

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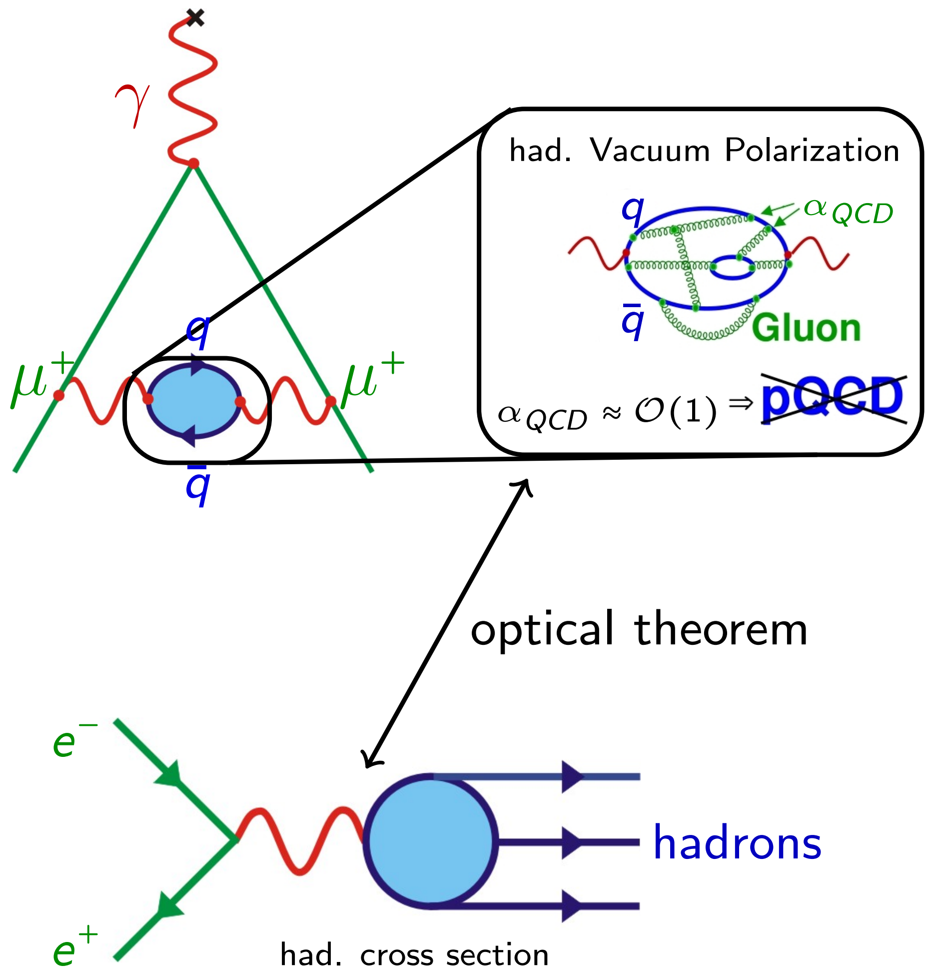
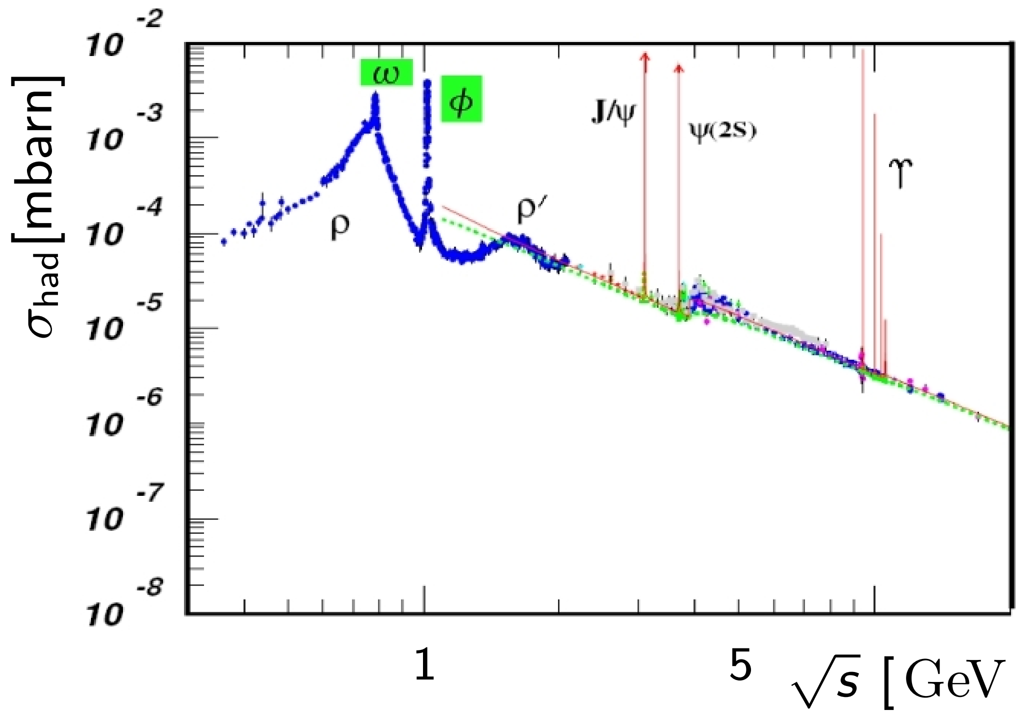
Hadronic Vacuum Polarization





$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds\ K(s)\ \sigma_{\text{had}}(s)$$

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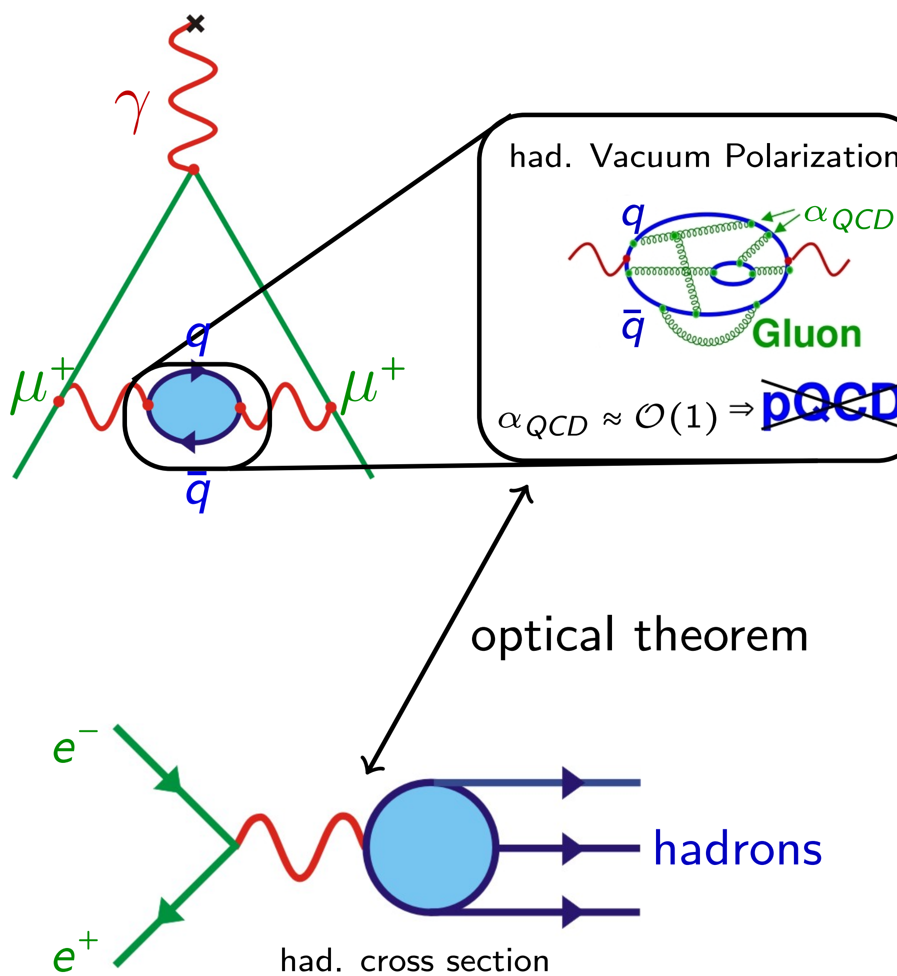
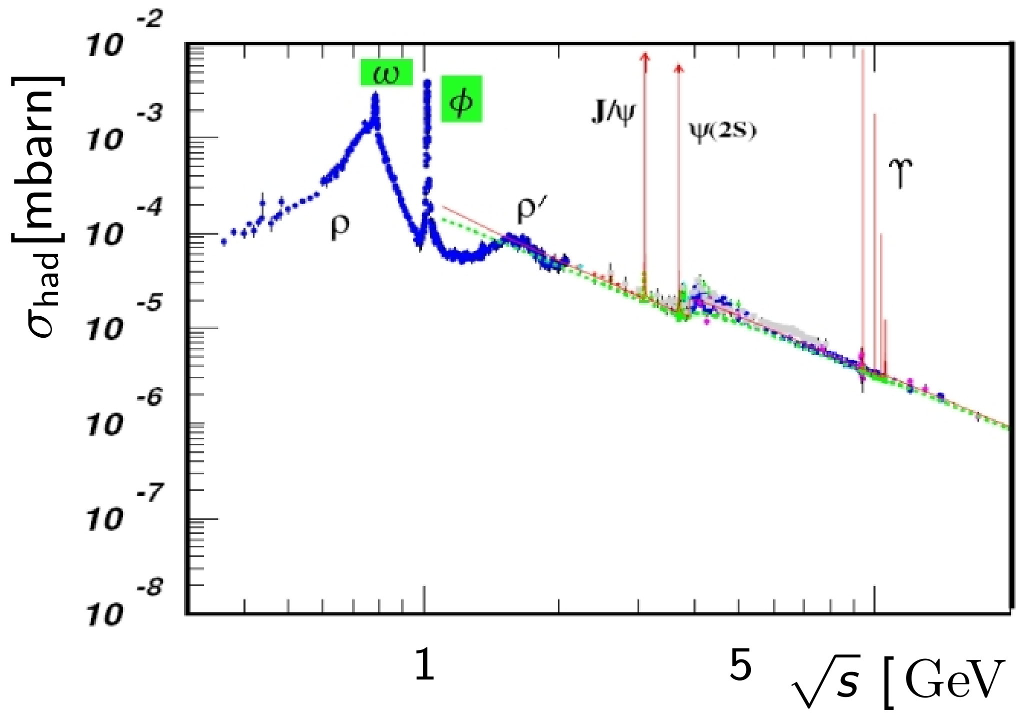




$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3}\int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{\text{had}}(s)$$

$$\sigma_{\text{had}}(s) \sim 1/s \quad \& \quad K(s) \sim 1/s$$

$$\Downarrow$$

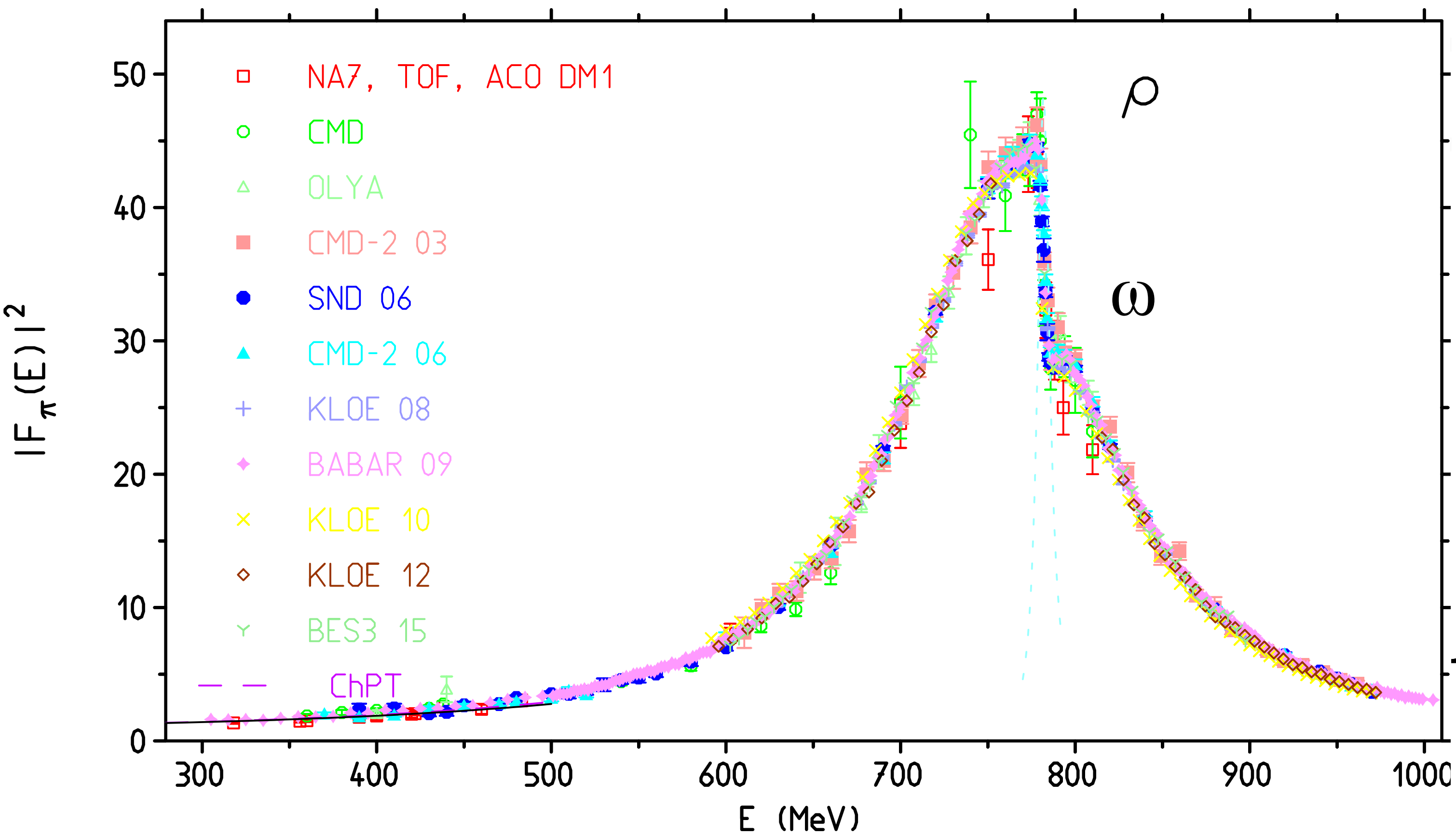
Low energy region important! $\sim 1/s^2$

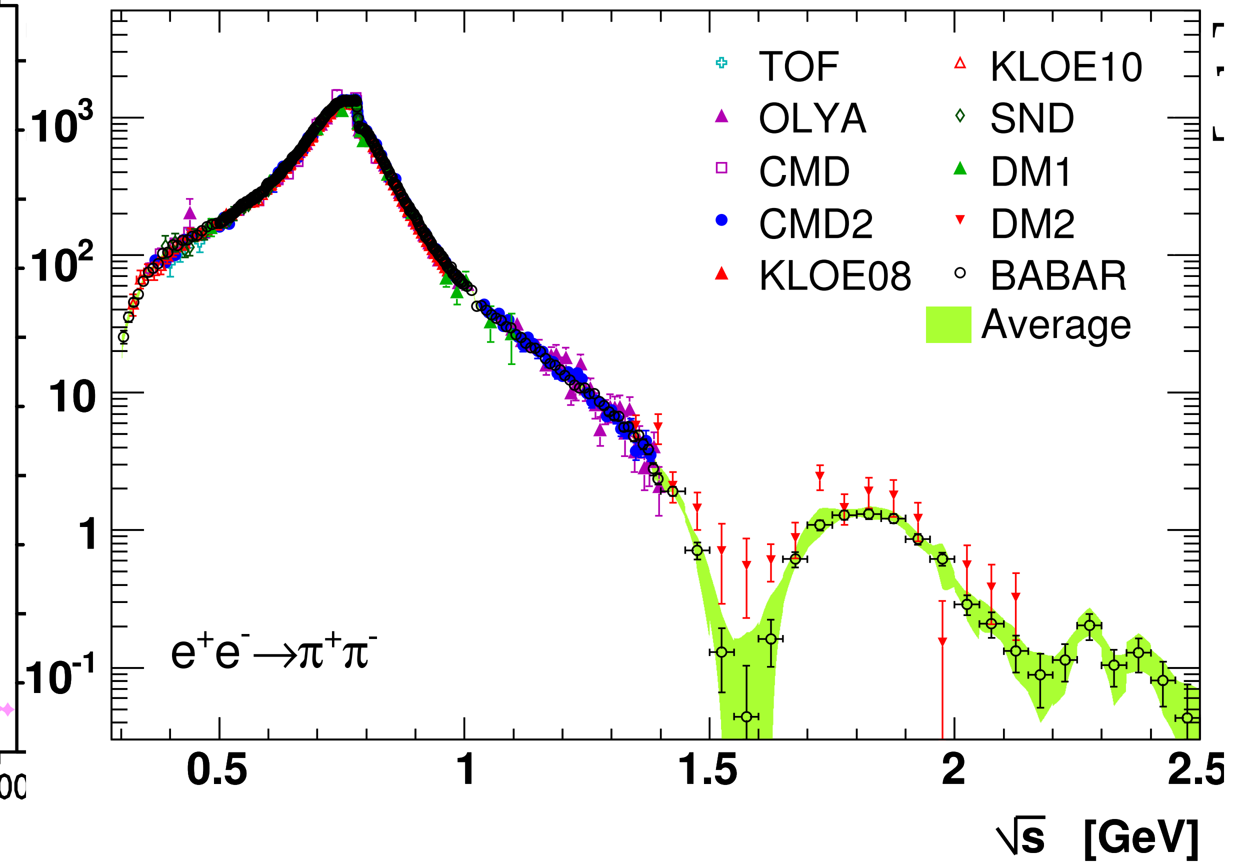
Sum of exclusive σ_{had}

$$\Downarrow$$

Hadronic contribution of a_μ

Hadronic Vacuum Polarization





- ρ peak
- ρ-ω interference
- Contribution to a_μ(VP): 75%
- Largest error from 1-2GeV

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- BNL E821: $11\,659\,209.1(6.3) \times 10^{-10}$ Bennet et al, PRD73,072003 (2006)

- Theory:

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NO HLBL

$$a_\mu^{\rm exp} - a_\mu^{\rm SM} = 41.7(7.9) \times 10^{-10} \Rightarrow 5.3\,\sigma$$ (2σ effect)

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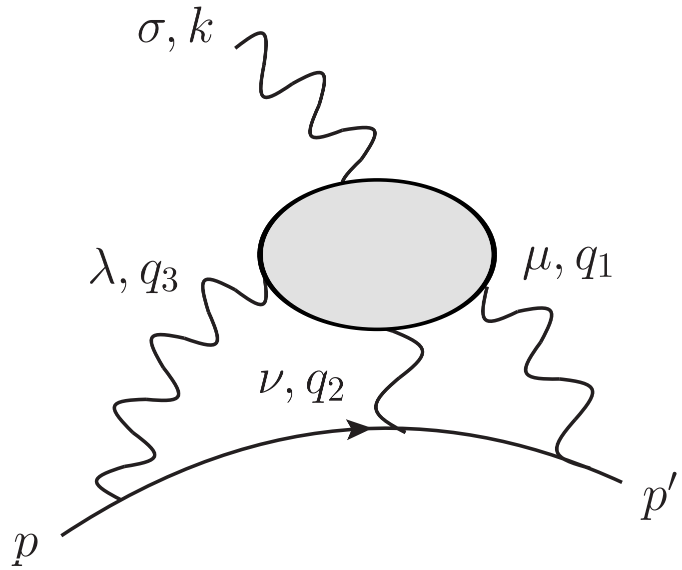
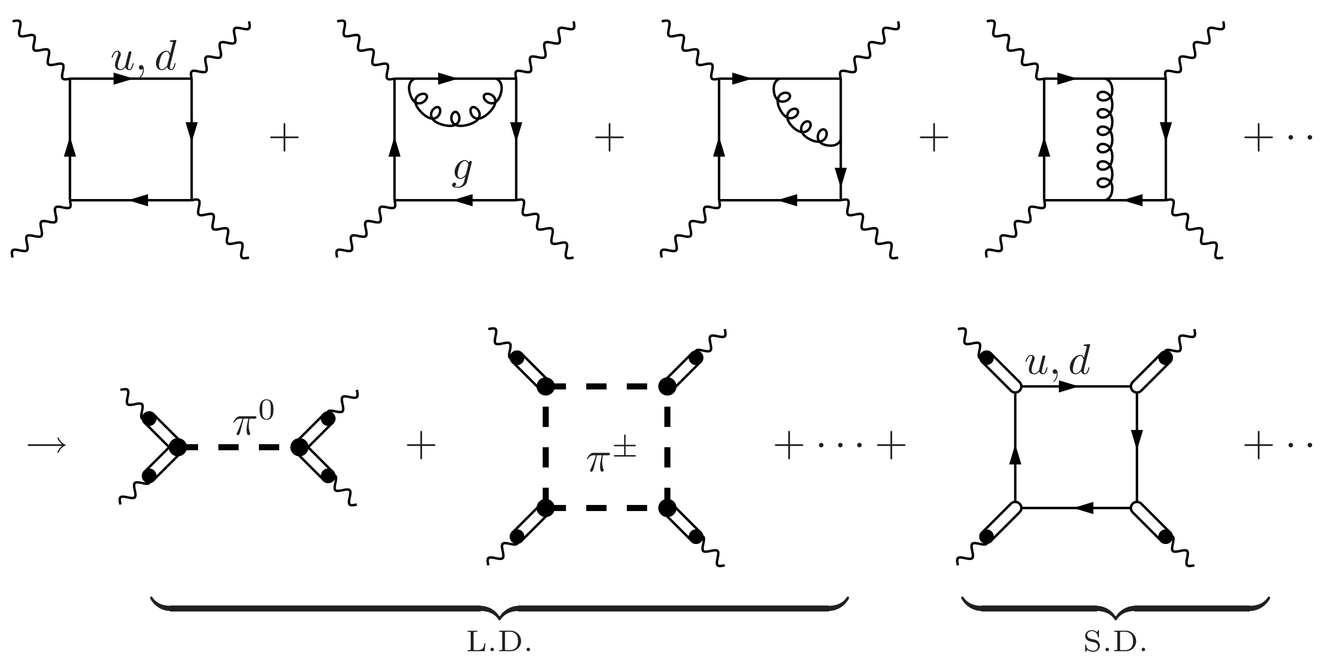
Forthcoming FNAL $\sim 1.6 \times 10^{-10}$ $\Longrightarrow$ from 5σ to 8σ, w/o HLBL: 5σ effect

We need to understand such numbers and errors

Hadronic light-by-light scattering in the muon g-2



order $O(\alpha^3)$ hadronic contribution



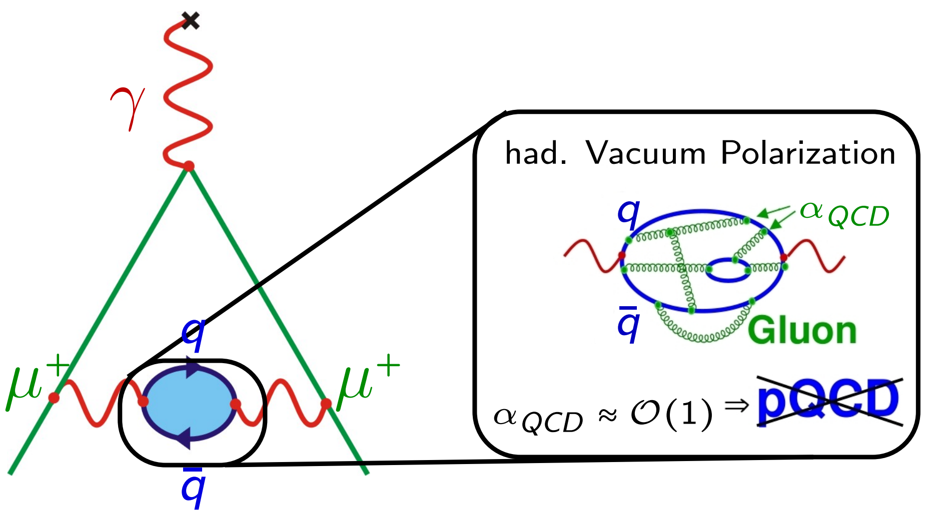
Model at low energies
(with exchange of resonances)

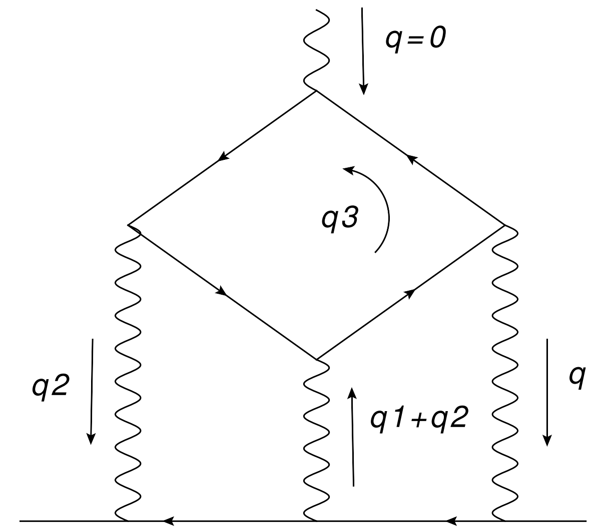
Model at high energies
(quark-loop)

The anomalous magnetic moment of the muon

Ballpark prediction: Order of magnitude?

Hadronic Vacuum Polarization vs Hadronic Light-by-Light





$$a_\mu^{\rm ferm}({\rm LO}) = I(m_q)\left(\frac{\alpha}{\pi}\right)^2$$

$$I(m_q) = \int_{4m_q^2}^{\infty} \frac{\rho_q(s)K(s)}{s} ds$$

$$\rho_q(s) = \frac{1}{3}\sqrt{1-\frac{4m_q^2}{s}}\left(1+\frac{2m_q^2}{s}\right)$$

(see Pivovarov '03)

$$a_\mu^{HVP} \sim 690 \times 10^{-10}$$

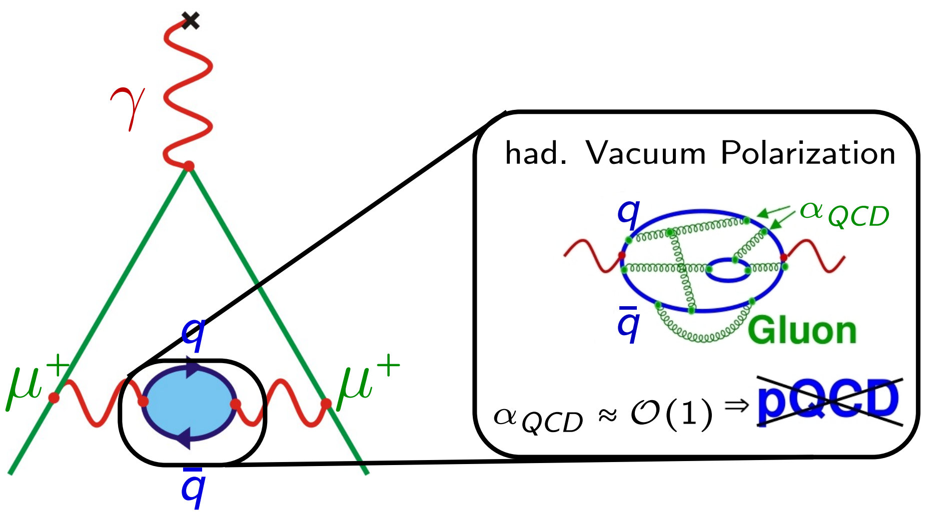
$$m_q \sim 0.160 - 0.180 \text{ GeV}$$

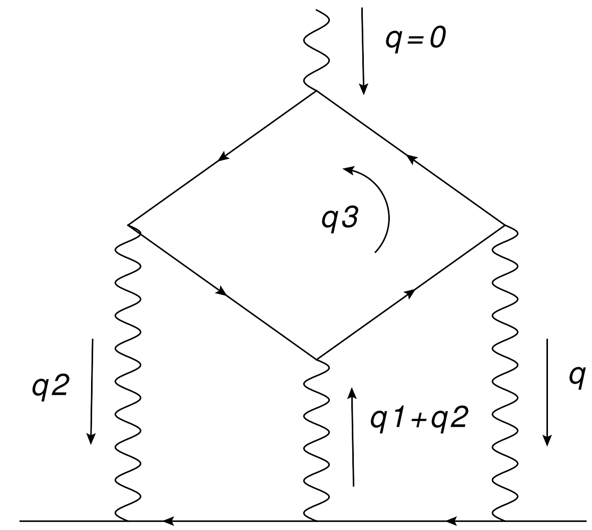
(Identify the constituent quark mass from HVP)

The anomalous magnetic moment of the muon

Quark models for a Ballpark prediction

Hadronic Vacuum Polarization vs Hadronic Light-by-Light





$$a_\mu^{HLBL}(M(Q)) = \left(\frac{\alpha}{\pi}\right)^3 N_c \left(\sum_{q=u,d,s} Q_q^4\right)\left[\left(\frac{3}{2}\zeta(3) - \frac{19}{16}\right)\frac{m_\mu^2}{M(Q)^2} + \mathcal{O}\left(\frac{m_\mu^4}{M(Q)^4}\log^2\frac{m_\mu^2}{M(Q)^2}\right)\right]$$

Laporta and Remiddi 1996

$a_\mu^{HLBL} \sim 14 \times 10^{-10}$ Pivovarov '03

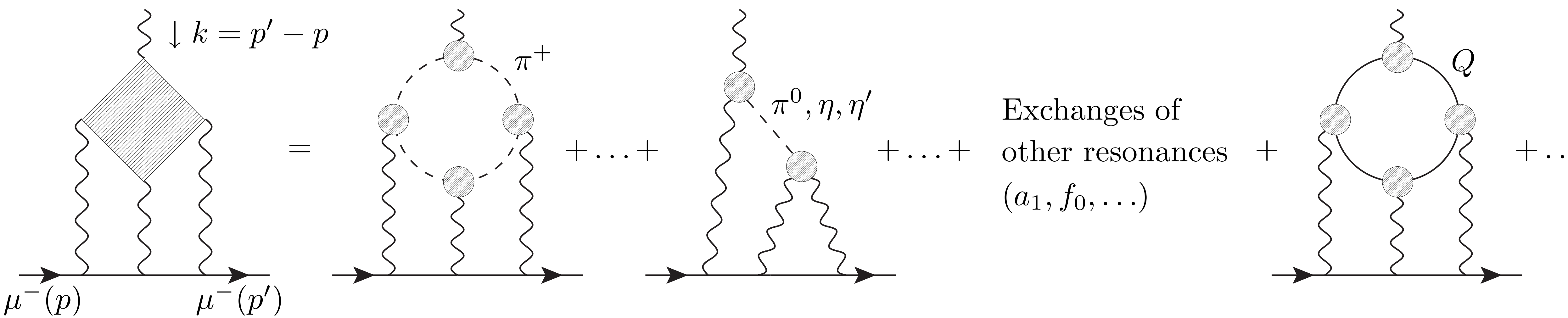
$a_\mu^{HLBL} < 15.9 \times 10^{-10}$ Erler and Toledo Sanchez '06 $m_q \sim 0.160 - 0.180$ GeV

$a_\mu^{HLBL} = 11.8 - 14.8 \times 10^{-10}$ Boughezal and Melnikov, '11

$a_\mu^{HLBL} = [10.5(2.0) \div 15.0(2.5)] \times 10^{-10}$ P.M, Vanderhaeghen 2012

Classification proposal by Eduardo de Rafael '94

Chiral Perturbation Theory counting (p^2)+large-Nc counting

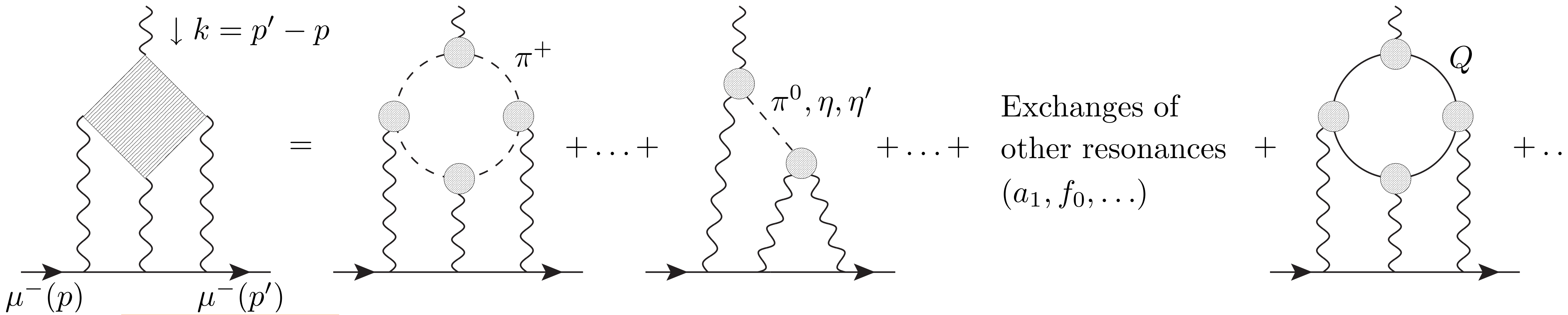


Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

Pesudoscalars: numerically dominant contribution (according to most models)

The anomalous magnetic moment of the muon

Reference numbers



						$a_\mu \times 10^{11}$
JN:	$+116(40)$	$-19(13)$	$+99(16)$	$+15(7)$	$+21(3)$	Large N_c, '09
PdRV:	$+105(26)$	$-19(19)$	$+114(13)$	$+8(12)$	0	Average, '09

JN: Jegerlehner and Nyffeler, Phys. Rep. 477 (2009) 1-110
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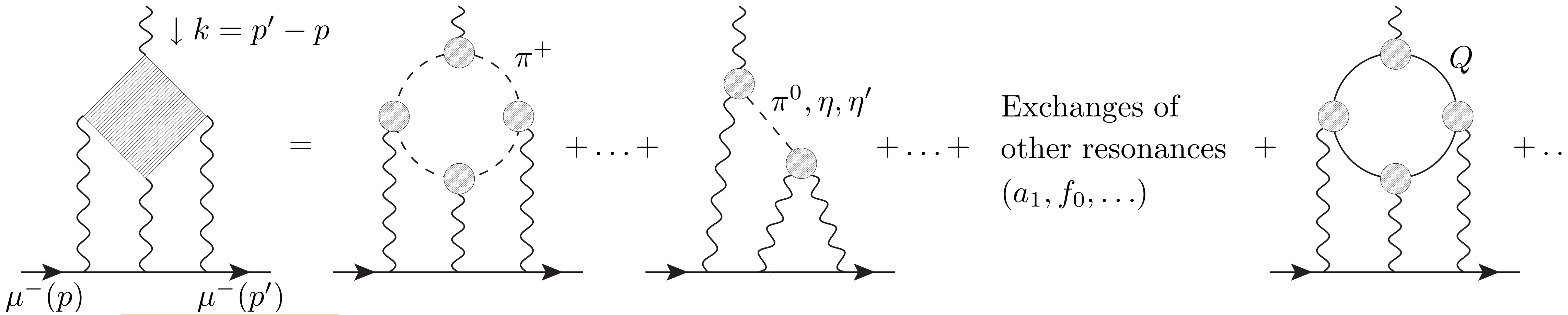

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- use the same model from Knecht and Nyffeler '01 and inputs for the PS (issue of pion-pole vs pion-exchange, i.e., how to correctly implement QCD constraints)
  - errors summed linearly in JN and in quadrature in PdRV
- lack of systematic error (large-Nc model, see P.M. and Vanderhaeghen '12)
- the model neither reproduce the new experimental data on PSTFF (see P.M in arXiv:1407.4021) nor the $\pi^0 \rightarrow e^+e^-$ (see P. Sanchez-Puertas in arXiv:1407.4021)
- On top, double counting (or correct overlap) + missing pieces (higher states...)

- All in all, need for more calculations, closer to data (if possible)

The anomalous magnetic moment of the muon

- Main current strategies
  - Lattice QCD
  - Data driven approaches

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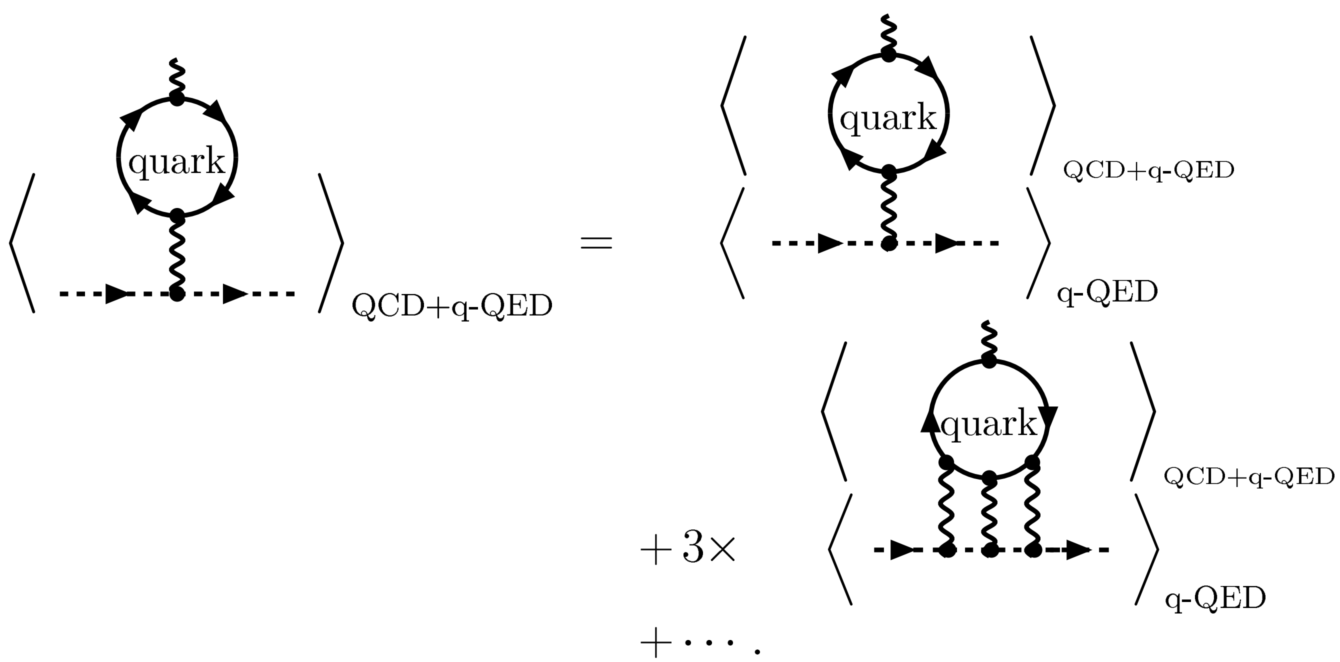
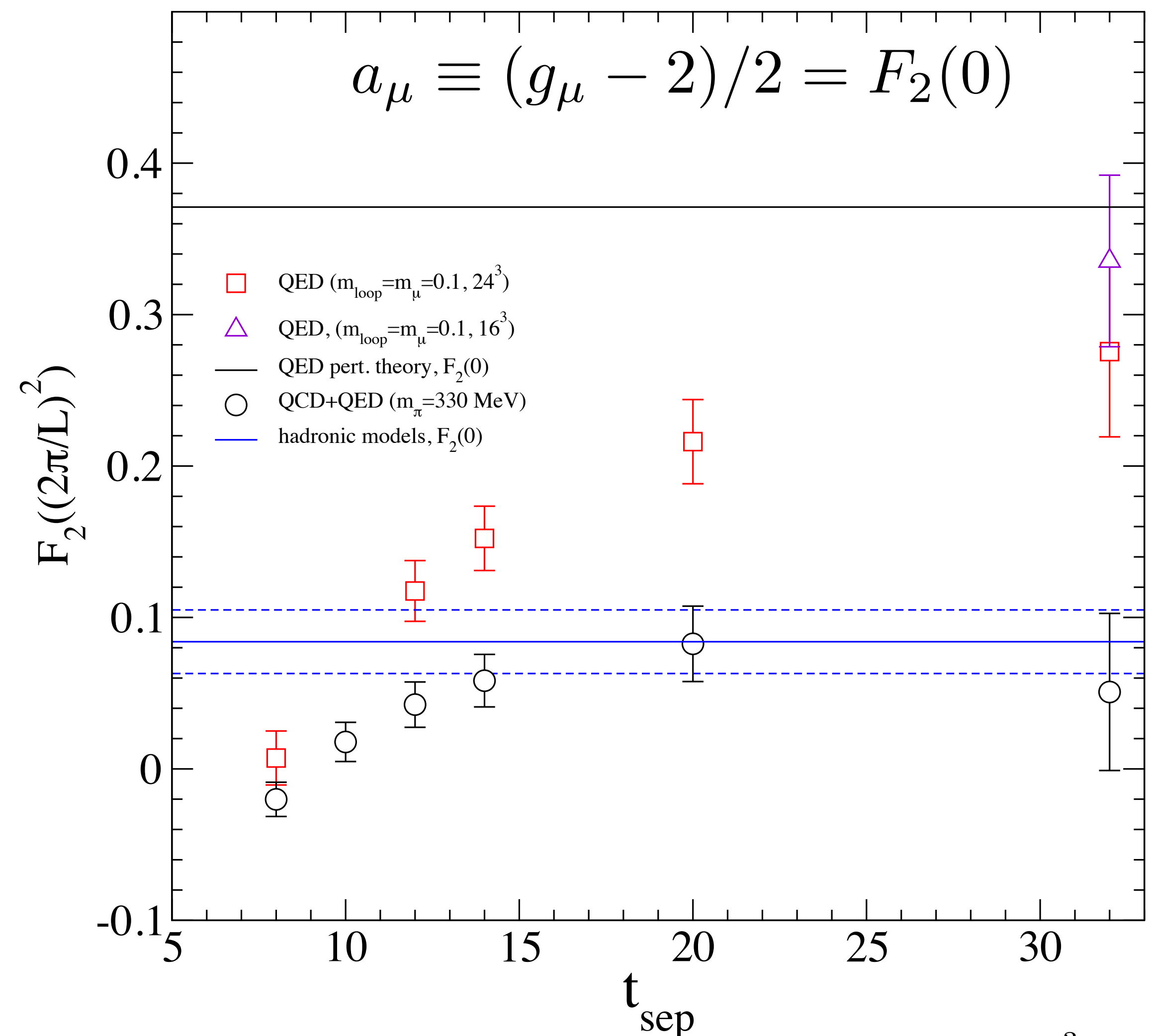
The anomalous magnetic moment of the muon

lattice QCD

Blum et al '14,'16,'17





The muon's magnetic form factor in units of $(\alpha/\pi)^3$

- Next step:
  - physical values
  - larger volume
  - controlled extrapolation to $Q^2=0$

The anomalous magnetic moment of the muon

- Main current strategies
  - Lattice QCD
  - Data driven approaches:
    - Dispersion relations for the low-energy region
    - Hadronic models for the different contributions

The anomalous magnetic moment of the muon

Dispersion relations for the low-energy region

$$\Pi^{\mu\nu\lambda\sigma}(q_1,q_2,q_3) = i^3\int d^4x\int d^4y\int d^4z\, e^{-i(x\cdot q_1+y\cdot q_2+z\cdot q_3)}\langle 0|T\{j^\mu(x)j^\nu(y)j^\lambda(z)j^\sigma(0)\}|0\rangle$$

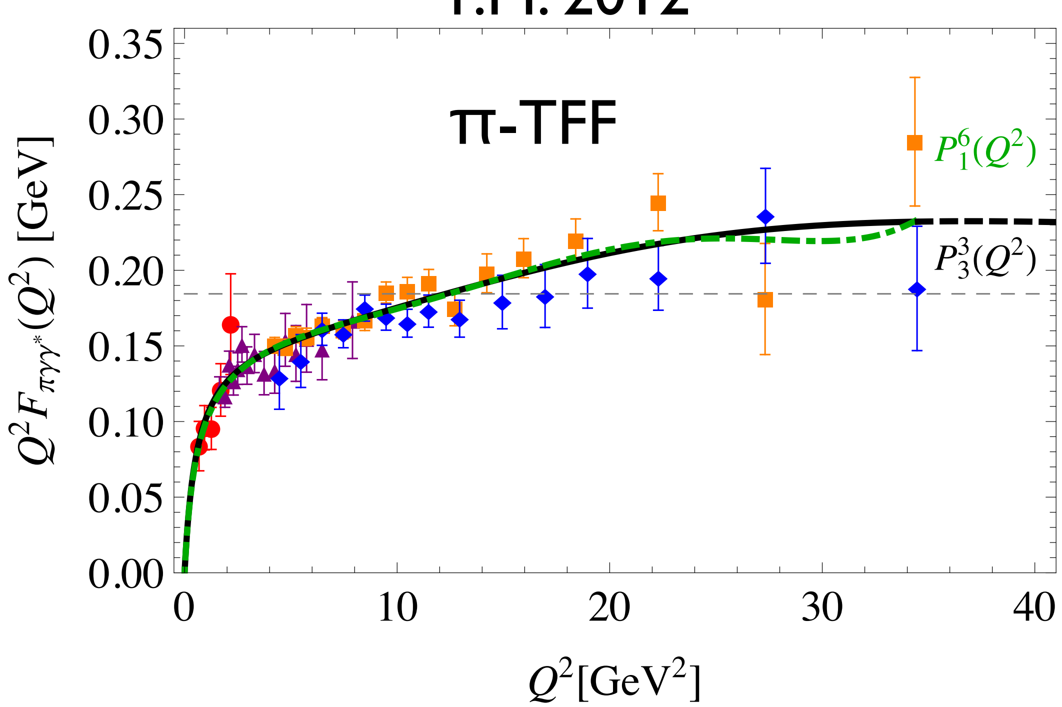
$$\Pi_{\mu\nu\lambda\sigma} = \Pi^{\pi^0\text{-pole}}_{\mu\nu\lambda\sigma} + \Pi^{\text{FsQED}}_{\mu\nu\lambda\sigma} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots\,,$$

(helicity amplitude decomposition)

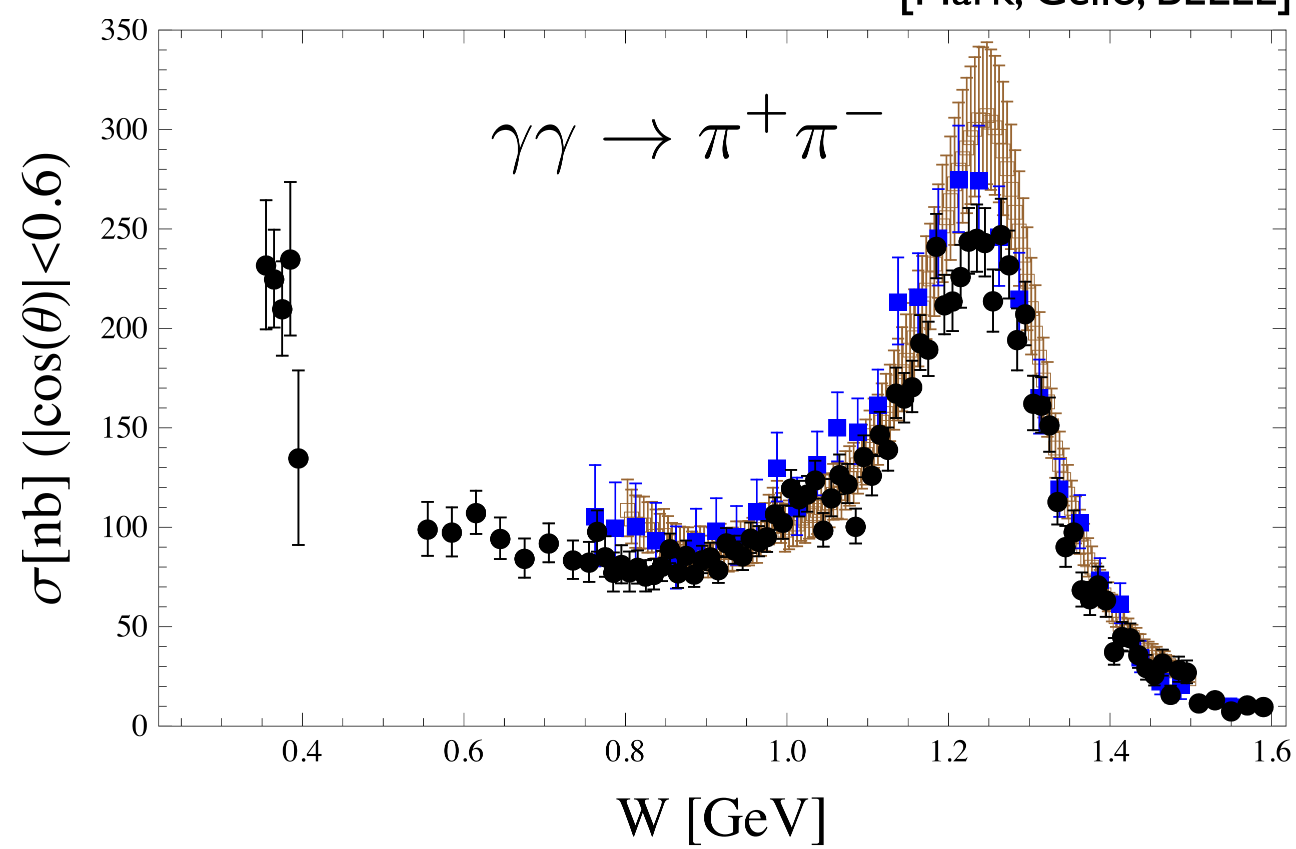
Colangelo et al, 1402.7081
Vanderhaeghen et al, 1403.7503

- no intermediate states
- all FF are on-shell, off-shell effects are included in subtraction constants
- need for input: π-TFF and $\gamma\gamma\to\pi^+\pi^-$

P.M. 2012



[Mark, Cello, BELLE]



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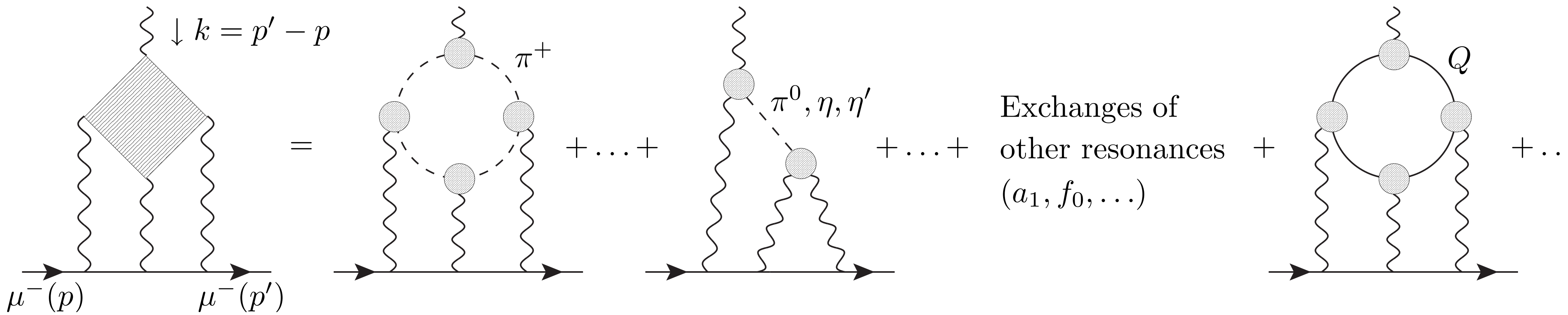
Hadronic models for the different contributions



$a_\mu^{HLBL} \times 10^{11}$

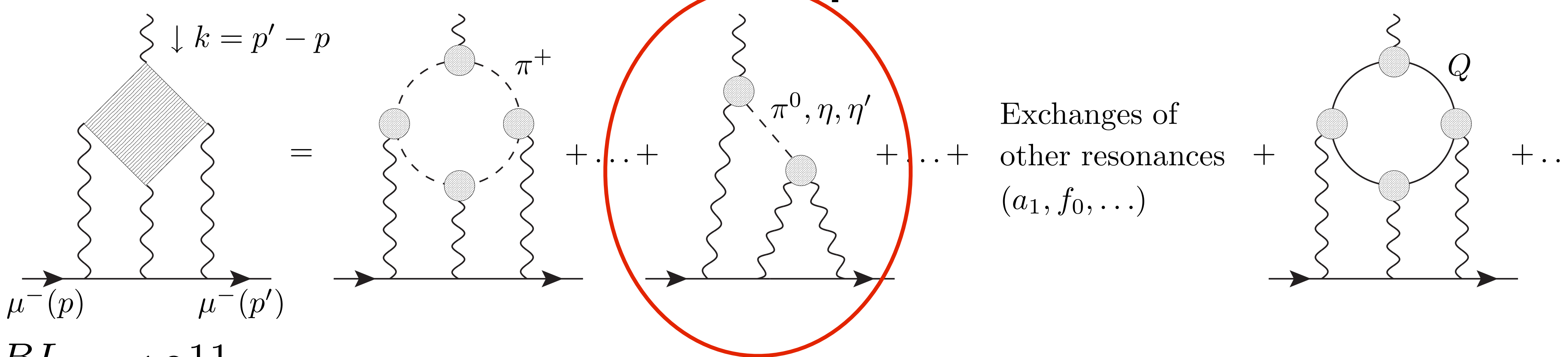
BPP:	$+83(32)$	$-19(13)$	$+85(13)$	$-4(3)$	$+21(3)$	ENJL, '95 '96 '02
HKS:	$+90(15)$	$-5(8)$	$+83(6)$	$+1.7(1.7)$	$+10(11)$	LHS, '98 '02
KN:	$+80(40)$		$+83(12)$			Large N_c, '02
MV:	$+136(25)$	$0(10)$	$+114(10)$	$+22(5)$	0	Large N_c, '04
JN:	$+116(40)$	$-19(13)$	$+99(16)$	$+15(7)$	$+21(3)$	Large N_c, '09
PdRV:	$+105(26)$	$-19(19)$	$+114(13)$	$+8(12)$	0	Average, '09
HK:	$+107$		$+107$			Holographic QCD, '09
DRZ:	$+59(9)$		$+59(9)$			Non-local q.m., '11
EMS:	$+107(20)$	$-19(13)$	$+90(7)$	$+15(7)$	$+21(3)$	Padé-data,'12 '12 '13
GLCR:	$+118(20)$	$-19(13)$	$+105(5)$	$+15(7)$	$+21(3)$	RχT, '14

should add the charm-quark contr. $\sim 2 \times 10^{-11}$

data+systematic error

The anomalous magnetic moment of the muon

The role of experimental data



$a_\mu^{HLBL} \times 10^{11}$

BPP:	+83(32)	−19(13)	+85(13)	−4(3)	+21(3)	ENJL, '95 '96 '02
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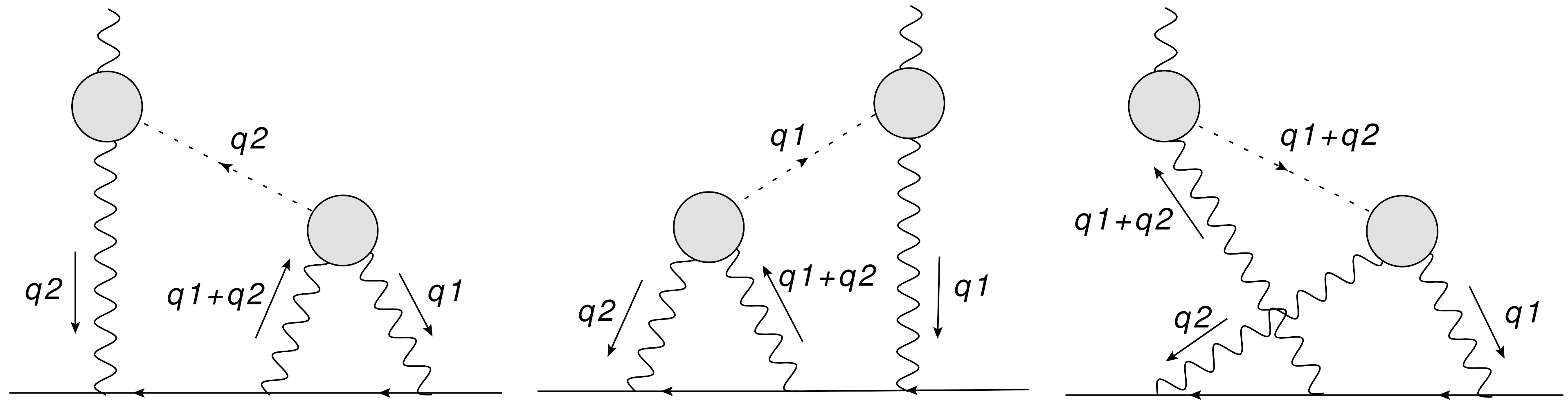
The anomalous magnetic moment of the muon

The role of experimental data



From Knecht and Nyffeler, '01:

$$a_\mu^{HLBL;\pi^0} = -e^6 \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_2}{(2\pi)^4} \frac{1}{q_1^2 q_2^2 (q_1+q_2)^2 [(p+q_1)^2 - m^2][(p-q_2)^2 - m^2]}$$

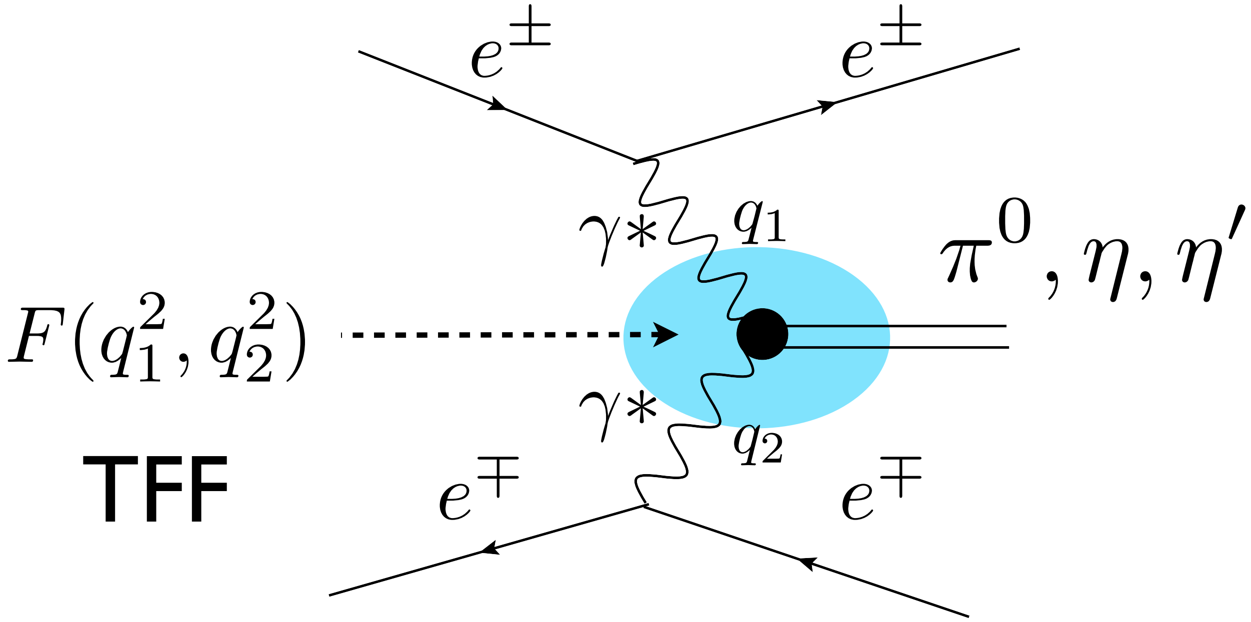
$$\times \left(\frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, (q_1+q_2)^2) F_{\pi^0\gamma^*\gamma^*}(q_2^2, 0)}{q_2^2 - M_\pi^2} T_1(q_1, q_2; p) \right.$$

Use data from
the pion Transition Form Factor

$$\left. + \frac{F_{\pi^0\gamma^*\gamma^*}(q_1^2, q_2^2) F_{\pi^0\gamma^*\gamma^*}((q_1+q_2)^2, 0)}{(q_1+q_2)^2 - M_\pi^2} T_2(q_1, q_2; p) \right)$$

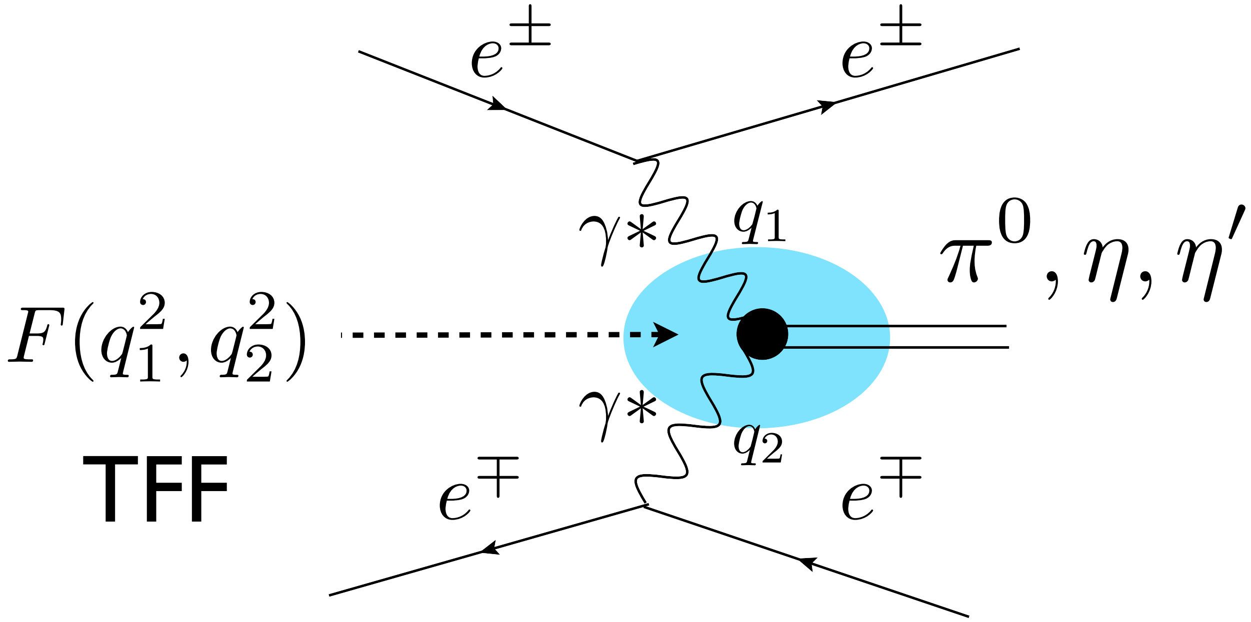
The anomalous magnetic moment of the muon

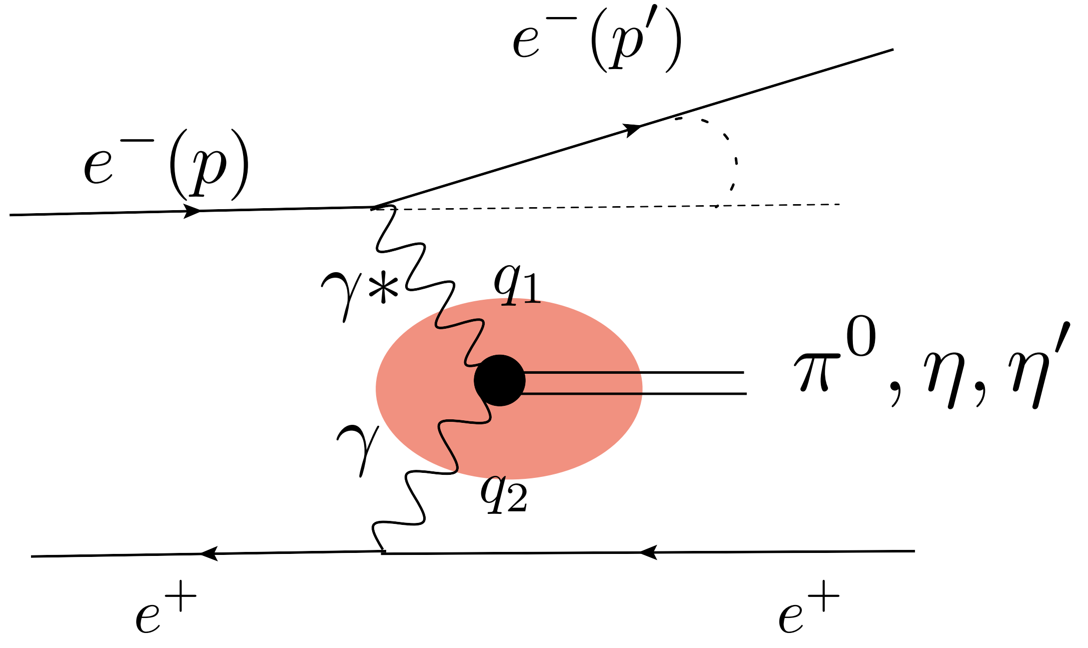
The role of experimental data



The anomalous magnetic moment of the muon

The role of experimental data





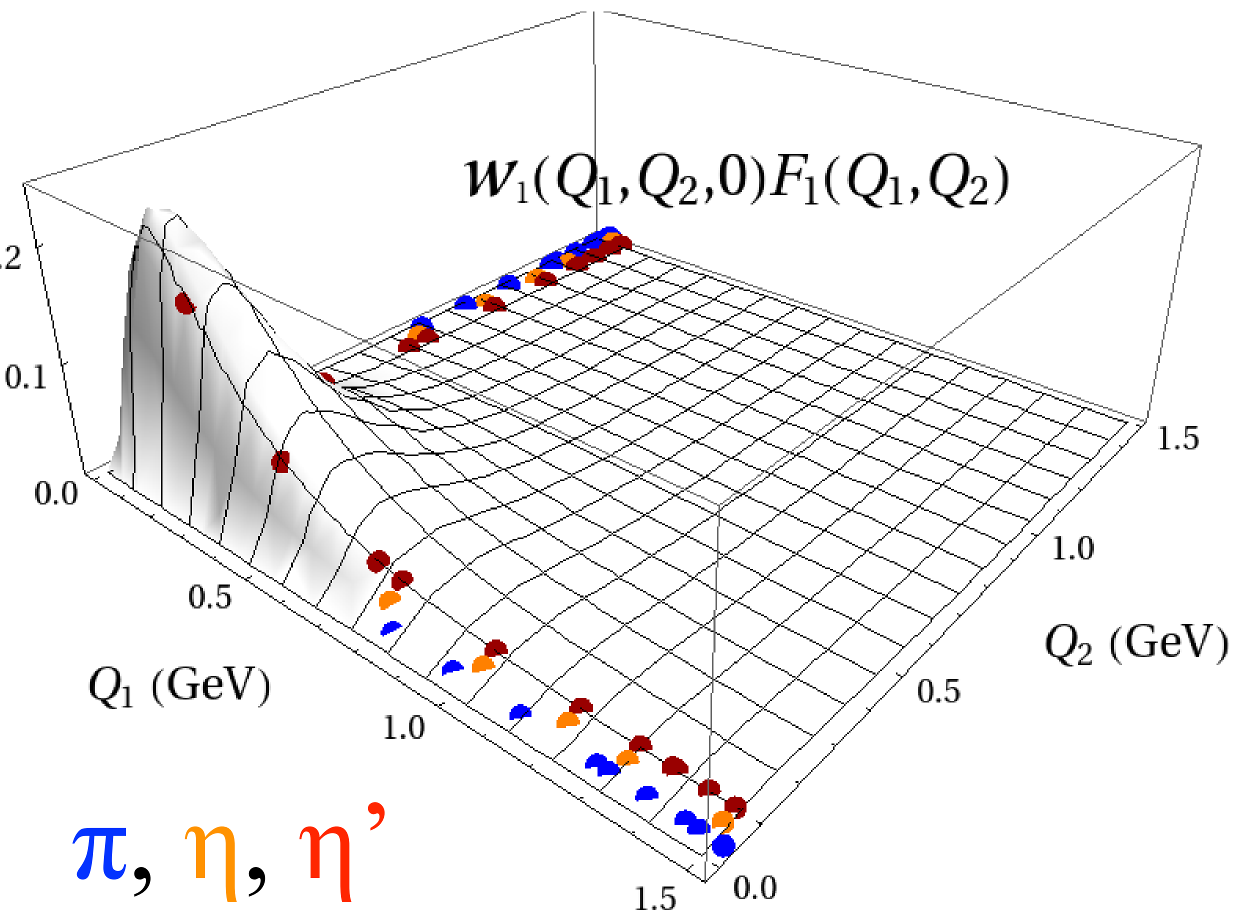
The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]

$$a_\mu^{HLBL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2)$$

Using $F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2)$



π, η, η'

(main energy range from 0 to 1.5 GeV^2)

The anomalous magnetic moment of the muon

The role of experimental data

Central value:

Model from Knecht and Nyffeler ’01 used in reference numbers

$$F_{\pi^0\gamma^*\gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:

$$F_\pi = 92.4 \text{ MeV}$$
$$m_\rho = 769 \text{ MeV}$$
$$m_{\rho'} = 1465 \text{ MeV}$$
$$h_1 = 0 \,(\text{ BL limit})$$
$$h_5 = 6.93 \text{ GeV}^4$$
$$h_2 = -10 \text{ GeV}^2$$

$$a_\mu^{\text{HLBL},\pi} = 6.3 \times 10^{-10}$$

The anomalous magnetic moment of the muon

The role of experimental data

Central value:

Model from Knecht and Nyffeler ’01 used in reference numbers

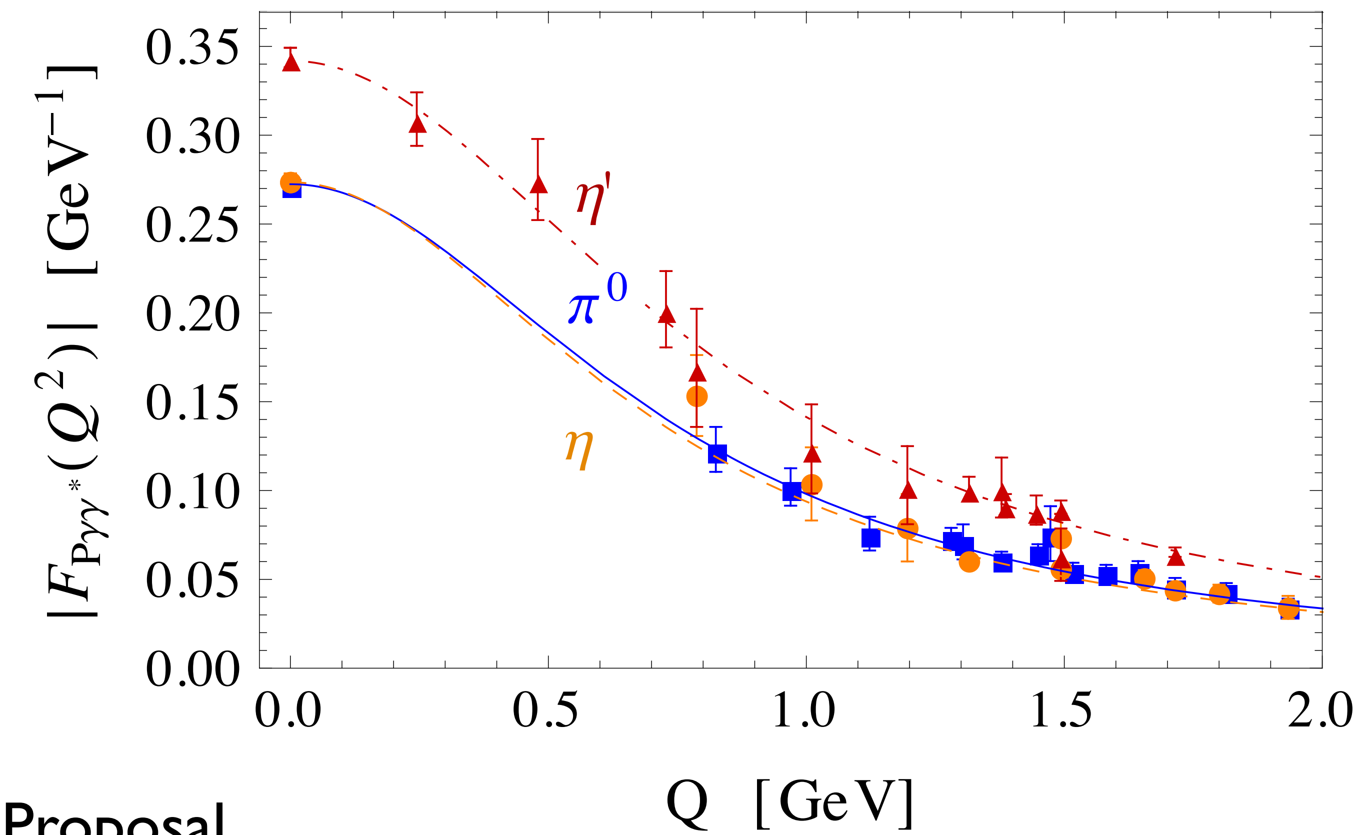
$$F^{LMD+V}_{\pi^0\gamma^*\gamma^*}(q_1^2,q_2^2) = \frac{f_\pi}{3}\frac{q_1^2q_2^2(q_1^2+q_2^2)+h_1(q_1^2+q_2^2)^2+h_2q_1^2q_2^2+h_5(q_1^2+q_2^2)+h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}$$

Publication:		Preliminary, using new exp data:
$F_\pi = 92.4$ MeV	→	$\Gamma_{\pi^0\to\gamma\gamma}$
$m_\rho = 769$ MeV		$m_\rho = 775$ MeV
$m_{\rho'} = 1465$ MeV	→	curvature TFF
$h_1 = 0$ (BL limit)		$h_1 = 0$ (BL limit)
$h_5 = 6.93\ \mathrm{GeV}^4$	→	slope TFF
$h_2 = -10\ \mathrm{GeV}^2$		$h_2 = -10\ \mathrm{GeV}^2$
$a_\mu^{\mathrm{HLBL},\pi} = 6.3\times10^{-10}$	→	$a_\mu^{\mathrm{HLBL},\pi} = 7.5\times10^{-10}$

The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]



At low energies:

$$F_{P\gamma^*\gamma^*}(Q_1^2, Q_2^2) = F_{P\gamma\gamma}(0,0)\left(1 - \frac{b_P}{m_P^2}(Q_1^2 + Q_2^2) + \frac{c_P}{m_P^4}(Q_1^4 + Q_2^4) + \frac{a_{P;1,1}}{m_P^4}Q_1^2 Q_2^2 + \ldots\right)$$

At high energies:

$$\lim_{Q^2\to\infty} F_{P\gamma^*\gamma}(Q^2, 0) = P_\infty Q^{-2} + \mathcal{O}(Q^{-4}),$$

$$\lim_{Q^2\to\infty} F_{P\gamma^*\gamma^*}(Q^2, Q^2) = \frac{P_\infty}{3}\left(\frac{1}{Q^2} - \frac{8}{9}\frac{\delta_P^2}{Q^4}\right) + \mathcal{O}(Q^{-6})$$

Proposal

- build a sequence of interpolators based on analyticity and unitary of amplitude: CANTERBURY approximants

$$C_1^0(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)}{1 + \frac{b_P}{m_P^2}(Q_1^2 + Q_2^2)},$$

$$C_2^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1 + \alpha_1(Q_1^2 + Q_2^2) + \alpha_{1,1}Q_1^2 Q_2^2)}{1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1}Q_1^2 Q_2^2 + \beta_{2,1}Q_1^2 Q_2^2(Q_1^2 + Q_2^2)}.$$

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The role of experimental data

[P.M., Sanchez-Puertas '17]

Using largest set ever:

- Space-like region

$$e^+e^- \to e^+e^- P$$

[L3,CLEO,CELLO,BABAR,BELLE]

- Time-like region

$$P \to \ell^+\ell^-$$

$$P \to \ell^+\ell^-\gamma$$

[NA48,A2,NA62+PDG]

$$P = \pi^0, \eta, \eta'$$
$$\ell = e, \mu$$

[13 different coll.]

$$a_\mu^{\mathrm{HLBL},\pi^0} = 81.8(1.7)[4.0] \cdot 10^{-11}$$

$$+ \quad a_\mu^{\mathrm{HLBL},\eta} = 27.1(1.8)[2.2] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{HLBL},\eta'} = 26.3(1.1)[4.6] \cdot 10^{-11}$$

$$a_\mu^{\mathrm{HLbL};P} = 135(11) \times 10^{-11}$$

adding the rest from *Glasgow Consensus*

$$a_\mu^{\mathrm{HLbL}} = 126(25) \times 10^{-11}$$

vs

$$a_\mu^{\mathrm{HLBL,GC}} = 105(26) \cdot 10^{-11}$$

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Anomalous magnetic moment a_μ (anomaly):

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	690.8 ± 4.7
HVP(NLO)	-9.93 ± 0.07
HVP(NNLO)	1.22 ± 0.01
HLBL (+NLO)	12.6 ± 2.9
EW	15.4 ± 0.1
Total	11659182.0 ± 5.5

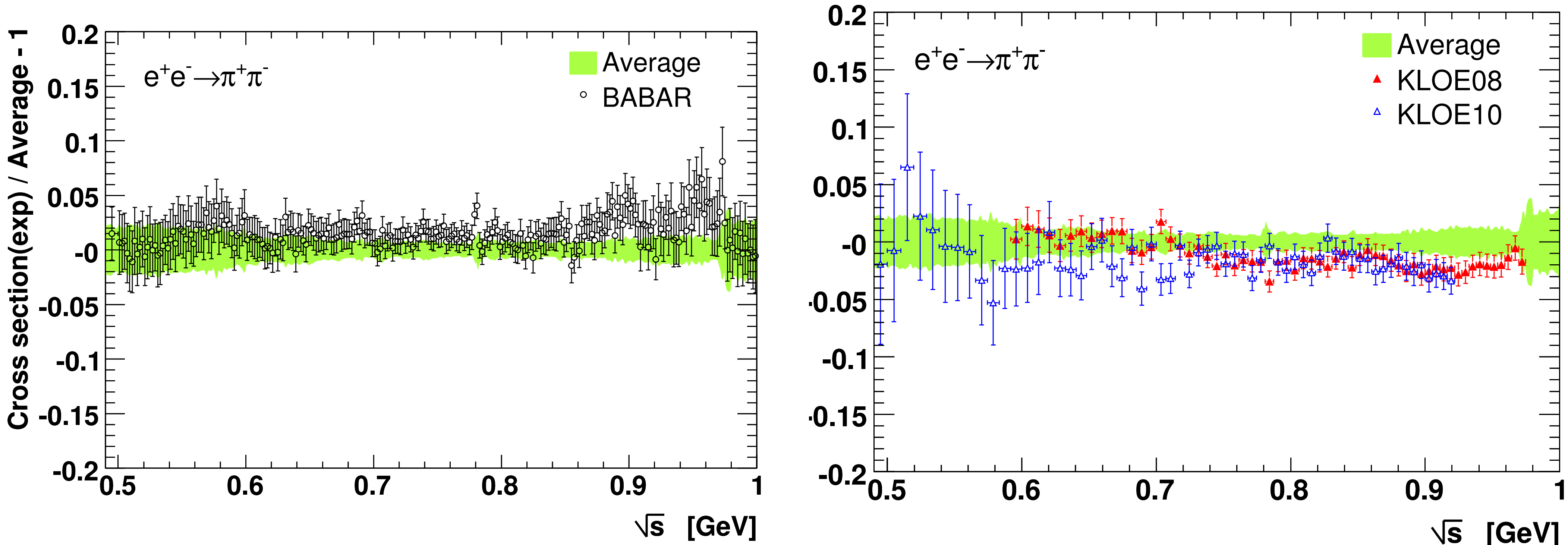
$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 27.1(8.4) \times 10^{-10} \Rightarrow 3.2\,\sigma$$

Outlook

- The reference numbers seem robust but...
- Still we need to understand the role of new and forthcoming data
  - decay constants, masses, form factors, rescattering
  - together with systematics (chiral and large-Nc)
- Ballparks show large numbers
- Lattice QCD is promising, still long way
- Still missing contributions: need more data on $\gamma\gamma \to$ hadrons (t-channel), and @mid-large energies

Thank you!

Hadronic Vacuum Polarization



- KLOE and BABAR dominates the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference
- Difference → large uncertainty in a_μ(VP)
- New measurement at BES-III lies in the middle, but shorter energy range (and lack of very-low energy region)

The anomalous magnetic moment of the muon

The role of experimental data

[P.M., Sanchez-Puertas '17]

Test with Toy models:

	Regge Model				Log Model			
	C_1^0	C_2^1	C_3^2	C_4^3	C_1^0	C_2^1	C_3^2	C_4^3
LE	55.2	59.7	60.4	60.6	56.7	64.4	66.1	66.8
OPE_0	65.7	60.8	60.7	60.7	65.7	67.3	67.5	67.6
OPE_1	–	60.6	60.7	60.7	65.7	67.3	67.5	67.6
OPE_2	–	60.8	60.7	60.7	65.7	67.3	67.5	67.6
Fact	54.6	57.3	57.4	57.5	54.6	60.3	61.3	61.6
Fit^{OPE}	66.3	62.7	61.1	60.8	79.6	71.9	69.3	68.4
Exact	60.7				67.6			

Regge Model:

$$F^{\text{Regge}}_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = \frac{F_{\pi^0\gamma^*\gamma^*}}{\psi^{(1)}(M^2/a)} \times \sum_{m=0}^{\infty} \frac{a^2}{(Q_1^2 + (M^2 + ma))(Q_2^2 + (M^2 + ma))}.$$

Log Model:

$$F^{\log}_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}}{M^2}\int_0^1 dx \frac{1}{xQ_1^2 + (1-x)Q_2^2 + M^2} = \frac{F_{P\gamma\gamma}M^2}{Q_1^2 - Q_2^2} \ln\left(\frac{1 + Q_1^2/M^2}{1 + Q_2^2/M^2}\right),$$

Observations:
- pattern of convergence
- better than factorization
- better than imposing high-energy alone