Status of hadronic light-by-light scattering and the muon (g-2)

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Work done in collaboration with Pablo Sanchez-Puertas

Outline

- Short Introduction
- Hadronic light-by-light scattering contribution to the muon g-2:
	- its role in the 3σ deviation
	- its framework and reference numbers
	- its new trends
- Outlook

The anomalous magnetic moment of the muon What is (*g^µ* 2) ?

The anomalous magnetic moment of the muon: the experiment

The anomalous magnetic moment of the muon: the experiment

depend on the direction of the muon momentum of *Pere Masjuan, CERN, PHOTON 17, May 2017* 5

• The E821 experiment at BNL

Bennet et al, PRD73,072003 (2006)

$$
a_{\mu+}^{\exp} = 11\,659\,204(6)(5) \times 10^{-10}
$$
\n
$$
a_{\mu-}^{\exp} = 11\,659\,215(8)(3) \times 10^{-10}
$$
\n[2001]

• Assuming CPT invariance $a_{\mu}^{\rm exp} = 11\,659\,209.1\,(5.4)(3.3)\times\!10^{-10}$ $\overline{(\begin{matrix} 6 & 3 \end{matrix})}$ (6*.*3) Bennet et al, PRD73,072003 (2006)

• The E821 experiment at BNL

$$
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• Assuming CPT invariance
Bennet et al, PRD73,072003 (2006) $a_\mu^{\rm exp} = 11\,659\,209.1(6.3)\times 10^{-10}$

Forthcoming exp: FNAL & J-PARC $\sim 1.6 \times 10^{-10}$

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Anomalous magnetic moment a_{μ} (anomaly):

$$
g_{\mu} = 2\left(1 + a_{\mu} = \frac{\alpha}{2\pi} + \cdots\right) \qquad \qquad a_{\mu}^{th} = a_{\mu}^{QED} + a_{\mu}^{weak} + a_{\mu}^{had}
$$

Schwinger 1948 *^µ*⃗ ⁼ *^g eh*̵ ²*mc* ⋅ *S*⃗ Petermann and Sommerfield 1958 Laporta and Remiddi 1996

Kinoshita *et al* 2012

Anomalous magnetic moment a_µ (anomaly): Motivation: Muon-Anomaly (*g* − 2)*^µ* Introduction and Direct Measurement

Anomalous magnetic moment a_{μ} (anomaly):

weak 15*.*4 ± 0*.*2 Pere Masjuan, CERN, PHOTON 17, May 2017

 \mathcal{A}, q_3

*µ, q*¹

 \sqrt{v}

11

 p^{\prime}

 $\sum_{i=1}^{n}$

 ν, q_2

 λ , q_3

p

Anomalous magnetic moment a_{μ} (anomaly):

Anomalous magnetic moment a_{μ} (anomaly):

Anomalous magnetic moment a_{μ} (anomaly):

$$
a_{\mu}^{\exp} - a_{\mu}^{\rm SM} = 28.0(8.8) \times 10^{-10} \Rightarrow 3.2 \sigma
$$

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Hints to NP

Very different contributions to *a*

(talk from Stöckinger)

$$
\text{generally:}\qquad C=\frac{\delta m_\mu(\text{N.P.})}{m_\mu},\quad \delta a_\mu(\text{N.P.})=\mathcal{O}(C)\left(\frac{m_\mu}{M}\right)^2
$$

classify new physics: *C* **very** model-dependent Very useful constraints on new physics

Anomalous magnetic moment a_{μ} (anomaly):

Contribution Result in 10^{-10} units $QED($ leptons $)$ 11658471*.*885 \pm 0*.*004 $HVP(leading order)$ 690*.8* \pm 4.7 $HVP(NLO)$ -9.93 ± 0.07 $HVP(NNLO)$ 1.22 \pm 0.01 $HLBL (+NLO)$ 11.7 \pm 4.0 EW 15.4 ± 0.1 Total 11659179.1 ± 6.2 $a_{\mu}^{\rm exp} - a_{\mu}^{\rm SM} = 28.0(8.8) \times 10^{-10} \Rightarrow 3.2 \, \sigma$

Hadronic Vacuum Polarization *e*
F UIdI IZAU

Hadronic Vacuum Polarization *e*
F UIdI IZAU

Hadronic Vacuum Polarization ↑ ● Crui Teation
———————————————

• ρ peak Figure 3. A compilation of the modulus square of the pion form factor in the pion factor in the α

 \sim 0.0 GeV, ∞

 $0.0 \text{ GeV}, \infty$

1.0 GeV

 $\boxed{\rho,\omega}$

• p-ω interference

 $\boxed{\rho,\omega}$

- **Contribution to a_μ(VP): 75%** $\mathbf{D} \cdot \mathbf{75\%}$
	- \bullet Largest error from $1-2GeV$

 $P_{0}H_{0}H_{0}H_{0}N_{0}U_{0}7$, May 2017

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• BNL E821: $\textbf{2021:} \boxed{11\,659\,209.1(6.3)\times 10^{-10}}$ Bennet et al, PRD73,072003 (2006)

NO HLBL (2σ effect) • Theory: Contribution Result in 10^{-10} units $QED($ leptons $)$ 11658471*.*885 \pm 0*.*004 $HVP(leading order)$ 690*.8* \pm 4.7 $HVP(NLO)$ -9.93 ± 0.07 $HVP(NNLO)$ 1.22 \pm 0.01 HLBL (+NLO) 11*.*7 *±* 4*.*0 EW 15.4 ± 0.1 Total 11659179.1 ± 6.2 HVP(leading order) 692*.*3 *±* 4*.*2 11659167*.*4 *±* 4*.*7 $a_{\mu}^{\rm exp} - a_{\mu}^{\rm SM} = 41.7(7.9) \times 10^{-10} \Rightarrow 5.3\,\sigma$

QED(leptons) 11658471*.*810 *±* 0*.*015

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(2σ effect) $a_\mu^{\rm exp} - a_\mu^{\rm SM} = 41.7(7.9) \times 10^{-10} \Rightarrow 5.3\,\sigma$

Forthcoming FNAL $\sim 1.6 \times 10^{-10} \implies$ from 5 σ to 8 σ , w/o HLBL: 5 σ effect

We need to understand such numbers and errors

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$k = p' - p$ Hadronic light-by-light scattering in the muon g-2 trivial task. As an illustration, while the dominant HVP $\frac{1}{3}$ scale, the Hubble involves 138 scalar functions ring in the muon σ-7 whereas large-*N^c* represents the only known perturba-

k

Need to calculate higher order *^O*(↵3) hadronic contribution to the muon *^g* 2:

 $\lambda \gg 1$

the pseudoscalar-pole terms, at order *^O*(*Nc, q*⁶). Follow-

tive approach to α approach to α approach to α approach to α approach to α

 μ (p)

 $\mu^-(p)$

Ballpark prediction: Order of magnitude?

Quark models for a Ballpark prediction

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Classification proposal by Eduardo de Rafael '94

Current approach to had to had to had to had to had. Labyl scattering the second second second second second s
Labyl scattering the second second

Pesudoscalars: numerically dominant contribution *(according to most models)* Pesudoscalars: numerically dominant contribution (Pesudoscalars: numerically dominant contribution (according to most models) Pesudoscalars: numerically dominant contribution (according to most models)

IN: legerlehner and Nyffeler. Phys. Rep. 477 (2009) 1-110 prong continuit and ryneier, myst rep. 177 (2007) 1-1110
PdRV: Prades, de Rafael, and Vainshtein, arXiv:0901.0306 (Glasgow White Paper) $\mathcal{L} = \mathcal{L} \cup \mathcal{L}$ JN: Jegerlehner and Nyffeler, Phys. Rep. 477 (2009) 1-110 PdRV: Prades, de Rafael, and Vainshtein, arXiv:0901.0306 (Glasgow White Paper)

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- \bullet use the same model from Knecht and Nyffeler '01 and inputs for the PS (issue of pio pion-exchange, i.e., how to correctly implement QCD constraints) \sim • use the same model from Knecht and Nyffeler UI and Inputs for the PS (issue of plon
pion exchange i.e. how to correctly implement OCD constraints) • use the same model from Knecht and Nyffeler '01 and inputs for the PS (issue of pion-pole vs
	- errors summed linearly in JN and in quadrature in PdRV
- lack of systematic error (large-Nc model, see P.M. and Vanderhaeghen '12)
- $\frac{1}{2}$ and the systematic enter this general model, see in mains variatinates it is $\frac{1}{2}$ ϵ and moder nergies reproduce and new experimental data on form (dee in finitivality) • the model neither reproduce the new experimental data on PSTFF (see P.M in arXiv:1407.4021) nor the $\pi^0\!\!\rightarrow\!$ e $^+$ e $\bar{}$ (see P. Sanchez-Puertas in arXiv:1407.4021)
- For the read of the contract consideration of the low-energy $\bigcap_{n=1}^{\infty}$ for the developed $\bigcup_{n=1}^{\infty}$ On top, double counting for correct overlappening preces finguer states...) • On top, double counting (or correct overlap) + missing pieces (higher states...)
- Δ ll in all nood for mare colculations closer to date (if possible) *t* an any need for filore carediacions, croser to data (ii possible) • All in all, need for more calculations, closer to data (if possible)

- Main current strategies
	- Lattice QCD
	- Data driven approaches

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	- Lattice QCD
	- Data driven approaches

The anomalous magnetic moment of the muon $\frac{1}{2}$ from anone-photon exchange between the lefton $\frac{1}{2}$ ic moment of the muon sistemation en die 243 rund eeu n.C. with the 243 results with the 243 results with the 243 results with the la
The 243 results with the lattice with the 243 results with the 243 results with the 243 results with the 243 r $\frac{1}{2}$ for the electric form factor $\frac{1}{2}$ $H\in$ anonialous in Eq. (1) die two $H\in$ averaging. For finite statistics, the delicate cancellation of the delicate cancellation of the delicate cancel s mamant of the muo states. The value for the largest separation (*t*sep = 32) is still somewhat below the continuum result, *F*2(0) = τ (*q*_{*e*)} (anonialous magnetic $s = \frac{1}{2}$ respectively. fact that the muon has charge 1 in units of *e >* 0.

- larger volume
- T_{eff} represented by the electronic contain for \sim \sim \sim \sim \sim \sim • controlled extrapolation to Q^2 =0 $\frac{1}{2}$ compact $\frac{1}{2}$ in the Feynman gauge, using $\frac{1}{2}$ and $\frac{1}{2}$ in the Feynman gauge, using $\frac{1}{2}$ • controlled extrapolation to Q^2 =0

bers, and *j^µ* is the electromagnetic current ¹. *k* is a Euclidean four-momentum, p is a three-momentum, each of the lattice size is $\mathsf{P} \mathsf{P} \mathsf{P} \mathsf{P}$. \overline{C} σ mass of the loop is equal to the muon mass of σ 2 In the pure Pere Masjuan, CERN, P Pere Masjuan, CERN, PHOTON 17, May 2017

To extract the form factors *F*¹ and *F*2, Eq. (1) is traced

- Main current strategies
	- Lattice QCD
	- Data driven approaches:
		- Dispersion relations for the low-energy region
		- Hadronic models for the different contributions

Dispersion relations for the low-energy region **BISPEI SION TERGONS TOT GIC TOW-CITELS** 2.1 Notation

$$
\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3) = i^3 \int d^4x \int d^4y \int d^4z \, e^{-i(x \cdot q_1 + y \cdot q_2 + z \cdot q_3)} \langle 0|T \{j^{\mu}(x)j^{\nu}(y)j^{\lambda}(z)j^{\sigma}(0)\} |0\rangle
$$

 $\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \cdots,$ Colangelo et al, 1402.7081 (INCILITY AMPROJE GELOMPORT) , and other higher-mass states are not negligible. (helicity amplitude decomposition) and the matrix element is to be evaluated in pure λ

 $\overline{}$ of model calculations (see, e.g. $\overline{}$) of the HLbL contributions to any, it is clear that that that $\overline{}$

Colangelo et al, 1402.7081 Vanderhaeghen et al, 1403.7503

[Mark, Cello, BELLE]

- no intermediate states • no intermediate states
- all FF are on-shell, off-shell effects king the configuration continues.
2 = 0. Contraction vectors the approximation vectors the approximation vectors that gives the approximation vectors the approximation vectors that is given by the approximation vectors the are included in subtraction constants
	-

 $\overline{0.50}$

1. 200 2. 200 2. 200 2. 200 2. 200 2.

data+systematic error chiral and large-N^c counting was proposed, see Figure 1. In general, all the interactions

should add the charm-quark contr. $\sim 2 \times 10^{-11}$

Pere Masjuan, CERN, PHOTON 17, May 2017 35 ρ mixing. Note that in the Feynman diagrams in F for σ in F for σ and σ shell form factors with off-shell form factors with off-shell form factors with off-shell form factors with order σ

From Knecht and Nyffeler, '01:

$$
a_{\mu}^{HLBL;\pi^{0}} = -e^{6} \int \frac{d^{4}q_{1}}{(2\pi)^{4}} \int \frac{d^{4}q_{2}}{(2\pi)^{4}} \frac{1}{q_{1}^{2}q_{2}^{2}(q_{1}+q_{2})^{2}[(p+q_{1})^{2}-m^{2}][(p-q_{2})^{2}-m^{2}]} \times \left(\frac{F_{\pi^{0}\gamma^{*}\gamma^{*}}(q_{1}^{2},(q_{1}+q_{2})^{2})F_{\pi^{0}\gamma^{*}\gamma^{*}}(q_{2}^{2},0)}{q_{2}^{2}-M_{\pi}^{2}} T_{1}(q_{1},q_{2};p) \right)
$$
\nUse data from

\nthe pion Transition Form Factor

\n
$$
+ \frac{F_{\pi^{0}\gamma^{*}\gamma^{*}}(q_{1}^{2},q_{2}^{2})F_{\pi^{0}\gamma^{*}\gamma^{*}}((q_{1}+q_{2})^{2},0)}{(q_{1}+q_{2})^{2}-M_{\pi}^{2}} T_{2}(q_{1},q_{2};p)
$$

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 $\overline{}$

The role of experimental data

The role of experimental data

The role of experimental data

[P.M., Sanchez-Puertas '17]

$$
a_{\mu}^{HLBL;\pi^0} = e^6 \int \frac{d^4Q_1}{(2\pi)^4} \int \frac{d^4Q_2}{(2\pi)^4} K(Q_1^2, Q_2^2) \quad \text{Using } F_{\pi^0 \gamma^* \gamma^*}(Q_1^2, Q_2^2)
$$

Γ complement of the annument contribution of Γ \overline{r} involves the full value \overline{r} 12 nom *f* Ω (*q*² ¹ *M*² *V*) *M*² *V M*2
2 *M*₂ The anomalous magnetic moment of the muon

p **erimental da** The role of experimental data

Dentral value µ ∴ 30™101101 Central value:

Model from Knecht and Nyffeler '01 used in reference numbers S

$$
F_{\pi^0 \gamma^* \gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3} \frac{q_1^2 q_2^2 (q_1^2+q_2^2) + h_1 (q_1^2+q_2^2)^2 + h_2 q_1^2 q_2^2 + h_5 (q_1^2+q_2^2) + h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}
$$

nant prior is on-shell prior is on-² *M*² *V*1 Publication: *V*2

$$
F_{\pi} = 92.4 \text{ MeV}
$$

\n
$$
m_{\rho} = 769 \text{ MeV}
$$

\n
$$
m_{\rho'} = 1465 \text{ MeV}
$$

\n
$$
h_1 = 0 \text{ (BL limit)}
$$

\n
$$
h_5 = 6.93 \text{ GeV}^4
$$

\n
$$
h_2 = -10 \text{ GeV}^2
$$

 $\overline{}$ and $\overline{}$ and $\overline{}$ $a_\mu^{\text{hildes}}, \alpha = 6.$ resonances just a finite set inspired by resonance saturation *FVMD* $\overline{10}$ (*q*² 10^{-10} of fixing the pole at *M*^r we could match the *MV* to reproduce $\overline{}$ and $\overline{}$ and $\overline{}$ $a_\mu^{\mathrm{HLBL},\pi} = 6.3 \times 10^{-10}$ ¹ (*MV* ⌘ *M*^r) with only one free

Γ complement of the annument contribution of Γ \overline{r} involves the full value \overline{r} 12 nom *f* Ω (*q*² ¹ *M*² *V*) *M*² *V M*2
2 *M*₂ The anomalous magnetic moment of the muon

p **erimental da** The role of experimental data

Dentral value µ ∴ 30™101101 Central value:

Model from Knecht and Nyffeler '01 used in reference numbers S

Preliminary, using new exp data:

$$
F_{\pi^0 \gamma^* \gamma^*}^{LMD+V}(q_1^2,q_2^2) = \frac{f_\pi}{3} \frac{q_1^2 q_2^2 (q_1^2+q_2^2) + h_1 (q_1^2+q_2^2)^2 + h_2 q_1^2 q_2^2 + h_5 (q_1^2+q_2^2) + h_7}{(q_1^2-M_{V_1}^2)(q_1^2-M_{V_2}^2)(q_2^2-M_{V_1}^2)(q_2^2-M_{V_2}^2)}
$$

nant prior is on-shell prior is on-² *M*² *V*1 Publication: *V*2

 $F_\pi=92.4$ i $m = 769$ $rac{1}{\sqrt{2}}$ is be a set of $rac{1}{\sqrt{2}}$ $m_{\rho'} = 1400$ $h_1 = 0$ (BL limit) on the consistent of the model-dependence of the model-dependence of the model-dependence would be model-dependency of the model-dependency of the model-dependency of the model-dependency of the model-dependence would be a $i_1i_2 = -10$ $\overline{}$ and $\overline{}$ and $\overline{}$ $a_\mu^{\text{hildes}}, \alpha = 6.$ resonances just a finite set inspired by resonance saturation Γ determined by experimental input or Γ or matching conditions. $\sim \pi \rightarrow \gamma \gamma$ authors of [?] obtained *^aLbyL*;p⁰ and 5*.*8(1*.*0)⇥10¹⁰ respectively. A way to ascribe a system $h_1 = 0$ (BL limit) \mathcal{A} and the difference between the results of one approximate \mathcal{A} $\frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}$ signals on the same sequence (see Refs. $h_2 = -10 \text{ GeV}^2$ *FVMD* p_{HIR} and the following one on the same set p_{HIR} (*q*² ¹ (*MV* ⌘ *M*^r) with only one free $p^{\text{m}} = 10^{-10}$ $\left| a_{\mu}^{\text{HLBL},\pi} = 7.5 \times 10^{-10} \right|$ of fixing the pole at *M*^r we could match the *MV* to reproduce $I'\pi - \partial Z$, 1 ivit $m_\rho=769\,\,{\rm MeV}$ $m_{\rho'} = 1465\,\, {\rm MeV}$ *h* $h_5 = 6.93\,\,{\rm GeV}^4$ $h_2 = -10 \text{ GeV}^2$ $a_\mu^{\mathrm{HLBL},\pi} = 6.3 \times 10^{-10}$ ¹ (*MV* ⌘ *M*^r) with only one free $F_{\pi}=92.4\,\, \mathrm{MeV}$

 $m_{\rho} = 775 \text{ MeV}$ $\Gamma_{\pi^0 \to \gamma\gamma}$ $h_2 = -10 \text{ GeV}^2$ slope TFF curvature TFF

The anomalous magnetic moment of the muon provide and halve shows magnetic *N*+1(*Q*² 1*, Q*² ²)-like fitting function with the appropriate OPE V. RESULTS OPE*ⁿ* rows incorporate high-energy constraints. The Fact row The anomalque magnet ρ and ρ and ρ and ρ and ρ and ρ and ρ in a show ρ ⁴⇡2*^F* tr(*Q*2*^P*)*.* (17) This expression is, strictly speaking, valid only at the strictly speaking, valid only at the strictly speaking, $\overline{}$ *Q*2!1 *Fhe anomalous magnetic moment of the mu* the double-virtual region, where α is available solution, where α is available solution, where α Summarizing, the present data subsequently

 $F_{P\gamma\gamma}(0,0)(1+\alpha_1(Q_1^2-\))$ $C_2^2(Q_1^2,Q_2^2)=\frac{1}{1+ \beta_1(Q^2+Q^2)+\beta_2(Q^4+Q^4)+q^4}$ $=$ $\frac{1}{2}$ (12) $\frac{1}{2}$ when $\frac{1}{2}$ when $\frac{1}{2}$ $\frac{f(u_1, 1 \vee 1 \vee 2)}{g(u_1, 1 \vee 1 \vee 2)}$ $b_1^2Q_2^2 + \beta_{2,1}Q_1^2Q_2^2(Q_1^2+Q_2^2)$ $C_0^1(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1+\alpha_1(Q_1^2+Q_2))}{(1+\alpha_1(Q_1^2))^2}$ p_1^2 (b) p_2^2 by $1 + \beta_1(Q_1^2 + Q_2^2) + \beta_2(Q_1^4 + Q_2^4) + \beta_{1,1}$ $\frac{1}{\sqrt{1-\frac{1}{2}}}\left(\frac{1}{2}, \frac{1}{2}\right)^2$ \mathcal{L} meromorphic function, for which converges \mathcal{L} $C_0^1(Q_1^2,Q_0^2) = \frac{F_{P\gamma\gamma}(0,0)(1+\alpha_1(0,0))}{2\gamma}$ $F = P_1 \cup P_2$ (4²) $-Q_2^2$ + $\beta_2(Q_1^4+Q_2^4)$ $(Q_2^2) + \alpha_{1,1} Q_1^2 Q$ *^F^P* ⇤⇤ (*Q*²*, Q*²) = *^P*¹ $C_2^1(Q_1^2,Q_2^2) = \frac{P(\gamma \gamma \sqrt{8},0)(1+\alpha)}{P(\gamma \sqrt{8},0)(1+\alpha)}$ $1 \mid \rho_1 \cup_2 1 \mid \nu_2 \rangle + \rho_2 \cup_1 1 \cup_2$ $\frac{\alpha_{2}}{\rho}$ $\frac{\alpha_{1,1}\alpha_{1}\alpha_{2}}{\rho}$ $\frac{\alpha_{2,1}}{\rho}$ $\frac{\alpha_{3,1}}{\rho}$ $\frac{\alpha_{2,1}}{\rho}$. $\mu_{1,1}Q_{1}^{T}Q_{2}^{T} + \mu_{2,1}Q_{1}^{T}Q_{2}^{T}(Q_{1}^{T} + Q_{2}^{T})$ $C_2^1(Q_1^2,Q_2^2) = \frac{F_{P\gamma\gamma}(0,0)(1+\alpha_1(Q_1^2+Q_2^2)+\alpha_{1,1}Q_1^2Q_2^2)}{1+Q_1(Q_1^2+Q_2^2)+Q_2(Q_1^2+Q_2^2)+Q_3(Q_2^2+Q_3^2)}$ $\mathcal{O}_2(\mathcal{Q}_1,\mathcal{Q}_2) = 1 + \beta_1(Q_1^2+Q_2^2) + \beta_2(Q_1^4+Q_2^4) + \beta_{1,1}Q_1^2Q_2^2 + \beta_{2,1}Q_1^2Q_2^2(Q_1^2+Q_2^2)$

. (21)

The anomalous magnetic moment of the muon ⌘ ⁰ 30*.*4(1*.*0)(0*.*5)[1*.*1] 26*.*8(1*.*1)[1*.*1] 25*.*8(0*.*7)(0*.*9)[1*.*1] ous magnetic moment of the muon sult. Accounting for the system of the s

The role of experimental data he role of experimental da

[P.M., Sanchez-Puertas '17] *^µ* in units of 10¹¹ according to the procedure in Ref. [19]. The errors and labeling are *^µ* = 135(11) ⇥ ¹⁰¹¹ *,* (E1)

Using largest set ever:
\n- Space-like region
\n
$$
e^+e^- \rightarrow e^+e^-P
$$

\n[L3,CLEO,CELLO,BABAR,BELLE]
\n- Time-like region
\n $P \rightarrow \ell^+\ell^-$
\n $P \rightarrow \ell^+\ell^- \gamma$
\n[NA48,A2,NA62+PDG]

$$
P = \pi^0, \eta, \eta'
$$

$$
\ell = e, \mu
$$

[13 different coll.]

$$
a_{\mu}^{\text{HLBL},\pi^{0}} = 81.8(1.7)[4.0] \cdot 10^{-11}
$$
\n+ $a_{\mu}^{\text{HLBL},\eta} = 27.1(1.8)[2.2] \cdot 10^{-11}$
\n $a_{\mu}^{\text{HLBL},\eta'} = 26.3(1.1)[4.6] \cdot 10^{-11}$
\n $a_{\mu}^{\text{HLbL};P} = 135(11) \times 10^{-11}$
\nadding the rest from Glasgow Consensus
\n $a_{\mu}^{\text{HLbL}} = 126(25) \times 10^{-11}$
\n $a_{\mu}^{\text{HLbL}} = 126(25) \times 10^{-11}$

estimates.

Anomalous magnetic moment a_{μ} (anomaly):

$$
a_\mu^{\rm exp} - a_\mu^{\rm SM} = 27.1(8.4) \times 10^{-10} \Rightarrow 3.2\,\sigma
$$

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Outlook

- The reference numbers seem robust but...
- Still we need to understand the role of new and forthcoming data
	- decay constants, masses, form factors, rescattering
	- together with systematics (chiral and large-Nc)
- Ballparks show large numbers
- Lattice QCD is promising, still long way
- Still missing contributions: need more data on

γγ→hadrons (t-channel), and @mid-large energies Thank you!

Hadronic Vacuu Hadronic Vacuum Polarization

- KLOE and BABAR dominates the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference

→ → Cross Section

- Systematic di↵erence, especially above ⇢ peak • Difference \rightarrow large uncertainty in $a_{\mu}(VP)$
- Difference rarge uncertainty in *a*_H(v)
• New measurement at BES-III lies in the middle, but shorter energy range (and lack of very-low energy region)

The role of experimental data

ERM., Sanchez-Puertas '17] The Regge model can be expressed as well as as an *infi*vergence, but still not such a good convergence as CAs

Test with Toy models: **T** (1)(*M*²*/a*) *a*2

weighted by the potential systematic residues, and the potential systematic residues, and the potential systematic residues, \mathbf{R}

$$
F_{\pi^0 \gamma^* \gamma^*}^{\text{Regge}}(Q_1^2, Q_2^2) = \frac{F_{\pi^0 \gamma^* \gamma^*}}{\psi^{(1)}(M^2/a)}
$$

\$\times \sum_{m=0}^{\infty} \frac{a^2}{(Q_1^2 + (M^2 + ma))(Q_2^2 + (M^2 + ma))}.

high-energy behavior is accounted for. More important, we find that the systematic uncertainty can be estimated uncertainty can be estimated uncertainty can be estimated uncertainty $\mathcal{L}(\mathbf{z})$ Log Model: $\frac{1}{1-\epsilon}$ included, assuming of course that the model parameters that the model parameters that the model parameters of ϵ $R = M \cdot d \cdot d \cdot k$

$$
F_{\pi^0 \gamma^* \gamma^*}^{\log}(Q_1^2, Q_2^2) = \frac{F_{P\gamma\gamma}}{M^2} \int_0^1 dx \frac{1}{xQ_1^2 + (1-x)Q_2^2 + M^2}
$$

=
$$
\frac{F_{P\gamma\gamma}M^2}{Q_1^2 - Q_2^2} \ln\left(\frac{1 + Q_1^2/M^2}{1 + Q_2^2/M^2}\right),
$$

TABLE I. The results for *a*HLbL;⇡⁰ **Observations:** and 1234 of the compared to th

- pattern of convergence
- better than factorization
- <u>Latter than imposing high-energy alone</u> row serves and imposing ing. The g/ which Der 50*.*8 57*.*8 59*.*4 59*.*9 Fit 55*.*9 67*.*2 58*.*3 65*.*4 - better than imposing high-energy alone