The Proton Radius Puzzle

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Introduction: The proton radius puzzle
Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

\[
\langle N(p_f)| \sum_q e_q \bar{q} \gamma^\mu q | N(p_i) \rangle = \bar{u}(p_f) \left[ \gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu \right] u(p_i)
\]

- Sachs electric and magnetic form factors

\[
G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \quad \quad \quad G_M(q^2) = F_1(q^2) + F_2(q^2)
\]

\[
G^p_E(0) = 1 \quad \quad \quad G^p_M(0) = \mu_p \approx 2.793
\]

- The slope of $G^p_E$

\[
\langle r^2 \rangle^p_E = 6 \left. \frac{dG_E^p}{dq^2} \right|_{q^2=0}
\]

determines the charge radius $r^p_E \equiv \sqrt{\langle r^2 \rangle^p_E}$

- The proton magnetic radius

\[
\langle r^2 \rangle^p_M = \frac{6}{G^p_M(0)} \left. \frac{dG_M^p(q^2)}{dq^2} \right|_{q^2=0}
\]
Lamb shift in muonic hydrogen [Pohl et al. Nature 466, 213 (2010)]
\[ r_E^P = 0.84184(67) \text{ fm} \]
more recently \[ r_E^P = 0.84087(39) \text{ fm} \] [Antognini et al. Science 339, 417 (2013)]

CODATA value [Mohr et al. RMP 80, 633 (2008)]
\[ r_E^P = 0.87680(690) \text{ fm} \]
more recently \[ r_E^P = 0.87750(510) \text{ fm} \] [Mohr et al. RMP 84, 1527 (2012)]
extracted mainly from (electronic) hydrogen

5\(\sigma\) discrepancy!

This is the proton radius puzzle
What could be the reason for the discrepancy?

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1) Problem with the electronic extraction? (Part 1 of this talk)
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2) Hadronic Uncertainty? (Part 2 of this talk)
What could be the reason for the discrepancy?

1) Problem with the electronic extraction? (Part 1 of this talk)

2) Hadronic Uncertainty? (Part 2 of this talk)

3) New Physics?
Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Backup slides: Connecting $\mu - p$ scattering and muonic hydrogen
- Conclusions and outlook
Part 1: Proton radii from scattering
Problem with the electronic extraction?

- Recent development: use of the $z$ expansion based on known analytic properties of form factors

- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C 74, 2890 (2014)]

- Now applied successfully to **baryon** form factors to extract $r_E^p$, $r_M^p$, $r_M^n$, $m_A$...
PDG 2016: $r^p_E$

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

---

**p CHARGE RADIUS**

This is the rms electric charge radius, $\sqrt{\langle r^2_E \rangle}$.

<table>
<thead>
<tr>
<th>VALUE (fm)</th>
<th>DOCUMENT ID</th>
<th>TECN</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8751 ± 0.0061</td>
<td>MOHR</td>
<td>RVUE</td>
<td>2014 CODATA value</td>
</tr>
<tr>
<td>0.84087 ± 0.00026 ± 0.00029</td>
<td>ANTOGNINI</td>
<td>LASR</td>
<td>$\mu_p$-atom Lamb shift</td>
</tr>
</tbody>
</table>

● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ●

0.895 ± 0.014 ± 0.014  
0.916 ± 0.024  
0.8775 ± 0.0051  
0.875 ± 0.008 ± 0.006  
0.879 ± 0.005 ± 0.006  
0.912 ± 0.009 ± 0.007  
0.871 ± 0.009 ± 0.003

0.84184 ± 0.00036 ± 0.00056  
0.8768 ± 0.0069  
0.844 ± 0.008 −0.004  
0.897 ± 0.018  
0.8750 ± 0.0068  
0.895 ± 0.010 ± 0.013

The Proton Radius Puzzle

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

**\( p \) MAGNETIC RADIUS**

This is the rms magnetic radius, \( \sqrt{\langle r_M^2 \rangle} \).

<table>
<thead>
<tr>
<th>VALUE (fm)</th>
<th>DOCUMENT ID</th>
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<th>COMMENT</th>
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</thead>
<tbody>
<tr>
<td>0.776 ± 0.034 ± 0.017</td>
<td>LEE 15</td>
<td>SPEC</td>
<td>Just 2010 Mainz data</td>
</tr>
<tr>
<td>• • • We do not use the following data for averages, fits, limits, etc. • • •</td>
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<td></td>
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<tr>
<td>0.914 ± 0.035</td>
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<td>SPEC</td>
<td>World data, no Mainz</td>
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<tr>
<td>0.87 ± 0.02</td>
<td>EPSTEIN 14</td>
<td>SPEC</td>
<td>Using ( ep, en, \pi \pi ) data</td>
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<tr>
<td>0.867 ± 0.009 ± 0.018</td>
<td>ZHAN 11</td>
<td>SPEC</td>
<td>Recoil polarimetry</td>
</tr>
<tr>
<td>0.777 ± 0.013 ± 0.010</td>
<td>BERNAUER 10</td>
<td>SPEC</td>
<td>( ep \rightarrow ep ) form factor</td>
</tr>
<tr>
<td>0.876 ± 0.010 ± 0.016</td>
<td>BORISYUK 10</td>
<td>SPEC</td>
<td>Reanalyzes old ( ep \rightarrow ep ) data</td>
</tr>
<tr>
<td>0.854 ± 0.005</td>
<td>BELUSHKIN 07</td>
<td></td>
<td>Dispersion analysis</td>
</tr>
</tbody>
</table>

\(^1\) Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD 90, 074027 (2014)]
[Lee, Arrington, Hill, PRD 92, 013013 (2015)]
Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

**n MAGNETIC RADIUS**

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

<table>
<thead>
<tr>
<th>VALUE (fm)</th>
<th>DOCUMENT ID</th>
<th>COMMENT</th>
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<tbody>
<tr>
<td>0.864 $^{+0.009}_{-0.008}$ OUR AVERAGE</td>
<td></td>
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<tr>
<td>0.89 $^{+0.03}_{-0.03}$</td>
<td>EPSTEIN 14</td>
<td>Using $e p$, $e n$, $\pi \pi$ data</td>
</tr>
<tr>
<td>0.862 $^{+0.009}_{-0.008}$</td>
<td>BELUSHKIN 07</td>
<td>Dispersion analysis</td>
</tr>
</tbody>
</table>

[Epstein, GP, Roy PRD 90, 074027 (2014)]
Part 2: Hadronic Uncertainty?

The bottom line

- Scattering:
  - World $e - p$ data [Lee, Arrington, Hill '15]
    \[ r_E^p = 0.918 \pm 0.024 \text{ fm} \]
  - Mainz $e - p$ data [Lee, Arrington, Hill '15]
    \[ r_E^p = 0.895 \pm 0.020 \text{ fm} \]
  - Proton, neutron and $\pi$ data [Hill, GP '10]
    \[ r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002 \text{ fm} \]

- Muonic hydrogen
  - [Pohl et al. Nature 466, 213 (2010)]
    \[ r_E^p = 0.84184(67) \text{ fm} \]
  - [Antognini et al. Science 339, 417 (2013)]
    \[ r_E^p = 0.84087(39) \text{ fm} \]

The bottom line:
using $z$ expansion scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen theory?
Muonic hydrogen theory

• Is there a problem with muonic hydrogen theory?

• Potentially yes!
  [Hill, GP PRL 107 160402 (2011)]

• Muonic hydrogen measures $\Delta E$ and translates it to $r_E^p$

    $\Delta E = 206.0573(45) - 5.2262(r_E^p)^2 + 0.0347(r_E^p)^3$ meV

    $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

• In both cases apart from $r_E^p$ need two-photon exchange

![Diagram of two-photon exchange]
Two photon exchange

In both cases apart from $r_E^p$ we have two-photon exchange

\[
\begin{align*}
W_{\mu\nu} = & \sum_s \int d^4x e^{iq \cdot x} \langle k, s | T \{ J^{\mu e} \cdot m \cdot J^{\nu e} \cdot m \} | k, s \rangle \\
= & \left(-g_{\mu\nu} + q_{\mu} q_{\nu} q^2\right) W_1 + \left(k_{\mu} - k \cdot q q_{\mu} q^2\right) \left(k_{\nu} - k \cdot q q_{\nu} q^2\right) W_2
\end{align*}
\]

Dispersion relations ($\nu = 2 \mathbf{k} \cdot \mathbf{q}$, $Q^2 = -q^2$)

\[
W_1(\nu, Q^2) = W_1(0, Q^2) + \nu^2 \pi \int_{\nu \text{cut}}^{\infty} (Q^2)^2 d\nu^\prime \text{Im} W_1(\nu^\prime, Q^2) \nu^\prime^2 \left(\nu^\prime^2 - \nu^2\right)
\]

\[
W_2(\nu, Q^2) = \pi \int_{\nu \text{cut}}^{\infty} (Q^2)^2 d\nu^\prime \text{Im} W_2(\nu^\prime, Q^2) \nu^\prime^2 - \nu^2
\]
Two photon exchange

In both cases apart from $r_E^p$ we have two-photon exchange

\[
W^{\mu\nu} = \frac{1}{2} \sum_s i \int d^4x \, e^{iq \cdot x} \langle k, s \mid T \{ J_{\text{e.m.}}^{\mu}(x) J_{\text{e.m.}}^{\nu}(0) \} \mid k, s \rangle
\]

\[
= \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1 + \left( k^\mu - \frac{k \cdot q q^\mu}{q^2} \right) \left( k^\nu - \frac{k \cdot q q^\nu}{q^2} \right) W_2
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Two photon exchange

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- Dispersion relations ($\nu = 2k \cdot q$, $Q^2 = -q^2$)

\[
W_1(\nu, Q^2) = W_1(0, Q^2) + \frac{\nu^2}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu' \frac{\text{Im} W_1(\nu', Q^2)}{\nu'2(\nu'^2 - \nu^2)}
\]

\[
W_2(\nu, Q^2) = \frac{1}{\pi} \int_{\nu_{\text{cut}}(Q^2)^2}^{\infty} d\nu' \frac{\text{Im} W_2(\nu', Q^2)}{\nu'^2 - \nu^2}
\]

- $W_1$ requires subtraction...
Two photon exchange

- In both cases apart from $r_E^p$ we have two-photon exchange

- Imaginary part of TPE related to data: form factors, structure functions
Two photon exchange

- In both cases apart from $r_E^p$ we have two-photon exchange

\[
\begin{array}{ccc}
  & l & l \\
\text{l} & \text{l} & \text{p} \\
\text{p} & \text{p} & \text{l}
\end{array}
\]

- Imaginary part of TPE related to data: form factors, structure functions

- Cannot reproduce it from its imaginary part: Dispersion relation requires subtraction

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

- Calculable in small $Q^2$ limit using NRQED
  \[\text{[Hill, GP, PRL 107 160402 (2011)]}\]
Two Photon exchange: small $Q^2$ limit

- Small $Q^2$ limit using NRQED [Hill, GP, PRL 107 160402 (2011)]
  The photon sees the proton “almost” like an elementary particle
Two Photon exchange: small $Q^2$ limit

- *Small* $Q^2$ limit using NRQED [Hill, GP, PRL 107 160402 (2011)]

The photon sees the proton “almost” like an elementary particle

$$W_1(0, Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2} \left\{ \frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3} \left[ (1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2 \right] \right\} + \mathcal{O} (Q^4)$$
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- $a_p = 1.793$, $\bar{\beta} = 2.5(4) \times 10^{-4}$ fm$^3$
- $r_M = 0.776(34)(17)$ fm,
- $r_E^H = 0.8751(61)$ fm or $r_E^{\mu H} = 0.84087(26)(29)$ fm

$$W_1(0, Q^2) = 13.6 + \frac{Q^2}{m_p^2} (-54 \pm 7) + O(Q^4)$$
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Two Photon Exchange: large $Q^2$ limit

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$
- Calculable in large $Q^2$ limit using Operator Product Expansion (OPE) [J. C. Collins, NPB 149, 90 (1979)]

The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = \frac{c}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)$$

- Result was used to estimate two photon exchange effects
- $c$ calculated in [J. C. Collins, NPB 149, 90 (1979)]

RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *

Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978
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RENORMALIZATION OF THE COTTINGHAM FORMULA

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Received 23 October 1978

- Was it?
Two Photon Exchange: large $Q^2$ limit

\[ W^\mu\nu = \frac{1}{2} \sum_s i \int d^4 x e^{iq \cdot x} \langle k, s | T \{ J^\mu_{\text{e.m.}}(x) J^\nu_{\text{e.m.}}(0) \} | k, s \rangle \]

\[ = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1 + \left( k^\mu - \frac{k \cdot q q^\mu}{q^2} \right) \left( k^\nu - \frac{k \cdot q q^\nu}{q^2} \right) W_2 \]

- $W_1(0, Q^2)$ is dimensionless

\[ W_1 \sim \frac{\langle \text{Proton} | O | \text{Proton} \rangle}{Q^2} + O \left( \frac{1}{Q^4} \right) \]

- $O$ is a dimension 4 operator:
  - Quarks: Spin 0: $m_q \bar{q} q$  Spin 2: $\bar{q} \left( iD^\mu \gamma^\nu + iD^\nu \gamma^\mu - \frac{1}{4} i[D] g^{\mu\nu} \right) q$
  - Gluons: must be color singlet: $G^\alpha_{a\beta} G^\rho_{a\sigma}$
  - What gluon operators can we have?
Gluons: must be color singlet $G_a^{\alpha\beta} G_a^{\rho\sigma}$

A product of $(E^i, B^i)$ and $(E^j, B^j)$ has $7 \times 6/2 = 21$ components:
Gluon operators

- Gluons: must be color singlet $G_\alpha^\beta G_\rho^\sigma$

  A product of $(E^i, B^i)$ and $(E^j, B^j)$ has $7 \times 6/2 = 21$ components:
  - 1 scalar: $G^{\mu\nu} G_{\mu\nu} = 2(\vec{B}^2 - \vec{E}^2)$
Gluon operators

- Gluons: must be color singlet \( G^{\alpha\beta}_a G^\rho\sigma_a \)

  A product of \((E^i, B^i)\) and \((E^j, B^j)\) has \(7 \times 6/2 = 21\) components:
  - 1 scalar: \( G^{\mu\nu} G_{\mu\nu} = 2(\vec{B}^2 - \vec{E}^2) \)
  - 1 pseudo scalar: \( \epsilon^{\alpha\beta\rho\sigma} G^{\alpha\beta}_a G^{\rho\sigma}_a = E \cdot B \): ruled out by parity
Gluon operators

- Gluons: must be color singlet $G^\alpha_\beta G^\rho_\sigma$
  
  A product of $(E^i, B^i)$ and $(E^j, B^j)$ has $7 \times 6/2 = 21$ components:

  - 1 scalar: $G^{\mu\nu} G_{\mu\nu} = 2(\vec{B}^2 - \vec{E}^2)$
  - 1 pseudo scalar: $\epsilon^\alpha_\beta_\rho_\sigma G^{\alpha_\beta} G^{\rho_\sigma} = E \cdot B$: ruled out by parity
  - 9 components of traceless symmetric tensor: $G^{\mu_\alpha} G^{\nu_\alpha} - \frac{1}{4} G^{\alpha_\beta} G^{\beta_\gamma} g^{\mu\nu}$
    chromomagnetic stress-energy tensor
  - What else?

- 10 components of $O^{\mu_\alpha_\beta_\gamma} = -\frac{1}{4} (\epsilon^\mu_{\alpha_\rho_\sigma} \epsilon^{\nu_\beta_\kappa_\lambda} + \epsilon^{\mu_\beta_\rho_\sigma} \epsilon^{\nu_\alpha_\kappa_\lambda}) G^\rho_\kappa G^\sigma_\lambda$
  
  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E_1 B_1 - E_3 B_3$

- For protons: $\langle \text{Proton} | O^{\mu_\alpha_\beta_\gamma} | \text{Proton} \rangle = 0$

- What about $\langle \text{Medium} | O^{\mu_\alpha_\beta_\gamma} | \text{Medium} \rangle$?

Solution looking for a problem...
Gluon operators

- Gluons: must be color singlet \( G^\alpha_\beta G^\rho_\sigma \)

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  - What else? 10 components of

\[ O_{\mu\alpha\nu\beta} = -\frac{1}{4} \left( \epsilon_{\mu\alpha\rho\sigma} \epsilon_{\nu\beta\kappa\lambda} + \epsilon_{\mu\beta\rho\sigma} \epsilon_{\nu\alpha\kappa\lambda} \right) G^\rho_\kappa G^\sigma_\lambda \]

For protons:
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What about \( \langle \text{Medium} | O_{\mu\alpha\nu\beta} | \text{Medium} \rangle \)?

Solution looking for a problem...
Gluon operators

- Gluons: must be color singlet $G_a^{\alpha\beta} G_a^{\rho\sigma}$
  A product of $(E^i, B^i)$ and $(E^j, B^j)$ has $7 \times 6/2 = 21$ components:
  - 1 scalar: $G^{\mu\nu} G_{\mu\nu} = 2(B^2 - E^2)$
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\[
O^{\mu\alpha\nu\beta} = -\frac{1}{4} \left( \epsilon^{\mu\alpha\rho\sigma} \epsilon_{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma} \epsilon_{\nu\alpha\kappa\lambda} \right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}
\]

For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
Gluon operators

- Gluons: must be color singlet \( G_\alpha^\beta G_\rho^\sigma \)
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    - 9 components of traceless symmetric tensor: \( G^{\mu\alpha} G^{\nu}_{\alpha} - \frac{1}{4} G^{\alpha\beta} G^{\alpha\beta} g^{\mu\nu} \)
      chromomagnetic stress-energy tensor
    - What else? 10 components of
      \[
      O^{\mu\alpha\nu\beta} = -\frac{1}{4} \left( \epsilon^{\mu\alpha\rho\sigma} \epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma} \epsilon^{\nu\alpha\kappa\lambda} \right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}
      \]
      For example \( O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3 \)
- For protons: \( \langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0 \)
  - What about \( \langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle \)?
  - Solution looking for a problem...
Summary: Possible operators

- In total we have four operators with non-zero proton matrix elements.

- **Quarks:**
  - Spin 0: $m_q \bar{q}q$
  - Spin 2: $\bar{q}(iD^\mu \gamma^\nu + iD^\nu \gamma^\mu - \frac{1}{4}i\not{D}g^{\mu\nu})q$

- **Gluons:**
  - Spin 0: $G^{\mu\nu}G_{\mu\nu}$
  - Spin 2: $G^{\mu\alpha}G^\nu_\alpha - \frac{1}{4}G^{\alpha\beta}G_{\alpha\beta}g^{\mu\nu}$
Summary: Possible operators

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  - Spin 0: $m_q \bar{q}q$
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- **Gluons:**
  - Spin 0: $G^{\mu\nu} G_{\mu\nu}$
  - Spin 2: $G^{\mu\alpha} G_{\alpha}^\nu - \frac{1}{4} G^{\alpha\beta} G_{\alpha\beta} g^{\mu\nu}$

**REnormalization of the Cottingham Formula**

John C. Collins *

*Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA*

Received 23 October 1978
Large $Q^2$ behavior

- In 1978 Collins calculated EM corrections to the nucleon mass with an emphasis on $m_n - m_p$
- The mass only depends on spin-0 operators ($q$ quark, $G^{\mu \nu}$ gluon)

$$\langle P | m_q \bar{q} q | P \rangle, \quad \langle P | G^{\mu \nu} G_{\mu \nu} | P \rangle$$

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- For $W_1(0, Q^2)$ you need also spin-2 operators
  
  \[ \langle P | \bar{q} \left( iD^\mu \gamma^\nu + iD^\nu \gamma^\mu - \frac{1}{4} i\gamma^\mu g^{\mu\nu} \right) q | P \rangle, \quad \langle P | G^{\mu\alpha} G^{\nu}_{\alpha} - \frac{1}{4} G^{\alpha\beta} G^{\alpha\beta}_{\alpha\beta} g^{\mu\nu} | P \rangle \]
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- Need to calculate the spin-2 contribution [Hill, GP arXiv:1611.09917]

- Collins’s result is not enough for muonic hydrogen!
Large $Q^2$ behavior

- Requires 1-loop calculation

\[ \text{Correct result:} \quad \sum q_{e_2} q = \left( \frac{2}{3} \right)^2 + \left( \frac{1}{3} \right)^2 + \left( \frac{1}{3} \right)^2 = \frac{2}{3} \]

Collins: \[ \sum q = 3 \]

Too large by a factor of 4.5...
Large $Q^2$ behavior

- Requires 1-loop calculation

Doing that, we found a mistake in Collins spin-0 calculation from 1978...
Large $Q^2$ behavior

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- Collins didn’t calculate the spin-0 gluon contribution directly
  He extracted it from another calculation
Large $Q^2$ behavior

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Doing that, we found a mistake in Collins spin-0 calculation from 1978...

- Collins didn’t calculate the spin-0 gluon contribution directly
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- For three light quark $u, d, s$
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...
Even worse, quark spin-0 and gluon spin-0 come with opposite signs. After correcting the mistake they largely cancel. \( W_1(0, Q^2) \) is dominated by spin-2 contribution.

Lesson: It is important to do a full calculation. Some good news: The mistake has no effect on \( m_n - m_p \) since gluon contribution is the same at lowest order in isospin breaking. Flip side: You cannot use \( m_n - m_p \) to constrain muonic hydrogen.

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- Flip side: You cannot use $m_n - m_p$ to constrain muonic hydrogen.
Large $Q^2$ behavior: Spin 0 contribution

- The correct spin 0 result

$$\frac{Q^2}{2m_p^2} W_1^{(\text{spin-0})}(0, Q^2) = -2 \sum_q e_q^2 f_q^{(0)} + \left( \sum_q e_q^2 \right) \frac{\alpha_s}{12\pi} \tilde{f}_g^{(0)}$$

- Quark and gluon matrix elements related by

$$1 = (1 - \gamma_m) \sum_q f_q^{(0)} - \frac{\beta}{2g} \tilde{f}_g^{(0)}$$

- Dashed blue: quark
- Dash-dotted red: gluon
- Vertical stripes: perturbative uncertainty $Q/2 < \mu < 2Q$
- Solid bands: hadronic uncertainties on matrix elements
Large $Q^2$ behavior: Spin 2 contribution

- The new spin 2 result

$$\frac{Q^2}{2m_p^2} W_1^{(\text{spin-2})}(0, Q^2) = 2 \sum_q e_q^2 f_q^{(2)}(\mu) + \left( \sum_q e_q^2 \right) \frac{\alpha_s}{4\pi} \left( -\frac{5}{3} + \frac{4}{3} \log \frac{Q^2}{\mu^2} \right) f_g^{(2)}(\mu)$$

- Quark and gluon matrix elements related by

$$\sum f_q^{(2)}(\mu) + f_g^{(2)}(\mu) = 1$$

![Graph showing the behavior of $Q^2 W_1^{(\text{spin-2})}(0, Q^2)$ with $Q^2 (\text{GeV}^2)$ on the x-axis and $Q^2 W_1^{(\text{spin-2})}(0, Q^2)/2 m_p^2$ on the y-axis.]

- Dashed blue: down quark
- Dash-dotted blue: up quark
- Red: Gluon contribution
- Vertical stripes: perturbative uncertainty $Q/2 < \mu < 2Q$
- Solid bands: hadronic uncertainty
Large $Q^2$ behavior: Total contribution

- The total contribution

- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature
Two Photon exchange: small $Q^2$ and large $Q^2$

- Using NRQED we have control over low $Q^2$

$$W_1(0, Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2} \left\{ \frac{2m_p^3\beta}{\alpha} - a_p - \frac{2}{3} \left[ (1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2 \right] \right\} + O(Q^4)$$

- Using OPE we now have control over the high $Q^2$

$$\frac{Q^2}{2m_p^2} W_1^{(\text{spin}-0)}(0, Q^2) = -2 \sum_q e_q^2 f_q^{(0)} + \left( \sum_q e_q^2 \right) \frac{\alpha_s}{12\pi} \tilde{f}^{(0)}$$

$$\frac{Q^2}{2m_p^2} W_1^{(\text{spin}-2)}(0, Q^2) = 2 \sum_q e_q^2 f_q^{(2)}(\mu) + \left( \sum_q e_q^2 \right) \frac{\alpha_s}{4\pi} \left( -\frac{5}{3} + \frac{4}{3} \log \frac{Q^2}{\mu^2} \right) f_g^{(2)}(\mu)$$

- The problem, like the joke, is how to make a whole fish from a head and a tail...

- Before this work we had only the low $Q^2$ knowing the large $Q^2$ allows to connect the dots
“Aggressive” modeling: use OPE for $Q^2 \geq 1$ GeV$^2$
- Model unknown $Q^4$: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV
Two Photon Exchange: Modeling

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- How to connect the curves?

![Graph showing the relation between $Q^2$ and $W_1(0,Q^2)$]
Two Photon Exchange: Modeling

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Interpolating:

![Graph showing $W_1(0, Q^2)$ vs $Q^2 (\text{GeV}^2)$](image)

- Energy contribution: $\delta E(2S)^{W_1(0, Q^2)} \in [-0.046 \text{ meV}, -0.021 \text{ meV}]$
  To explain the puzzle need this to be $\sim -0.3$ meV
- Caveats: OPE might be only valid for larger $Q^2$
  $W_1(0, Q^2)$ might be different than the interpolated lines
Experimental test

- How to test?
- New experiment: \( \mu - p \) scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen
  can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD 87 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD 94 013006 (2016)]
Conclusions
Conclusions

- Proton radius puzzle: > 5σ discrepancy between
  - $r_p^E$ from muonic hydrogen
  - $r_p^E$ from hydrogen and $e - p$ scattering

- Recent muonic deuterium results find similar discrepancies
  [Pohl et al. Science 353, 669 (2016)]

- After 6 years the origin is still not clear
  1) Is it a problem with the electronic extraction?
  2) Is it a hadronic uncertainty?
  3) Is it new physics?

- Motivates a reevaluation of our understanding of the proton
Conclusions

- Discussed three topics:
  1. Extraction of proton radii from scattering: Use an established tool of the $z$ expansion. Studies disfavor the muonic hydrogen value.
  2. The first full and correct evaluation of large $Q^2$ behavior of forward virtual Compton tensor. Can improve the modeling of two photon exchange effects.
  3. Direct connection between muon-proton scattering and muonic hydrogen using a new effective field theory: QED-NRQED. Much more work to do!
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Thank you
Gil Paz (Wayne State University)
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- Much more work to do!

- Thank you
Backup: Connecting muon-proton scattering and muonic hydrogen
MUSE

- Muonic hydrogen:
  Muon momentum $\sim m_{\mu}c\alpha \sim 1$ MeV
  Both proton and muon non-relativistic

- MUSE:
  Muon momentum $\sim m_{\mu} \sim 100$ MeV
  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:
  - Use QED for muon
  - Use NRQED for proton
    $m_{\mu}/m_p \sim 0.1$ as expansion parameter

- A new effective field theory suggested in
  [Hill, Lee, GP, Mikhail P. Solon, PRD 87 053017 (2013)]]
QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD 94 013006 (2016)]

- Consider muon-proton scattering $\mu(p) + p(k) \rightarrow \mu(p') + p(k')$
  - At lowest order in $1/m_p$: $p^0 = p'^0 \Rightarrow \delta(p^0 - p'^0)$
  - At the proton rest frame $k = (m_p, \vec{0}) \Rightarrow k^0 = 0$ in NRQED

- NRQED propagator:
  \[
  \frac{1}{l^0 - l^2/2M + i\epsilon}
  \]

- In total
  \[
  \delta(p^0 - p'^0) \delta(L^0 - p^0) = \delta(L^0 - p^0) \delta(L^0 - p'^0)
  \]
The amplitude

\[ iM(2\pi)^4\delta^4(k + p - k' - p') = Z^2e^4 \int \frac{d^4L}{(2\pi)^4} \frac{1}{(L - p)^2(L - p')^2} \]

\[ \times \bar{u}(p')\gamma^0 \frac{i}{\not{L} - m} \gamma^0 u(p)\chi^\dagger \chi(2\pi)\delta(L^0 - p^0)(2\pi)\delta(L^0 - p'0) \]

\[ \times (2\pi)^3\delta^4(\vec{\rho} - \vec{\rho}' - \vec{k}') \]

The cross section

\[ \frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^24E^2}{\vec{q}^4} \left(1 - \nu^2\sin^2\frac{\theta}{2}\right) \left[1 + \frac{Z\alpha\pi\nu\sin\frac{\theta}{2}(1 - \sin\frac{\theta}{2})}{1 - \nu^2\sin^2\theta}\right] \]

\[ Z = 1, \ E = \text{muon energy}, \ \nu = |\vec{p}|/E, \ q = p' - p, \ \theta \text{ scattering angle} \]
QED-NRQED Effective Theory

QED-NRQED result

\[
\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2 (1 - \nu^2 \sin^2 \theta/2)}{q^4} \left[ 1 + \frac{Z\alpha\pi \nu \sin \theta/2 (1 - \sin \theta/2)}{1 - \nu^2 \sin^2 \theta} \right]
\]

Same result as scattering relativistic lepton off static \(1/r\) potential [Dalitz, Proc. Roy. Soc. Lond. 206, 509 (1951)] reproduced in [Itzykson, Zuber, “Quantum Field Theory”]

Same result as \(m_p \to \infty\) of “point particle proton” QED scattering (For \(m_p \to \infty\) only proton charge is relevant)
QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections

- Example: one photon exchange $\mu + p \to \mu + p$:
  
  QED-NRQED = $1/m_p$ expansion of form factors

  [Steven P. Dye, Matthew Gonderinger, GP, PRD 94 013006 (2016)]
Connecting muon-proton scattering to muonic hydrogen

- Matching

\[
\text{QED, QCD} \quad G_{E,M}, \text{Structure func.}, \quad W_1(0, Q^2)
\]

Scale: \( m_p \sim 1 \text{ GeV} \)

\[
\text{QED-NRQED: } MUSE \quad r_E^p, \bar{\mu} \gamma^0 \mu \psi_p^\dagger \psi_p
\]

Scale: \( m_\mu \sim 0.1 \text{ GeV} \)

\[
\text{NRQED-NRQED: } \text{muonic } H \quad r_E^p, \psi_\mu^\dagger \psi_\mu \psi_p^\dagger \psi_p
\]

- Need to match QED-NRQED contact interaction, e.g. \( \bar{\mu} \gamma^0 \mu \psi_p^\dagger \psi_p \)
  to NRQED-NRQED contact interaction, e.g. \( \psi_\mu^\dagger \psi_\mu \psi_p^\dagger \psi_p \)

[Dye, Gonderinger, GP in progress]
Connecting muon-proton scattering to muonic hydrogen

To do list:

1) Relate QED-NRQED contact interactions to NRQED contact interactions and $\mathcal{W}_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \rightarrow \mu + p)$ and asymmetry in terms of $r_E^p$ and $d_2$

3) Direct relation between $\mu-p$ scattering and muonic $H$