Photon Structure Functions
past, present, future

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Prologue

- e^+e^- collider experiments

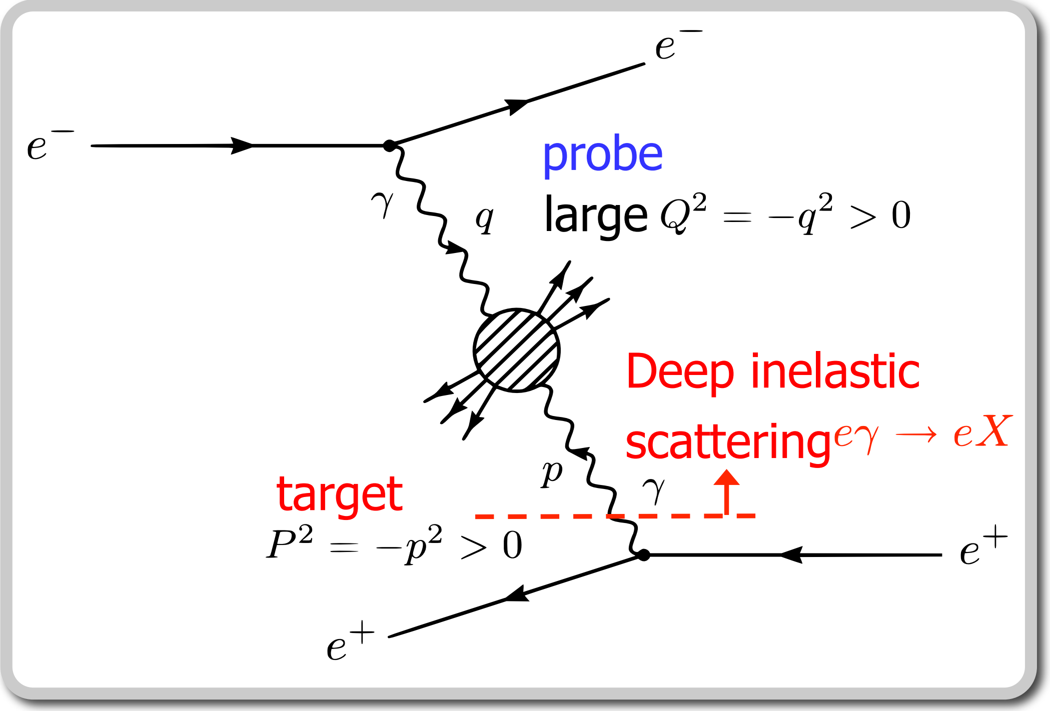
Two-photon process $\quad e^+e^- \to (e^+e^-\gamma\gamma) \to e^+e^-X$

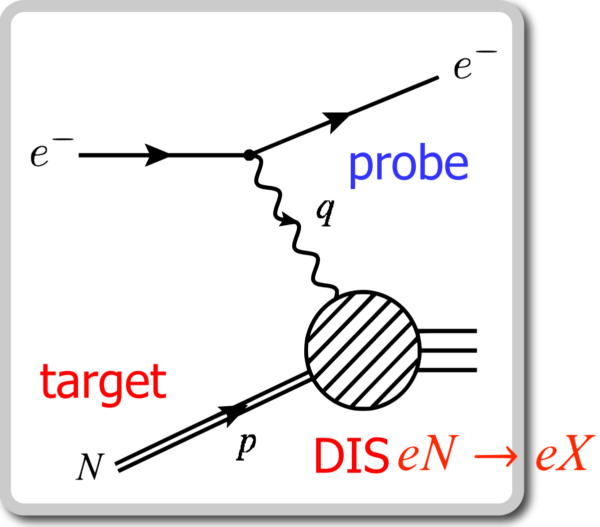
Viewed as a deep-inelastic electron-photon scattering

Brodsky-Kinoshita-Terazawa, Walsh (1971)

We can study the structures of photon

⇨ F_2^γ and F_L^γ





Two-photon process

- Differential cross section

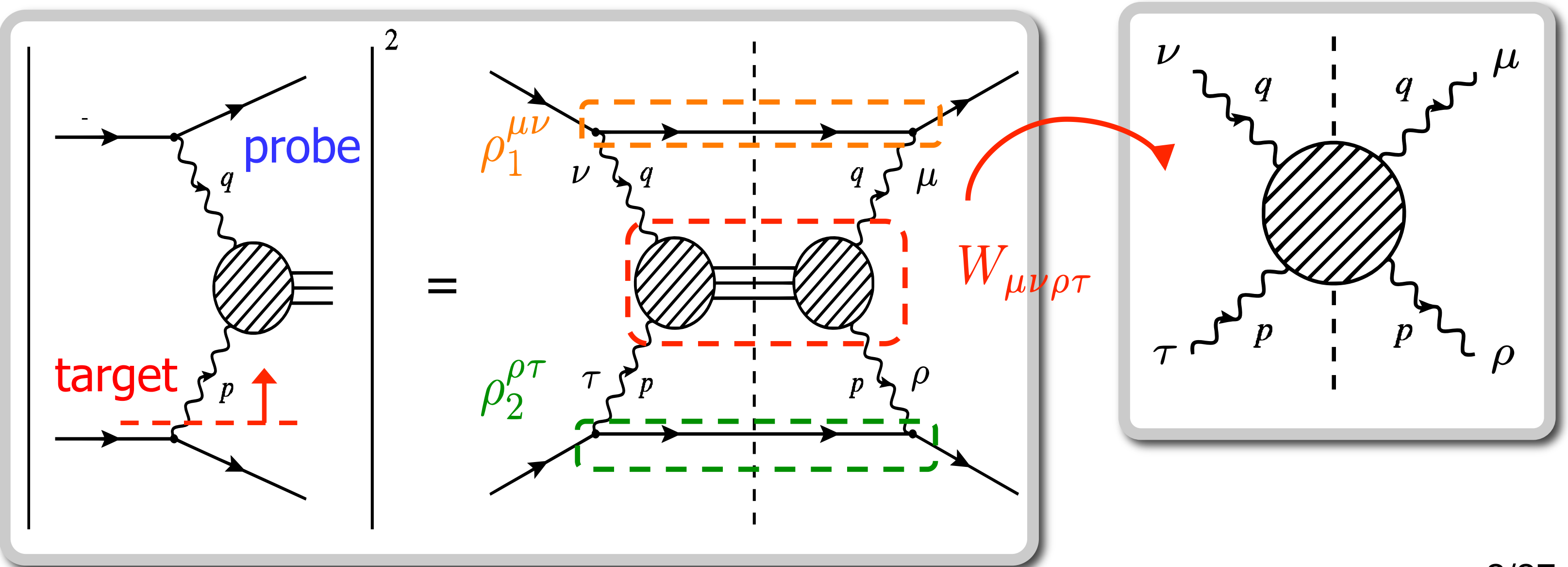
$$d\sigma = \frac{d^3 l_1' d^3 l_2'}{2E_1' 2E_2' (2\pi)^5} \frac{(4\pi\alpha)^3}{p^2 q^2} \frac{1}{4\sqrt{(l_1 \cdot l_2)^2 - m_e^2}} \rho_1^{\mu\nu} \rho_2^{\rho\tau} W_{\mu\nu\rho\tau}$$

  - Leptonic part $\rho_1^{\mu\nu}$ and $\rho_2^{\rho\tau}$

    $e \to e + \gamma$ QED vertex

  - Hadronic part $W_{\mu\nu\rho\tau}$ photon structure tensor

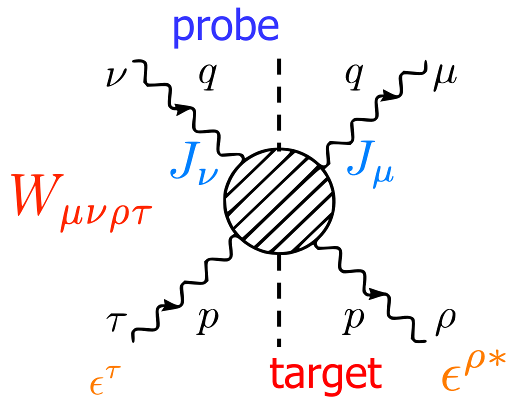
    $\gamma\gamma \to X$



Spin-averaged photon structure functions

- Spin-averaged structure tensor

$$W^{\gamma}_{\mu\nu}(p,q) = \frac{1}{2}\sum_{\lambda}\epsilon^{\rho *}_{(\lambda)}(p) W_{\mu\nu\rho\tau}\epsilon^{\tau}_{(\lambda)}(p)$$
$$= \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle\gamma(p)|J_\mu(x)J_\nu(0)|\gamma(p)\rangle_{\text{spin ave.}}$$



- Structure functions F_2^γ and F_L^γ $\quad F_L = F_2 - xF_1$

$$W^{\gamma}_{\mu\nu} = e_{\mu\nu}\frac{1}{x}F_L^{\gamma}(x,Q^2,P^2) + d_{\mu\nu}\frac{1}{x}F_2^{\gamma}(x,Q^2,P^2)$$

In analogy with the nucleon

$$e_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \qquad d_{\mu\nu} = -g_{\mu\nu} + \frac{q_\mu p_\nu + p_\mu q_\nu}{p\cdot q} - \frac{p_\mu p_\nu q^2}{(p\cdot q)^2}$$

$x = \dfrac{Q^2}{2p\cdot q}$:Bjorken variable $\quad 0 \le x \le 1$

$-Q^2 = q^2 \le 0$:mass squared of the probe photon

$-P^2 = p^2 \le 0$:mass squared of the target photon

Measuring the photon structure functions

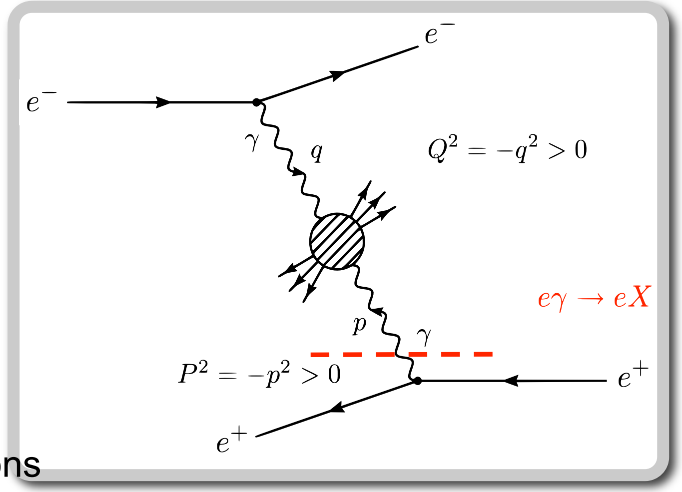
- Single tag

Real photon structure functions

$$\frac{d^3\sigma}{dQ^2dxdz} = \frac{2\pi\alpha^2}{xQ^4}\left\{\left[1+(1-y)^2\right]F_2^\gamma - y^2F_L^\gamma\right\} \times \frac{dN_\gamma^T}{dz}$$

$$\frac{dN_\gamma^T}{dz} = \frac{\alpha}{2\pi}\left[\frac{1+(1-z)^2}{z}\ln\frac{P^2_{\rm max}}{P^2_{\rm min}} - 2m_e^2 z\left(\frac{1}{P^2_{\rm min}} - \frac{1}{P^2_{\rm max}}\right)\right]$$

Flux of collinear real photons



$z = (E_2 - E_2')/E_2$ $\qquad$ $y = 1 - (E_1'/E_1)\cos^2\theta_1/2$

- Double tag

Virtual photon structure functions

$$F^\gamma_{\rm eff} \simeq F_2^\gamma + \frac{3}{2}F_L^\gamma$$

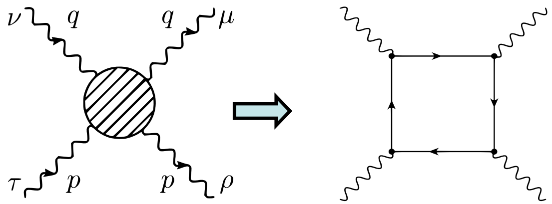
Photon structure functions--Past

- For real photon target $(P^2 \approx 0)$

  Structure function $F_2^\gamma(x, Q^2)$

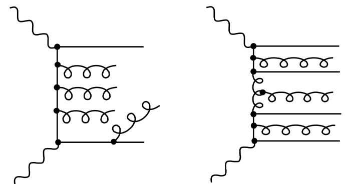
  - QPM calculation: Walsh-Zerwas (1973)

    Point-like contribution dominates $\sim \ln Q^2$



  - pQCD calculation (OPE+RGE)

Pointlike piece
calculable in pQCD



$$F_2^\gamma(x, Q^2) = \alpha \left[\frac{1}{\alpha_s(Q^2)} A + B + B' + \mathcal{O}(\alpha_s) \right]$$

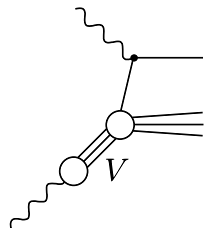
$\sim \ln \frac{Q^2}{\Lambda^2}$

(LO) (NLO)

Hadronic piece
not calculable in pQCD

Witten (1977)

Bardeen-Buras (1979)



Moments of $F_2^\gamma(x, Q^2)$

$$\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2)|_{\rm pointlike} = \alpha\left\{\frac{1}{\alpha_s(Q^2)}a_n + b_n + \mathcal{O}(\alpha_s(Q^2))\right\}$$

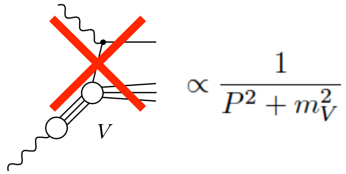
$$\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2)|_{\rm hadronic} = \alpha h_n(\alpha_s(Q^2)) \ ,$$

- For n>2, the hadronic moments $h_n(\alpha_s(Q^2))$ vanish in the large Q^2 limit
- At n=2, the hadronic energy-momentum tensor operator comes into play
- b_n shows a singularity at n=2, and $h_{n=2}(\alpha_s(Q^2))$ does not vanish at large Q^2

Definite information on the NLO second moment is missing

To predict $F_2^\gamma(x,Q^2)$, we need to rely on some nonperturbative methods

- For highly virtual photon target ($\Lambda^2 \ll P^2 \ll Q^2$)



$$\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2,P^2) = \alpha\left\{\frac{1}{\alpha_s(Q^2)}\tilde{a}_n + \tilde{b}_n + \mathcal{O}(\alpha_s(Q^2))\right\}$$

Λ : QCD scale parameter

(LO) (NLO) Uematsu-Walsh (1981,1982)

Hadronic piece can also be dealt with perturbatively

Definite prediction of $F_2(x,Q^2,P^2)$, its shape and magnitude, is possible

Real photon vs. Virtual photon

- Moments of F_2^γ with arbitrary P^2 but $P^2 \ll Q^2$

$$\int_0^1 dx x^{n-2} F_2^\gamma(x, Q^2, P^2) = \sum_{j=S,G,NS,\gamma} C_n^j\left(Q^2/\mu^2, \bar{g}(\mu^2), \alpha\right) \langle \gamma(p)|O_n^i(\mu^2)|\gamma(p)\rangle$$

Photon matrix elements

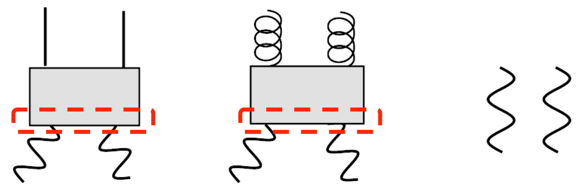
RG improved coefficients

Calculated in QCD with massless quarks with n_f flavors

hadronic ops. $\vec{O}_n = (O_n^S, O_n^G, O_n^{NS})$ photon op. O_n^γ

quark : O_n^S (flavor singlet)   gluon : O_n^G

O_n^{NS} (flavor non-singlet)



- Note: $\langle \gamma(p)|O_n^\gamma(\mu^2)|\gamma(p)\rangle = 1$
- Photon matrix elements of hadronic ops.

  renormalized at $\mu^2 = Q_0^2$ with $\Lambda^2 \ll Q_0^2 \ll Q^2$

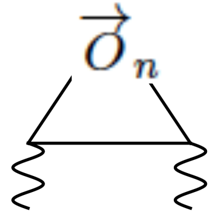
$$\langle \gamma(p)|\vec{O}_n(\mu^2)|\gamma(p)\rangle\Big|_{\mu^2=Q_0^2} = \frac{\alpha}{4\pi}\vec{A}_n(Q_0^2; P^2)$$

Real photon vs. Virtual photon

- When photon state becomes far off-shell and P^2 approaches Q_0^2

  $\vec{A}_n(Q_0^2; P^2)$: considered to be point-like and calculable perturbatively

$$\vec{A}_n(Q_0^2; P^2 = Q_0^2) \equiv \vec{A}_n^{(1)}$$



At one-loop

- For P^2 in the range $0 \le P^2 \le Q_0^2$, we devide $\vec{A}_n(Q_0^2; P^2)$ into two pieces

$$\vec{A}_n(Q_0^2; P^2) = \vec{\widetilde{A}}_n(Q_0^2; P^2) + \vec{A}_n^{(1)}$$

- $\vec{\widetilde{A}}_n(Q_0^2; P^2)$ contains nonperturbative contributions for $0 \le P^2 \le Q_0^2$

  and $\vec{\widetilde{A}}_n(Q_0^2; P^2 = Q_0^2) = \vec{0}$

Real photon vs. Virtual photon

- Moments of $F_2^\gamma(x,Q^2,P^2)$ at NLO for arbitrary P^2 with $0 \le P^2 \le Q_0^2$:

$$\int_0^1 dx x^{n-2} F_2^\gamma(x,Q^2,P^2)/\left(\frac{\alpha}{4\pi}\right) = \frac{4\pi}{\alpha_s(Q^2)} \sum_i \frac{\mathcal{L}_i^n}{1+d_i^n}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{1+d_i^n}\right] \quad \text{LO}$$

$$+\sum_i \frac{\mathcal{A}_i^n}{d_i^n}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d_i^n}\right] + \sum_i \frac{\mathcal{B}_i^n}{1+d_i^n}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{1+d_i^n}\right] + \mathcal{C}^n \quad \text{NLO}$$

$$+\vec{\widetilde{A}}_n(Q_0^2;P^2)\cdot \sum_i P_i^n \vec{C}_n(1,0)\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d_i^n} \qquad \text{with} \quad i=+,-,NS$$

nonperturbative contributions

Uematsu-Walsh (1982)
Gluck-Reya (1983)

- $P^2 \to 0$ real photon target $F_2^\gamma(x,Q^2)$
  Problem of a singularity at n=2 is solved: $\frac{1}{d_-^{n=2}}\left[1-\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{d_-^{n=2}}\right] \to \text{finite}$ as $d_-^{n=2} \to 0$

- $P^2 \to Q_0^2$ virtual photon target $F_2^\gamma(x,Q^2,Q_0^2)$
  nonperturbative contributions disappear

Measurements of F_2^γ

From talk of R. Nisius at Photon 2009

- 1981: The first measurement of F_2^γ by PLUTO
- 1986: The first extraction of Λ (or α_s) from F_2^γ data
- 1990: Start of F_2^γ measurements at TRISTAN
- 1994: Start of F_2^γ measurements at LEP
- 2002: NLO extraction of α_s , based on a large set of data

  Albino-Klasen-Soldner-Rembold

- 2005: The final LEP2 results were published

In the talk at Photon 2015, F. Kapusta said

" LEP, The Lord of the Collider Rings, was dismantled abruptly in 2000, leaving physicists without additional high energy e^+e^- collider data and the possibility to develop a demonstrator of real $\gamma\gamma$ collisions. And still now we miss a high energy e^+e^- collider"

Without new data, interest on photon structure functions has diminished gradually

Review reports: Nisius(2000), Krawczyk-Zembrzuski-Stanszel(2001), Klasen(2002), Schienbein(2002)

Recent review, "Photon Structure Function Revisited" Ch. Berger(2015)

Photon structure functions--Present

- First attempt for NNLO analysis of photon structure

Moch-Vermaseren-Vogt (2002)

They obtained

- Photonic coefficient function at order $\alpha\alpha_s$
- The low integer moments of photon-quark and photon-gluon splitting functions at order $\alpha\alpha_s^2$
  Later, compact parametrization of these functions (2006)

They analyzed parton structure in photon at NNLO

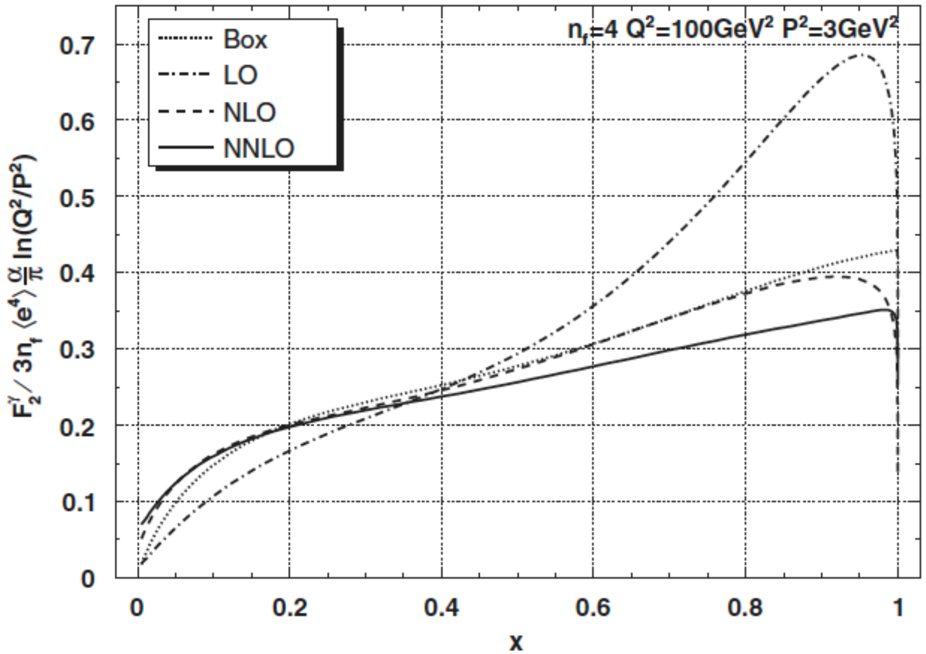
● NNLO $(\mathcal{O}(\alpha\alpha_s))$ analysis
of virtual photon target $F_2^\gamma(x, Q^2, P^2)$

Ueda-Uematsu-K.S. (2007)

Motivated by the calculation of 3-loop anomalous dimensions
Vogt-Moch-Vermaseren (2004,2006)

Definite prediction
its shape and magnitude

NNLO corrections reduce
$F_2^\gamma(x, Q^2, P^2)$ at larger x



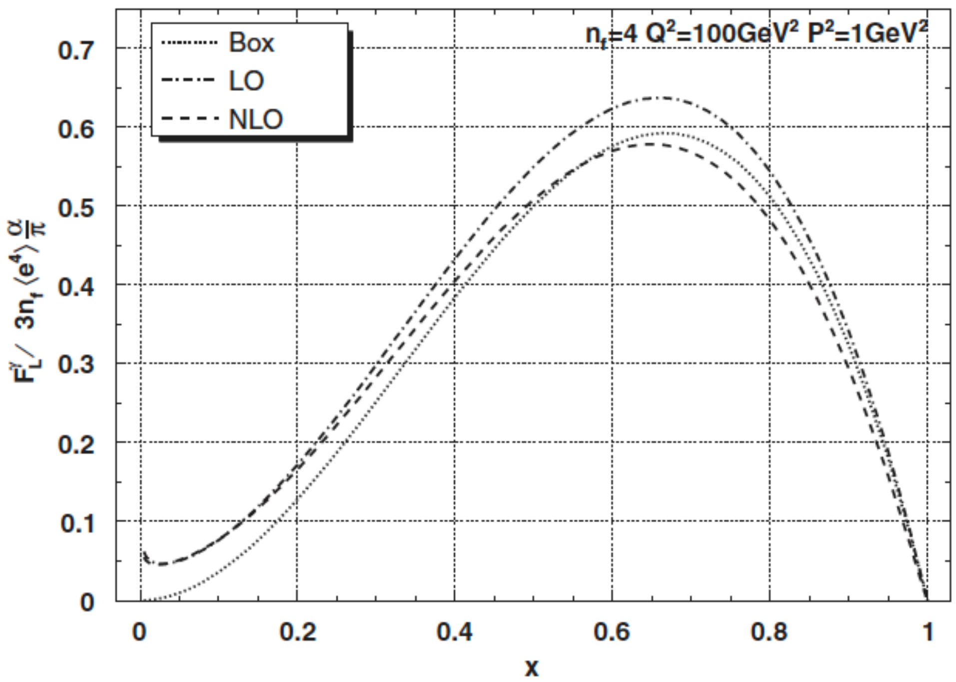
$$F_2^{\gamma(\text{box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} n_f \langle e^4 \rangle \Big\{ x[x^2 + (1-x)^2] \ln\frac{Q^2}{P^2} - 2x[1 - 3x + 3x^2 + (1 - 2x + 2x^2)\ln x] \Big\}$$

Longitudinal photon sf. F_L^γ

- LO --- of order α ; NLO ---- of order $\alpha\alpha_s$

  - Real photon target
    LO Witten (1977)
    Bardeen-Buras (1979)

  - Virtual photon target
    LO Uematsu-Walsh (1982)
    NLO Ueda-Uematsu-K.S. (2007)



$$F_L^{\gamma(\text{box})}(x, Q^2, P^2) = \frac{3\alpha}{\pi} n_f \langle e^4 \rangle \{4x^2(1-x)\}$$

Parton distribution functions in photon

- QCD improved parton model

$$F_2^\gamma(x,Q^2,P^2) = \sum_i C^i \otimes q_i^\gamma + C^G \otimes G^\gamma + C^\gamma \otimes \Gamma^\gamma$$

- DGLAP parton evolution eqs.

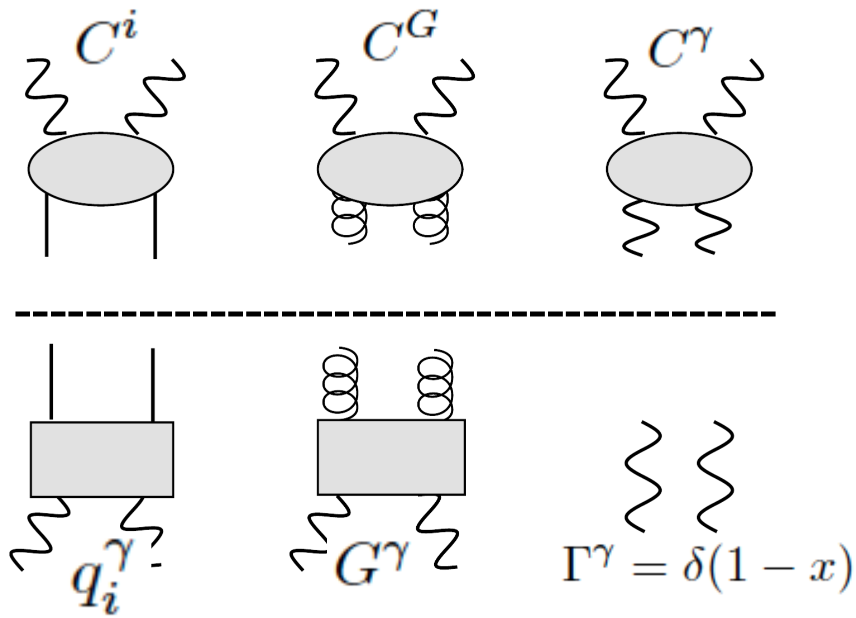
$$\frac{dq_i^\gamma(x,Q^2,Q_0^2)}{d\ln Q^2} = \frac{\alpha}{2\pi}P_{q_i\gamma} + \frac{\alpha_s}{2\pi}\left\{\sum_k P_{q_iq_k}\otimes q_k^\gamma + P_{q_iG}\otimes G^\gamma\right\}$$

$$\frac{dG^\gamma(x,Q^2,Q_0^2)}{d\ln Q^2} = \frac{\alpha}{2\pi}P_{G\gamma} + \frac{\alpha_s}{2\pi}\left\{\sum_k P_{Gq_k}\otimes q_k^\gamma + P_{GG}\otimes G^\gamma\right\}$$

- Solve DGLAP eqs. with initial conditions

$q_i^\gamma(x,Q_0^2,Q_0^2)$ $\quad G^\gamma(x,Q_0^2,Q_0^2)$

⇨ Parton distributions in photon in $\overline{\rm MS}$ factorization scheme



Parton distribution functions in photon

- DIS_γ factorization scheme Gluck-Reya-Vogt(1992)

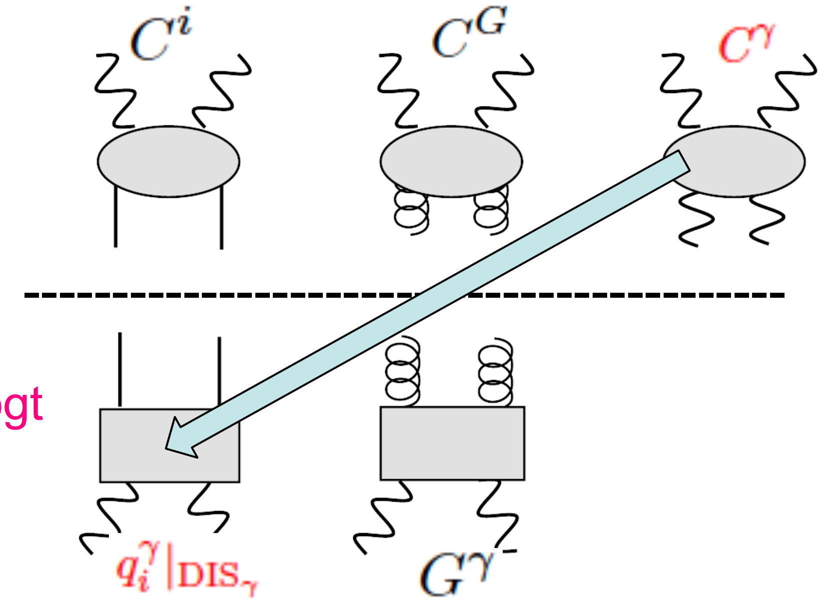
Photonic coefficient function C^γ is absorbed into quark distributions

C^γ Is negative and diverges in large x

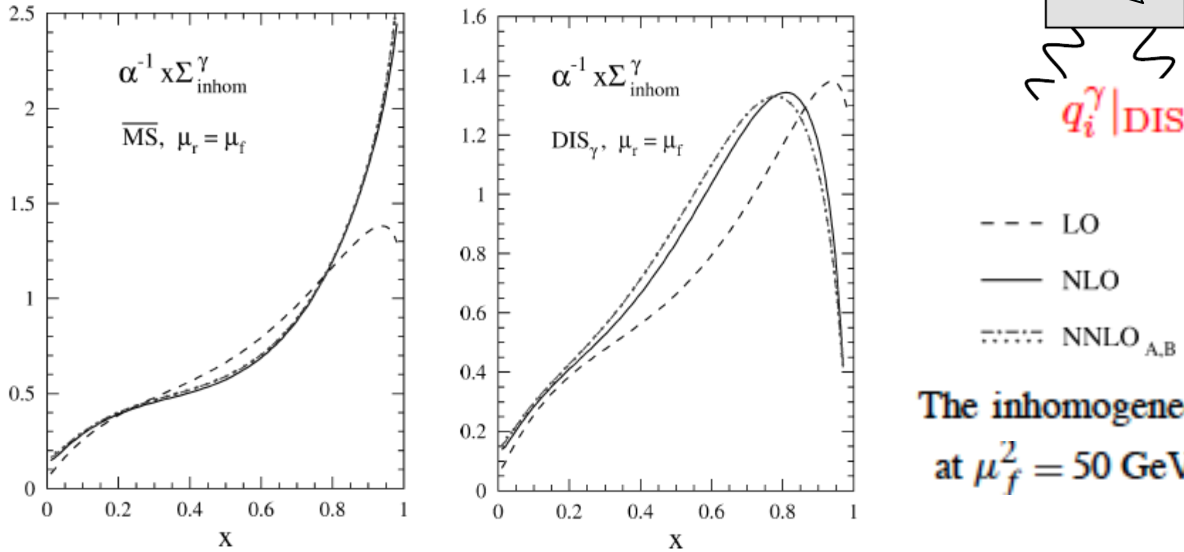
$$C_2^\gamma = \frac{\alpha}{4\pi} C_2^{\gamma(1)}(x) + \frac{\alpha\alpha_s}{(4\pi)^2} C_2^{\gamma(2)}(x) + \cdots$$

as $x \to 1$

$$C_2^{\gamma(l)} \sim A_l \ln^{2l-1}(1-x) \qquad \text{with} \quad A_l > 0$$



- Real photon : NNLO (2002) Moch-Vermaseren-Vogt



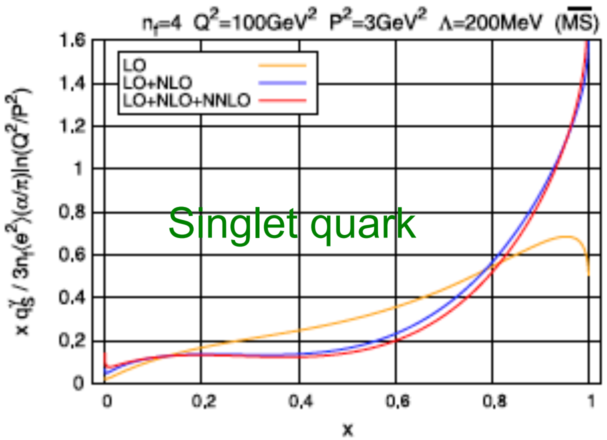
The inhomogeneous LO, NLO and NNLO contributions at $\mu_f^2 = 50\ \mathrm{GeV}^2$ for $N_f = 3$

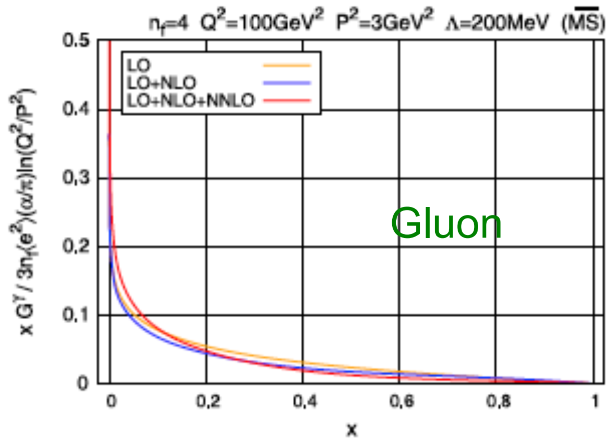
Parton distribution functions in photon

- Virtual photon

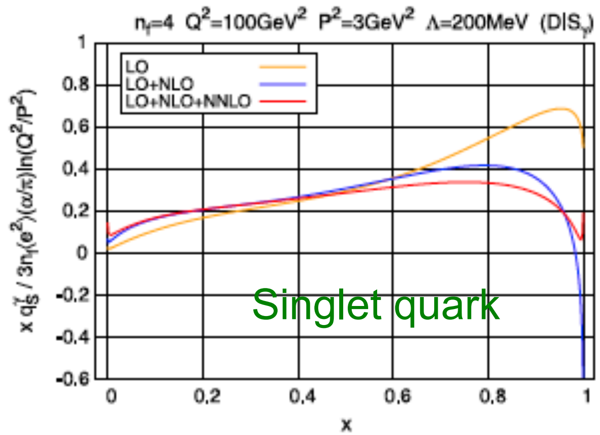
NLO: Gluck-Reya-Stratmann (1995)
NNLO: Ueda-Uematsu-K.S. (2009)

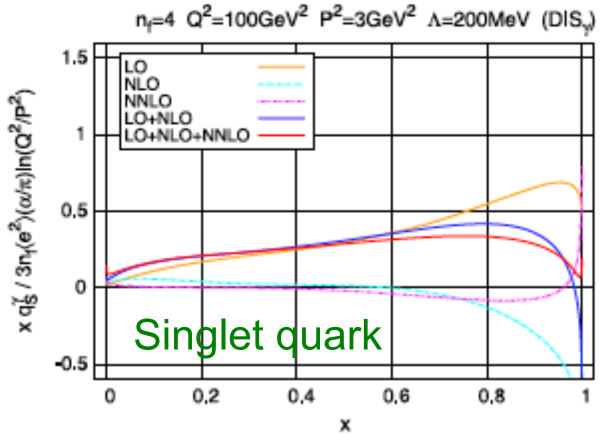
$\overline{\mathrm{MS}}$ scheme





DIS_γ scheme





$$\delta q^{\gamma(2)}_{NS,n}\big|_{\mathrm{DIS}_\gamma} \propto \left(c^{\gamma(2)}_{2,n}\big|_{\overline{\mathrm{MS}}} - c^{\gamma(1)}_{2,n}\big|_{\overline{\mathrm{MS}}} c^{q(1)}_{2,n}\big|_{\overline{\mathrm{MS}}}\right)$$

Parton distribution functions in photon

- Quark distribution shows quite different behaviors in two scheme, especially in large-x region

- In DIS_γ scheme, NLO and NNLO contributions to quark distribution are small for moderate x --- appropriate behaviors from the viewpoint of "perturbative stability"

- Near x=1, NLO quark contribution $q_S^{\gamma(1)}|_{\mathrm{DIS}_\gamma}$ negatively diverges as $\ln(1-x)$ while NNLO quark contribution $q_S^{\gamma(2)}|_{\mathrm{DIS}_\gamma}$ positively diverges as $[-\ln^3(1-x)]$. This may hint the necessity of considering the resummation near x=1

- Gluon distribution is very small in absolute value except in small-x region

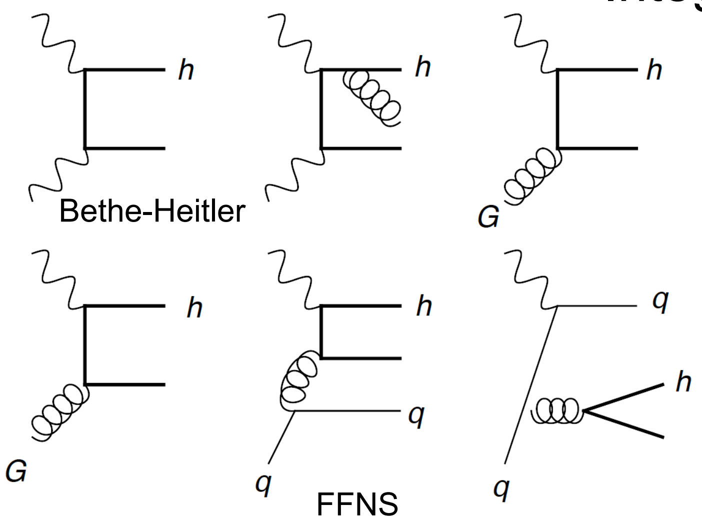
- Keep in mind that parton distribution functions in photon are not physical quantities and factorization scheme dependent

Heavy quark mass effects

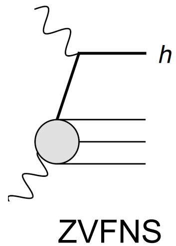
- Heavy quark contribution starts at $W^2 = Q^2\left(\frac{1}{x} - 1\right) > (2m_h)^2$

- Two standard schemes
  - FFNS: Heavy quarks appear only in the final state
    not appropriate when $Q^2 \gg m_h^2$
  - ZVFNS: For $Q^2 > m_h^2$ a new massless parton density is added
    $q_h(x, Q^2 \le m_h^2) = 0$ not appropriate when $Q^2 \approx m_h^2$
  - Combined scheme—ACOT(χ)
    Introduce the parameter $\chi_h \equiv x\left[1 + \frac{4m_h^2}{Q^2}\right] < 1$
    Integration range of convolution is modified

$$\int_x^1 \frac{dy}{y} f(y) C\left(\frac{x}{y}\right) \Longrightarrow \int_{\chi_h}^1 \frac{dy}{y} f(y) C\left(\frac{x}{y}\right)$$



Laenen, Riemersma
Smith, van Neerven(1994)



Heavy quark mass effects

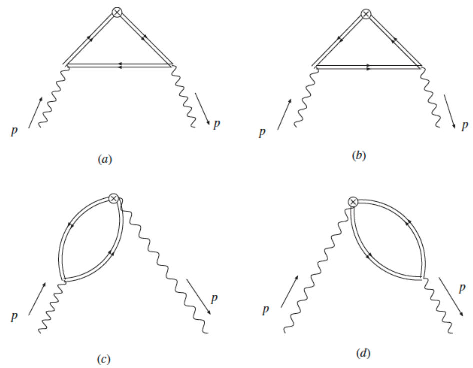
- Data analyses with heavy quark mass effects

  - Dortmund group: Gluck, Reya, Vogt, Stratmann, Schienbein (1992), (1995), (1999)
  - Laenen, Riemersma, Smith, van Neerven (1994)
  - Cornet, Jankowski, Krawczyk, Lorca (2003) (2004)
  - Aurenche, Fontannaz, Guillet (1994) (2005)
  - Slominski, Abramowicz, Levy (2006)

For more details, see
”Experimental Review of Photon Structure Function Data”
R. Nisius at Photon 2009

  - Kitadono, Ueda, Uematsu, K.S. (2010)

$$\left[\boldsymbol{q}^{\gamma(0)}\right]_H(0) \simeq -\frac{\alpha}{2\pi} k_H^{(0)} \ln\frac{m^2}{P^2} \quad \Longrightarrow q_h(x, Q^2 = m_h^2) \approx 0$$



We need more data to refine models for the treatment of heavy quark mass effects

Behaviors near x=0 and x=1

- Near x=1
  - n^{th}-th order splitting functions and coefficient functions in DIS

    $(\alpha_s)^n \ln^{2n-l}(1-x)$ as $x \to 1$

    ⇨ these logarithms require an all-order resummation

    Vogt and others

- Near x=0
  - A new approach for study of photon structure functions at small x

    A. Watanabe and Hsiang-nan Li (2015)

    - adopt the Pomeron exchange picture
      in the framework of holographic QCD
      (AdS/CFT correspondence)
    - realization of the vector meson dominance

Photon structure functions--Future

- Build a new e^+e^- collider machine ----

CLIC, ILC
FCC-ee, CEPC
……

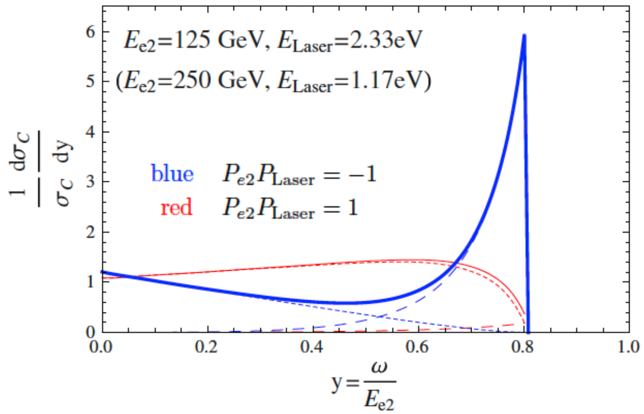
- Before e+ beams are ready,
  other options are possible:
  - an e^-e^- option
  - an $e^-\gamma$ option

    use one e- beam to produce high energy photons

I.Gizburg, G.Kotkin, V.Serbo, V.Telnov
A.De Roeck: hep-ph/0311138v1(2003)
V.Telnov: Photon 2015

- Highly polarized beams may be possible
  Then, we can study the spin structure of photon

Polarized photon structure function g_1^γ



Energy spectra of the laser light backward scatterng

Recent review
"Polarized structure functions and two photon physics at Super-B"
Shore (2013)

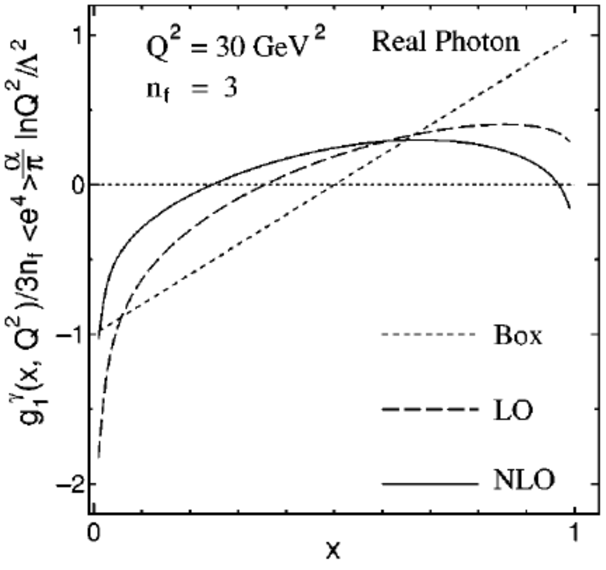
Polarized photon structure function g_1^γ

- QCD analysis of g_1^γ
  - Real photon target $g_1^\gamma(x, Q^2)$

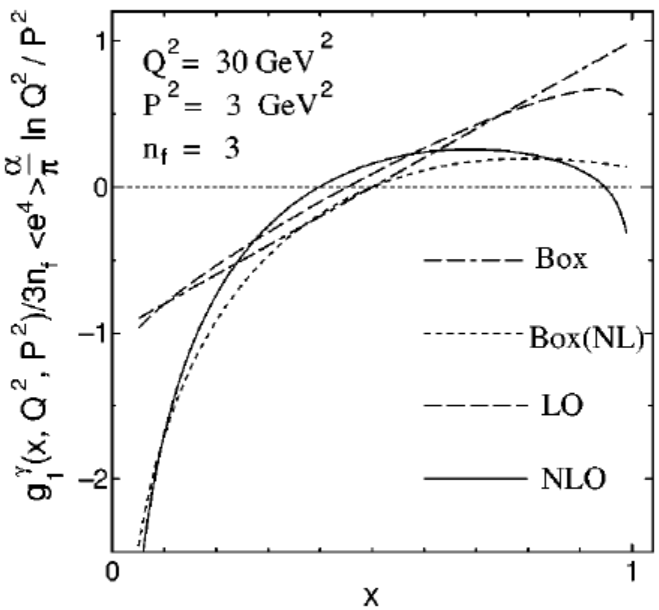
    LO: K.S.(1980) NLO: Stratmann-Vogelsang (1996)

  - Virtual photon target $g_1^\gamma(x, Q^2, P^2)$

    NLO: Uematsu-K.S. (1999)



Pointlike piece



Polarized photon structure function g_1^γ

- First moment of g_1^γ
  - Real photon target $g_1^\gamma(x,Q^2)$

$$\int_0^1 dx g_1^\gamma(x,Q^2) dx = 0$$

by gauge invariance

Bass (1992)
Narison, Shore, Veneziano (1993)
Freund, Sehgal (1994)
Bass, Brodsky, Schmidt (1998)

  - Virtual photon target $g_1^\gamma(x,Q^2,P^2)$

$$\int_0^1 dx g_1^\gamma(x,Q^2,P^2) dx = -\frac{3\alpha}{\pi}\left[\Sigma_{i=1}^{n_f} e_i^4\left(1-\frac{\alpha_s(Q^2)}{\pi}\right) - \frac{2}{\beta_0}\left(\Sigma_{i=1}^{n_f} e_i^2\right)^2\left(\frac{\alpha_s(P^2)}{\pi}-\frac{\alpha_s(Q^2)}{\pi}\right)\right]$$

LO
QED axial anomaly

NLO
QCD axial anomaly

Narison, Shore, Veneziano (1993)

NNLO

Ueda, Uematsu, K.S. (2006)

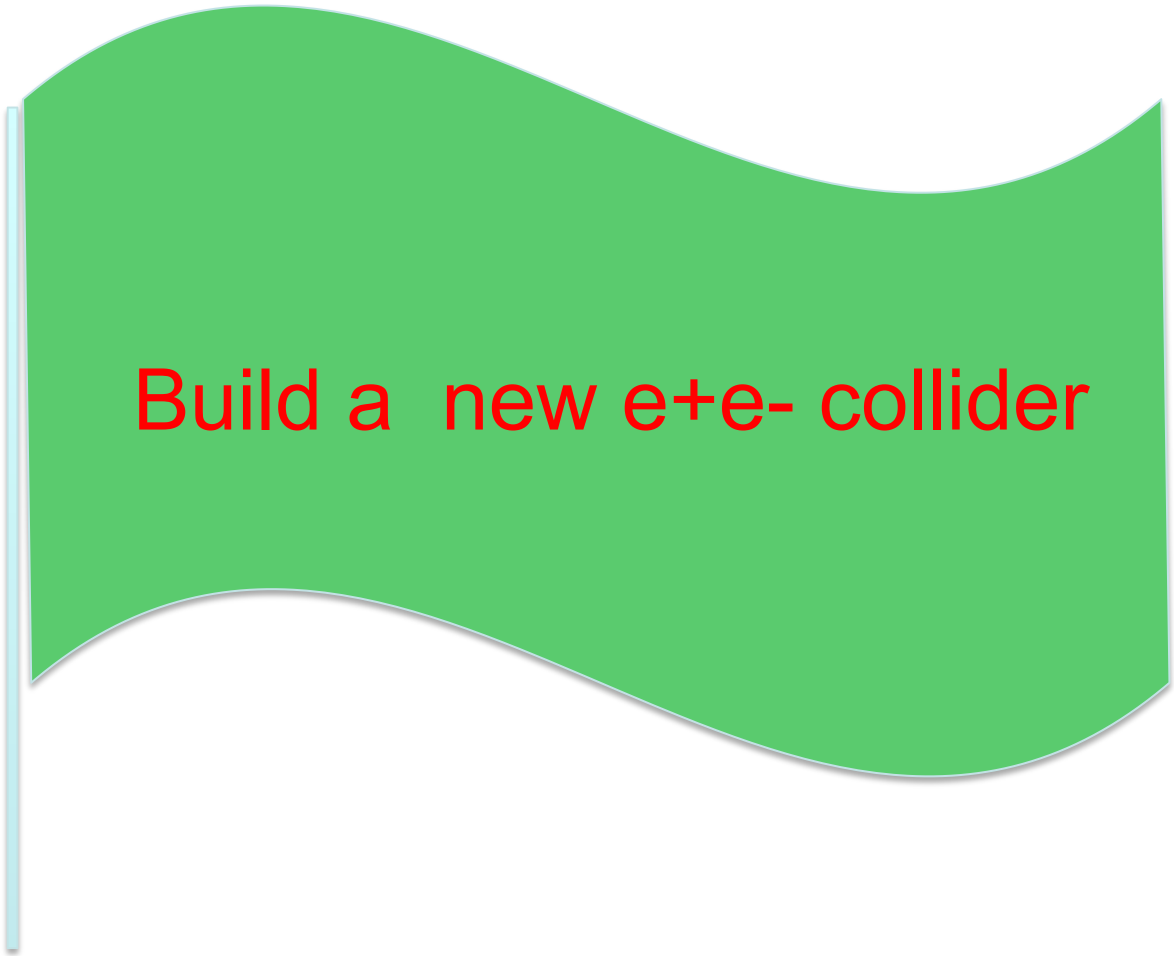
Epilogue

- Future investigation on photon structure

  We still need to understand

  - hadronic contributions to photon
  - heavy quark mass effects
  - transition from real to virtual photon target
  - factorization scheme dependence
  - behaviors near x=0 and x=1
  - spin structure of photon

To that end new data are essential !!!!



International Collaboration