



Isolated photons at NNLO

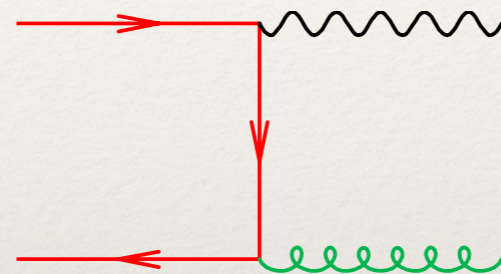
John Campbell, Keith Ellis,
and Ciaran Williams

1612.04333: Direct photon production at next-to-next-to-leading order

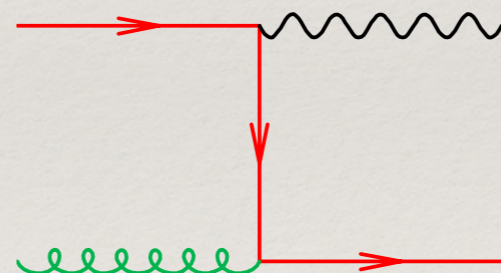
1703.10109: Driving Miss Data: Going up a gear to NNLO

Motivation for Direct Photon Studies

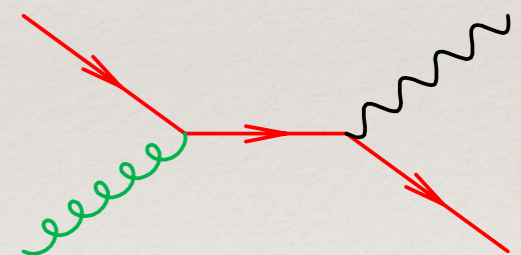
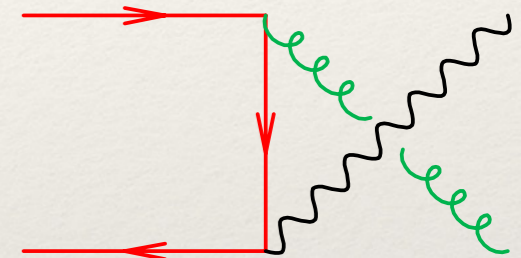
- ❖ Direct photon and photon + jet are interesting:-
 - ❖ In their own right as tests of the standard model
 - ❖ As a proxy for Z+jet processes, to estimate SM sources of jets+missing energy, especially at large jet p_T
 - ❖ As a probe of parton distributions, especially the gluon.



(a)

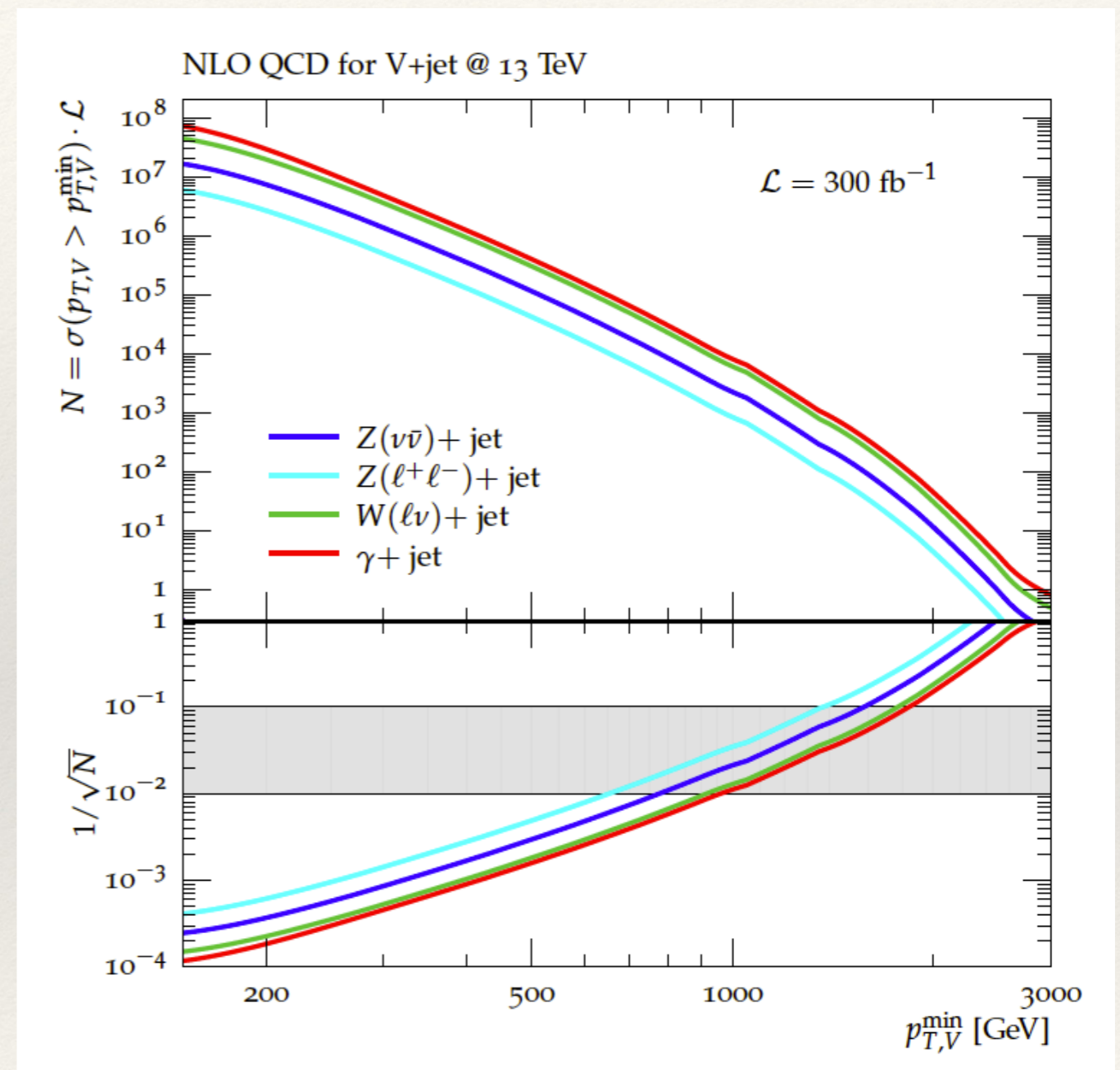


(b)



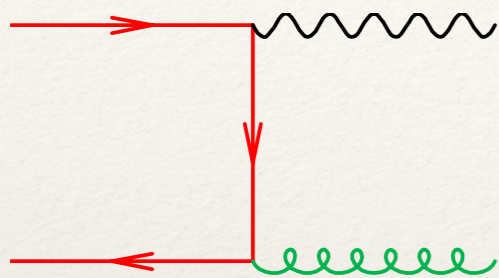
Production rate vs minimum p_T

- ❖ Photon production is used to determine the shape of the MET spectrum (in the high energy region where the $Z \rightarrow \mu^+ \mu^-$ data runs out).
- ❖ Lower panel shows the expected statistical uncertainty in the data, which sets the goal for the theoretical uncertainty we should aim for at each p_T

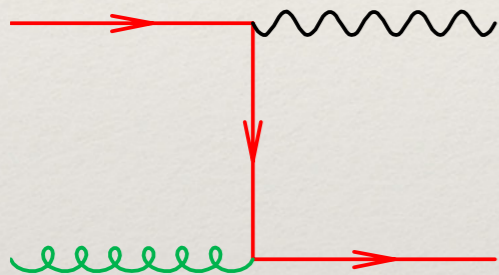
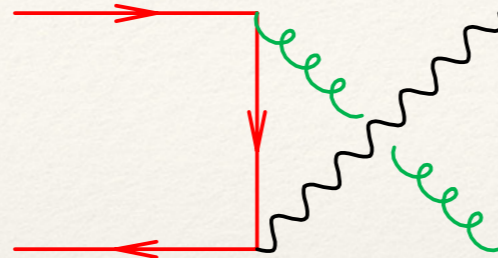


Lindert et al., 1705.04664

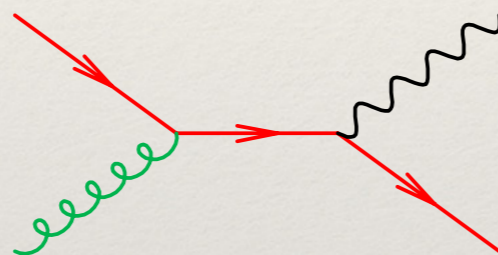
Direct Photon and Parton PDFs



(a)



(b)



The photon (+jet) process depends at LO on the gluon PDF. Additionally the process has high statistics and good phase space control (with control of photon p_T and rapidities), makes it a candidate for PDF fitting.

CERN-PH-TH/2013-006

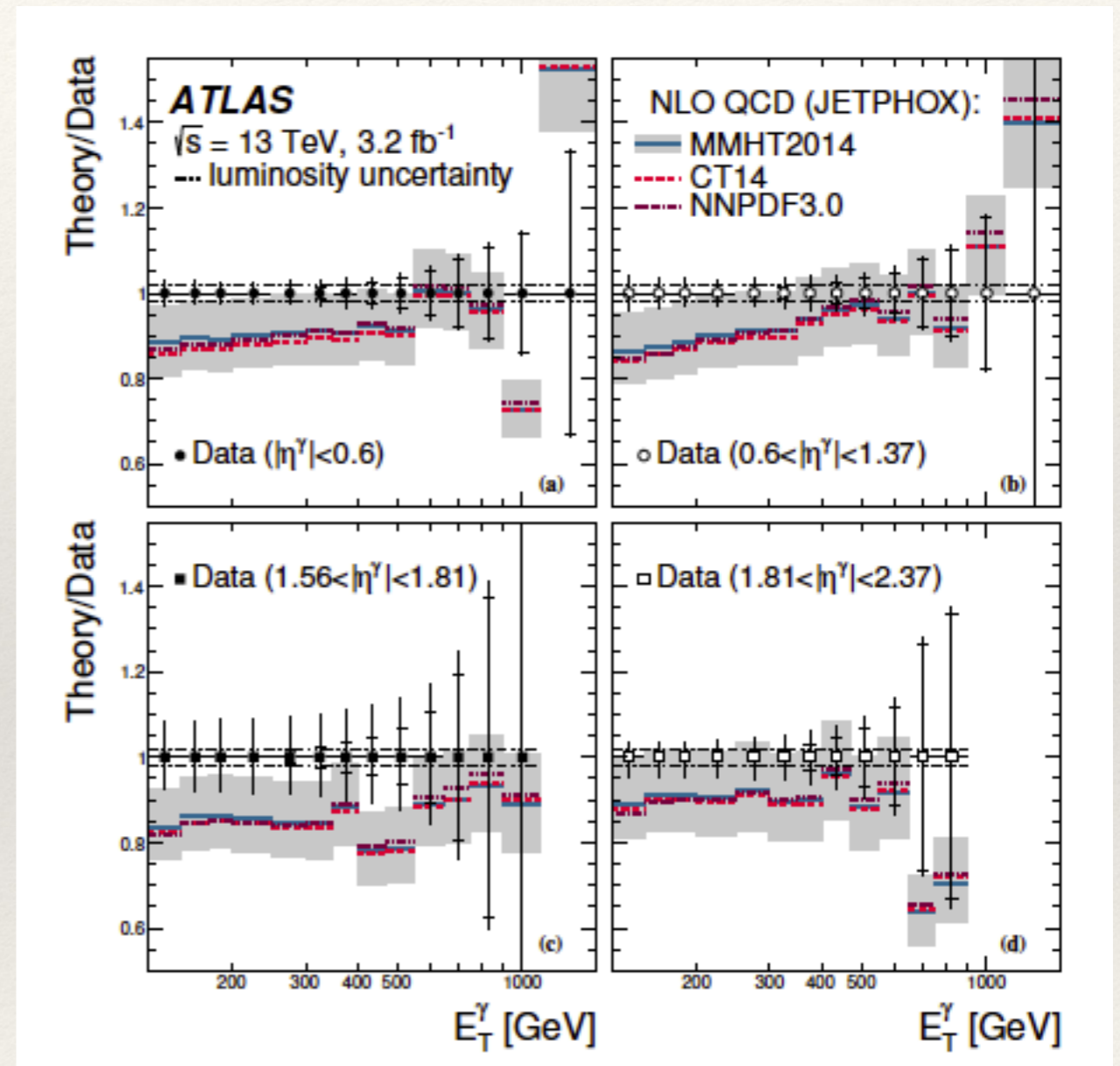
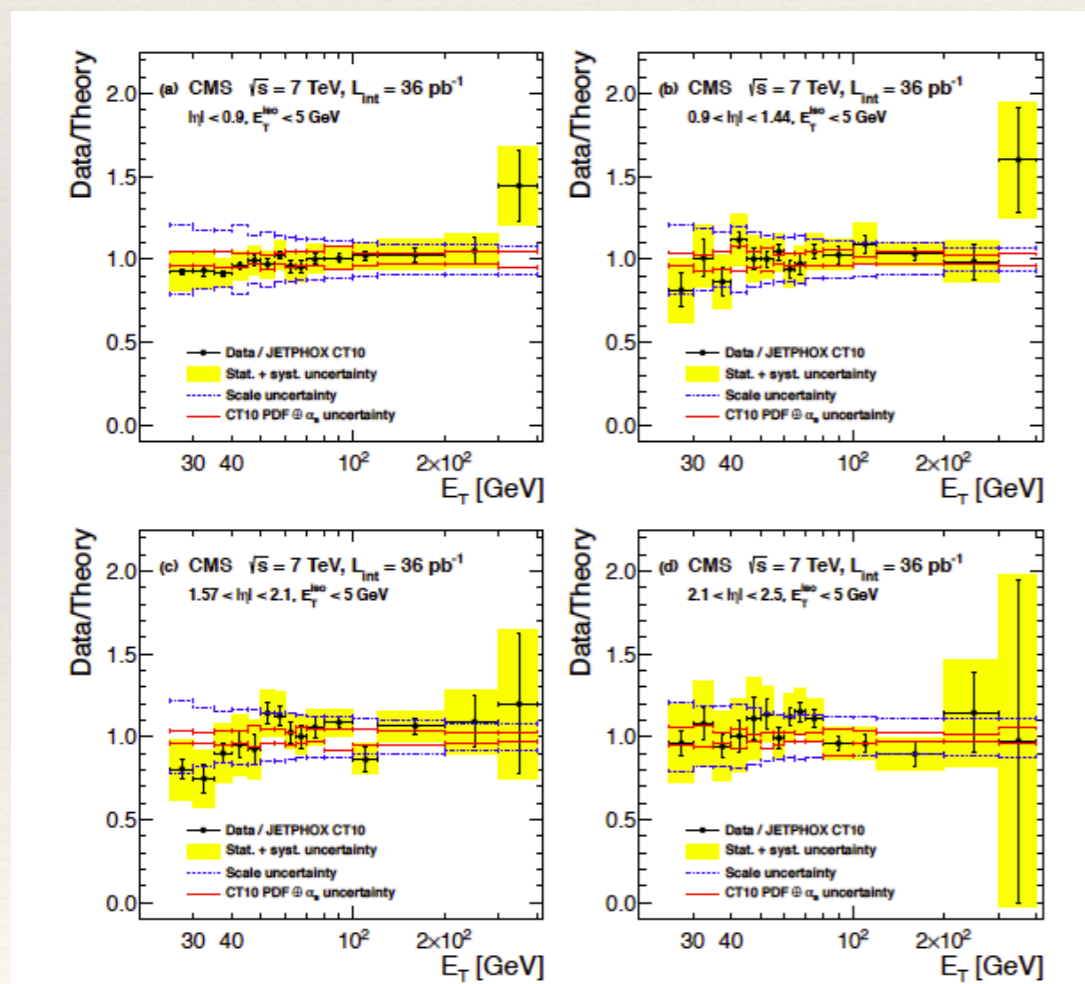
Sensitivity of the LHC isolated- γ +jet data to the parton distribution functions of the proton

1212.5511

L. Carminati^{1,2}, G. Costa¹, D. d'Enterria³, I. Koletsou¹,
G. Marchiori⁴, J. Rojo⁵, M. Stockton⁶, F. Tartarelli¹

Comparisons of data with NLO+PS

- ❖ In most bins the experimental accuracy is higher than the theoretical error
- ❖ At NLO theoretical error is dominated by scale variation.



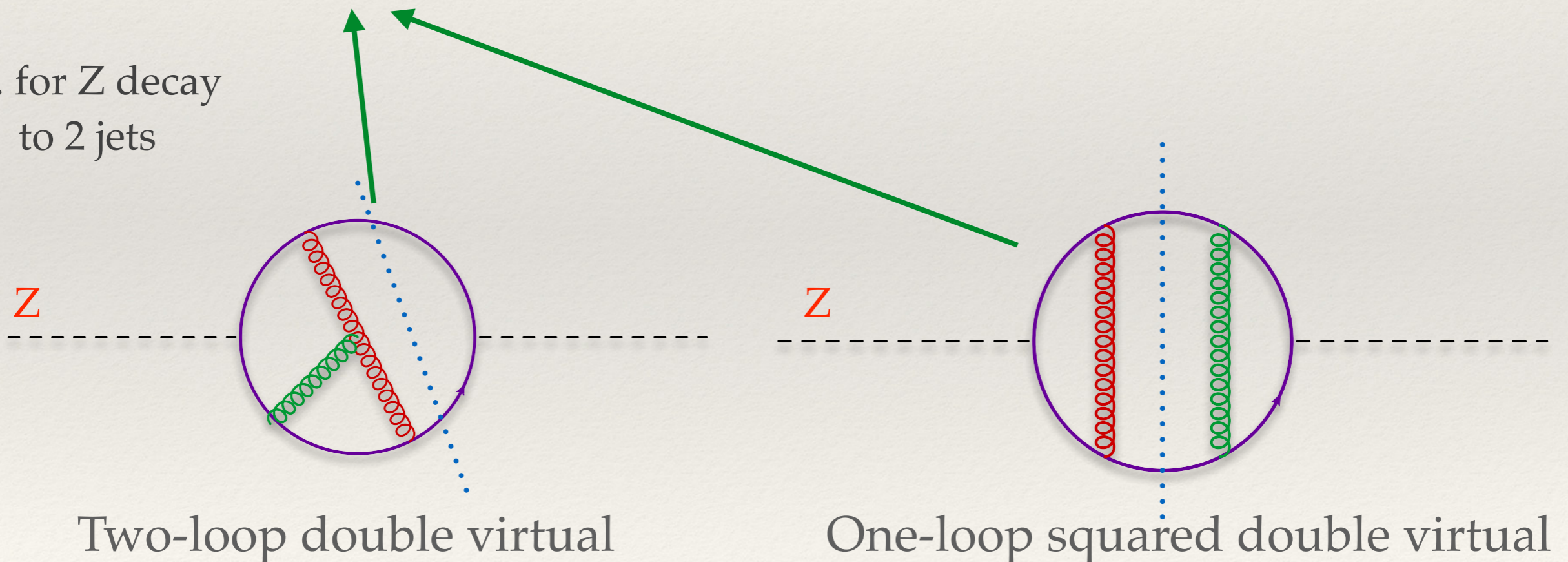
Atlas Measurements at 7,8 and 13 TeV, 1311.1440,1605,03495,1701.06882

Ingredients of a NNLO calculation

At NNLO we have to include three final state phase spaces of different dimensionality, (VV,RV,RR)

$$\sigma_{NNLO} = \int |\mathcal{M}_{VV}|^2 d^m \Phi + \int |\mathcal{M}_{RV}|^2 d^{m+1} \Phi + \int |\mathcal{M}_{RR}|^2 d^{m+2} \Phi$$

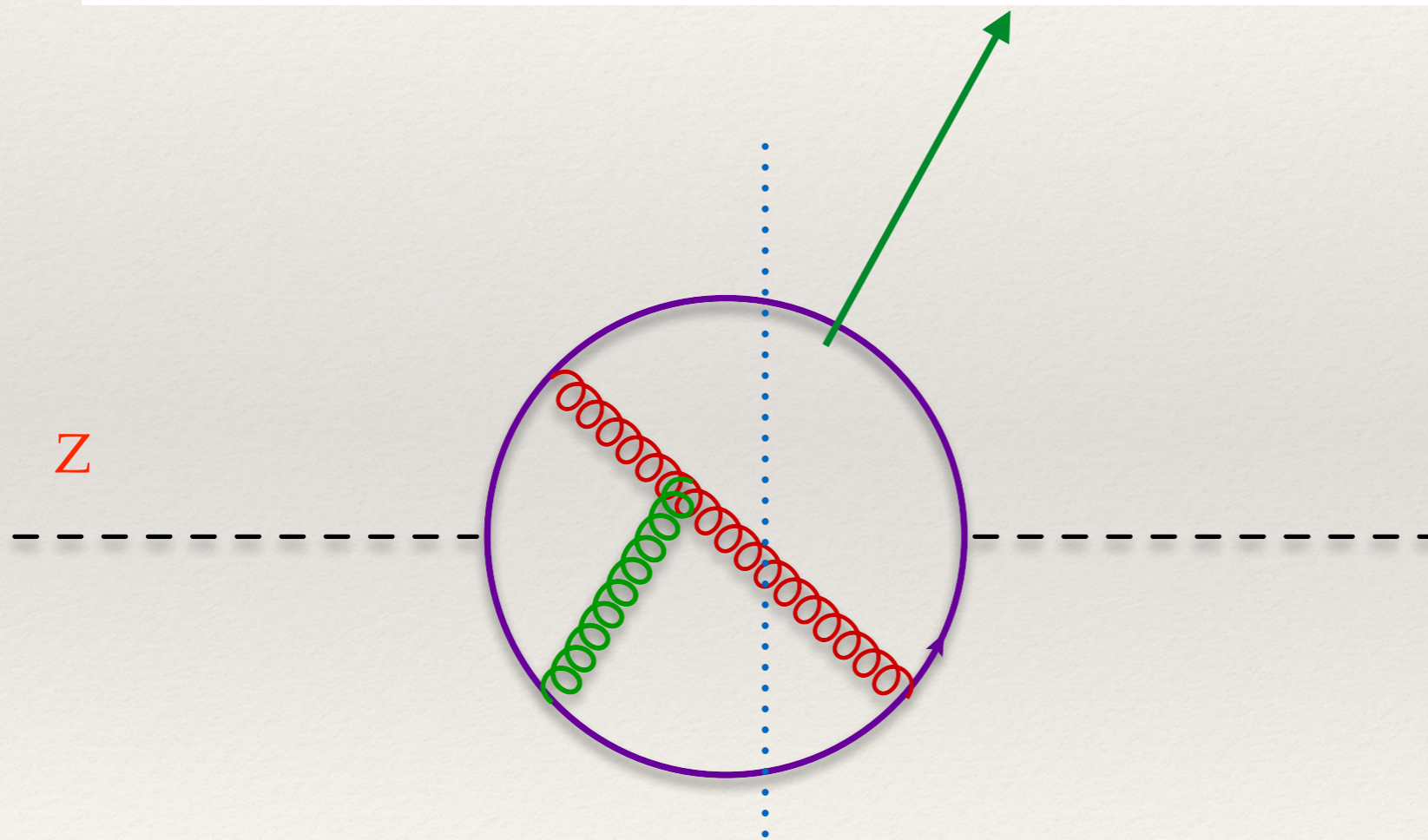
eg. for Z decay
to 2 jets



Ingredients of a NNLO calculation

At NNLO we have three types of final state phase spaces

$$\sigma_{NNLO} = \int |\mathcal{M}_{VV}|^2 d^m \Phi + \int |\mathcal{M}_{RV}|^2 d^{m+1} \Phi + \int |\mathcal{M}_{RR}|^2 d^{m+2} \Phi$$

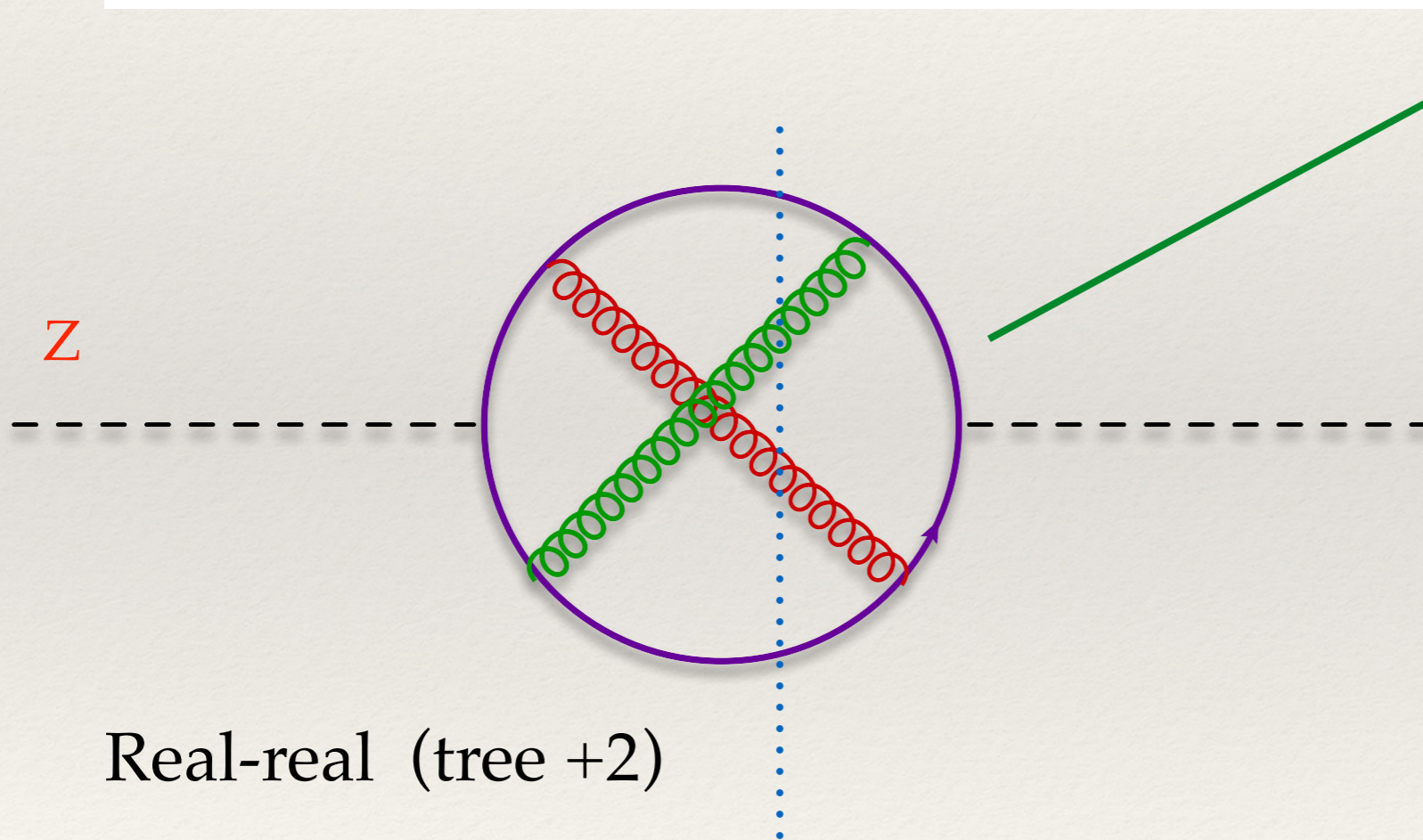


Real-virtual (one-loop +1) x (real + 1)

Ingredients of a NNLO calculation

At NNLO we have three types of final state phase spaces

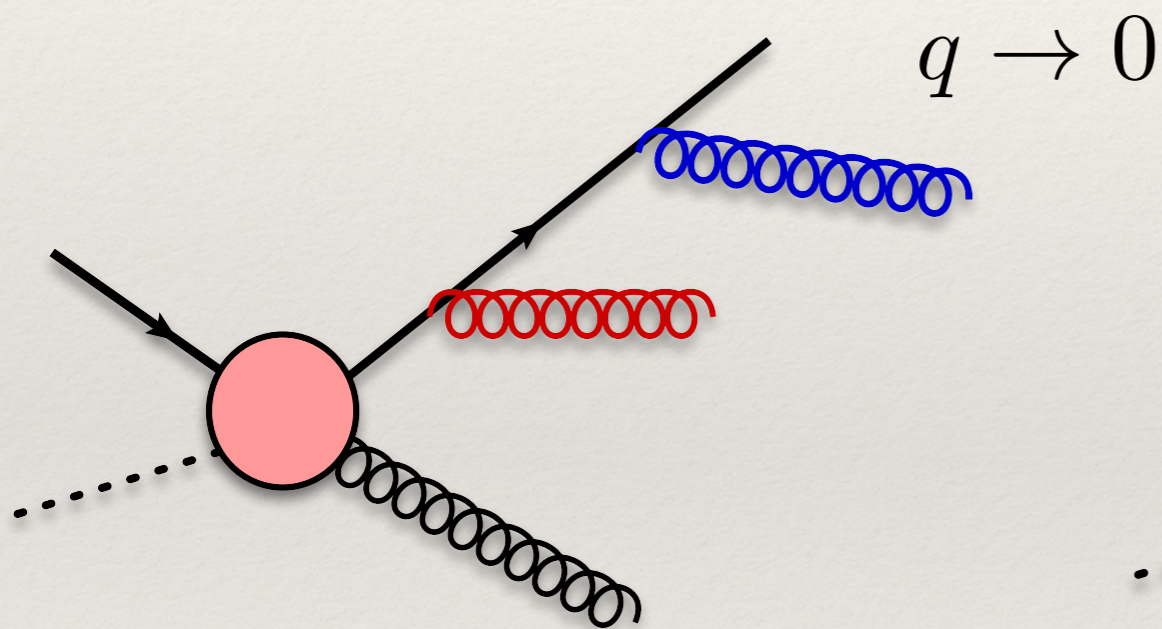
$$\sigma_{NNLO} = \int |\mathcal{M}_{VV}|^2 d^m \Phi + \int |\mathcal{M}_{RV}|^2 d^{m+1} \Phi + \int |\mathcal{M}_{RR}|^2 d^{m+2} \Phi$$



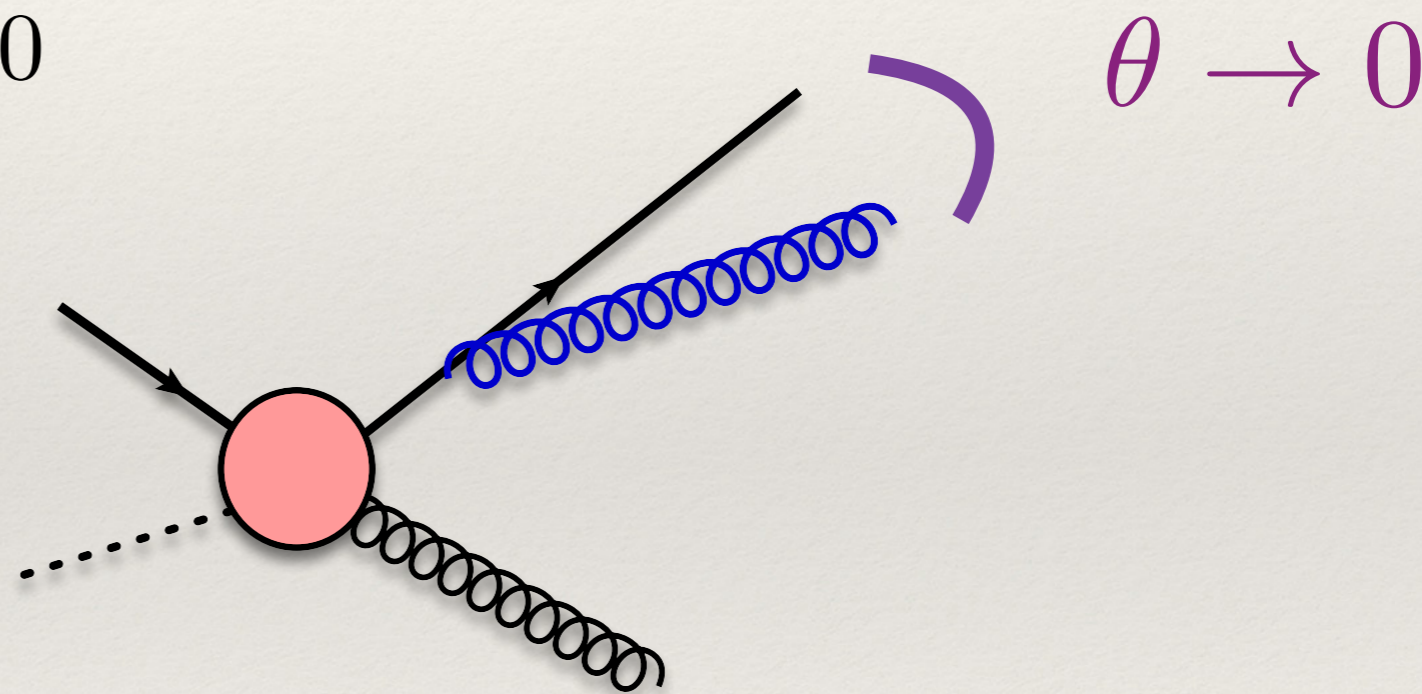
Divergences

All of our contributions (VV, RV, RR) are divergent in the soft and collinear regions.

There are two types of singularities in real matrix elements,



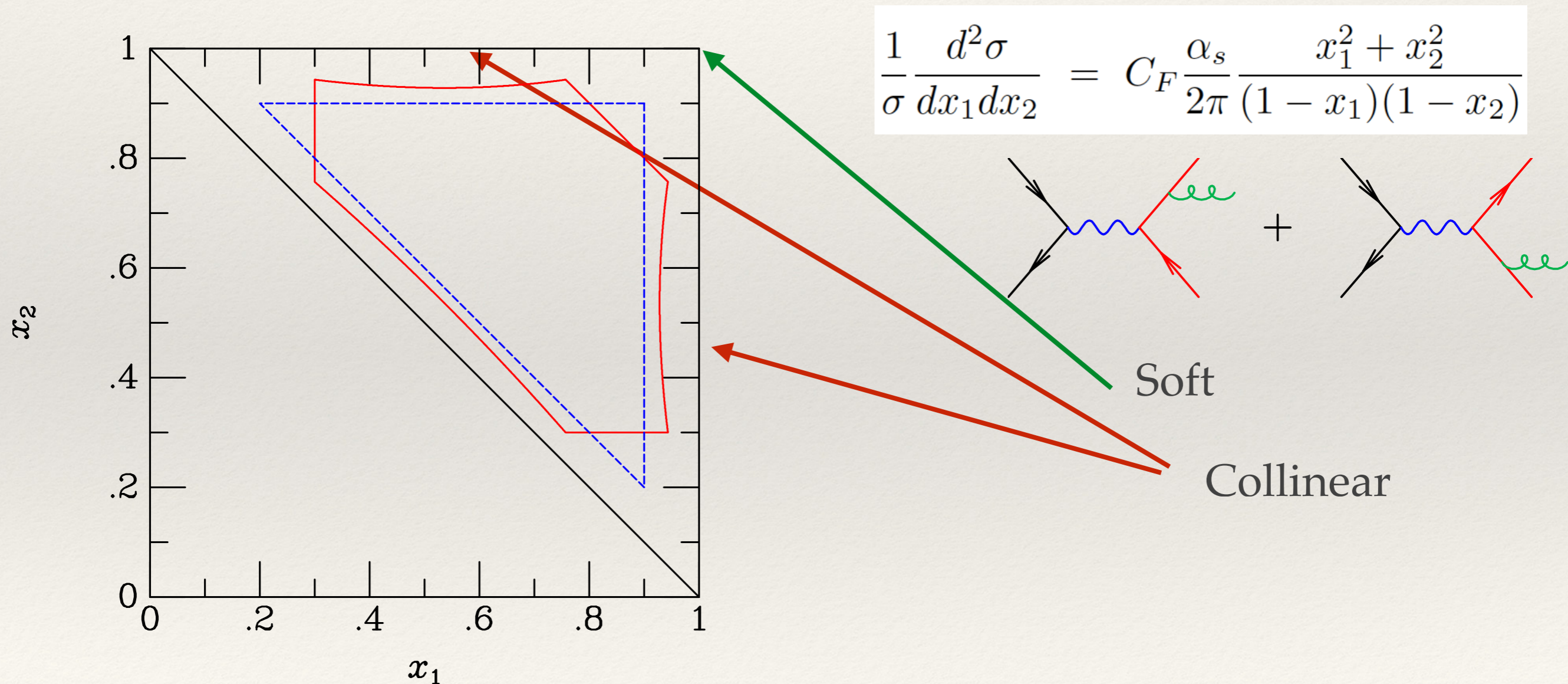
Soft (particle momenta vanishes)



Collinear (angle between two massless particles vanishes)

Slicing methods

A simple way of dealing with the IR singularities is phase space slicing
(eg. Sterman-Weinberg 1977)



Colour neutral final states

For color neutral final states the transverse momentum of the recoiling EW particles determines the double and singly unresolved regions of phase space. (Catani Grazzini 07)

$$\sigma_{NNLO} = \int dq_T \frac{d\sigma}{dq_T} \theta(q_T^{cut} - q_T) + \int dq_T \frac{d\sigma}{dq_T} \theta(q_T - q_T^{cut})$$

Obtained from the Collins-Soper-Sterman factorization theorem for small q_T

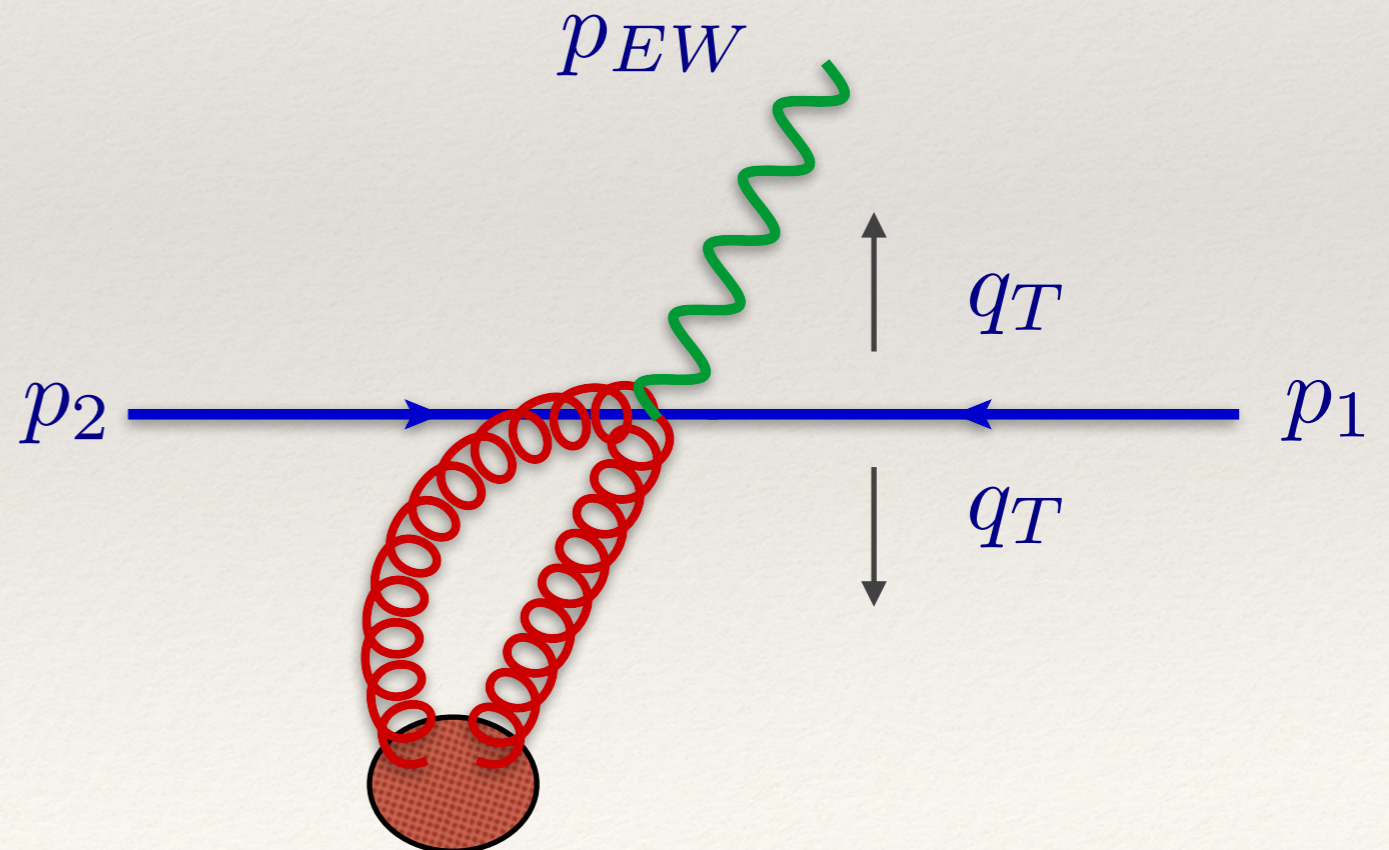
This is an NLO cross section for one additional parton extrapolated to q_T^{cut}

Q_T slicing

For color neutral final states the transverse momentum of the recoiling EW particles determines the double and singly unresolved regions of phase space. (Catani Grazzini hep-ph/0703012)

$$\sigma_{NNLO} = \int dq_T \frac{d\sigma}{dq_T} \theta(q_T^{cut} - q_T) + \int dq_T \frac{d\sigma}{dq_T} \theta(q_T - q_T^{cut})$$

This method fails for processes with colored partons in the final state, so we have consider slicing using the global event-shape jet-veto parameter, called N-jettiness.



N-jettiness

N-jettiness is an global event shape variable, designed to veto final state jets (Stewart, Tackmann, Waalewijn)

$$\mathcal{T}_N(\Phi_M) = \sum_{k=1}^M \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

The diagram illustrates the components of the N-jettiness formula. A blue arrow points from the text 'N=Number of final state jets' to the subscript 'N' in the formula. A red arrow points from 'M=Number of final state partons' to the subscript 'M' in the formula. A blue arrow points from 'Momentum of final state jets and two beam momenta' to the 'p_k' term in the numerator. A red arrow points from 'A hard scale (e.g. Energy of jets)' to the 'Q_i' term in the denominator. A blue arrow points from 'All final state partons' to the 'q_i' term in the numerator.

N =Number of final state jets

M =Number of final state partons

Momentum of final state jets and two beam momenta

All final state partons

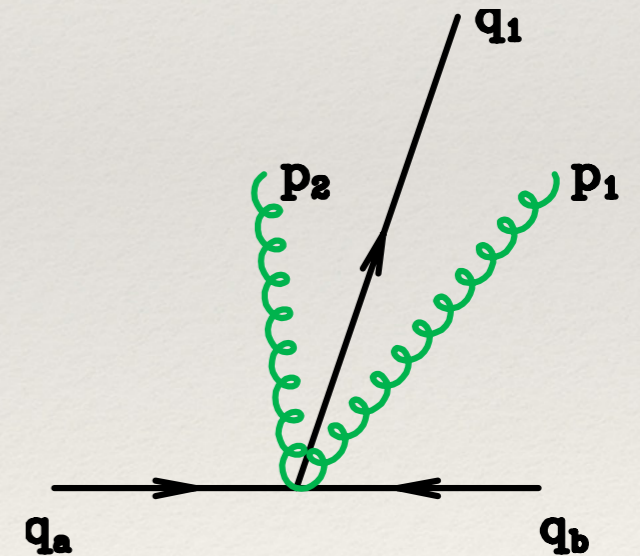
A hard scale (e.g. Energy of jets)

Zero and one jet cases, τ_0, τ_1

- ❖ Here we are concerned mainly with τ_0, τ_1 .
- ❖ Direction q_1 defined by a jet algorithm.

$$\tau_0 = \sum_{k=1}^M \min \left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b} \right\}$$

$$\tau_1 = \sum_{k=1}^M \min \left\{ \frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_1 \cdot p_k}{Q_1} \right\}$$



N-jettiness slicing

The method can be used as a regularization scheme, (Boughezal et al, 1505.03893, Gaunt et al, 1505.04794) using N-jettiness to separate the doubly and singly unresolved regions.

$$\begin{aligned} \sigma_{NNLO} = & \int d\Phi_N |\mathcal{M}_N|^2 + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^< \\ & + \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^< + \int d\Phi_{N+1} |\mathcal{M}_{N+1}|^2 \theta_N^> \\ & + \int d\Phi_{N+2} |\mathcal{M}_{N+2}|^2 \theta_N^> \end{aligned}$$



= Below the cut (can use factorization theorem)



= Above the cut (can use NLO code)

Below cut region

Factorization theorem valid in the below cut region based on SCET,
(Stewart et al, 1004.2489).

Beam functions, describes radiation collinear to initial state

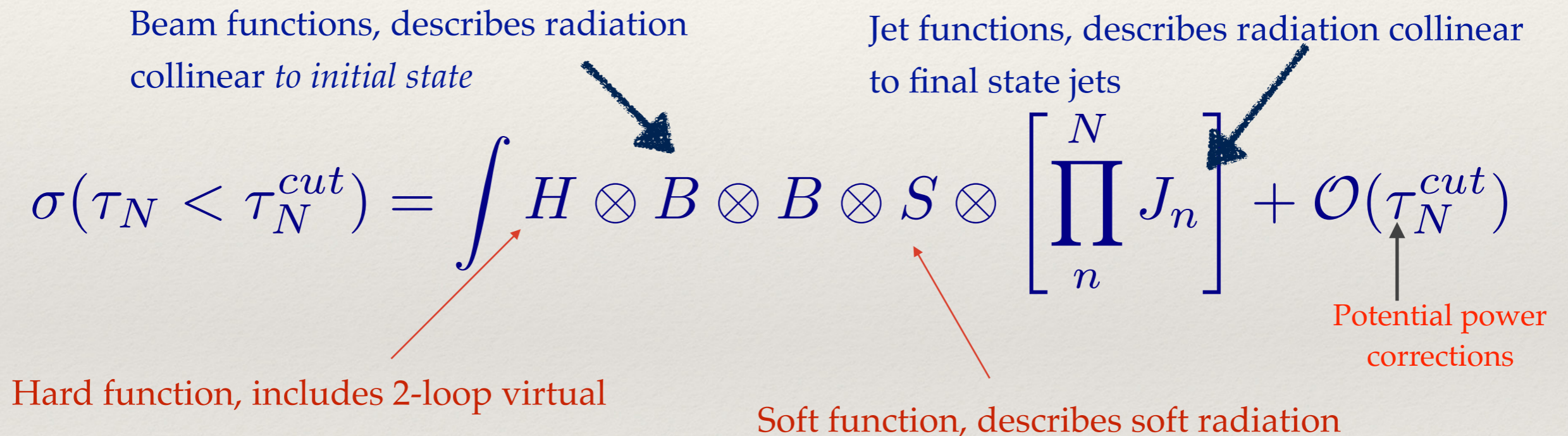
Jet functions, describes radiation collinear to final state jets

$$\sigma(\tau_N < \tau_N^{cut}) = \int H \otimes B \otimes B \otimes S \otimes \left[\prod_n^N J_n \right] + \mathcal{O}(\tau_N^{cut})$$

Hard function, includes 2-loop virtual

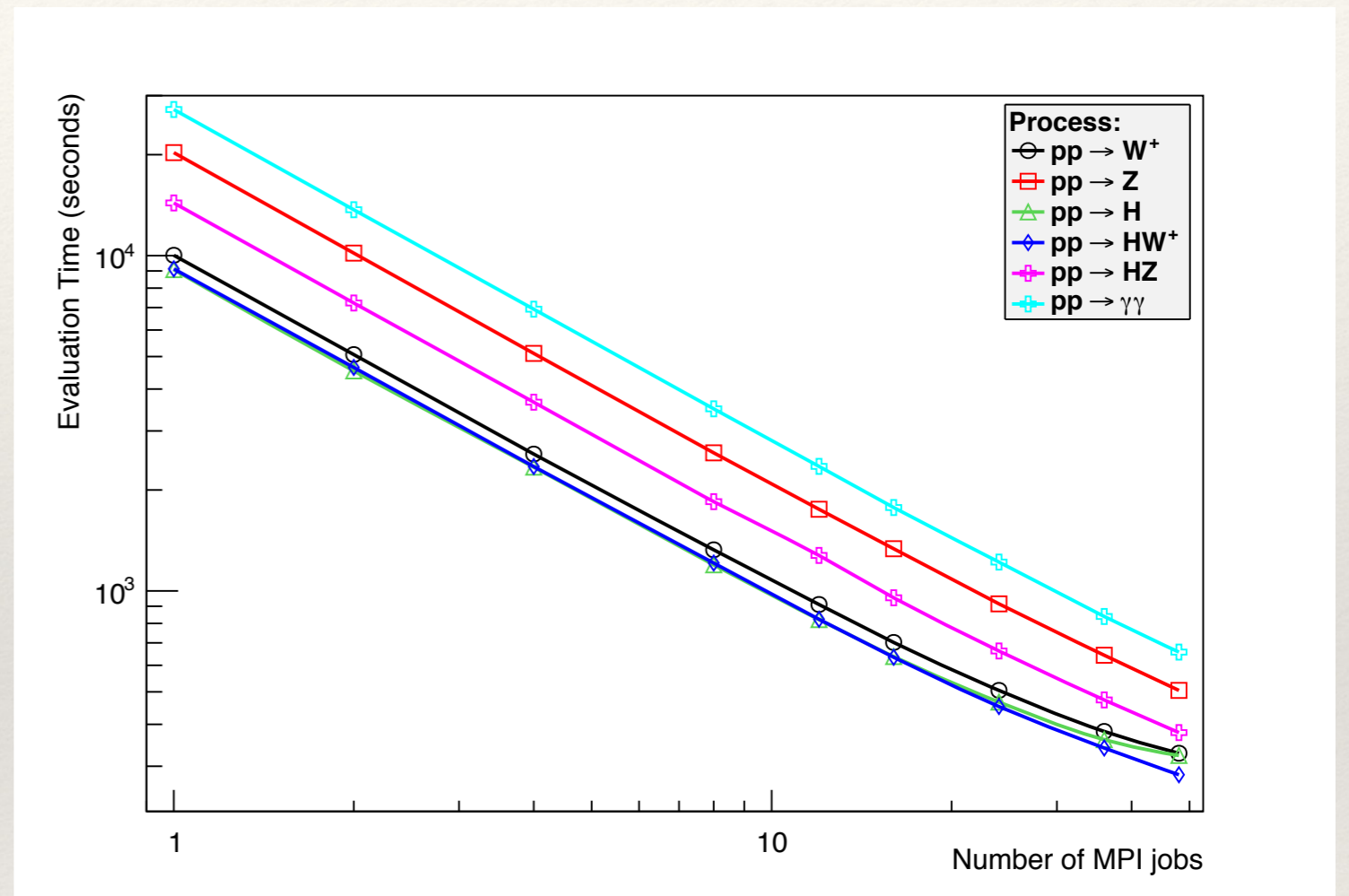
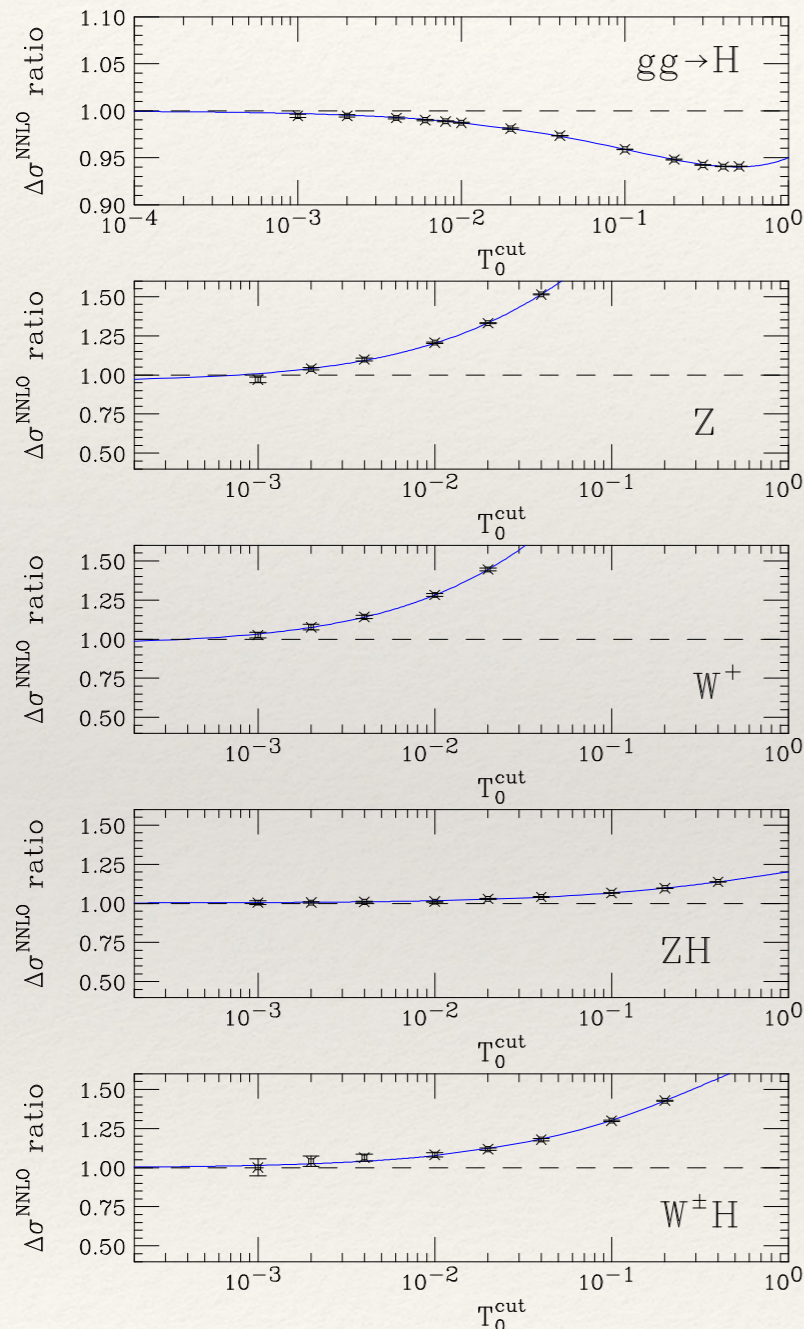
Soft function, describes soft radiation

Potential power corrections



- ❖ **B@NNLO** : Gaunt, Stahlhofen, Tackmann (1401.5478,1405.1044)
- ❖ **S@NNLO** : Boughezal, Liu, Petriello (1504.02540)
- ❖ **J@NNLO** : Becher Neubert (hep-ph/0607228), Becher, Bell (1008.1936)

Proof of principle with known colour singlet production processes



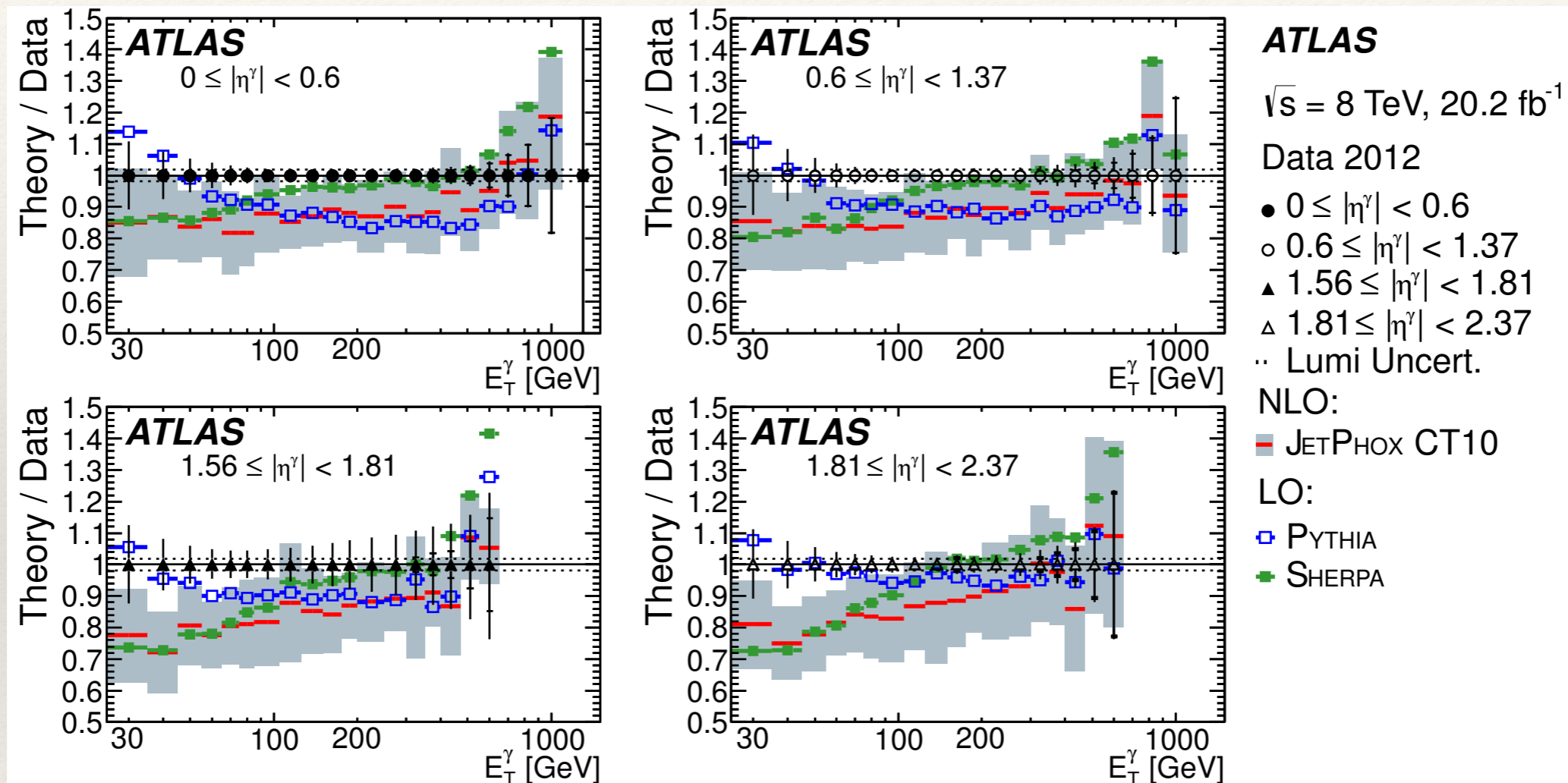
Code is public and can be downloaded from mcfm.fnal.gov

Results can be compared with numerically more inclusive results.

Inclusive photons

- ❖ This is somewhat more challenging than say, Z production because the existence of a photon at large p_T , mandates a colored parton in the final state.
- ❖ So we use a hybrid of τ_0 and τ_1 .

Comparison of 8 TeV data with LO & NLO

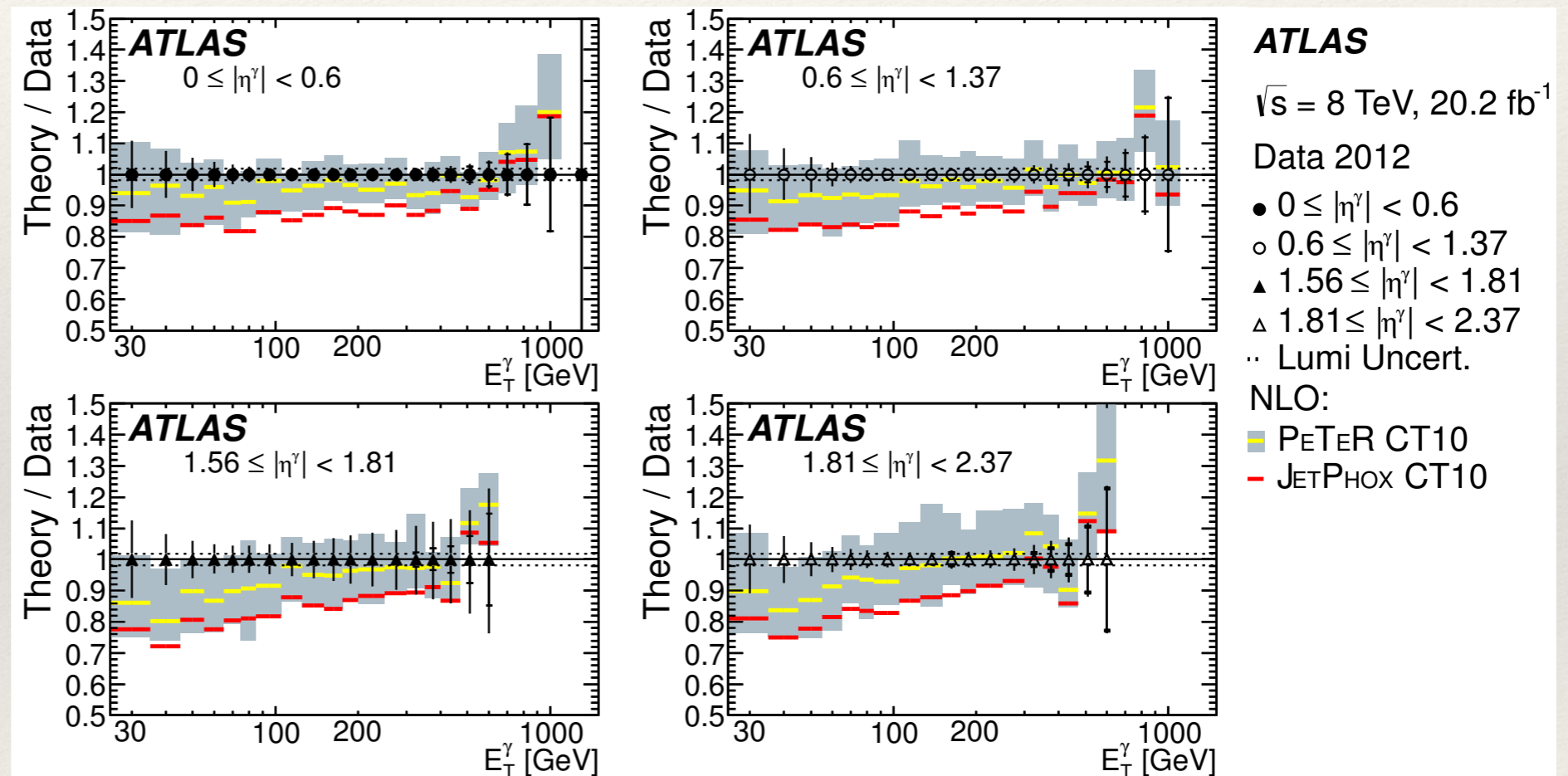


NLO uncertainty is large, but tension with data appears.

Adding threshold resummation+EW

Including threshold resummation + EW corrections improves things slightly, but theoretical errors are still large, compared to ATLAS errors.

PeTeR:Becher,
Lorentzen,
Schwartz,1206.6115



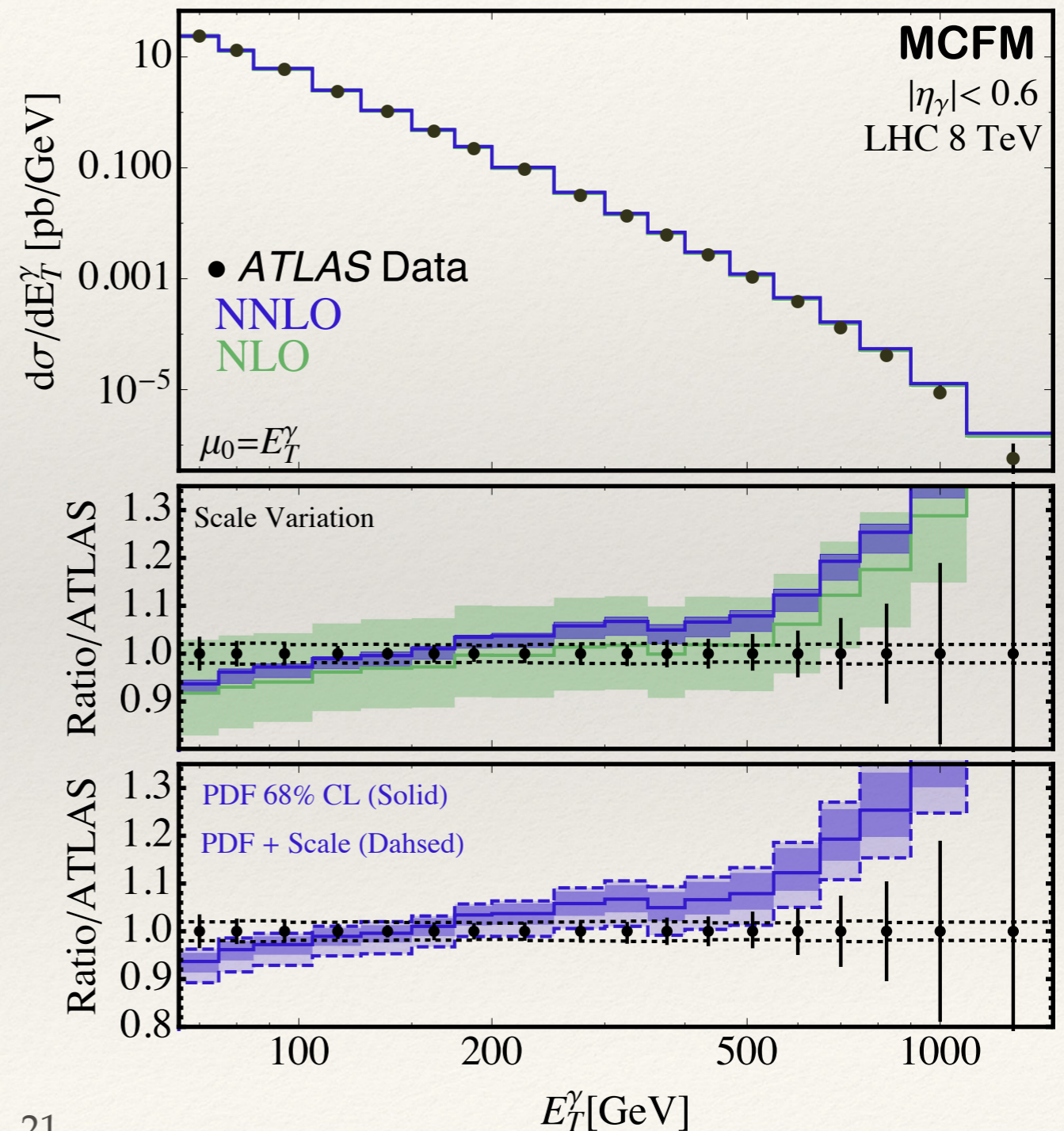
$$\alpha_{PETER} = \frac{1}{127.9}$$

Warning:

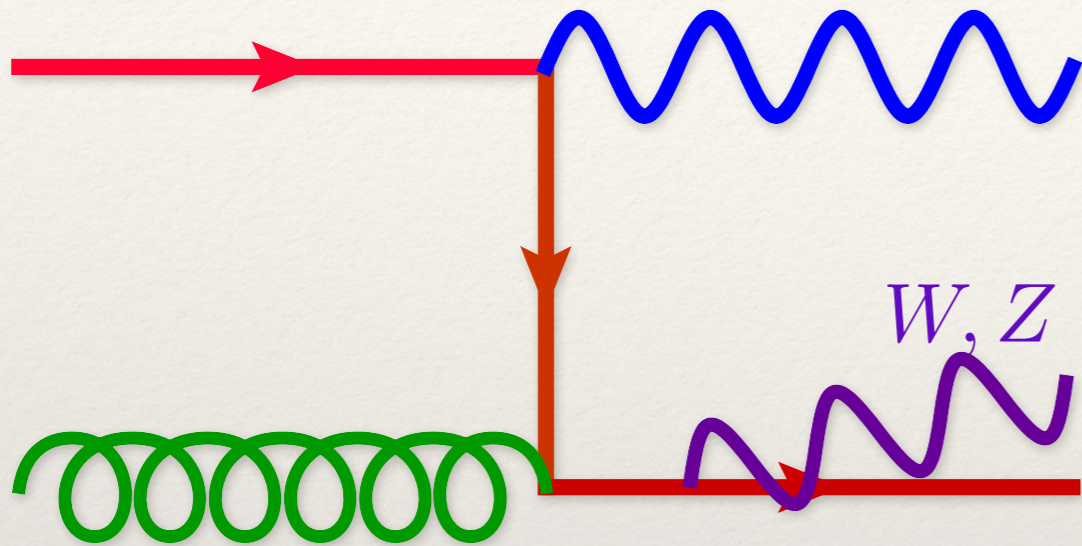
$$\alpha_{JETPHOX} = \frac{1}{137}$$

Comparison of data with MCFM

- ❖ Comparison at NLO and NNLO.
- ❖ With larger alpha NLO does a much better job than JETPHOX.
- ❖ Scale variation at NNLO is now comparable to data error.
- ❖ However at NNLO the shape is not so well described, especially at the highest p_T 's.



Electroweak effects

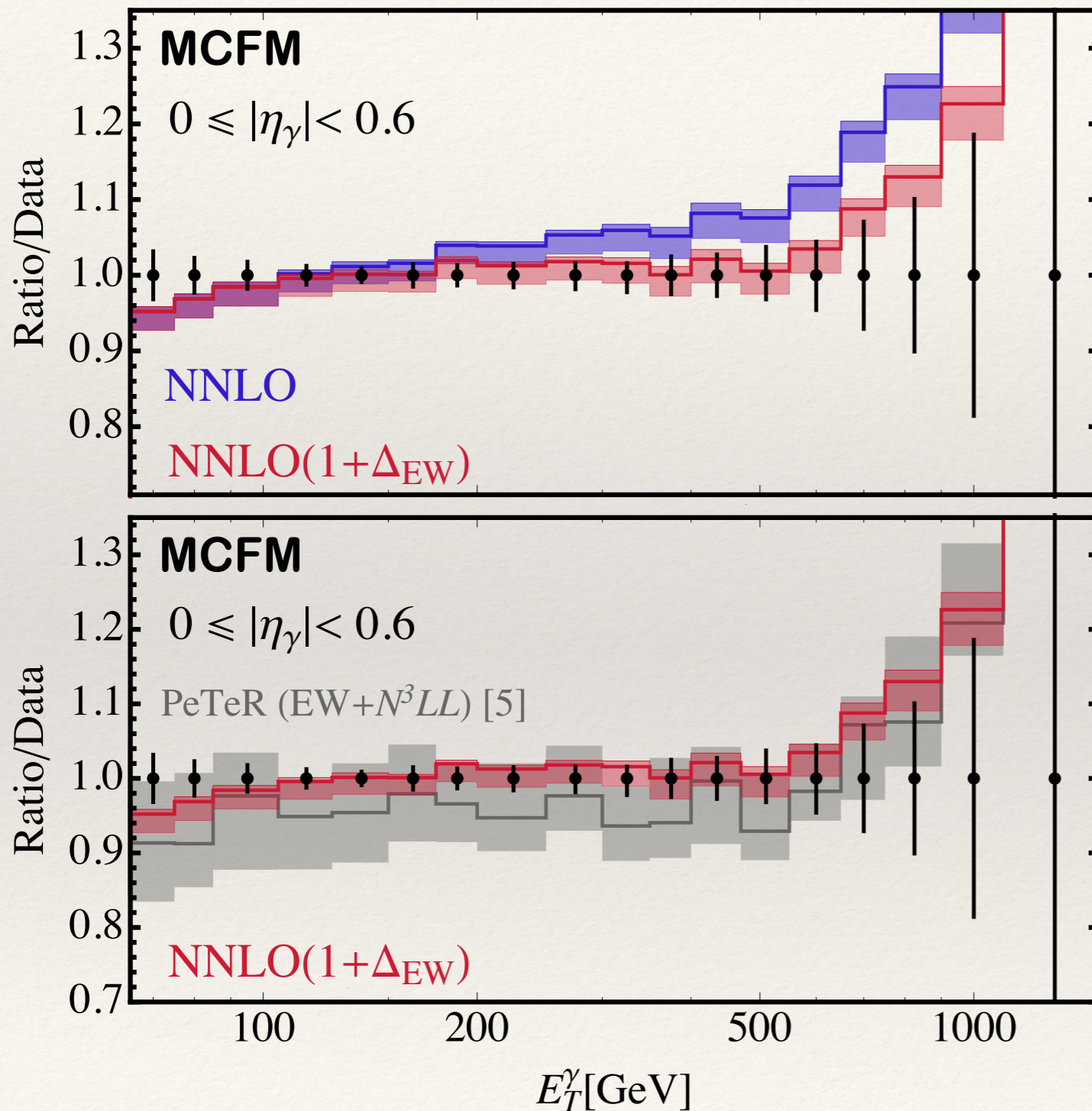


If the scale of the process is > 1 TeV,
then EW effects should be
important.

We take the parameterization for the photon p_T spectrum of the
LL Sudakov, EW corrections presented by [Kuhn, Kulesza,
Pozzorini, and Schulze 05'](#)

This allows us to do a fair comparison to PeTeR, which
includes the same EW resummation.

MCFM+EW vs PeTeR

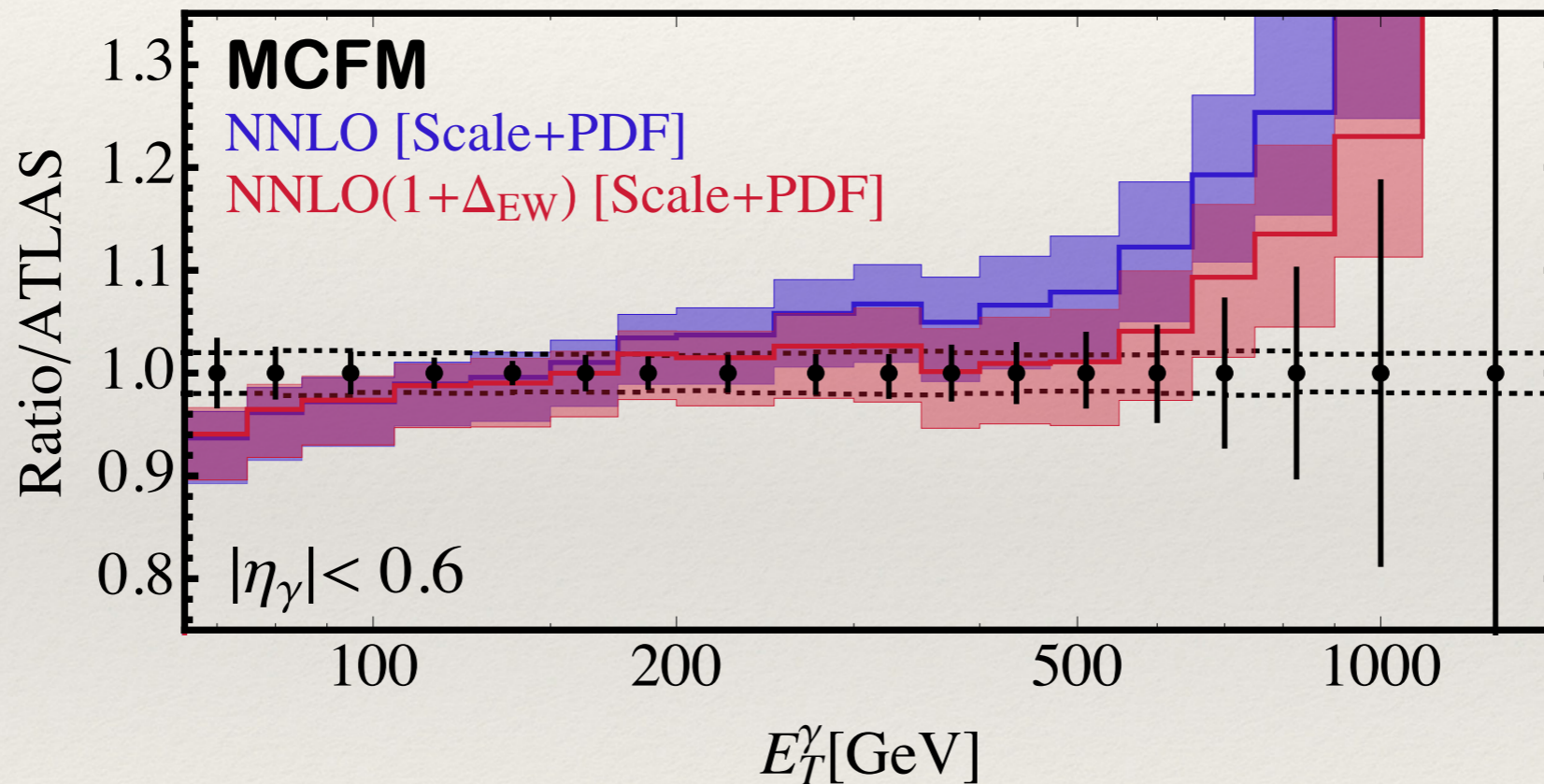


NNLO and EW together
do a better job of
describing the data

The estimate of the
theoretical error is now
comparable to
experiment
and better than that
obtained with N³LL.

PDF errors

- ❖ NNLO+EW is in good agreement with the data



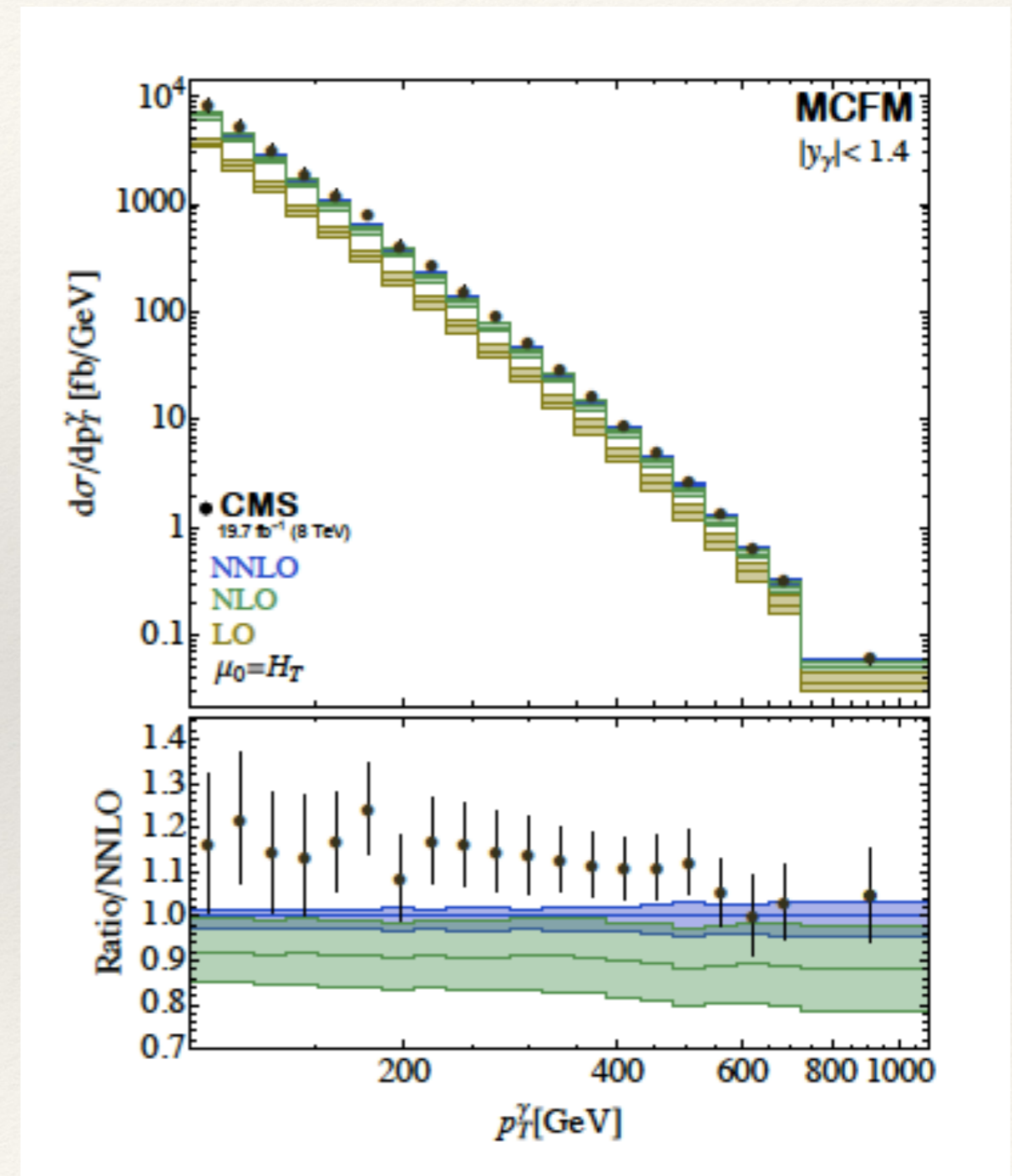
- ❖ the large PDF errors highlight how useful this channel will be to constrain PDFs at LHC

“Driving miss data”

- ❖ The aim of this work is to investigate the role of γ +jet data as a proxy for Z+jet, to estimate rates for MET+Jet, especially at large p_T
- ❖ We first investigate agreement of theory with γ +jet data
- ❖ Ratio $(\gamma+2j)/(\gamma+j)$
- ❖ And then finally, the ratio $(l^+l^-+j)/(\gamma+j)$

γ +jet data

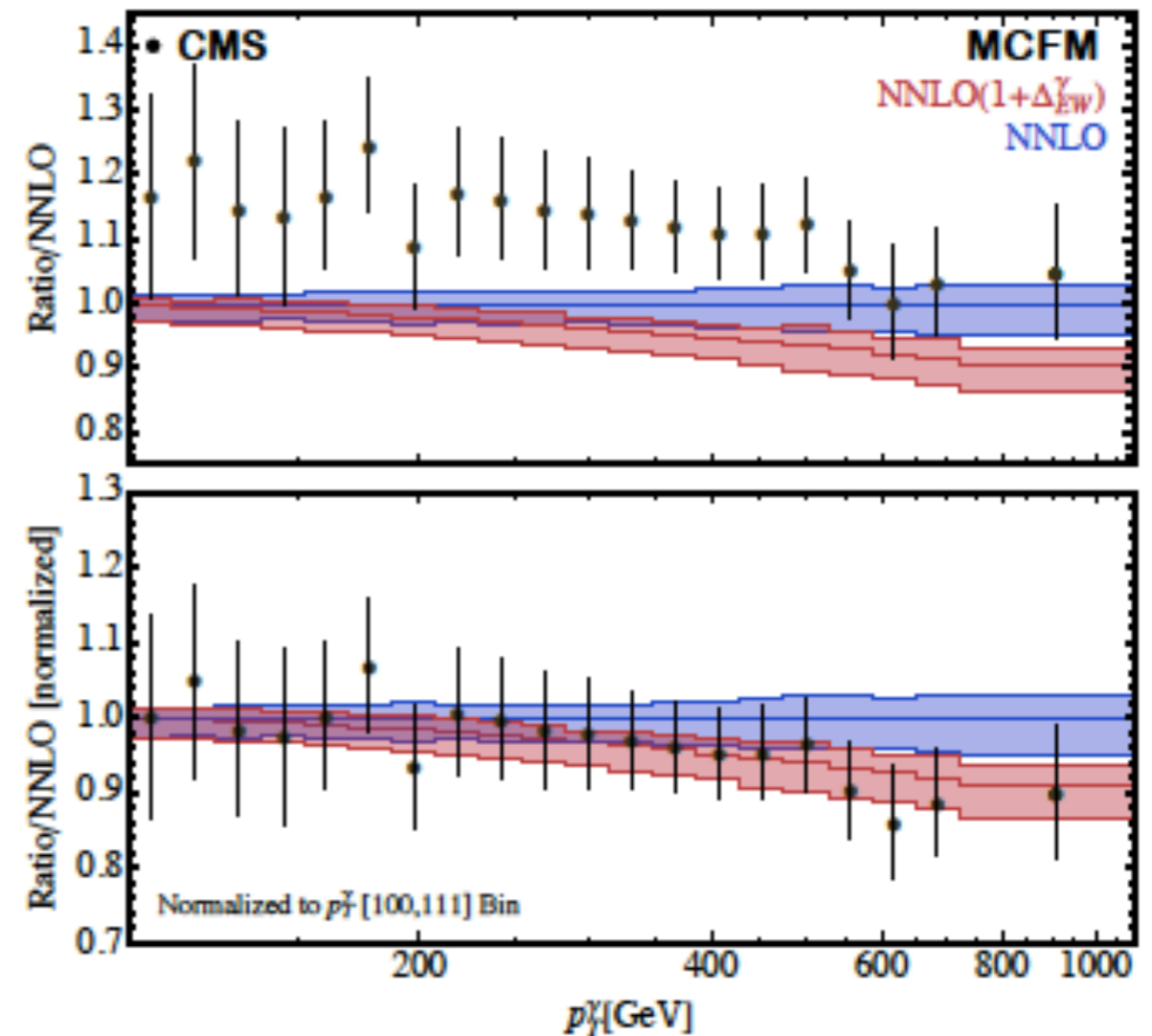
- ❖ Comparison to CMS data
1505.06520
- ❖ small p_T dependence in
NNLO/NLO K-factor
- ❖ Scale variation of order 2-3%
at NNLO, vs 8-10% at NLO



γ +jet data +EW effects

5

- ❖ Adding electroweak effects,
(Becher et al, 1305.4202,1509.01961)
normalization worsens, but
shape improves

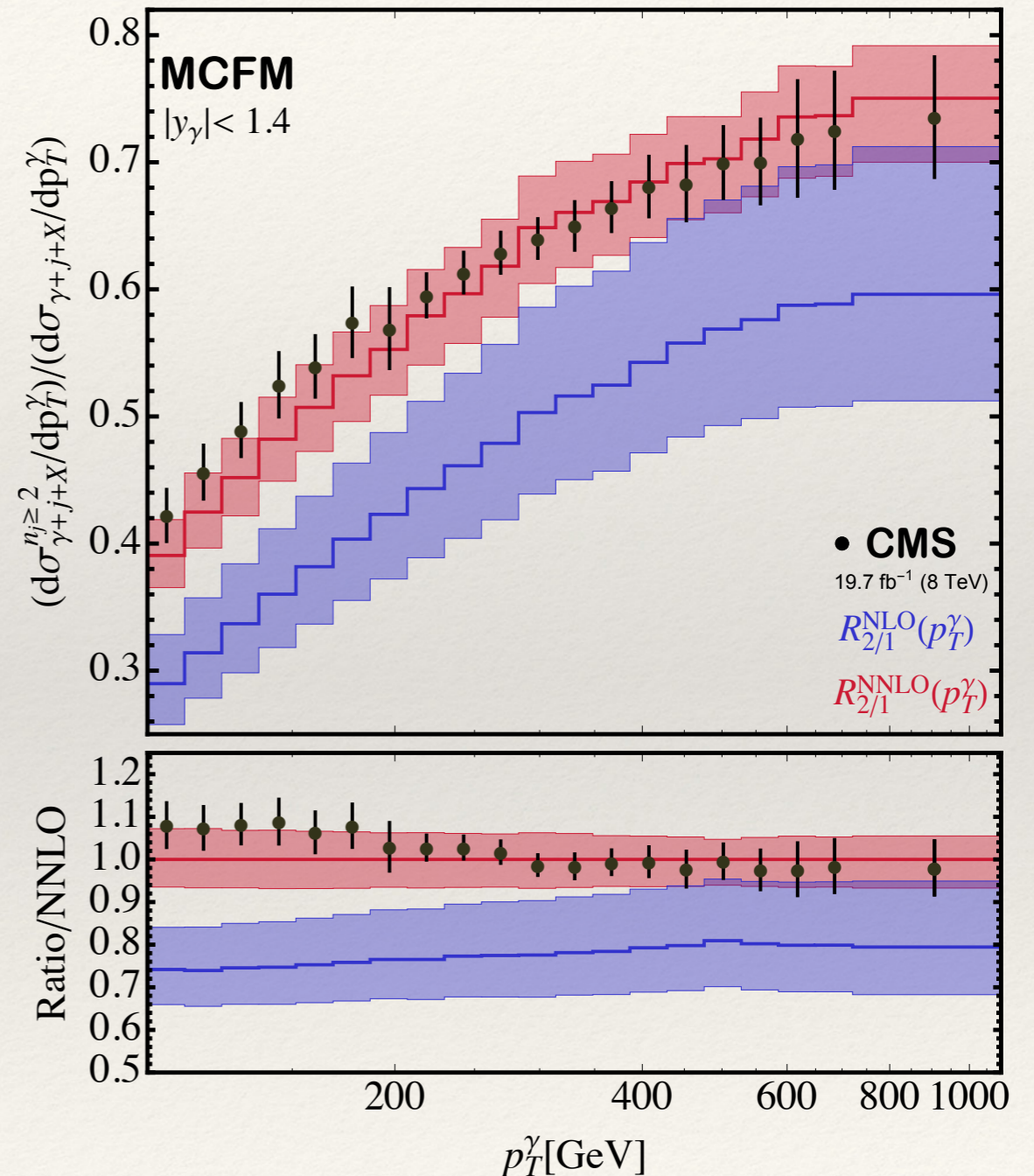


$$(\gamma+2j)/(\gamma+j)$$

- Using our NLO and NNLO calculation for $\gamma+j$ we can calculate,

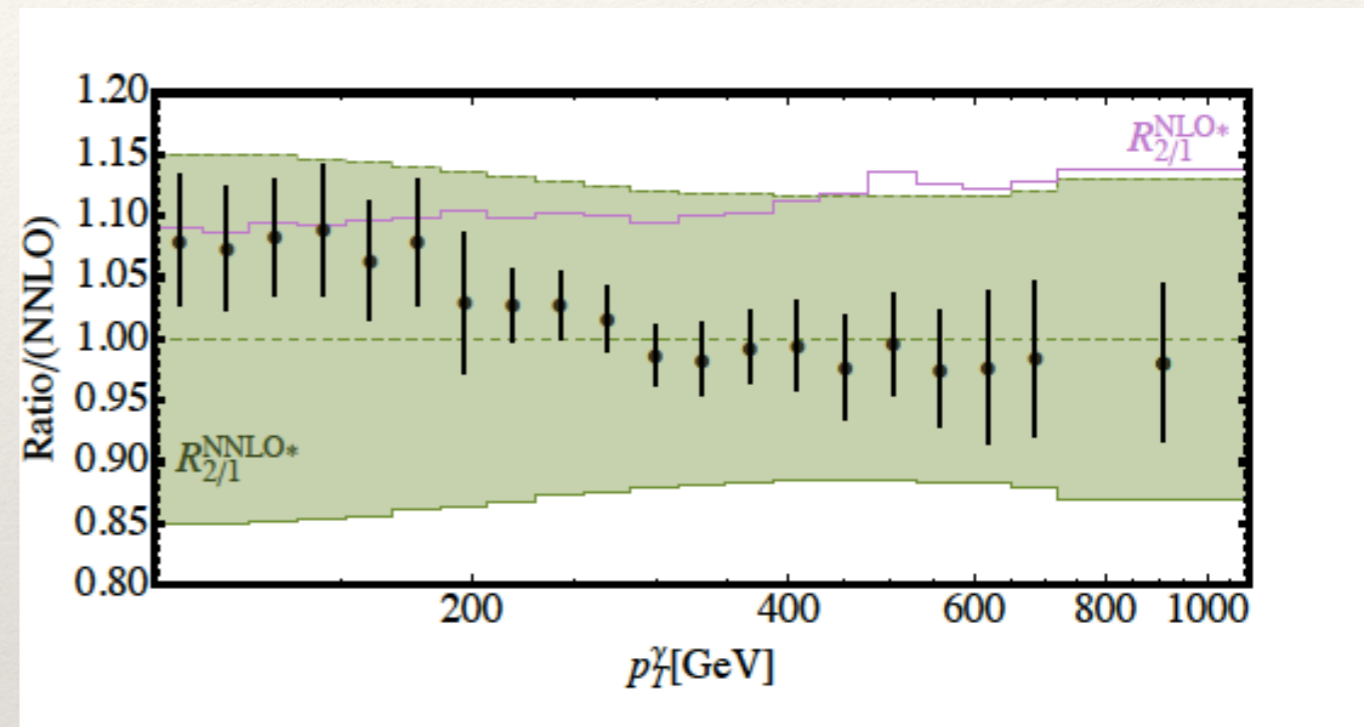
$$R_{2/1}(p_T^\gamma) = \frac{\alpha_s^2 \sum_{k=0}^{n_2} \alpha_s^k d\sigma_{\gamma+2j}^{(k)}/dp_T^\gamma}{\alpha_s \sum_{k=0}^{n_1} \alpha_s^k d\sigma_{\gamma+j}^{(k)}/dp_T^\gamma}.$$

- for $n_1=1$ (NLO) and $n_2=2$ (NNLO), where $n_2=n_1-1$



$$(\gamma + 2j) / (\gamma + j)$$

- ❖ Introducing an estimate for the uncalculated NNLO term in the two jet rate we find that the expected range encompasses both the NLO prediction and the data.



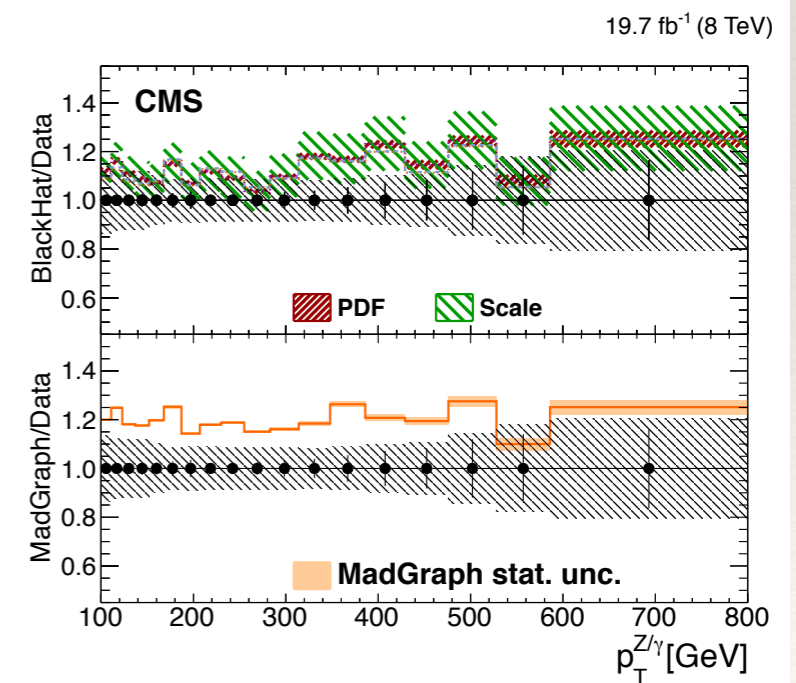
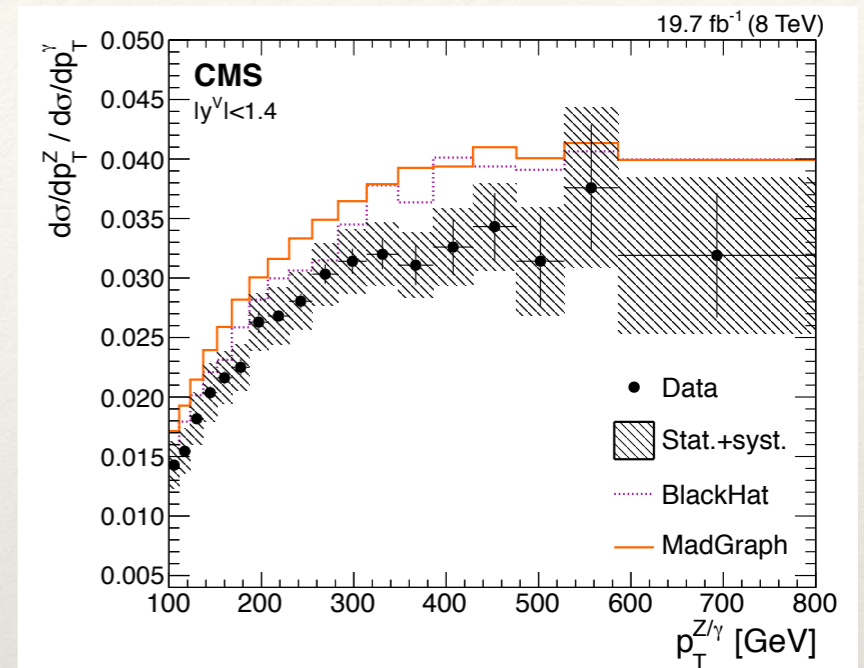
$$(l^+l^-+j)/(\gamma+j)$$

$$R_{Z/\gamma} = \left(R_u + \frac{R_d - R_u}{1 + \frac{Q_u^2 \langle u \rangle}{Q_d^2 \langle d \rangle}} \right) [\text{Br}(Z \rightarrow \ell^- \ell^+) \times \mathcal{A}]$$

$$R_q = \frac{v_q^2 + a_q^2}{4 \sin^2 \theta_w \cos^2 \theta_w Q_q^2}$$

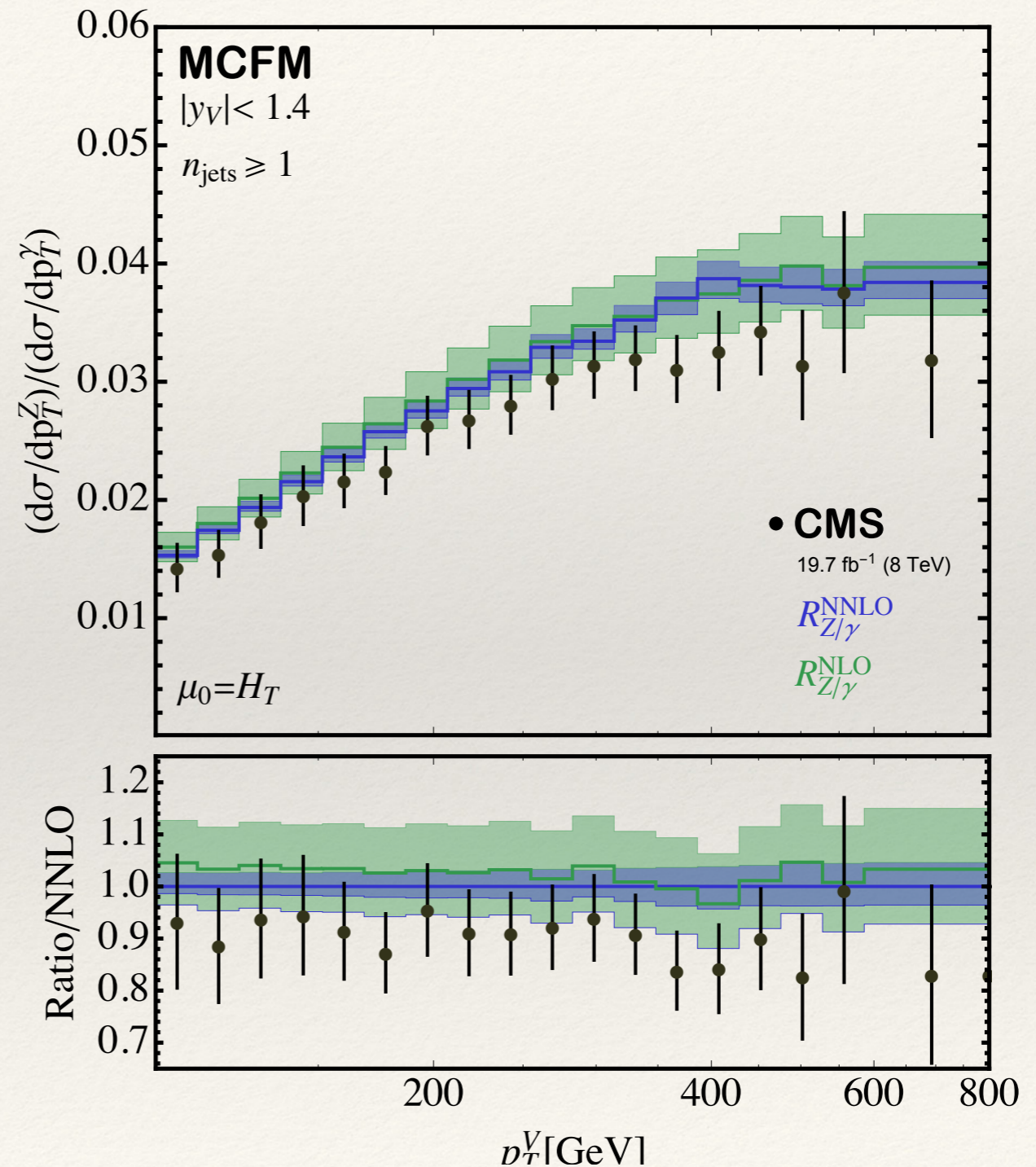
- At high p_T ,
($x \rightarrow 1, \langle d \rangle / \langle u \rangle \rightarrow 0$),
ratio is expected to
reach an asymptotic
value proportional
to R_u

$$R_u \sim 0.90, \quad R_d \sim 4.7$$



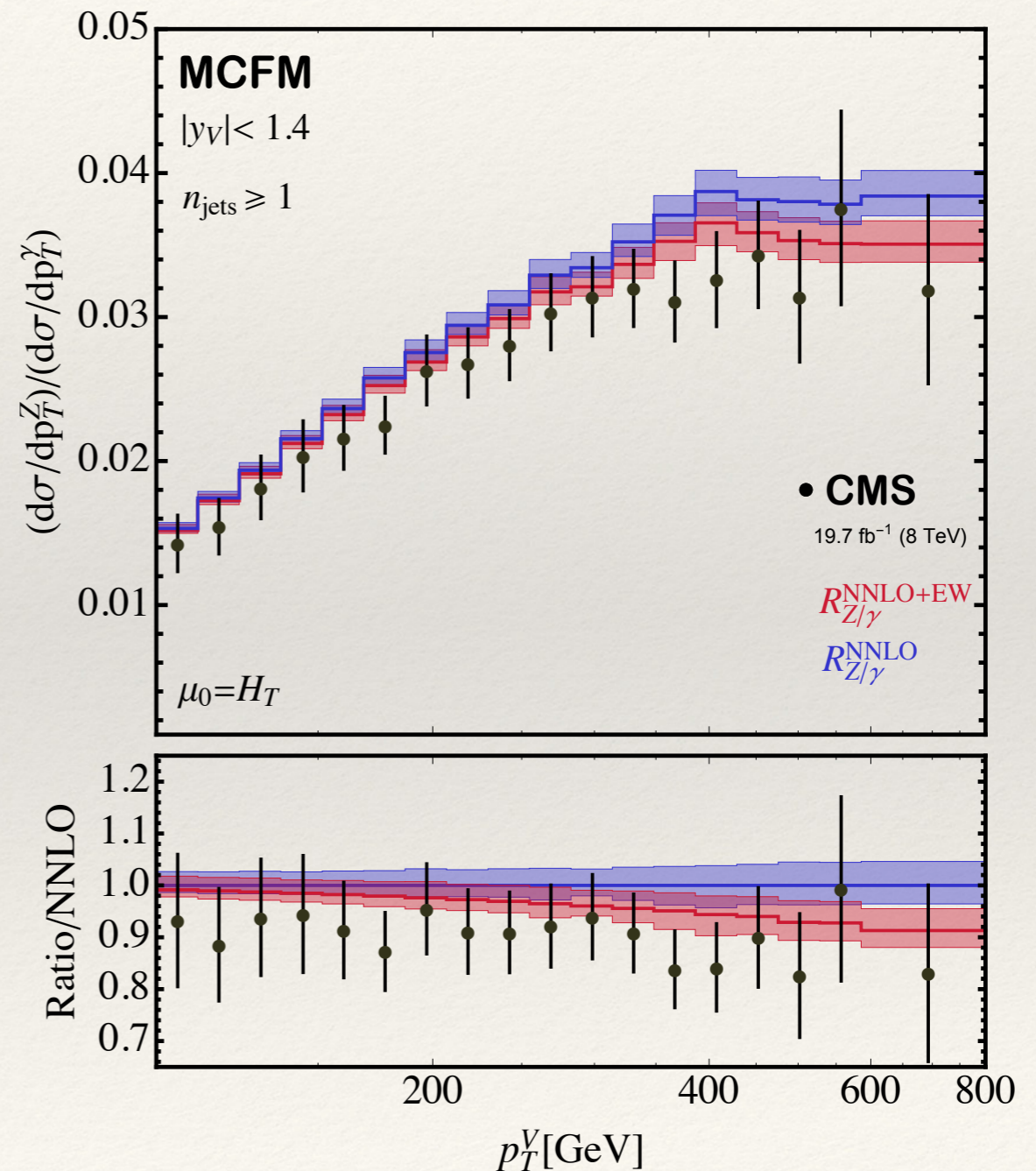
$$(1^+1^-+j)/(\gamma+j)$$

- ❖ NNLO effects reduce the ratio, especially at lower values of p_T



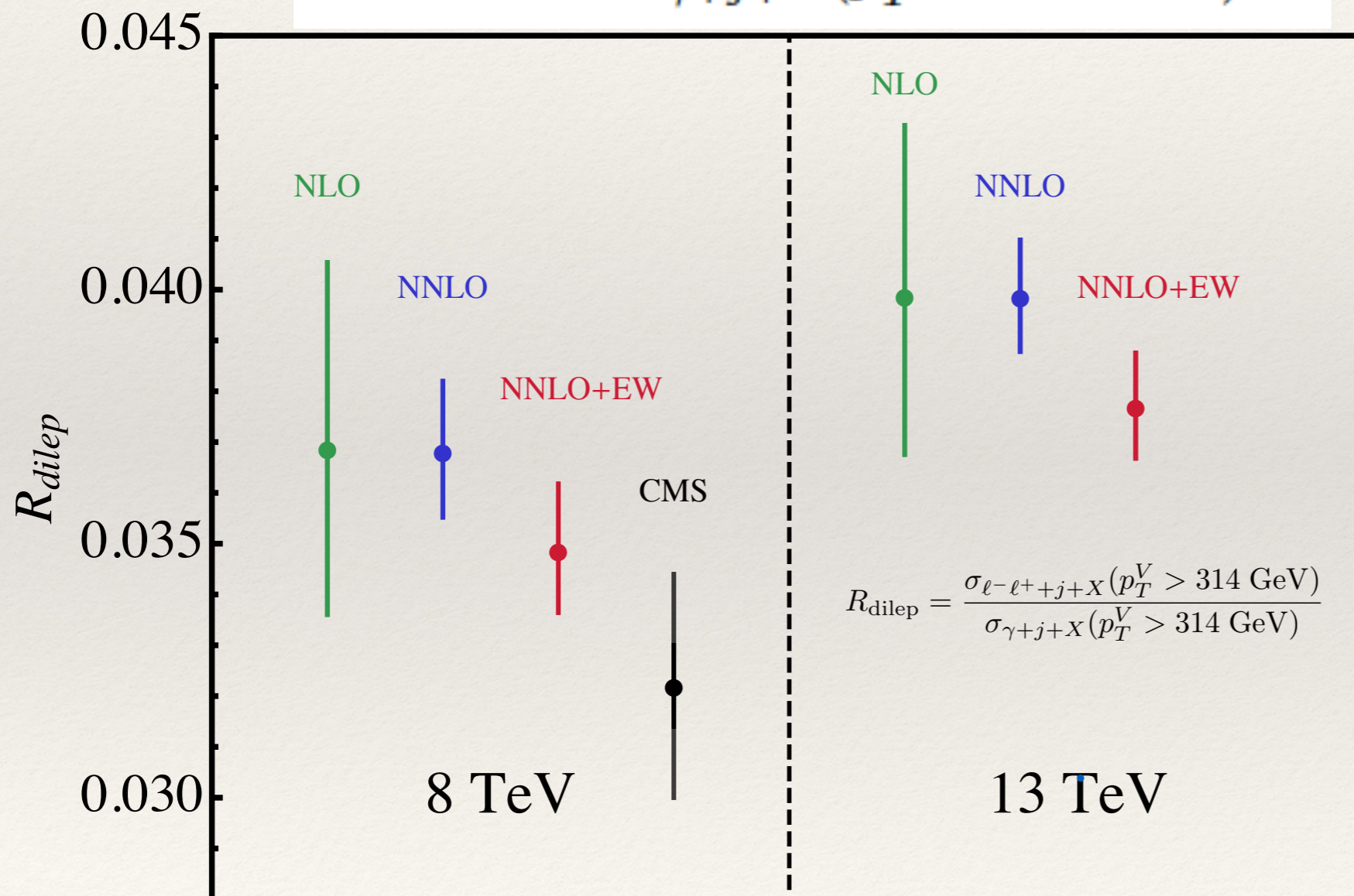
$$(1+1_{-}+j)/(\gamma+j)+EW$$

- ❖ Include EW effects
- ❖ Since electroweak corrections do not change the $Z+j$ and $\gamma+j$ processes in the same way, the ratio is modified, by as much as 7% in the high p_T bins



Asymptotic estimate for ratio

$$R_{\text{dilep}} = \frac{\sigma_{\ell-\ell'+j+X}(p_T^V > 314 \text{ GeV})}{\sigma_{\gamma+j+X}(p_T^V > 314 \text{ GeV})}$$



❖ Projection for this ratio at 13 TeV

Conclusions

- ❖ The precision of NNLO is needed by the data in direct photon studies and allows for interesting phenomenology to be undertaken.
- ❖ NNLO QCD is becoming the standard for 2->2 processes at the LHC, albeit with a few caveats.
- ❖ I have presented NNLO predictions for photon processes $\gamma + X$, $\gamma + j + X$ and considered the effect of electroweak corrections.
- ❖ Also computed the $(Z+j)/(\gamma + j)$ ratio at NNLO+EW and compared to 8 TeV CMS data. This ratio can be used to extract the MET+jets shape in searches.